MRES | MATHEMATICS

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This handbook and the following are available on the School's website:  
http://www.birmingham.ac.uk/MathsPG

Although every effort is made to ensure accuracy, the School of Mathematics reserves the right to modify or cancel any statement in this handbook in light of events occurring after its production.
This booklet contains descriptions of the level M courses available to postgraduate students. Students studying for the MRes degree must take 60 credits worth of courses in their first year. Each course is normally 20 credits (we call it a double module, it runs in the autumn and the spring), though there are also some 10 credit single modules. Students are required to select courses, within the discipline they are studying. For example a student studying for an MRes in Applied Mathematics must select 60 credits worth of modules normally from the Applied Mathematics provision. These modules should be selected in consultation with your supervisor and decisions must be made, at the very latest, in the first week of term as the courses will all have started by then. A good time to see your supervisor is the day after the Induction meeting. Students studying for the MRes may well be recommended additional credits for “thesis preparation”. These courses will generally be chosen from our undergraduate provision. These credits will not form part of the 60 credits of course work.

The MRes programme aims to train students up to the frontiers of research in selected mathematical areas by attendance for 7 months on appropriate courses and research seminars, while at the same time developing work on a research topic to be continued for a further 5 months.
5 School Code of Conduct

Students in the School of Mathematics are expected to conduct themselves in a manner consistent with University regulations. They are expected to make themselves fully aware of their responsibilities and obligations and to meet these on time.

Information on University regulations, guidance notes and codes of practice can be found in the "for students" section of the Web pages of the University of Birmingham.

Many of the lecture classes in the School, all first and second year classes in particular are large. Noise and casual chatter, if they occur, can cause difficulties to the majority of students and be distracting to the lecturer, possibly resulting in a decrease in quality of the lecture.

During lectures, only discussions and conversations led by the lecturer are permitted. Any student failing to abide by this rule or causing disruption may be required to leave the lecture room and subsequently disciplinary action regarding the incident may be taken.

Mobile phones must be switched off during lectures and other classes.

Several large computer clusters are provided in the Poynting Building (R13 on the campus map) and in the Learning Centre which will be used by several courses on your programme. You have free access to these clusters when they are not booked by a class. If they are booked by a class, you will need to seek and obtain permission from whoever is supervising the class before you can use a computer. This holds even if there are many computers not being used.

These clusters are provided for work related to your degree programme, so at all times disturbances of any sort need to be avoided. These include playing personal stereos (even with earphones if the volume is high), loud conversations, etc.
6 | Office hours

All members of staff (unless they are on study leave) have allocated a total of one and a half hours per week during term-time (known as Office Hours) when they guarantee to be available to see their tutees and other students. Office Hours are displayed prominently on office doors. If you wish to see a member of staff during his/her office hours but cannot because of a clash with lectures, or other good reason, then email the member of staff to arrange an appointment.

Students are strongly encouraged to take advantage of lecturers' office hours. Feel free to turn up either individually or in small groups. As well as providing you with the opportunity to discuss mathematics with a lecturer on a one-to-one basis, office hours also give lecturers valuable feedback on how his/her course is progressing.

Should you require additional feedback on a piece of continuous assessment, then please see the lecturer during his/her office hours.

Progress review tutorials

All students have three progress review tutorials with their personal tutor, one towards the end of each of the Autumn, Spring and Summer Terms. The attendance at these is mandatory. Students will be sent, by email, information about progress review tutorials a week or two before they are due to take place.
Learning Outcomes and Objectives
By the end of the module the student should be able to: describe the deformation and stress in an elastic solid or fluid; solve simples problems involving these continua; realise which effects play a role in the motion of a fluid; describe the flow (velocities and pressures) in certain configurations, for example, the flow around a cylinder, sphere and aerofoil. By the end of the module, students should also be able to explore these topics beyond the taught syllabus.

Module Description
Elsewhere in the programme, a study of particle and rigid body dynamics was made. In this module, we consider the case when the number of particles tends to infinity, with relative motion between the particles. This course will develop the idea of a continuum and study the deformation, strain and stress of both elastic and fluid continua. A preliminary survey of the physical properties of common elastic solids and fluids will be followed by a development of the appropriate strain and stress tensors and general conservation laws. Simple examples of deformation such as uniaxial tension and torsion in elastic solids will be given. This module also contains the derivation and solution of the equations which govern the motion of fluids. The module uses the solution of partial differential equations and exploits complex variable theory in order to describe some quite involved flows. This module will stand as an introduction to the subject in its own right and also as an introduction to the more advanced material at Level M.

Teaching Methods
44 hours of lectures
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
Learning Outcomes and Objectives
By the end of this module, students should be able to: classify a partial differential equation, and make an intelligent decision concerning which methods could be exploited in order to solve the problem; (in certain cases) find an analytical solution to the problem, and realise when this is not viable; understand how to model reaction and diffusion processes; formulate reaction-diffusion models of chemical processes and simple models of epidemics; motivate reaction-diffusion models of nerve signals; analyse the governing equations and interpret the results. By the end of this module, students should also be able to explore these topics beyond the taught syllabus.

Module Description
This module exposes students to a variety of techniques available to solve partial differential equations. Solutions can be constructed by combining known solutions to the problem or by direct analytical calculations. Numerous chemical and biological phenomena are modelled by non-linear ODEs, for example chemical reactions and voltage clamped nerve cells. Incorporating spatial heterogeneity in such models yields parabolic PDE’s with non-linear interaction kinetics, often referred to as reaction-diffusion equations. In this module, such equations and their application are investigated in detail.

Teaching Methods
44 of hours lectures
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
9 | Perturbation Theory & Chaos

MSM4A05 | 06 22517 | 20 Credits | Autumn & Spring Term

Learning Outcomes and Objectives
By the end of the module students should be able to: demonstrate an understanding of the methods used to obtain asymptotic expansions of integrals; demonstrate an understanding of the methods used to obtain asymptotic expansions of solutions to ordinary differential equations; find the limiting behaviour of certain dynamical systems; determine whether or not a dynamical system has chaotic traits. By the end of the module, students should also be able to explore these topics beyond the taught syllabus.

Module Description
This module provides an introduction to the concepts and techniques of perturbation analysis and its applications. This module is also designed to allow students to meet some of the concepts used to analyse Dynamical Systems and maps. The idea of what is meant by a system being chaotic is explored. Examples are given of systems which, although they appear simple, can yield incredibly beautiful tee shirt designs.

Teaching Methods
44 hours of lectures
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
10 | Viscous Fluid Mechanics with Applications

MSM4A06 | 06 22945 | 20 Credits | Autumn and Spring Term

Learning Outcomes and Objectives
By the end of the module students should be able to: develop a more comprehensive understanding of viscous fluid mechanics which will be coupled to visualization of flows provided on video and CD; demonstrate an ability to apply theoretical developments in viscous fluid mechanics to a range of problems drawn from science, engineering, medicine and industry.

Module Description
The main areas covered by this module is introductory level in viscous fluid mechanics where the importance of the viscosity of the fluid has a direct impact on the motion of the fluid. The method of matched asymptotic expansions will be exploited to understand both highly and weakly viscous flows; in the latter case using singular perturbation techniques. Other area of this module covers the use the methods developed for very low Reynolds numbers, high values of the Womersley parameter and intermediate Reynolds numbers in applications drawn from biological and physiological fluid mechanics, micro-fluidics and a range of applications from industry.

Teaching Methods
44 hours of lectures
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
11 Waves

MSM4A07 | 06 22518 | 20 Credits | Autumn and Spring Term

Learning Outcomes and Objectives
By the end of this module, the student will be able to: develop mathematical models of waves; solve differential equations arising from waves, including the use of the method of characteristics; understand and apply the physics of waves in various media.

Module Description
This module examines waves in various media, including water waves, sound waves and waves in elastic bodies. Various applications will be considered including earthquakes, ship waves and shock waves in gases and liquids. Also a selection of mathematical models for waves and various mathematical methods for the solution of the resulting problems will be studied.

Teaching Methods
44 hours of lectures
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
Learning Outcomes and Objectives
By the end of the module the student should be able to: demonstrate understanding of the theory of the computational solution of ordinary and partial differential equations; implement the numerical solution of differential equations using a suitable computer package or computer language; demonstrate knowledge of recent developments in computational methods.

Module Description
In the Autumn Term, this module examines the computational solution of ordinary and partial differential equations. A mix of computational methods will be examined, including, for example, the use of the finite difference method and the finite element method. The implementation of these methods will also be examined in a suitable computer package or computer language, such as for example in MATLAB. In the Spring Term, members of staff actively involved in research in computational mathematics will describe the current state of the art in their fields. There may be several topics covered, relating to the actual research interests of the staff involved.

Teaching Methods*
22 hours of lectures
44 hours of computer based laboratories
* depending on the actual topic changes may apply, typically in the ratio of 1 lecture to 2 hours computer laboratories

Assessment
70% based on a 3hr written examination
30% based on course work during term time
13 | Computational Methods & Programming

MSM4A12 | 06 22295 | 20 Credits | Autumn & Spring Term

Learning Outcomes and Objectives
By the end of the module the student should be able to: demonstrate knowledge of standard numerical methods for ordinary and partial differential equations; apply a range of numerical methods to standard differential equations and derive the relevant error analysis; have an understanding of the programming issues associated with numerical schemes; understand the basic principles of scientific and engineering programming; understand key attributes of the following elements of programming languages: variables; data types; data arrays; assignment statements; iteration statements; develop and implement numerically stable and accurate algorithms for the basic tasks of computational science and engineering; develop stable algorithms for matrix algebra, numerical solution of ordinary differential equations and for finding roots of non-linear equations.

Module Description
In the Autumn term, a selection of numerical methods for differential equations will be reviewed, together with some of the existing analysis. The material is divided into two main parts, each concerned, respectively, with the numerical solution of ordinary differential equations (ODE) and partial differential equations (PDE). Part I introduces numerical methods for first-order ODE and is limited to one-step methods, with emphasis on Runge-Kutta methods. It includes a derivation of standard error analysis (stability, order, accuracy, consistency) for both implicit and explicit methods. It also considers the solution of linear systems and questions related to stability of dynamical systems. Examples are drawn from Hamiltonian mechanics. Some examples of nonlinear ODE are also considered. Part II is devoted to the finite element method (FEM) for the discretization of PDE. The presentation starts with a review of function spaces with emphasis on Sobolev spaces and polynomial approximation in these spaces. It then includes the variational formulation of boundary value problems, definition and construction of finite element spaces. Standard error analysis is also included: a priori and a posteriori error estimation together with theoretical questions regarding accuracy, stability, reliability and adaptivity. In the Spring term, fundamental concepts necessary in order to think like a programmer will be presented. This course has been developed for students with no previous programming experience as well as for those already familiar with main ideas in programming. The computer language (C++) is introduced from first principles. The students are supposed to learn general techniques for solving programming problems and to understand important concepts of programming such as algorithms and code debugging. The programming techniques introduced in the course will be implemented for numerical solution of several basic scientific and engineering problems.

Teaching Methods
38 hours of lectures
22 hours of computer based laboratories
11 hours of back-up classes

Assessment
70% based on a 3hr written examination
30% based on course work during term time
Learning Outcomes and Objectives
By the end of the module students should be able to: apply successfully core ideas in the mathematical modelling of molecules, cells and organisms; formulate models of new problems using the ideas presented in the course, make informed decisions about the analysis and solution of these models; link mathematical models to experimental work, particularly in the estimation of parameters and the testing and refining of models; apply mathematical models to make useful predictions regarding biological and biomedical problems.

Module Description
Mathematical models are used increasingly to understand complex phenomena in biology and medicine, and have been used to explain phenomena at a wide range of scales, from genes, proteins and metabolites, cells, tissues and organs, to organisms, populations and ecosystems. This module builds on the students’ knowledge of mathematical nonlinear differential and difference equations to explore the paradigm models in mathematical biology, particularly microbiology, developmental biology and physiology. The mathematical models will be linked to experimental work and biomedical science, in particular focusing on the importance of experiment in testing and refining models, in estimating parameters, and finally the application of models in making useful predictions.

Teaching Methods
22 hours of lectures
4 hours of computer sessions
5 hours of back-up

Assessment
85% based on a 1hr 30min written examination
10% based on course work during term time
5% based on assessed computer laboratories
Learning Outcomes and Objectives
By the end of the module the student should be able to: write down the governing partial differential equations and boundary conditions for a range of financial derivative problems; solve the relevant partial differential equations arising from the study of some financial derivative problems using analytical and computational methods; demonstrate an understanding of how mathematics and in particular discrete mathematics is used in the financial sector of the economy; demonstrate an understanding of interest calculations, asset return and investment types such as bonds, futures and options, and of how investment portfolios of risky assets should be composed in order to obtain a desired return with minimum risk; explore these topics beyond the taught syllabus.

Module Description
Financial derivatives will be examined using a continuous-time approach, examining the relevant partial differential equations and boundary conditions in a number of different problems. The solution method will also be examined, using a mix of analytical and computational methods. A range of discrete time financial models will be analysed. This will include mainly (but not exclusively) the return of assets and their volatility, two-asset and multi-asset portfolio optimisation and various investment models such as options, futures and bonds.

Teaching Methods
44 hours of lectures (2 hours weekly)
10 hours of back-up

Assessment
90% based on a 3 hour written examination
10% based on course work during term time
Learning Outcomes and Objectives
By the end of the module the student should be able to: demonstrate knowledge and understanding of interest rate derivative products and Bonds, and other advanced topics in Mathematical Finance; solve mathematical problems in the pricing of various derivative products; explore these topics beyond the taught syllabus.

Module Description
To introduce the student to the pricing of more advanced derivative products in Mathematical Finance, and to examine more advanced solution methods. Advanced derivative products of the kind often used in reality will be examined in more detail. This will include examining options not considered previously, but also looking in more detail at derivative products already studied in the Mathematical Finance module. This may include topics such as: energy, weather and insurance derivatives; credit derivatives; market crashes; multi-asset options; interest rate derivatives. More advanced numerical methods for the solution of option pricing problems will also be examined. This module should better prepare students for a career in Mathematical Modelling in Finance, including a broader background in the trading of derivatives and options.

Teaching Methods
22 hours of lectures
5 hours of example classes

Assessment
90% based on a 1hr 30min written examination
10% based on continuous assessment
Learning Outcomes and Objectives
Students will understand that integer programming problems arise in many fields such as engineering, economics, and management. They will understand basic theory and algorithms for integer programs and understand why integer programs are hard to handle in general. They will know how to use the basic techniques of integer programming (branch-bound, relaxation, cutting-plane, heuristics, etc.) to solve integer programming models for a variety of managerial and other practical problems. They will be able to solve an IP problem efficiently by taking into account the particular structure of the problem and interpret the results obtained. They will be able to explain the difference between discrete and continuous optimisation, and understand the uniqueness of the techniques for solving integer programs. By the end of the module students should be able to explore these topics beyond the taught syllabus.

Module Description
Many practical problems such as train and airline scheduling, vehicle routing, production planning, resource management and telecommunications network design can be modelled as integer (or mixed-integer) programs. This module presents a comprehensive theory of and exact and approximate algorithms for integer programming and a wide variety of its applications starting from formulations and illustrative examples of integer programs to optimality, relaxation to methods such as branch and bound, cutting planes and valid inequalities, Lagrangean relaxation and heuristic methods. Basic computational complexity theory will be also introduced. Modern semi-definite programming (SDP) technique dealing with integer programming is optional and at the discretion of the lecturer in charge.

Teaching Methods
2 hours of lectures weekly
1 hour of problem discussion fortnightly

Assessment
90% based on a 1hr 30min examination
10% based on course work during term time
Combinatorial Optimisation

MSM4M02b | 06 20442 | 10 Credits | Spring Term

Learning Outcomes and Objectives
The students should be able to formulate practical problems as discrete optimisations and apply exact or approximate algorithms to problems like shortest path, network flows, transportation, and knapsack and travelling salesman problems; demonstrate an understanding of computational complexity and its reduction and of proofs that certain problems are NP hard, demonstrate an understanding of the role of matroids in optimisation. By the end of the module, the students should be able to explore this topic beyond the taught syllabus.

Module Description
This module develops some of the ideas introduced in Management Mathematics, and will continue to encourage interest in practical problems. The module presents a systematic survey of methods of optimisation for problems with discrete features and relates them to practical problems such as finding the cheapest route through a transportation network or efficiently assigning resources to objectives. The concept of computational complexity leads to a classification of problems into grades of hardness and to the concept of the efficiency of an algorithm.

Teaching Methods
2 hours lectures weekly
1 hour problem discussion fortnightly

Assessment
90% based on a 1hr 30min examination
10% based on course work during term time
Learning Outcomes and Objectives
By the end of this module, students should be able to understand and use the main concepts of game theory like optimal strategies, stable sets and the core of the game; formulate games from economic narratives; use a range of methods for solving these games. By the end of the module students should be able to explore these topics beyond the taught syllabus.

Module Description
This module presents game theory as the study of decision-making in competitive situations. It provides an introduction to the theory of finite and infinite games with a particular emphasis on 2-person games. All results will be presented in a rigorous way and accompanied, wherever possible, by showing economic applications. This module also demonstrates that the results and concepts of other branches of mathematics (like the fixed point theorem, convexity, duality) have practical interpretation and use.

Teaching Methods
22 hours of lectures
5 hours of back-up

Assessment
90% based on a 1hr 30min written examination
10% based on course work during term time
Learning Outcomes and Objectives
Students will be able to formulate management problems using conic programming techniques, understand and deal with the difficulties which may arise in practice (formulation, implementation, etc.), use and develop their own judgement in attempting to address issues including some research topics.

Module Description
The course focuses on techniques of conic programming, in particular, conic quadratic and semidefinite programming. The main topics will include:
(a) Convex conic programming: Convex sets, cones. From LP to conic programming. Quadratically Constrained Quadratic Programming (QCQP);
(b) Semidefinite Programming: Definition. What can be expressed via SDP. Duality and optimality conditions;
(c) Robust optimisation: What it is good for. Robust counterparts to LP, QCQP and SDP;

The choice of the topics may vary from year to year, but at least two topics from each part will be covered.

Teaching Methods
22 hours of lectures
5 hours of back-up

Assessment
90% based on a 1hr 30min written examination
10% based on course work during term
Learning Outcomes and Objectives
By the end of the module, the student should be able to: understand and explain the basic concepts of nonlinear programming; understand and explain first and second order optimality conditions; understand and explain the role of convexity in optimisation. By the end of the module, the student should be able to explore this topic beyond the taught syllabus.

Module Description
Many decision problems arising in managerial decision making in the public as well as in the private sector are inherently nonlinear, and the same holds for various problems occurring in science and engineering tackling highly realistic nonlinear problems leads to solution methods totally different from those of, say, linear programming. In this course, the essential ideas as well as some of the most important solution algorithms for nonlinear decision problems are studied.

Teaching Methods
22 hours of lectures
5 hours of back-up

Assessment
90% based on a 1hr 30min written examination
10% based on course work during term time
Learning Outcomes and Objectives
By the end of the module, the students should be able to: understand and explain further concepts of nonlinear programming; discuss several different solution methods, apply them, and understand their limitation in practice; discuss recent trends and latest developments in nonlinear programming.

Module Description
Many decision problems arising in managerial decision making in the public as well as in the private sector are inherently nonlinear, and the same holds for various problems occurring in science and engineering tackling highly realistic nonlinear problems leads to solution methods totally different from those of, say, linear programming. In this course, further theory as well as several highly innovative solution methods for nonlinear decision problems are studied. This course uses and further develops knowledge gained in Nonlinear Programming I of both constrained and unconstrained optimisation.

Teaching Methods
22 hours of lectures
5 hours of back-up

Assessment
90% based on a 1hr 30min written examination
10% based on course work during term time
23 | Heuristic Optimisation

MSM4M12a | 06 19611 | 10 Credits | Autumn Term

Learning Outcomes and Objectives
By the end of the module, the student should be able to: understand and explain why and when heuristic optimization techniques are useful in Management mathematics; understand and explain the basic concepts of classical heuristic optimization techniques; design data structure for the computer code and apply rules of heuristics for that problem. By the end of the module students should be able to explore these topics beyond the taught syllabus.

Module Description
Most problems from management mathematics (discrete or continuous) are NP-hard. In other words, optimization problems that arise in industry or in public sector could not be solved exactly in reasonable computing time, even with modern computers. Therefore, when traditional mathematics techniques fail to give fast answer, one should relay on near-optimal solution methods or heuristics. Ideas of classical heuristics (greedy, constructive, relaxation, local search, Lagrangean, etc.) will be studied first. A modern heuristics (metaheuristics) or general frameworks for building heuristics, usually give rules of escaping from the so-called “local optima trap”. Such methods are Genetic algorithms, Tabu search, Variable neighbourhood search, etc.

Teaching Methods
22 hours of lectures
5 hours of back-up

Assessment
90% based on a 1hr 30min written examination
10% based on course work during term time
Learning Outcomes and Objectives
By the end of the module, the student should be able to understand and explain the basic concepts of multicriteria optimisation formulate real-world problems from management and science as multicriteria optimisation problems, discuss different solution concepts.

Module Description
In multicriteria or multiobjective optimisation, several conflicting objectives have to be optimised simultaneously: stock portfolios have to be chosen such that the portfolio maximises the return and simultaneously minimises the risk; health care has to be managed such that the service is efficient yet not too costly; hazardous material has to be transported such that risk as well as costs incurred are minimal; bridges and buildings have to be designed such that they can be build cheap but still with maximum stability, etc. As such, multicriteria problems are of prime importance for decision makers in the private as well as the public sector. This module brings together various ideas from geometry and analysis in studying solutions and solution methods for multicriteria problems in management and science.

Teaching Methods
22 hours of lectures
5 hours of back-up

Assessment
90% based on a 1hr 30min written examination
10% based on course work during term time
Learning Outcomes and Objectives
That the student be able to: analyze Diophantine equations by factorizations in appropriate rings and the use of modular arithmetic; understand modular arithmetic and the techniques involved in proving; Gauss’ Law of Quadratic Reciprocity; use this law to find quadratic residues. By the end of the module, students should be able to explore these topics beyond the taught syllabus.

Module Description
A spectacular development in mathematics is Wiles’ proof of Fermat’s Last Theorem: if $n>2$ then $x^n+y^n=z^n$ has no nontrivial integer solutions. A high point of the module is a proof of Fermat’s Last Theorem for $n=3$. Ideas relating to integer and primes are generalized to other number systems e.g. the Gaussian integers $\mathbb{Z}[i] = \{x + iy \mid x \text{ and } y \text{ integers}\}$. An analogue of the Fundamental Theorem of Arithmetic is proved for $\mathbb{Z}[i]$. Concrete numerical examples illustrate to concepts involved. Modular arithmetic is studied. The high point being Gauss’ celebrated Law of Quadratic Reciprocity concerning the existence of square roots modulo a prime. Time permitting, other topics may be studied, e.g. Fermat’s Last Theorem for $n=5$, Mersenne primes, the abc-conjecture, recent advances.

Teaching Methods
44 hours of lectures
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
26 | Group Theory & Galois Theory

MSM4P08 | 06 22519 | 20 Credits | Autumn & Spring Term

Learning Outcomes and Objectives
By the end of the module students should be able to: handle and apply group actions, the Sylow theorems and the Jordan-Hölder theorem; understand the Galois group of a field extension; calculate Galois groups of some polynomials; understand the Galois correspondence. By the end of the module students should be able to explore these topics beyond the taught syllabus.

Module Description
The highlight of this Group Theory Module is Sylow’s Theorem, which is probably the most fundamental result about the structure of finite groups. The Jordan-Hoelder Theorem shows that simple groups are the building blocks from which other groups are built, while the alternating groups are also provided as the first examples of nonabelian simple groups. Galois theory studies field extensions, particularly those arising from adjoining roots of polynomials. To each field extension we associate the Galois group. The structure of the group is related to the structure of the field extension via the Fundamental Theorem of Galois Theory.

Teaching Methods
44 hours of lectures
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
Learning Outcomes and Objectives
On completion of this module the student will be able to: demonstrate an understanding of the material delivered; apply their acquired knowledge to the solution of relevant problems; appreciate a number of areas of modern Pure Mathematics and contemporary research in this field.

Module Description
This module is devoted to Algebraic Number Theory. In the first semester, after reviewing the necessary facts from the Galois Theory and introducing norms and traces, we will concentrate on the theory of Dedekind domains, including ideal factorization, units, etc. This theory will be illustrated on the examples from number fields and on Kummer’s approach to the Fermat’s Last Theorem.
In the second semester, we switch to a more geometric point of view and develop the theory of valuations, which will be applied to examples from function fields. The connection to algebraic curves (Riemann surfaces) will be made. Time permitting we will discuss the Riemann-Roch Theorem and possibly show a glimpse of the more modern techniques involved in Wiles' proof of the Fermat's Last Theorem.

Teaching Methods
44 hours of lectures and 10 hours of back-up

Assessment
80% based on a 3hr written examination in the Summer Term
20% based on course work during term-time
Learning Outcomes and Objectives
By the end of the module, the student should be familiar with: basic graph parameters and their relationships; understand and be able to apply fundamental results on graphs; understand how some problems can be modelled as algorithmic graph problems and should know approaches to solve them; be able to apply the basic techniques introduced in the lectures. By the end of the module, the student should also be able to explore these topics beyond the taught syllabus.

Module Description
The module will give an introduction to fundamental results and concepts in Graph theory. Topics are likely to include Hamilton cycles, graph matching, connectivity, graph colourings, planar graphs and extremal graph problems. For example, a fundamental result in Graph Theory is Dirac’s theorem. It gives a condition which ensures that a graph has a Hamilton cycle (i.e. a cycle which contains all vertices of the graph). Another example is the four colour theorem. It states that every planar map can be coloured with at most 4 colours such that adjacent regions have different colours (this can be translated into a graph colouring problem). Some of the ideas will also be applied to algorithmic problems for graphs.

Teaching Methods
44 hours of lectures
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
Learning Outcomes and Objectives
By the end of the module, the student should: be familiar with basic combinatorial structures; understand and be able to apply fundamental results on finite sets, codes and communication over a noisy channel; be able to apply the basic techniques introduced in the lectures. By the end of the module, the student should also be able to explore these topics beyond the taught syllabus.

Module Description
The Combinatorics part of the module will give an introduction to several combinatorial structures. In particular, the module will give an introduction to Ramsey theory, which asserts (loosely speaking) that complete disorder is impossible. An easy example of a result in Ramsey theory is that in a group of six people, there are always three people who know each other or three people who are strangers to each other. Another typical example is that whenever the integers are coloured with two colours, then one can find integers x, y, z of the same colour so that x+y=z. Another result discussed in the course is that any sequence of n^2 real numbers contains either an increasing subsequence of length n or a decreasing one of length n. Other topics discussed in this module may include directed graphs as well as Positional Game Theory and its relation to Ramsey Theory.

The Communication Theory part of the module consists of an introduction to information theory and coding theory. Here one is concerned with transmitting messages efficiently over a communication channel. First the noiseless channel will be considered (i.e. the channel transmits information reliably). Here the aim is to encode messages in such a way that redundancy is eliminated, which ensures fast transmission of information. Then the noisy channel will be considered (i.e. it can make errors when transmitting a message). In this case one wants to transmit an encoded version of the message which includes some redundancy. This allows the message to be decoded correctly even if a limited number of errors have been made in transmission. However, too much redundancy will slow down the information transmission, so a suitable balance between the two aims of speed and reliability has to be found.

Teaching Methods
44 hours of lectures
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
Learning Outcomes and Objectives
By the end of the module, the student should be able to: analyse simple algorithms, determining relevant features such as their outputs, whether or not the algorithm halts, and if so what resources are required for this; write programs for the simple machine models studied in the course; demonstrate an understanding of some of the applications of the mathematical theory of these machines; to construct formal proofs in the propositional and predicate calculus; to understand the connections between semantic notions in logic and syntactic notions; apply the Soundness Theorem to demonstrate non-provability of certain statements; apply the completeness theorem and/or compactness theorem to demonstrate the existence of ‘new’ mathematical structures. By the end of the module, the student should also be able to explore these topics beyond the taught syllabus.

Module Description
Computability is concerned with the mathematics of computation on idealized ‘machines’, such as Turing machines, register machines, etc. These machine models are mathematical objects, and as such are the subject of an important mathematical theory which has many practical applications, and applications to many areas of pure mathematics, as well as applications to computer science. In this module, one of these machine models will be taken and studied in depth, and the theory will be developed far enough for the student to appreciate some of the wide range of applications available. Logic is concerned with the consistency of sets of axioms, and setting up precise rules for deriving theorems from these axioms. Two significant theorems (the Soundness Theorem and the Completeness Theorem) will be proved. The Soundness and Completeness Theorems have important applications for mathematics. In one direction, they are applied to give rigorous proofs that certain statements cannot be proved from given axioms. In the other direction, they are applied to provide new interesting mathematical structures such as ‘non-standard’ versions of the integers which behave like the usual finite integers in many ways but which nevertheless contain infinite numbers. Familiarity with Level I Linear Algebra may be beneficial.

Teaching Methods
44 hours of lectures
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
31 | Advanced Topics in Combinatorics

MSM4P18 | 06 20439 | 10 Credits | Autumn Term

Learning Outcomes and Objectives
By the end of the module, the student should be able to: demonstrate an understanding of the material delivered; apply their knowledge to the solution of relevant problems.

Module Description
This module will expose students to advanced topics in Combinatorics, which could include hypergraphs, graph colouring problems, graph minors and subdivisions (both of which generalize the notion of a subgraph), probabilistic Combinatorics, extremal questions as well as Ramsey theory. This module exploits the research expertise present in the department in this area at the time of delivery.

Teaching Methods
22 hours of lectures
10 hours of back-up

Assessment
90% based on a 1hr 30min written examination
10% based on course work during term time
Module Notes
This will be taught in the MAGIC room using access grid facilities and there will be other participants from universities within the MAGIC consortium.

Learning Outcomes and Objectives
On completion of this fundamental module students will have been exposed the major results in the representation theory of finite groups and should understand the relationship between representations of groups and modules; be able to construct character table of small group and deduce properties of the groups from its character table; appreciate how techniques from linear algebra can be used to study groups.

Module Description
Representation theory studies groups by their action on vector spaces. Representations are introduced and studied. The important concept of irreducibility is introduced and Schur's Lemma is proved. The complete reducibility theorem of Maschke is also presented. The course then continues with the theory of characters and character tables are then introduced, the classical orthogonality relations between irreducible characters are developed. Examples of character tables of small groups will be constructed. It will be shown how properties of groups can be derived from their character tables. Time allowing further applications of characters will be presented, such as the famous Burnside Theorem on groups whose order is divisible by at most two primes.

Teaching Methods
22 hours of lectures
10 hours of back-up

Assessment
90% based on a 1hr 30min written examination in the Summer Term;
10% based on course work during term-time
Linear Analysis

MSM4P21 | 06 22791 | 20 Credits | Autumn and Spring Term

Learning Outcomes and Objectives
By the end of the module the student should be able to work in infinite dimensional Hillbert and Banach spaces, finding bases of Hilbert space, and using different types of convergence; Combine ideas from algebra and analysis to solve problems in functional analysis; Understand the limitations of Riemann integration and how the Lebesgue integral overcomes many of these; Manipulate abstract sets to prove results on measurability and measure; Apply the main theorems on Lebesgue integration to solve problems of convergence of functions. By the end of this module, students should also be able to explore these topics beyond the taught syllabus.

Module Description
Linear Analysis underpins much modern mathematics, from partial differential equations to probability theory. One half of this course introduces Hilbert space, Banach spaces, dual spaces, and linear operators, and explores the interaction between linear algebra and analysis in the study of infinite dimensional spaces. The other half examines integration in more detail, and develops the theory of measurability, measure and the Lebesgue integral on euclidean spaces. The concepts of measure and integral are then extended to the more abstract context of measure spaces, and the monotone and dominated convergence theorems for the Lebesgue integral are proved.

Teaching Methods
44 hours of lectures
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
Learning Outcomes and Objectives
By the end of the module the student should be able to understand better complex valued functions of a complex variable; understand conformal mappings, in particular Möbius transformations, and apply them in particular examples; evaluate a wider range of contour integrals, in particular integrals around branch points; understand and apply various results in the theory of analytic functions such as the principle of the argument; understand the definitions of metric and topological spaces and verify these definitions in examples; manipulate open and closed sets; understand the definitions of compactness and connectedness and prove basic results involving these properties. By the end of this module, students should also be able to explore these topics beyond the taught syllabus.

Module Description
This module continues the study of complex-valued functions of a complex variable begun in MSM2B. Analytic functions have more structure that their real counterparts, the differentiable functions, and this extra structure results in a fascinating theory and widespread applications in other areas of mathematics and beyond. The course touches on both these aspects: conformal maps, which have many applications, are discussed, and the techniques of contour integration are extended to deal with functions involving logarithms and roots, while the theory and structure of analytic functions are used to give simple and elegant proofs of the fundamental theorem of algebra.

This module also introduces metric and topological spaces as abstract settings for the study of analytical concepts such as convergence and continuity. This generalization allows one, for example, to regard functions as points of a space and to consider various ways in which the function can be the limit of other functions. Extra structure is introduced: compactness, for instance, is shown to be the proper generalization of the closed bounded intervals often used in analysis on the real line. The course may end with an application: for example, using topological techniques, one can prove the existence of continuous on the real line which fail to be differentiable at any point.

Teaching Methods
44 hours of lectures
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
Learning Outcomes and Objectives
By the end of the module, the students should be able to understand statistical association and analyse data by building appropriate models; appreciate the use of statistics in various subject areas; analyse discrete, continuous and multi-factor data; appreciate statistical aspects in model building. The students will also learn to implement various data analysis methods using some standard statistical software. By the end of this module, the students should also be able to explore these topics beyond the taught syllabus.

Module Description
This module focuses on data analysis using multiple regression models. A unifying theme is the transferability of statistical ideas. Topics covered include linear regression models, general linear models, prediction problems, sensitivity analysis, analysis of incomplete data, robust regression, multiple comparisons. To emphasise the importance of properly designed studies in statistics, basics of experimental design will be introduced. Randomization, blocking and confounding, factorial designs, fractional factorial designs, incomplete block designs will also be discussed. In the later part, generalized linear models for continuous and discrete data will be introduced. Logistic regression, log-linear models, conditional and quasi likelihoods will be discussed. Every data analysis topic discussed in the module will be complemented by computing sessions with R statistical package.

Teaching Methods
44 hours of lectures
6 hours of computer practicals
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
Learning Outcomes and Objectives
By the end of the module the students should be able to appreciate how medical statistics plays a fundamental role in understanding health and diseases in the population; understand why medical statistics informs clinical practice and the screening, diagnosis, and treatment of patients; understand the rationale for and design of cohort studies, case-control studies, clinical trials, and systematic reviews; understand, apply and interpret a range of statistical methods in the context of epidemiological studies, diagnostic tests, clinical trials, survival analysis, and systematic reviews; apply mathematical theory to derive unbiased estimates of disease risk in different groups of patients, and to interpret their uncertainty by calculating confidence intervals and p-values; understand and interpret regression models for continuous, binary and survival data; understand the differences between Bayesian and classical statistical inference, and perform Bayesian statistical analyses of medical data, understand new developments in medical statistics.

Module Description
This module will give a comprehensive introduction to medical statistics and show why it is an important application of mathematical and statistical theory. The module will use statistical methods learnt in other modules, and also introduce some new statistical concepts and methods, with application to real medical problems and healthcare research. The module is split into 5 sections. In Epidemiology, students will cover the analysis of diagnostic test studies; the design and analysis of cohort studies and casecontrol studies; the use and interpretation of linear and logistic regression models; and the problem of confounding factors. In Survival Analysis students will consider how to analyse censored time-to-event data; the specification and interpretation of hazard and survival functions; the estimation of Kaplan-Meier curves; and the use and interpretation of Cox regression models. In Clinical Trials students will consider the rationale and design of randomised controlled trials; the derivation and application of sample size equations; and the analysis of parallel group trials and cross-over trials. In Systematic Reviews students will consider how to synthesise multiple studies and produce evidence-based results; show the differences between fixed-effect and random-effects meta-analysis models; show how to measure, account for, and explain between-study heterogeneity; and appreciate how to assess publication bias. Finally, in Bayesian Statistics students will cover the differences between Bayesian and frequentist statistical inference; work through the Bayesian analysis of a clinical trial and derive and interpret posterior probability distributions.

Teaching Methods
44 hours of lectures
10 hours of tutorial sessions

Assessment
90% based on a 3hr written examination
10% based on coursework during term time (including a 10% mini-project)
Learning Outcomes and Objectives
Upon completion of this module, the student will be able to: understand the nature of statistical inferential procedures involved in analysing economic data; formulate models to solve some empirical economic problems; apply appropriate statistical methods and techniques to understand relationships among variables; gain hands-on experience in using statistical computing program of SAS; understand the power and limitations of applied statistical analysis; perform and present research by using relevant data and statistical tools. By the end of the module students should be able to explore these topics beyond the taught syllabus.

Module Description
This course is designed for students with limited or no prior economic theory background. It emphasizes the understanding of quantitative methods, model evaluations, and the techniques for empirical studies in economics. This module starts with an introduction to general economic concepts, then it will cover the basics and extension of ordinary least square methods, heteroscedasticity, autocorrelation, multicollinearity, model specifications, simultaneous equation models, binary and discrete choice models, qualitative and limited dependent variable models, time series analysis, panel data models, and nonparametric analysis with their applications in Economics. Students will gain hands-on experience formulating and estimating models, interpreting results, and making forecasts.

Teaching Methods
44 hours of lectures
10 hours of back-up

Assessment
90% based on a 3hr written examination
10% based on course work during term time
Learning Outcomes and Objectives
By the end of the module the students should be able to: demonstrate a comprehensive knowledge and understanding of mathematical and computer models relevant to the mechanical and related engineering disciplines, and an appreciation of their limitations; apply mathematical and computer-based models for solving problems in engineering, and assess the limitations of particular cases; demonstrate a thorough understanding of current practice and its limitations and some appreciation of likely new developments.

Module Description
The aim of the module is to give students the mathematical theory related to curve and surface representation for computer aided engineering. Syllabus includes:

a) motivation for and introduction to vector valued parametric polynomial curve and surface representations
b) parametric Cubic curve segments including Ferguson, Hermite, Bézier and B-spline forms
c) curve evaluation and interrogation including De Casteljau recursive subdivision
d) parameteric Bicubic Bézier surfaces, interrogation and continuity
e) rational Bézier cubic curves and conic representation
f) advanced interrogations with curves and surfaces using bounding boxes and subdivision

Teaching Methods
16 hours of lectures
4 hours of tutorials
80 hours of associated private study

Assessment
100% based on a 1hr 30min written examination.
### Suggested reading material

<table>
<thead>
<tr>
<th>Course Code</th>
<th>Title</th>
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<tbody>
<tr>
<td>MSM4A03</td>
<td><em>Elementary Fluid Dynamics</em> by D J Acheson, Oxford University Press 1990&lt;br&gt;<em>An Introduction to Fluid Dynamics</em> by G K Batchelor, Cambridge University Press 1997&lt;br&gt;<em>Vector Calculus</em> by P C Matthews</td>
</tr>
<tr>
<td>MSM4A04a</td>
<td><em>Partial Differential Equations: An Introduction</em> by W A Strauss</td>
</tr>
<tr>
<td>MSM4A06a</td>
<td><em>Incompressible Flow</em> by Ronald L Panton, John Wiley &amp; Sons Inc.</td>
</tr>
<tr>
<td>MSM4A06b</td>
<td><em>Elementary Fluid Dynamics</em> by D J Acheson, Oxford University Press 1990</td>
</tr>
<tr>
<td>MSM4A07</td>
<td><em>Elementary Fluid Dynamics</em> by D J Acheson, Oxford University Press 1990</td>
</tr>
<tr>
<td>MSM4A13</td>
<td><em>Mathematical Biology</em> by J D Murray</td>
</tr>
<tr>
<td>MSM4M10</td>
<td><em>Introduction to Game Theory</em> by Peter Morris, Springer 1994</td>
</tr>
<tr>
<td>MSM4M12a</td>
<td><em>How to Solve It: Modern Heuristics</em> by Z Michalkiewicz and D B Fogel, Springer</td>
</tr>
<tr>
<td>MSM4P08a</td>
<td><em>The Theory of Finite Groups</em> by Hans Kurzweil and B Stellmacher, Springer 2004</td>
</tr>
<tr>
<td>MSM4A10b</td>
<td><em>Computational Grids: Generation, Adaptation and Solution Strategies</em> by G F Carey, Taylor and Francis 1997</td>
</tr>
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</table>
Suggested reading material

MSM4P17a  Introduction to Languages & the Theory of Computation by J Martin. McGraw Hill
MSM4P17b  The Mathematics of Logic by R W Kaye. CUP, 2007
MSM4P18   Graph Theory by R Diestel, Springer Verlag, 3rd Edition
MSM4P21   Beginning Functional Analysis by Karen Saxe, Springer Verlag
MSM4P22   Complex Analysis by John M Howie, Springer
           Introduction to Topology by Bert Mendelson, Dover
MSM4S05a  Regression Analysis By Example, 3rd Edition by S Chatterjee, A S Hadi and B Price 2000, Wiley
           Statistics for Experimenters: Design, Innovation, and Discovery, 2nd Edition by G E P Box, J S
           Hunter and W G Hunter, 2005, Wiley
           York 2000
MSM4S09   Newbold by Carlson and Thorne 2010
           Statistics for Business and Economics, Pearson, McClave, Benson and Sincich 2009
           for Business and Economics, South-Weston