International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

Mathematics

Higher level

Specimen papers 1 and 2

For first examinations in 2014

CONTENTS

International Baccalaureate®
Baccalauréat International
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MATHEMATICS
HIGHER LEVEL
PAPER 1

SPECIMEN

2 hours

Candidate session number

0	0							

Examination code

X	X	X	X	–	X	X	X	X

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics HL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The angle θ lies in the first quadrant and $\cos\theta = \frac{1}{3}$.

(a) Write down the value of $\sin\theta$. *[1 mark]*

(b) Find the value of $\tan 2\theta$. *[2 marks]*

(c) Find the value of $\cos\left(\frac{\theta}{2}\right)$, giving your answer in the form $\frac{\sqrt{a}}{b}$ where $a, b \in \mathbb{Z}^+$. *[3 marks]*

2. *[Maximum mark: 7]*

Consider the equation $9x^3 - 45x^2 + 74x - 40 = 0$.

(a) Write down the numerical value of the sum and of the product of the roots of this equation. *[1 mark]*

(b) The roots of this equation are three consecutive terms of an arithmetic sequence. Taking the roots to be α, $\alpha \pm \beta$, solve the equation. *[6 marks]*

3. *[Maximum mark: 6]*

A bag contains three balls numbered 1, 2 and 3 respectively. Bill selects one of these balls at random and he notes the number on the selected ball. He then tosses that number of fair coins.

(a) Calculate the probability that no head is obtained. *[3 marks]*

(b) Given that no head is obtained, find the probability that he tossed two coins. *[3 marks]*

4. *[Maximum mark: 6]*

The continuous variable X has probability density function

$$f(x)=\begin{cases}12x^2(1-x), & 0\le x\le 1\\ 0, & \text{otherwise.}\end{cases}$$

(a) Determine $\mathrm{E}(X)$. *[3 marks]*

(b) Determine the mode of X. *[3 marks]*

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5. *[Maximum mark: 7]*

The function f is defined, for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, by $f(x) = 2\cos x + x\sin x$.

(a) Determine whether f is even, odd or neither even nor odd. *[3 marks]*

(b) Show that $f''(0) = 0$. *[2 marks]*

(c) John states that, because $f''(0) = 0$, the graph of f has a point of inflexion at the point $(0, 2)$. Explain briefly whether John's statement is correct or not. *[2 marks]*

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6. *[Maximum mark: 7]*

In the triangle ABC, $AB = 2\sqrt{3}$, $AC = 9$ and $B\hat{A}C = 150^\circ$.

(a) Determine BC, giving your answer in the form $k\sqrt{3}$, $k \in \mathbb{Z}^+$. *[3 marks]*

(b) The point D lies on (BC), and (AD) is perpendicular to (BC). Determine AD. *[4 marks]*

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7. *[Maximum mark: 8]*

Consider the following system of equations:

$$x + y + z = 1$$
$$2x + 3y + z = 3$$
$$x + 3y - z = \lambda$$

where $\lambda \in \mathbb{R}$.

(a) Show that this system does not have a unique solution for any value of λ. *[4 marks]*

(b) (i) Determine the value of λ for which the system is consistent.

(ii) For this value of λ, find the general solution of the system. *[4 marks]*

8. *[Maximum mark: 6]*

The vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ satisfy the equation $\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}=\boldsymbol{0}$. Show that $\boldsymbol{a}\times\boldsymbol{b}=\boldsymbol{b}\times\boldsymbol{c}=\boldsymbol{c}\times\boldsymbol{a}$.

9. *[Maximum mark: 7]*

The function f is defined on the domain $x \geq 0$ by $f(x) = \mathrm{e}^x - x^\mathrm{e}$.

(a) (i) Find an expression for $f'(x)$.

(ii) Given that the equation $f'(x) = 0$ has two roots, state their values. *[3 marks]*

(b) Sketch the graph of f, showing clearly the coordinates of the maximum and minimum. *[3 marks]*

(c) Hence show that $\mathrm{e}^\pi > \pi^\mathrm{e}$. *[1 mark]*

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

10. *[Maximum mark: 12]*

Consider the complex numbers $z_1 = 2\operatorname{cis}150^\circ$ and $z_2 = -1 + \mathrm{i}$.

(a) Calculate $\frac{z_1}{z_2}$ giving your answer both in modulus-argument form and Cartesian form. *[7 marks]*

(b) Using your results, find the exact value of $\tan 75^\circ$, giving your answer in the form $a + \sqrt{b}$, a, $b \in \mathbb{Z}^+$. *[5 marks]*

11. *[Maximum mark: 19]*

(a) Find the value of the integral $\int_0^{\sqrt{2}} \sqrt{4 - x^2}\,\mathrm{d}x$. *[7 marks]*

(b) Find the value of the integral $\int_0^{0.5} \arcsin x\,\mathrm{d}x$. *[5 marks]*

(c) Using the substitution $t = \tan\theta$, find the value of the integral

$$\int_0^{\frac{\pi}{4}} \frac{\mathrm{d}\theta}{3\cos^2\theta + \sin^2\theta}.$$ *[7 marks]*

12. *[Maximum mark: 15]*

The function f is defined by $f(x) = \mathrm{e}^x \sin x$.

(a) Show that $f''(x) = 2\mathrm{e}^x \sin\left(x + \frac{\pi}{2}\right)$. *[3 marks]*

(b) Obtain a similar expression for $f^{(4)}(x)$. *[4 marks]*

(c) Suggest an expression for $f^{(2n)}(x)$, $n \in \mathbb{Z}^+$, and prove your conjecture using mathematical induction. *[8 marks]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 14]*

The function f is defined by

$$f(x) = \begin{cases} 2x - 1, & x \le 2 \\ ax^2 + bx - 5, & 2 < x < 3 \end{cases}$$

where $a, b \in \mathbb{R}$.

(a) Given that f and its derivative, f', are continuous for all values in the domain of f, find the values of a and b. *[6 marks]*

(b) Show that f is a one-to-one function. *[3 marks]*

(c) Obtain expressions for the inverse function f^{-1} and state their domains. *[5 marks]*

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MARKSCHEME

SPECIMEN

MATHEMATICS

Higher Level

Paper 1

12 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question. Award the marks as usual and then write –1(****MR****) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled ***(d)*** *and a brief* ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

  Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

  $f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$ ***A1***

  Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

The method of dealing with accuracy errors on a whole paper basis by means of the Accuracy Penalty ***(AP)*** *no longer applies.*

Instructions to examiners about such numerical issues will be provided on a question by question basis within the framework of mathematical correctness, numerical understanding and contextual appropriateness.

The rubric on the front page of each question paper is given for the guidance of candidates. The markscheme (MS) may contain instructions to examiners in the form of "Accept answers which round to n significant figures (sf)". Where candidates state answers, required by the question, to fewer than n sf, award ***A0****. Some intermediate numerical answers may be required by the MS but not by the question. In these cases only award the mark(s) if the candidate states the answer exactly or to at least 2sf.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

SECTION A

1. (a) $\sin\theta = \frac{\sqrt{8}}{3}$ ***A1***

[1 mark]

(b) $\tan 2\theta = \frac{2\times\sqrt{8}}{1-8} = -\frac{2\sqrt{8}}{7} \quad \left(-\frac{4\sqrt{2}}{7}\right)$ ***M1A1***

[2 marks]

(c) $\cos^2\left(\frac{\theta}{2}\right) = \frac{1+\frac{1}{3}}{2} = \frac{2}{3}$ ***M1A1***

$\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{6}}{3}$ ***A1***

[3 marks]

Total [6 marks]

2. (a) sum $=\frac{45}{9}$, product $=\frac{40}{9}$ ***A1***

[1 mark]

(b) it follows that $3\alpha = \frac{45}{9}$ and $\alpha(\alpha^2-\beta^2) = \frac{40}{9}$ ***A1A1***

solving, $\alpha = \frac{5}{3}$ ***A1***

$\frac{5}{3}\left(\frac{25}{9}-\beta^2\right) = \frac{40}{9}$ ***M1***

$\beta = (\pm)\frac{1}{3}$ ***A1***

the other two roots are $2, \frac{4}{3}$ ***A1***

[6 marks]

Total [7 marks]

3. (a) $\text{P}(\text{no heads from } n \text{ coins tossed}) = 0.5^n$ ***(A1)***

$\text{P}(\text{no head}) = \frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{4}+\frac{1}{3}\times\frac{1}{8}$ ***M1***

$= \frac{7}{24}$ ***A1***

[3 marks]

(b) $\text{P}(2 \mid \text{no heads}) = \frac{\text{P}(2 \text{ coins and no heads})}{\text{P}(\text{no heads})}$ ***M1***

$= \frac{\frac{1}{12}}{\frac{7}{24}}$ ***A1***

$= \frac{2}{7}$ ***A1***

[3 marks]

Total [6 marks]

4. (a) $\text{E}(X) = \int_0^1 12x^3(1-x)\,\text{d}x$ ***M1***

$= 12\left[\frac{x^4}{4}-\frac{x^5}{5}\right]_0^1$ ***A1***

$= \frac{3}{5}$ ***A1***

[3 marks]

(b) $f'(x) = 12(2x-3x^2)$ ***A1***

at the mode $f'(x) = 12(2x-3x^2) = 0$ ***M1***

therefore the mode $= \frac{2}{3}$ ***A1***

[3 marks]

Total [6 marks]

5. (a) $f(-x) = 2\cos(-x) + (-x)\sin(-x)$ ***M1***

$= 2\cos x + x\sin x \quad \left(= f(x)\right)$ ***A1***

therefore f is even ***A1***

[3 marks]

(b) $f'(x) = -2\sin x + \sin x + x\cos x \quad (= -\sin x + x\cos x)$ ***A1***

$f''(x) = -\cos x + \cos x - x\sin x \quad (= -x\sin x)$ ***A1***

so $f''(0) = 0$ ***AG***

[2 marks]

continued …

Question 5 continued

(c) John’s statement is incorrect because
either; there is a stationary point at $(0, 2)$ and since f is an even function and therefore symmetrical about the y-axis it must be a maximum or a minimum
or; $f''(x)$ is even and therefore has the same sign either side of $(0, 2)$ ***R2***

[2 marks]

Total [7 marks]

6. (a) $BC^2 = 12 + 81 + 2\times 2\sqrt{3}\times 9\times \frac{\sqrt{3}}{2} = 147$ ***M1A1***

$BC = 7\sqrt{3}$ ***A1***

[3 marks]

(b) area of triangle $ABC = \frac{1}{2}\times 9\times 2\sqrt{3}\times \frac{1}{2}\ \left(= \frac{9\sqrt{3}}{2}\right)$ ***M1A1***

therefore $\frac{1}{2}\times AD\times 7\sqrt{3} = \frac{9\sqrt{3}}{2}$ ***M1***

$AD = \frac{9}{7}$ ***A1***

[4 marks]

Total [7 marks]

7. (a) using row operations, ***M1***
to obtain 2 equations in the same 2 variables ***A1A1***
for example $y - z = 1$

$2y - 2z = \lambda - 1$

the fact that one of the left hand sides is a multiple of the other left hand side indicates that the equations do not have a unique solution, or equivalent ***R1AG***

[4 marks]

(b) (i) $\lambda = 3$ ***A1***

(ii) put $z = \mu$ ***M1***
then $y = 1 + \mu$ ***A1***
and $x = -2\mu$ or equivalent ***A1***

[4 marks]

Total [8 marks]

8. taking cross products with $\boldsymbol{a}$, *M1*

$\boldsymbol{a}\times(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})=\boldsymbol{a}\times\boldsymbol{0}=\boldsymbol{0}$ *A1*

using the algebraic properties of vectors and the fact that $\boldsymbol{a}\times\boldsymbol{a}=\boldsymbol{0}$, *M1*

$\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{a}\times\boldsymbol{c}=\boldsymbol{0}$ *A1*

$\boldsymbol{a}\times\boldsymbol{b}=\boldsymbol{c}\times\boldsymbol{a}$ *AG*

taking cross products with $\boldsymbol{b}$, *M1*

$\boldsymbol{b}\times(\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c})=\boldsymbol{0}$

$\boldsymbol{b}\times\boldsymbol{a}+\boldsymbol{b}\times\boldsymbol{c}=\boldsymbol{0}$ *A1*

$\boldsymbol{a}\times\boldsymbol{b}=\boldsymbol{b}\times\boldsymbol{c}$ *AG*

this completes the proof

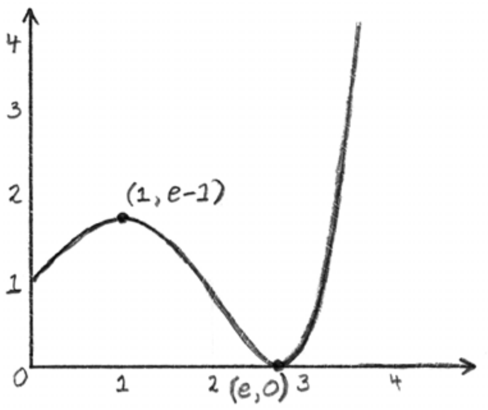
[6 marks]

9. (a) (i) $f'(x)=\mathrm{e}^{x}-\mathrm{e}x^{\mathrm{e}-1}$ *A1*

(ii) by inspection the two roots are 1, e *A1A1*

[3 marks]

(b)



A3

Note: Award ***A1*** for maximum, ***A1*** for minimum and ***A1*** for general shape.

[3 marks]

(c) from the graph: $\mathrm{e}^{x}>x^{\mathrm{e}}$ for all $x>0$ except $x=\mathrm{e}$ *R1*

putting $x=\pi$, conclude that $\mathrm{e}^{\pi}>\pi^{\mathrm{e}}$ *AG*

[1 mark]

Total [7 marks]

SECTION B

10. (a) in Cartesian form

$z_1 = 2 \times -\frac{\sqrt{3}}{2} + 2 \times \frac{1}{2}\mathrm{i}$ ***M1***

$= -\sqrt{3} + \mathrm{i}$ ***A1***

$\frac{z_1}{z_2} = \frac{-\sqrt{3} + \mathrm{i}}{-1 + \mathrm{i}}$

$= \frac{\left(-\sqrt{3} + \mathrm{i}\right)}{(-1 + \mathrm{i})} \times \frac{(-1 - \mathrm{i})}{(-1 - \mathrm{i})}$ ***M1***

$= \frac{1 + \sqrt{3}}{2} + \frac{\left(\sqrt{3} - 1\right)}{2}\mathrm{i}$ ***A1***

in modulus-argument form

$z_2 = \sqrt{2}\,\mathrm{cis}135^\circ$ ***A1***

$\frac{z_1}{z_2} = \frac{2\,\mathrm{cis}150^\circ}{\sqrt{2}\,\mathrm{cis}135^\circ}$

$= \sqrt{2}\,\mathrm{cis}15^\circ$ ***A1A1***

[7 marks]

(b) equating the two expressions for $\frac{z_1}{z_2}$

$\cos 15^\circ = \frac{1 + \sqrt{3}}{2\sqrt{2}}$ ***A1***

$\sin 15^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}}$ ***A1***

$\tan 75^\circ = \frac{\cos 15^\circ}{\sin 15^\circ} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$ ***M1***

$= \frac{\left(\sqrt{3} + 1\right)\left(\sqrt{3} + 1\right)}{\left(\sqrt{3} - 1\right)\left(\sqrt{3} + 1\right)}$ ***A1***

$= 2 + \sqrt{3}$ ***A1***

[5 marks]

Total [12 marks]

11. (a) let $x = 2\sin\theta$ ***M1***

$dx = 2\cos\theta d\theta$ ***A1***

$I = \int_0^{\frac{\pi}{4}} 2\cos\theta \times 2\cos\theta \, d\theta \quad \left(= 4\int_0^{\frac{\pi}{4}} \cos^2\theta \, d\theta\right)$ ***A1A1***

Note: Award ***A1*** for limits and ***A1*** for expression.

$= 2\int_0^{\frac{\pi}{4}} (1 + \cos 2\theta) \, d\theta$ ***A1***

$= 2\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{4}}$ ***A1***

$= 1 + \frac{\pi}{2}$ ***A1***

[7 marks]

(b) $I = \left[x \arcsin x\right]_0^{0.5} - \int_0^{0.5} x \times \frac{1}{\sqrt{1 - x^2}} dx$ ***M1A1A1***

$= \left[x \arcsin x\right]_0^{0.5} + \left[\sqrt{1 - x^2}\right]_0^{0.5}$ ***A1***

$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$ ***A1***

[5 marks]

(c) $dt = \sec^2\theta \, d\theta, \left[0, \frac{\pi}{4}\right] \to [0, 1]$ ***A1(A1)***

$I = \int_0^1 \frac{\frac{dt}{(1+t^2)}}{\frac{3}{(1+t^2)} + \frac{t^2}{(1+t^2)}}$ ***M1(A1)***

$= \int_0^1 \frac{dt}{3 + t^2}$ ***A1***

$= \frac{1}{\sqrt{3}}\left[\arctan\left(\frac{x}{\sqrt{3}}\right)\right]_0^1$ ***A1***

$= \frac{\pi}{6\sqrt{3}}$ ***A1***

[7 marks]

Total [19 marks]

12. (a) $f'(x) = e^x \sin x + e^x \cos x$ ***A1***

$f''(x) = e^x \sin x + e^x \cos x + e^x \cos x - e^x \sin x$ ***A1***

$= 2e^x \cos x$ ***A1***

$= 2e^x \sin\left(x + \frac{\pi}{2}\right)$ ***AG***

[3 marks]

(b) $f'''(x) = 2e^x \sin\left(x + \frac{\pi}{2}\right) + 2e^x \cos\left(x + \frac{\pi}{2}\right)$ ***A1***

$f^{(4)}(x) = 2e^x \sin\left(x + \frac{\pi}{2}\right) + 2e^x \cos\left(x + \frac{\pi}{2}\right) + 2e^x \cos\left(x + \frac{\pi}{2}\right) - 2e^x \sin\left(x + \frac{\pi}{2}\right)$ ***A1***

$= 4e^x \cos\left(x + \frac{\pi}{2}\right)$ ***A1***

$= 4e^x \sin(x + \pi)$ ***A1***

[4 marks]

(c) the conjecture is that

$f^{(2n)}(x) = 2^n e^x \sin\left(x + \frac{n\pi}{2}\right)$ ***A1***

for $n = 1$, this formula gives

$f''(x) = 2e^x \sin\left(x + \frac{\pi}{2}\right)$ which is correct ***A1***

let the result be true for $n = k$, $\left(\textit{i.e. } f^{(2k)}(x) = 2^k e^x \sin\left(x + \frac{k\pi}{2}\right)\right)$ ***M1***

consider $f^{(2k+1)}(x) = 2^k e^x \sin\left(x + \frac{k\pi}{2}\right) + 2^k e^x \cos\left(x + \frac{k\pi}{2}\right)$ ***M1***

$f^{(2(k+1))}(x) = 2^k e^x \sin\left(x + \frac{k\pi}{2}\right) + 2^k e^x \cos\left(x + \frac{k\pi}{2}\right) + 2^k e^x \cos\left(x + \frac{k\pi}{2}\right) - 2^k e^x \sin\left(x + \frac{k\pi}{2}\right)$

A1

$= 2^{k+1} e^x \cos\left(x + \frac{k\pi}{2}\right)$ ***A1***

$= 2^{k+1} e^x \sin\left(x + \frac{(k+1)\pi}{2}\right)$ ***A1***

therefore true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for $n = 1$
the result is proved by induction. ***R1***

Note: Award the final ***R1*** only if the two ***M*** marks have been awarded.

[8 marks]

Total [15 marks]

13. (a) f continuous $\Rightarrow \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ ***M1***

$4a + 2b = 8$ ***A1***

$f'(x) = \begin{cases} 2, & x < 2 \\ 2ax + b, & 2 < x < 3 \end{cases}$ ***A1***

f' continuous $\Rightarrow \lim_{x\to 2^-} f'(x) = \lim_{x\to 2^+} f'(x)$

$4a + b = 2$ ***A1***

solve simultaneously ***M1***

to obtain $a = -1$ and $b = 6$ ***A1***

[6 marks]

(b) for $x \le 2$, $f'(x) = 2 > 0$ ***A1***

for $2 < x < 3$, $f'(x) = -2x + 6 > 0$ ***A1***

since $f'(x) > 0$ for all values in the domain of f, f is increasing ***R1***

therefore one-to-one ***AG***

[3 marks]

(c) $x = 2y - 1 \Rightarrow y = \frac{x+1}{2}$ ***M1***

$x = -y^2 + 6y - 5 \Rightarrow y^2 - 6y + x + 5 = 0$ ***M1***

$y = 3 \pm \sqrt{4 - x}$

therefore

$f^{-1}(x) = \begin{cases} \frac{x+1}{2}, & x \le 3 \\ 3 - \sqrt{4-x}, & 3 < x < 4 \end{cases}$ ***A1A1A1***

Note: Award ***A1*** for the first line and ***A1A1*** for the second line.

[5 marks]

Total [14 marks]

International Baccalaureate®
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MATHEMATICS
HIGHER LEVEL
PAPER 2

SPECIMEN

2 hours

Candidate session number

0	0							

Examination code

X	X	X	X	–	X	X	X	X

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics HL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

Answer ***all*** *questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

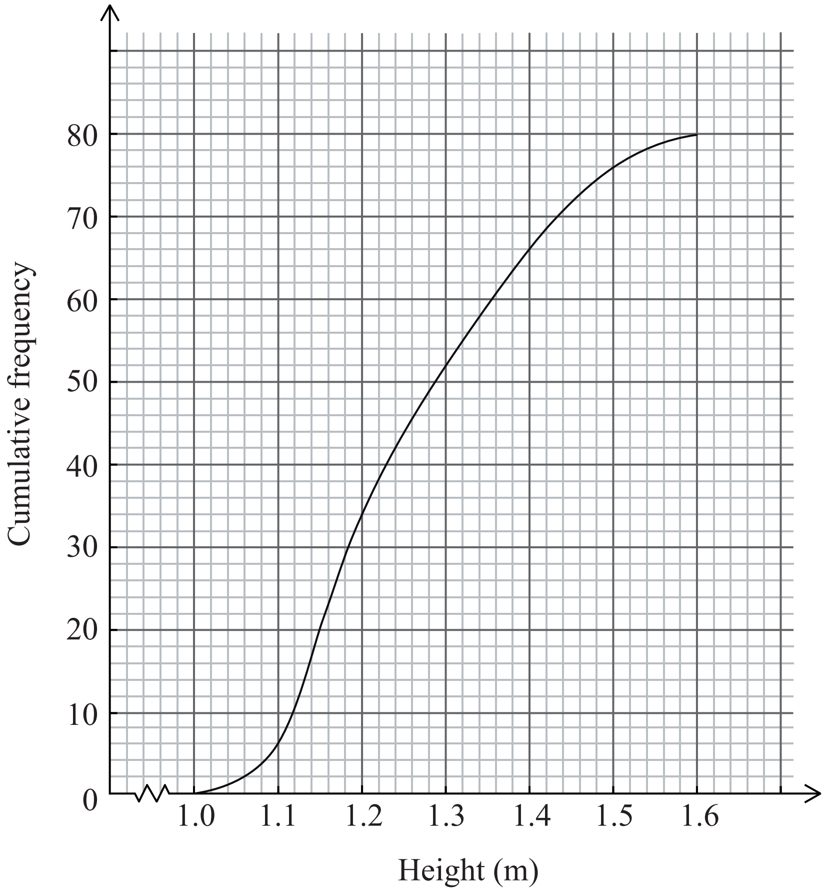
Given that $(x-2)$ is a factor of $f(x)=x^3+ax^2+bx-4$ and that division of $f(x)$ by $(x-1)$ leaves a remainder of -6, find the value of a and the value of b.

2. *[Maximum mark: 5]*

The first term and the common ratio of a geometric series are denoted, respectively, by a and r where $a, r \in \mathbb{Q}$. Given that the third term is 9 and the sum to infinity is 64, find the value of a and the value of r.

3. *[Maximum mark: 6]*

The heights of all the new boys starting at a school were measured and the following cumulative frequency graph was produced.



(a) Complete the grouped frequency table for these data. *[2 marks]*

Interval	**Frequency**
]1.0, 1.1]	
]1.1, 1.2]	
]1.2, 1.3]	
]1.3, 1.4]	
]1.4, 1.5]	
]1.5, 1.6]	

(This question continues on the following page)

(Question 3 continued)

(b) Estimate the mean and standard deviation of the heights of these 80 boys. *[2 marks]*

(c) Explain briefly whether or not the normal distribution provides a suitable model for this population. *[2 marks]*

4. *[Maximum mark: 6]*

The complex number $z = -\sqrt{3} + \mathrm{i}$.

(a) Find the modulus and argument of z, giving the argument in degrees. *[2 marks]*

(b) Find the cube root of z which lies in the first quadrant of the Argand diagram, giving your answer in Cartesian form. *[2 marks]*

(c) Find the smallest positive integer n for which z^n is a positive real number. *[2 marks]*

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5. *[Maximum mark: 6]*

The particle P moves along the x-axis such that its velocity, v m s^{-1}, at time t seconds is given by $v = \cos(t^2)$.

(a) Given that P is at the origin O at time $t = 0$, calculate

(i) the displacement of P from O after 3 seconds;

(ii) the total distance travelled by P in the first 3 seconds. *[4 marks]*

(b) Find the time at which the total distance travelled by P is 1 m. *[2 marks]*

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6. *[Maximum mark: 6]*

The function f is of the form $f(x)=\dfrac{x+a}{bx+c}$, $x \neq -\dfrac{c}{b}$. Given that the graph of f has asymptotes $x=-4$ and $y=-2$, and that the point $\left(\dfrac{2}{3}, 1\right)$ lies on the graph, find the values of a, b and c.

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7. *[Maximum mark: 9]*

A ship, S, is 10 km north of a motorboat, M, at 12.00pm. The ship is travelling northeast with a constant velocity of $20\ \text{km hr}^{-1}$. The motorboat wishes to intercept the ship and it moves with a constant velocity of $30\ \text{km hr}^{-1}$ in a direction θ degrees east of north. In order for the interception to take place, determine

(a) the value of θ; *[4 marks]*

(b) the time at which the interception occurs, correct to the nearest minute. *[5 marks]*

8. *[Maximum mark: 9]*

OABCDE is a regular hexagon and $\boldsymbol{a}$, $\boldsymbol{b}$ denote respectively the position vectors of A, B with respect to O.

(a) Show that $\text{OC} = 2\text{AB}$. *[2 marks]*

(b) Find the position vectors of C, D and E in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. *[7 marks]*

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9. *[Maximum mark: 7]*

A ladder of length 10 m on horizontal ground rests against a vertical wall. The bottom of the ladder is moved away from the wall at a constant speed of $0.5\ \text{m s}^{-1}$. Calculate the speed of descent of the top of the ladder when the bottom of the ladder is 4 m away from the wall.

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*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

10. *[Maximum mark: 12]*

The points A and B have coordinates $(1, 2, 3)$ and $(3, 1, 2)$ relative to an origin O.

(a) (i) Find $\overrightarrow{OA} \times \overrightarrow{OB}$.

(ii) Determine the area of the triangle OAB.

(iii) Find the Cartesian equation of the plane OAB. *[5 marks]*

(b) (i) Find the vector equation of the line L_1 containing the points A and B.

(ii) The line L_2 has vector equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$.

Determine whether or not L_1 and L_2 are skew. *[7 marks]*

*Do **NOT** write solutions on this page.*

11. *[Maximum mark: 13]*

A bank offers loans of $\$P$ at the beginning of a particular month at a monthly interest rate of I. The interest is calculated at the end of each month and added to the amount outstanding. A repayment of $\$R$ is required at the end of each month. Let $\$S_n$ denote the amount outstanding immediately after the n^{th} monthly repayment.

(a) (i) Find an expression for S_1 and show that

$$S_2 = P\left(1+\frac{I}{100}\right)^2 - R\left(1+\left(1+\frac{I}{100}\right)\right).$$

(ii) Determine a similar expression for S_n. Hence show that

$$S_n = P\left(1+\frac{I}{100}\right)^n - \frac{100R}{I}\left(\left(1+\frac{I}{100}\right)^n - 1\right).$$

[7 marks]

(b) Sue borrows \$5000 at a monthly interest rate of 1 % and plans to repay the loan in 5 years (*i.e.* 60 months).

(i) Calculate the required monthly repayment, giving your answer correct to two decimal places.

(ii) After 20 months, she inherits some money and she decides to repay the loan completely at that time. How much will she have to repay, giving your answer correct to the nearest \$?

[6 marks]

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 17]*

The weights, in kg, of male birds of a certain species are modelled by a normal distribution with mean μ and standard deviation σ.

(a) Given that 70 % of the birds weigh more than 2.1 kg and 25 % of the birds weigh more than 2.5 kg, calculate the value of μ and the value of σ. *[4 marks]*

(b) A random sample of ten of these birds is obtained. Let X denote the number of birds in the sample weighing more than 2.5 kg.

(i) Calculate $\mathrm{E}(X)$.

(ii) Calculate the probability that exactly five of these birds weigh more than 2.5 kg.

(iii) Determine the most likely value of X. *[5 marks]*

(c) The number of eggs, Y, laid by female birds of this species during the nesting season is modelled by a Poisson distribution with mean λ. You are given that $\mathrm{P}(Y \geq 2) = 0.80085$, correct to 5 decimal places.

(i) Determine the value of λ.

(ii) Calculate the probability that two randomly chosen birds lay a total of two eggs between them.

(iii) Given that the two birds lay a total of two eggs between them, calculate the probability that they each lay one egg. *[8 marks]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 18]*

The function f is defined on the domain $[0, 2]$ by $f(x) = \ln(x+1)\sin(\pi x)$.

(a) Obtain an expression for $f'(x)$. *[3 marks]*

(b) Sketch the graphs of f and f' on the same axes, showing clearly all x-intercepts. *[4 marks]*

(c) Find the x-coordinates of the two points of inflexion on the graph of f. *[2 marks]*

(d) Find the equation of the normal to the graph of f where $x = 0.75$, giving your answer in the form $y = mx + c$. *[3 marks]*

(e) Consider the points $\mathrm{A}(a, f(a))$, $\mathrm{B}(b, f(b))$ and $\mathrm{C}(c, f(c))$ where a, b and c $(a < b < c)$ are the solutions of the equation $f(x) = f'(x)$. Find the area of the triangle ABC. *[6 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

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MARKSCHEME

SPECIMEN

MATHEMATICS

Higher Level

Paper 2

12 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Write the marks in red on candidates' scripts, in the right hand margin.

- Show the **breakdown** of individual marks awarded using the abbreviations ***M1***, ***A1***, *etc*.
- Write down the total for each **question** (at the end of the question) and **circle** it.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*Award **N** marks for **correct** answers where there is **no** working.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 Implied marks

Implied marks appear in ***brackets e.g. (M1)****, and can only be awarded if* ***correct*** *work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 Follow through marks

*Follow through (****FT****) marks are awarded where an incorrect answer from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s). To award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part.*

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent *A*** marks can be awarded, but ***M*** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (****MR****). Apply a* ***MR*** *penalty of 1 mark to that question. Award the marks as usual and then write –1(****MR****) next to the total. Subtract 1 mark from the total for the question. A candidate should be penalized only once for a particular mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled ***(d)*** *and a brief* ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2\sin(5x-3)$, the markscheme gives:

$f'(x) = \left(2\cos(5x-3)\right)5 \quad \left(=10\cos(5x-3)\right)$ ***A1***

Award ***A1*** for $\left(2\cos(5x-3)\right)5$, even if $10\cos(5x-3)$ is not seen.

10 Accuracy of Answers

*The method of dealing with accuracy errors on a whole paper basis by means of the Accuracy Penalty **(AP)** no longer applies.*

Instructions to examiners about such numerical issues will be provided on a question by question basis within the framework of mathematical correctness, numerical understanding and contextual appropriateness.

*The rubric on the front page of each question paper is given for the guidance of candidates. The markscheme (MS) may contain instructions to examiners in the form of "Accept answers which round to n significant figures (sf)". Where candidates state answers, required by the question, to fewer than n sf, award **A0**. Some intermediate numerical answers may be required by the MS but not by the question. In these cases only award the mark(s) if the candidate states the answer exactly or to at least 2sf.*

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

SECTION A

1. $f(2) = 8 + 4a + 2b - 4 = 0$ ***M1***

$\Rightarrow 4a + 2b = -4$ ***A1***

$f(1) = 1 + a + b - 4 = -6$ ***M1***

$\Rightarrow a + b = -3$ ***A1***

solving, $a = 1$, $b = -4$ ***A1A1***

[6 marks]

2. we are given that $ar^2 = 9$ and $\frac{a}{1-r} = 64$ ***A1***

dividing, $r^2(1-r) = \frac{9}{64}$ ***M1***

$64r^3 - 64r^2 + 9 = 0$ ***A1***

$r = 0.75$, $a = 16$ ***A1A1***

[5 marks]

3. (a)

Interval	Frequency
]1.0, 1.1]	6
]1.1, 1.2]	28
]1.2, 1.3]	18
]1.3, 1.4]	14
]1.4, 1.5]	10
]1.5, 1.6]	4

A2

[2 marks]

(b) $\mu = 1.26$, $\sigma = 0.133$ ***A1A1***

[2 marks]

(c) no because the normal distribution is symmetric and these data are not ***R2***

[2 marks]

Total [6 marks]

4. (a) $\text{mod}(z) = 2$, $\arg(z) = 150^\circ$ ***A1A1***

[2 marks]

(b) $z^{\frac{1}{3}} = 2^{\frac{1}{3}}(\cos 50^\circ + \text{i}\sin 50^\circ)$ ***(M1)***

$= 0.810 + 0.965\text{i}$ ***A1***

[2 marks]

(c) we require to find a multiple of 150 that is also a multiple of 360, so by any method, ***M1***

$n = 12$ ***A1***

Note: Only award 1 mark for part (c) if $n = 12$ is based on $\arg(z) = -30$.

[2 marks]

Total [6 marks]

5. (a) (i) displacement $= \int_0^3 v \, dt$ ***(M1)***

$= 0.703$ (m) ***A1***

(ii) total distance $= \int_0^3 |v| \, dt$ ***(M1)***

$= 2.05$ (m) ***A1***

[4 marks]

(b) solving the equation $\int_0^t \left|\cos(u^2)\right| du = 1$ ***(M1)***

$t = 1.39$ (s) ***A1***

[2 marks]

Total [6 marks]

6. vertical asymptote $x = -4 \Rightarrow -4b + c = 0$ ***M1***

horizontal asymptote $y = -2 \Rightarrow \frac{1}{b} = -2$ ***M1***

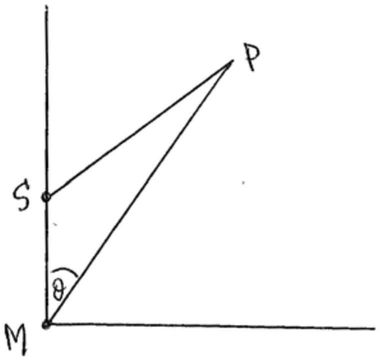
$b = -\frac{1}{2}$ and $c = -2$ ***A1A1***

$1 = \frac{\frac{2}{3} + a}{-\frac{1}{2} \times \frac{2}{3} - 2}$ ***M1***

$a = -3$ ***A1***

[6 marks]

7.



(a) let the interception occur at the point P, t hrs after 12:00
then, $\text{SP} = 20t$ and $\text{MP} = 30t$ ***A1***
using the sine rule,

$$\frac{\text{SP}}{\text{MP}} = \frac{2}{3} = \frac{\sin\theta}{\sin 135}$$ ***M1A1***

whence $\theta = 28.1$ ***A1***

[4 marks]

(b) using the sine rule again,

$$\frac{\text{MP}}{\text{MS}} = \frac{\sin 135}{\sin(45 - 28.1255...)}$$ ***M1A1***

$$30t = 10 \times \frac{\sin 135}{\sin 16.8745...}$$ ***M1***

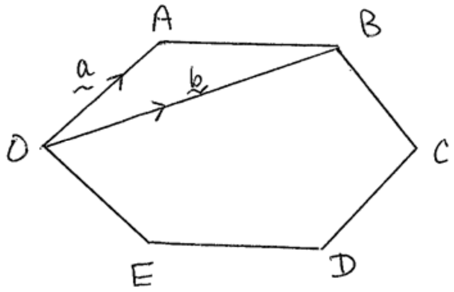
$t = 0.81199...$ ***A1***

the interception occurs at 12:49 ***A1***

[5 marks]

Total [9 marks]

8.



(a) $\text{OC} = \text{AB} + \text{OA}\cos 60 + \text{BC}\cos 60$ ***M1***

$= \text{AB} + \text{AB} \times \frac{1}{2} + \text{AB} \times \frac{1}{2}$ ***A1***

$= 2\text{AB}$ ***AG***

[2 marks]

(b) $\vec{\text{OC}} = 2\vec{\text{AB}} = 2(\boldsymbol{b} - \boldsymbol{a})$ ***M1A1***

$\vec{\text{OD}} = \vec{\text{OC}} + \vec{\text{CD}}$ ***M1***

$= \vec{\text{OC}} + \vec{\text{AO}}$ ***A1***

$= 2\boldsymbol{b} - 2\boldsymbol{a} - \boldsymbol{a} = 2\boldsymbol{b} - 3\boldsymbol{a}$ ***A1***

$\vec{\text{OE}} = \vec{\text{BC}}$ ***M1***

$= 2\boldsymbol{b} - 2\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{b} - 2\boldsymbol{a}$ ***A1***

[7 marks]

Total [9 marks]

9. let x, y (m) denote respectively the distance of the bottom of the ladder from the wall and the distance of the top of the ladder from the ground
then,

$x^2 + y^2 = 100$ ***M1A1***

$2x\frac{\text{d}x}{\text{d}t} + 2y\frac{\text{d}y}{\text{d}t} = 0$ ***M1A1***

when $x = 4$, $y = \sqrt{84}$ and $\frac{\text{d}x}{\text{d}t} = 0.5$ ***A1***

substituting, $2 \times 4 \times 0.5 + 2\sqrt{84}\frac{\text{d}y}{\text{d}t} = 0$ ***A1***

$\frac{\text{d}y}{\text{d}t} = -0.218 \text{ m s}^{-1}$ ***A1***

(speed of descent is 0.218 ms^{-1})

[7 marks]

SECTION B

10. (a) (i) $\overrightarrow{OA} \times \overrightarrow{OB} = \boldsymbol{i} + 7\boldsymbol{j} - 5\boldsymbol{k}$ ***A1***

(ii) area $= \frac{1}{2}|\boldsymbol{i} + 7\boldsymbol{j} - 5\boldsymbol{k}| = \frac{5\sqrt{3}}{2}$ (4.33) ***M1A1***

(iii) equation of plane is $x + 7y - 5z = k$ ***M1***

$x + 7y - 5z = 0$ ***A1***

[5 marks]

(b) (i) direction of line $= (3\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}) - (\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) = 2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$ ***M1A1***

equation of line is

$\boldsymbol{r} = (\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) + \lambda(2\boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k})$ ***A1***

(ii) at a point of intersection,

$1 + 2\lambda = 2 + \mu$

$2 - \lambda = 4 + 3\mu$ ***M1A1***

$3 - \lambda = 3 + 2\mu$

solving the 2nd and 3rd equations, $\lambda = 4$, $\mu = -2$ ***A1***

these values do not satisfy the 1st equation so the lines are skew ***R1***

[7 marks]

Total [12 marks]

11. (a) (i) $S_1 = P\left(1+\frac{I}{100}\right) - R$ ***A1***

$S_2 = P\left(1+\frac{I}{100}\right)^2 - R\left(1+\frac{I}{100}\right) - R$ ***M1A1***

$= P\left(1+\frac{I}{100}\right)^2 - R\left(1+\left(1+\frac{I}{100}\right)\right)$ ***AG***

(ii) extending this,

$S_n = P\left(1+\frac{I}{100}\right)^n - R\left(1+\left(1+\frac{I}{100}\right)+\ldots+\left(1+\frac{I}{100}\right)^{n-1}\right)$ ***M1A1***

$= P\left(1+\frac{I}{100}\right)^n - \frac{R\left(\left(1+\frac{I}{100}\right)^n - 1\right)}{\frac{I}{100}}$ ***M1A1***

$= P\left(1+\frac{I}{100}\right)^n - \frac{100R}{I}\left(\left(1+\frac{I}{100}\right)^n - 1\right)$ ***AG***

[7 marks]

(b) (i) putting $S_{60} = 0$, $P = 5000$, $I = 1$ ***M1***

$5000 \times 1.01^{60} = 100R(1.01^{60} - 1)$ ***A1***

$R = (\$)111.22$ ***A1***

(ii) putting $n = 20$, $P = 5000$, $I = 1$, $R = 111.22$ ***M1***

$S_{20} = 5000 \times 1.01^{20} - 100 \times 111.22(1.01^{20} - 1)$ ***A1***

$= (\$)3652$ ***A1***

which is the outstanding amount

[6 marks]

Total [13 marks]

12. (a) we are given that

$2.1 = \mu - 0.5244\sigma$

$2.5 = \mu + 0.6745\sigma$ ***M1A1***

$\mu = 2.27,\ \sigma = 0.334$ ***A1A1***

[4 marks]

(b) (i) let X denote the number of birds weighing more than 2.5 kg

then X is $\mathrm{B}(10,\ 0.25)$ ***A1***

$\mathrm{E}(X) = 2.5$ ***A1***

(ii) 0.0584 ***A1***

(iii) to find the most likely value of X, consider

$p_0 = 0.0563\ldots,\ p_1 = 0.1877\ldots,\ p_2 = 0.2815\ldots,\ p_3 = 0.2502\ldots$ ***M1***

therefore, most likely value $= 2$ ***A1***

[5 marks]

(c) (i) we solve $1 - \mathrm{P}(Y \le 1) = 0.80085$ using the GDC ***M1***

$\lambda = 3.00$ ***A1***

(ii) let $X_1,\ X_2$ denote the number of eggs laid by each bird

$\mathrm{P}(X_1 + X_2 = 2) = \mathrm{P}(X_1 = 0)\mathrm{P}(X_2 = 1) + \mathrm{P}(X_1 = 1)\mathrm{P}(X_2 = 1) + \mathrm{P}(X_1 = 2)\mathrm{P}(X_2 = 0)$ ***M1A1***

$= \mathrm{e}^{-3} \times \mathrm{e}^{-3} \times \frac{9}{2} + (\mathrm{e}^{-3} \times 3)^2 + \mathrm{e}^{-3} \times \frac{9}{2} \times \mathrm{e}^{-3} = 0.0446$ ***A1***

(iii) $\mathrm{P}(X_1 = 1,\ X_2 = 1 \mid X_1 + X_2 = 2) = \frac{\mathrm{P}(X_1 = 1,\ X_2 = 1)}{\mathrm{P}(X_1 + X_2 = 2)}$ ***M1A1***

$= 0.5$ ***A1***

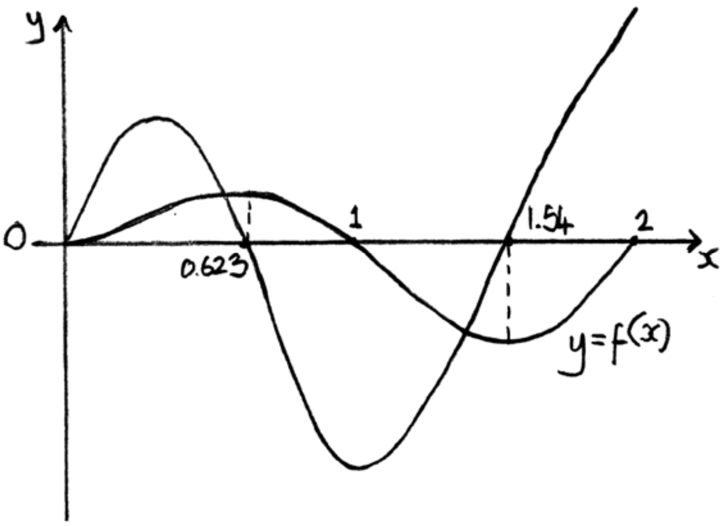
[8 marks]

Total [17 marks]

13. (a) $f'(x) = \frac{1}{x+1}\sin(\pi x) + \pi \ln(x+1)\cos(\pi x)$ ***M1A1A1***

[3 marks]

(b)



A4

Note: Award ***A1A1*** for graphs, ***A1A1*** for intercepts.

[4 marks]

(c) $0.310, 1.12$ ***A1A1***

[2 marks]

(d) $f'(0.75) = -0.839092$ ***A1***

so equation of normal is $y - 0.39570812 = \frac{1}{0.839092}(x - 0.75)$ ***M1***

$y = 1.19x - 0.498$ ***A1***

[3 marks]

(e) $\text{A}(0, 0)$

$\text{B}(\overbrace{0.548\ldots}^{c}, \overbrace{0.432\ldots}^{d})$ ***A1***

$\text{C}(\overbrace{1.44\ldots}^{e}, \overbrace{-0.881\ldots}^{f})$ ***A1***

Note: Accept coordinates for B and C rounded to 3 significant figures.

area $\Delta\text{ABC} = \frac{1}{2}\left|(c\boldsymbol{i} + d\boldsymbol{j}) \times (e\boldsymbol{i} + f\boldsymbol{j})\right|$ ***M1A1***

$= \frac{1}{2}(de - cf)$ ***A1***

$= 0.554$ ***A1***

[6 marks]

Total [18 marks]