MSci/BSc Project Descriptions
2012–2013

March 3, 2013

Contents

Applied Mathematics

Supervisor: **Dr Rosemary Dyson**

Title: **Models of plant root growth**

Description: Understanding plant root growth is essential to promote healthy plant growth in normal and stressed (e.g. during a drought) environments. A root grows through the elongation of some of its cells, pushing the root forward through the soil. These cells differ from animal cells through the presence of a tough cell wall surrounding the cell, which maintains a high internal turgor pressure whilst allowing significant growth. It is the (varying) mechanical properties of this cell wall which control growth, whilst the driving force for expansion is provided by the turgor pressure.

Potential projects include investigating how bending of the root, for example as displayed under a gravity stimulus, is generated via differential wall properties across the root tissue, or modelling the movement of water through the root cross-section required to maintain the turgor pressure within the root cells as well provide a source of water for the rest of the plant.

This project will be in collaboration with colleagues at the Centre for Plant Integrative Biology, University of Nottingham.

Prerequisites: MSM3A03 Continuum mechanics, MSM3A05a Perturbation Theory and Asymptotics likely, depending on actual project.

Supervisor: **Dr Rosemary Dyson**

Title: **Modelling in mathematical medicine**

Description: The use of mathematical modelling within medical research is widespread, covering topics such as cancer, inflammation, neuroscience, physiological fluid mechanics and many others. The first two of these are the focus of the new UoB Systems Science for Health (SSfH) initiative, which brings together researchers with complementary skills in modelling, bioinformatics and experimental biology from across the university to improve our understanding of health and disease. This project will involve working closely with relevant colleagues in other Schools to tackle a specfic biological question of relevance to SSfH through formulating, analysing and solving (using analytical and/or numerical techniques) mathematical models. It is likely to involve models of how cells interact with each other or their environment as this is a common feature of both cancer and inflammation.

Pre- and co-requisites: MSM3A03 Continuum mechanics, MSM3A05a Perturbation Theory and Asymptotics, MSM4A06 Viscous Fluid Mechanics with Applications likely depending on actual project.

Supervisor: **Dr Sara Jabbari**

Title: **Mathematical modelling of competing variants of the hepatitis C virus**

Description: The hepatitis C virus is a leading cause of liver failure and represents a significant global health burden, infecting an estimated 170 million people worldwide. With no vaccine currently available, it is imperative that the mechanisms governing infection are understood as fully as possible in order to optimise treatment.

Mathematical modelling is increasingly being employed as an informative method to glean insights into viral dynamics which would not be accessible with experimental research alone. Based on data generated at the Universities of Birmingham and Nottingham, this project will involve ordinary differential equation modelling of two competing variants of the hepatitis C virus. Both numerical and asymptotic approaches will be employed to analyse the emerging dynamics, the effect of a range of therapies and the host immune response to infection.

Pre/co-requisites: MSM4A13 (Mathematical Biology) and MSM3A05a (Perturbation Theory and Asymptotics) desirable

References: [1] A.S. Perelson, Modelling viral and immune system dynamics. Nature Reviews Immunology 2: 28-36 (2002).

Supervisor: **Dr Sara Jabbari and Dr Jan Kreft (Biosciences)**

Title: **Incorporating gene regulation networks in individual-based models**

Description: All cellular processes are governed by networks of genes which detect extra-, intra- and inter-cellular signals to determine a cell's behaviour at any particular time (genes are not active all the time, instead they are 'switched on' when the cell requires it). It is clear, therefore, that understanding the underlying network will yield important insights into the biological processes that it governs. Given the complexity and size of many of these networks, computational modelling is invaluable in the visualisation and prediction of their dynamics.

The conventional approach to model gene regulation networks is to employ ordinary and, where spatial structure is a consideration, partial differential equations. However, individual-based modelling is an alternative strategy to predict the emergent behaviour of a population of cells on a spatial domain based on the activities of the individual cells making up the population. Using computer programming, the cells' behaviours are described by rules which they follow with assigned probabilities (e.g. if (hunger > hungerThreshold) then eat) and/or by differential equations describing intracellular dynamics such as gene regulation. This project will seek to combine the above approaches, investigating the influence of gene regulation on a cell's decision at any particular step in time, by implementing an ODE solver embedded in each individual cell. The results will be compared to more traditional reaction-diffusion systems.

The prevalence of gene regulation networks throughout biology means that any number of examples could be modelled. However, this project will focus on populations of bacteria and the influence of cell-cell signalling (quorum sensing) on biofuel production (by *Clostridium acetobutylicum*), sporulation (*Bacillus subtilis*) or disease mechanisms (MRSA and *C. difficile*), and can therefore be tailored to the student's interests.

Pre/co-requisites: MSM4A13 (Mathematical Biology) and MSM3A05a Perturbation Theory and Asymptotics desirable, programming skills essential (e.g. Matlab, Java)

References:
[1] H. De Jong, Modeling and simulation of genetic regulatory systems: a literature review. Journal of Computational Biology 9: 67-103 (2002).

[2] A.B. Goryachev, Understanding bacterial cell-cell communication with computational modeling. Chemical Reviews 111: 238-250 (2011).

[3] L.A. Lardon, B.V. Merkey, S. Martins, A. Dötsch, C. Picioreanu, J.U. Kreft, B.F. Smets, iDynoMiCS: next-generation individual-based modelling of biofilms Environmental Microbiology 13: 2416-2434 (2011).

Supervisor: **Dr John Leach**

Title: **An initial-value problem for the generalized Burgers' equation.**

Description: This project considers an initial-value problem for the generalized Burgers' equation. Specifically, the case when the initial data has a discontinuous expansive step is considered. The aim will be to develop, via the method of matched asymptotic coordinate expansions, the complete large-time asymptotic structure of the solution to this problem.
Burgers' equation is a canonical equation combining both nonlinearity and diffusion and as such arises in the modelling of many physical phenomenon and is one of the fundamental model equations in fluid dynamics.

Prerequisites: None

Supervisor: **Dr D Leppinen**

Title: **Disappearing plumes**

Description: A localised heat source in an otherwise quiescent, homogeneous environment will generate a plume which carries heat upwards. The dynamics of a single phase plume are such that the temperature, velocity and width of the plume evolve with height above the source in a manner which conserves the total flux of buoyancy across any horizontal plane above the source. A multiphase plume, on the other hand, does not necessarily have to conserve buoyancy. Consider the example of a bubbly plume rising through a liquid column and suppose that the bubbles are dissolving as they rise. Provided the liquid column is tall enough to allow all of the bubbles to completely dissolve, then the initially bubbly plume will eventually turn into a non-buoyant jet.

The purpose of this project is to incorporate mass transfer effects into multiphase plume theory by coupling existing multiphase plume models with simplified models for mass transfer from isolated bubbles, droplets or particles. A number of simplifying assumptions will be required to make the problem tractable and these assumptions should be stated and justified. The project should begin by reviewing the existing multiphase plume models listed below and then identifying the physical processes which can lead to buoyancy loss/gains in multiphase plumes.

References:
Brevik, I. & Killie, R. (1996). Phenomenological description of axisymmetric air-bubble plume. *Int. J. Multiphase Flow*, **22**, 535-549.

Leitch, A. M. & Baines, W. D. (1989). Liquid volume flux in a weak bubble plume. *J. Fluid Mech.*, **205**, 77-98.

McDougall, T. J. (1978). Bubble plumes in stratified environments. *J. Fluid Mech.*, **85**, 655-672.

Milgram, J. H. (1983). Mean flow in round bubble plumes. *J. Fluid Mech.*, **133**, 345-376.

Woods, A. W. & Phillips, J. C. (1999). Turbulent bubble plumes and CO_2 driven lake eruptions. *J. Volcanology and Geothermal Research*, **92**, 259-270.

Clift, R. Grace, J. R. & Weber M. E. Bubbles, drops and particles. Academic Press, New York, 1978.

Pre-requisites: Continuum Mechanics

Supervisor: **Dr D Leppinen**

Title: **How to design a calculator**

Description: The arithmatic operations of addition, subtraction and multiplication are very straight forward. How would you design a calculator to perform division? What is the most efficient way to evaluate the functions $\sin(x)$, $\exp(x)$, $\log(x)$, ..., etc?
We now take calculators (and computers, and mobile phones, and ...) for granted, but what are the underlying mathematics associated with a calculator? What is the most efficient way of designing a calculator (or a computer)? What is the most accurate?

Supervisor: **Daniel Loghin**

Title: **Domain decomposition methods**

Description: Domain decomposition methods are standard solution methods for large scale calculations arising in the area of numerical solution of PDE. The approach is to divide up the computational domain into smaller domains leading to a set of smaller, independent problems. The advantages are obvious: (i) the problems are smaller and thus easier to solve numerically and (ii) they can be solved in parallel. The downside is that this re-formulation of the problem is only possible through knowledge and suitable discretisation of boundary operators defined on the interior of the domain. This is a hard problem which may not have a known solution in general. The aim of the project is to review existing domain decomposition techniques for elliptic equations and to generalise and analyse some standard choices of interface operators to the case of elliptic problems with variable coefficients.

Prerequisites: Partial Differential Equations, Computational Mathematics and Research Frontiers, Numerical Methods in Linear Algebra.

References:

1. A. Quarteroni and A. Valli, Domain Decomposition Methods for Partial Differential Equations, Oxford University Press, 1999.
2. T. F. Chan, Domain Decomposition Algorithms, Acta Numerica, 1994, pp. 61-143.

Supervisor: **Daniel Loghin**

Title: **Continuation methods for non-Newtonian flows**

Description: Non-Newtonian fluids are unusual materials such as paint, molten plastics, foam, but also toothpaste and chocolate. They are also remarkable in that hard, nonlinear systems of PDE are required for their modeling. In general, a whole suite of computational techniques is needed to provide a numerical solution. The project will focus on only one of these: continuation methods. These are methods that, given a varying parameter, trace the resulting solution curve. Thus, one can use the solution (or information) from a nearby problem to find the solution to the problem at hand. The application of interest is shear-thinning power-law fluids in extrusion flow for the limiting case when the exponent goes to zero [1]. The aim is to devise a suitable continuation technique, analyze it and compare it with existing analytical or numerical methods.

Prerequisites: Continuum Mechanics, Computational Methods, Numerical Methods in Linear Algebra.

References:

1. S. J. Chapman, A. D. Fitt and C. P. Please, Extrusion of Power-Law Shear-Thinning Fluids with Smal l Exponent, Int. J. Non-Linear Mech. (1997), Vol. 32, No. 1, pp. 187-199.

2. E. L. Allgower and K. Georg, Numerical Continuation Methods, Springer-Verlag, 1990.

3. H-C. Huang, Z-H. Li and A. S. Usmani, Finite Element Analysis of Non-Newtonian Flow, Springer, 1999.

Supervisor: Dr D Loghin

Title: **Fluid mechanics of sperm (finite element methods).**

Description: Sperm swimming is vital to life; the events surrounding the journey of the successful sperm to the egg are a subject of great scientific interest, in addition to being of potential importance in the design of new treatments and diagnostics for infertility. The propulsion of sperm by their beating tails has been one of the most important problems in the development of 20th Century biological fluid mechanics, and continues to pose challenges for mathematicians. This project will involve learning about and helping to adapt finite element methods for modelling sperm swimming, which take into account complex effects such as non-Newtonian rheology or fluid-structure interaction. The computations will be implemented in parallel on a multi-processor cluster. There will also be the opportunity to work with real experimental data generated by colleagues in the Medical School and Birmingham Women's Hospital IVF clinic, and the opportunity to meet clinical researchers and learn more about interdisciplinary science.

Prerequisites: MSM3A03 Continuum mechanics

Desirable: MSM2G09, Each of MSM4A10, MSM4A03, MSM4A06 as a co-module and Matlab programming skills and An interest in biology and/or non-Newtonian flows

References:
Smith, D. J., Gaffney, E. A., Blake, J. R., Kirkman-Brown, J. C., 2009. Human sperm accumulation near surfaces: a simulation study. J. Fluid Mech. 621, 289–320
Smith, D. J., Gaffney, E. A., Gadhela, H., Kapur, N., Kirkman-Brown, J. C., 2009. Bend propagation in the flagella of migrating human sperm, and its modulation by viscosity. Cell Motility and the Cytoskeleton 66, 220-236 DOI: 10.1002/cm.20345.
Finite Elements and Fast Iterative Solvers with Applications in Incompressible Fluid Dynamics H. C. Elman, D. J. Silvester and A. J. Wathen OUP 2005

Supervisor: **Dr Rachel Nicks**

Title: **Speciation and symmetry-breaking bifurcations**

Description: A classic example of speciation is "Darwin's finches' in The Galapagos Islands, where over a period of about 5 million years a single species of finch has diversified into 14 species. One possible mechanism for the generation of new species is symmetry-breaking - where the single species state losses stability to a multiple species state.
In this project you will study a model in which speciation is represented as a form of spontaneous symmetry-breaking in a system of coupled nonlinear differential equations. You will learn how the model was developed, how to analyse the model and interpret the solutions to draw biological conclusions. To achieve this you will learn some of the techniques used to study symmetry-breaking bifurcations, including a little representation theory of the relevant symmetry groups and some of the results of equivariant bifurcation theory. In particular, you will see how the symmetry of a system alone can be used to determine the normal form of the bifurcation equations. You will find the primary branches of equilibria of these equations created at the bifurcation and investigate the stability of these solutions. You will then interpret the results in terms of speciation.
This project will involve mainly analytical techniques however there is scope for numerical work, including finding numerical solutions of the symmetric differential equations. Depending on your interests, you could go on to look the solutions created at secondary bifurcations (i.e. secondary speciation where a subspecies again losses stability to multiple species), or symmetry-breaking in all-to-all coupled systems in a more generic framework.

Prerequisites: MSM201 Applied Mathematics II is essential. Symmetry and Groups from MSM2D would be desirable.

References:
[1] I. Stewart, T. Elmhirst, J. Cohen, Symmetry-breaking as an Origin of Species. In Bifurcations, Symmetry and Patterns, 3 – 54 (2003) Birkhauser.
[2] M. Golubitsky and I. Stewart, The Symmetry Perspective: From Equilibrium to Chaos in Phase Space and Physical Space, (2000) Birkhauser.
[3] I. Stewart, Speciation: A case study in symmetric bifurcation theory, Univ. Iagellonicae Acta Math. 41 (2003) 67 – 88.

Supervisor: **Dr Rachel Nicks**

Title: **Symmetry in animal gaits and coupled nonlinear oscillators**

Description: Animal locomotion typically employs several distinct time-periodic patterns of leg movements, known as gaits (e.g. walk, run, gallop etc.). It has long been observed that most gaits possess a degree of symmetry. In this project you will focus on the gaits of four-legged animals. This provides an excellent introduction to spatio-temporal symmetries of coupled networks of nonlinear oscillators. Here a nonlinear oscillator is a system of nonlinear ordinary differential equations with a stable periodic solution.

You will begin by investigating the time-periodic symmetries of standard animal gaits. These gaits also have spatial symmetries corresponding to permutations of the legs. Thus the symmetries of animal gaits are spatio-temporal symmetries. You will see that symmetry constrains the gaits which can exist and allows for model-independent predictions about animal gaits.

One biological theory is that an animal's gait patterns are governed by a network of neurons called the Central Pattern Generator (CPG) that runs independently of the brain. The CPG can be thought of as a network of oscillators with symmetric coupling. You will observe that there are six assumptions that the CPG must satisfy which allow the structure of the CPG network to be deduced. You will see that these assumptions require that the CPG network has twice the number of cells as the animal has legs. You will observe why this is the case and interpret the predictions of the eight-cell model concerning quadruped locomotion. This could be extended to investigate the gaits of six legged animals.

For this project the symmetries of animal gaits will be specified in terms of a group of transformations which preserve the gait pattern. You will learn how to describe spatio-temporal symmetries and use techniques from symmetric bifurcation theory to classify gaits as primary or secondary noting that bifurcations represent transitions between different gaits.

This project can be extended to study the dynamics of rings of oscillators or other topics in coupled cell network dynamics.

Prerequisites: MSM201 Applied Mathematics II is essential. Symmetry and Groups from MSM2D would be desirable.

References:
[1] M. Golubitsky, I. Stewart, P.L. Buono, and J.J. Collins, Symmetry in locomotor central pattern generators and animal gaits, Nature 401 (1999) 693–695.

[2] J. J. Collins and I. N. Stewart, Coupled nonlinear oscillators and the symmetries of animal gaits, Journal of Nonlinear Science 3.1 (1993): 349–392.

[3] M. Golubitsky and I. Stewart, The Symmetry Perspective: From Equilibrium to Chaos in Phase Space and Physical Space, (2000) Birkhauser.

[4] P.L. Buono and M. Golubitsky. Models of central pattern generators for quadruped locomotion: I. primary gaits, J. Math. Biol. 42 (2001) 291–326.

Supervisor: **Dr N Petrovskaya**

Title: **Estimating pest insect population density from trap counts**

Description: In ecological studies, populations are usually described in terms of the population density or population size. Having these values known over a period of time, conclusions can be made about a given species, community, or ecosystem as a whole. In particular, in pest management, the information gained about pest abundance in a given field or area is then used to make a decision about pesticide application. To avoid unjustified decisions and unnecessary losses, the quality of the information about the population density is therefore a matter of primary importance. However, the population density is rarely measured straightforwardly, e.g. by direct counting of the individuals. In the case of insects, their density is often estimated based on trap counts. The problem is that, once the trap counts are collected, it is not always clear how to use them in order to obtain an estimate of the population density in the field. The aim of this project is to overcome current limitations of trapping methods used in ecological studies through developing a theoretical and computational framework that enables a direct estimate of populations from trap counts. A 'mean-field' diffusion model will be considered to investigate if it is capable of revealing the generic relationship between trap catches and population density.

Prerequisites: MSM2A Analytical Techniques, MSM2C Linear algebra and Programming, MSM3A04 PDEs & Reaction Diffusion Systems.

References:

[1] Crank, J., 1975. The Mathematics of Diffusion (2nd edition). Oxford University press, Oxford.

[2] Kot, M., 2001. Elements of Mathematical Ecology. Cambridge University Press, Cambridge.

[3] Okubo, A., Levin SA, 2001. Diffusion and Ecological Problems: Modern Perspectives. Springer, Berlin.

Supervisor: **Dr Chris Sangwin**

Title: **Intervals and inequalities**

Description: Some sets of real numbers can be represented as either collections of intervals e.g. $([1,2] \cup [3,4])$ or as collections of intervals $((1 \leq x \text{ and } x \leq 2) \text{ or } (3 \leq x \text{ and } x \leq 4))$. The goal of this project is to (i) understand which sets of real numbers can be represented in this way, (ii) investigate decision procedures for establishing equivalence of sets of real numbers, (iii) writing Maxima code to manipulate representations to transform sets into equivalent forms, including canonical forms.

Prerequisites: MSM2C

References: Bundy, A. (1983), The Computer Modelling of Mathematical Reasoning, Academic Press.

Supervisor: **Dr Chris Sangwin**

Title: **Computation and proof: how can CAS track side conditions?**

Description: The simple algebraic transformation $ax = a \Rightarrow x = 1$ misses the condition $a \neq 0$. Computer algebra systems are notorious for ignoring such special cases. What has, or could, be done to better automate mathematical knowledge?

Prerequisites: MSM2C

References: (1) Stoutemyer, D. R., (1991) Crimes and misdemeanors in the computer algebra trade. Notices of the American Mathematical Society, 1991, 38(7), 778–785. (2) Bundy, A. (1983), The Computer Modelling of Mathematical Reasoning, Academic Press.

Supervisor: **Dr Chris Sangwin**

Title: **The nonlinear elastic pendulum**

Description: This project reviews the mathematical models developed for the elastic pendulum in three dimensions.

Prerequisites: Applied Mathematics II, MSM3A05 (Chaos)

References: Lynch, P. (2002) Resonant motions of the three-dimensional elastic pendulum. International Journal of Non-Linear Mechanics, 37(2), 345–367.

Supervisor: **Professor Y. D. Shikhmurzaev**

Title: **Dynamics of microbubbles**

Description: The nucleation of bubbles and their subsequent evolution is the key element in the dynamics of boiling and a number of other applications. Although the general area of bubble dynamics is well developed, the very transition in the topology of the flow domain as bubbles start to appear and their subsequent dynamics are poorly understood. In particular, the large surface-to-volume ratio that characterizes microbubbles makes their dynamics differ from that of macroscopic bubbles that have been the subject of numerous works. The main objective of this project is to understand the main features of the dynamics of a microbubble as it size goes down.
This project can be seen as a preparation for a subsequent study at a Ph.D. level.

Prerequisites: MSM3A03 Fluid mechanics, MSM3A04 Partial differential equations.

Co-requisites: MSM4A07 Waves.

Supervisor: **Dr G. Sisoev**

Title: **Melted film flows**

Description: Film flow on a melted surface is observed in many engineering problems with intensive heat and mass transfer. In particular, such flows are appears on surfaces of meteoroids and meteorites as well as returning space vehicles. These flow regimes controlled by the viscous forces, the capillary forces and the external drag forces. The project deals with identifying the flow instabilities in the framework of the linear stability analysis.

Prerequisites: MSM3A04a PDE's, MSM3A03 Continnuum Mechanics.

Co-requisites: MSM4A06 Viscous Fluid Mechanics with Applications, MSM3A07 Waves.

References: [1] Acheson, D.J.; *Elementary Fluid Mechanics*, Clarendon Press, 1990. [2] Shkadov, V.Y.; Sisoev, G.M.; Wave induced by instability in falling films of finite thickness., Fluid Dynamics Research, 35 (2004), 357-389.

Supervisor: **Dr W. R. Smith**

Title: **Solidification of a sessile drop**

Description: Containerless solidification is one of the most common methods by which very pure materials can be produced.
This project addresses the small-time analytical solution when a drop of water is placed on a cold aluminium plate. The water at the base of the drop solidifies, but the drop is held in place by its own surface tension.

Pre-requisites: Continuum Mechanics (MSM3A03) Partial Differential Equations (MSM3A04a) Perturbation Theory (MSM3A05a)

Supervisor: **Dr W. R. Smith**

Title: **Asymptotic Analysis of the Stationary Breather**

Description: Breathers are localised periodic solutions of the sine-Gordon equation or the nonlinear Schrödinger equation. The name originates in that they are localised in space and oscillate in time. In this project, we consider the perturbed sine-Gordon equation of the form

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} + \sin(u) = \epsilon F, \qquad (1)$$

with the boundary conditions that

$$u \text{ decays to zero exponentially as } x \to \pm\infty, \qquad (2)$$

in which $0 < \epsilon \ll 1$ and F is an arbitrary perturbation function. Furthermore we assume that the boundary value problem (1)-(2) with $\epsilon = 0$ has time periodic solutions. The method of Kuzmak-Luke is used to analyse this perturbed sine-Gordon equation.

Prerequisites: MSM3A04, MSM3A05a

References:

G.E. Kuzmak. Asymptotic solutions of nonlinear second order differential equations with variable coefficients. *Prikl. Mat. Mekh.* **23**, 515–526 (Russian). *J Appl Math Mech*, **23**, 730–744, (1959).

J.C. Luke. A perturbation method for nonlinear dispersive wave problems. *Proc Roy Soc Lond A*, **292**, 403–412, (1966).

Supervisor: **Dr Q Wang**

Title: **Nonlinear acoustic cavitation in biomedical applications**

Description: Ultrasonically-driven cavitation can be used as a noninvasive means for drug therapy and the treatment of cancer tumours. In the former application, bubble pulsations enhance the uptake of drugs or genes into surrounding tissues, whereas in the latter case, cavitation enables real-time imaging and localized heating that causes tumour destruction. Bubbles driven by ultrasound undergo complex, even chaotic, oscillations. An understanding of the interaction between the incident sound waves and the bubble dynamics is crucial to optimizing biomedical treatments utilizing cavitation.

In this project, the student will analyse a model based on a nonlinear ordinary differential equation that describes the oscillatory behaviour of acoustically-driven spherical bubbles. The dynamics of bubbles as a function of the frequency and strength of the incident sound waves will be explored and stable and inertial cavitation regimes will be identified. Computer programming (e.g., Matlab, FORTRAN) is required along with the ability to numerically solve ordinary differential equations.

Pre-requisites: MSM 3A03 Continuum Mechanics

Desirable:
MSM 2G09 Linear Algebra: Theory and Numerical Methods
MSM 4A10 Computational Methods and Frontiers (as a co-module)
MSM 4A03 Continuum Mechanics (as a co-module)
MSM 4A06 Viscous Fluid Mechanics and High Speed Flow (as a co-module)
Matlab programming skills
An interest in biology and/or Newtonian flows

References: C. C. Coussios and R. A. Roy (2008). Applications of acoustics and cavitation to noninvasive therapy and drug delivery. *Annual Review of Fluid Mechanics* **40**, 395-420.

Supervisor: **Dr Q Wang**

Title: **Unsteady potential flow between two moving circular cylinders**

Description: The flow around two parallel circular cylinders is a classical problem in fluid mechanics. Lagally found the exact solution to the potential flow induced by two stationary circular cylinders in a uniform steam in 1929. Wang (2004) obtained the exact solution to the potential flow around two circular cylinders moving arbitrarily. In this project, the student will consider the potential flow between two moving circular cylinders, which has important applications in analyzing a train penetrating a tunnel. The mathematical model of the problem is the two dimensional Laplace equation in the region bounded by an external circle and an inner circle, which will be solved using conformal mapping. Through the project, the student will appreciate the beauty of mathematics applied to a classical problem. She/he will get experience to perform dynamic analysis, get physical insights to this problem and draft research report.

Pre-requisites: Fluid Mechanics, Partial Differential Equations, Conformal Mapping

References: Q X Wang 2004 Interaction of two circular cylinders in inviscid fluid. Phys. Fluids 16 (12), 4412-4425.

Supervisor: **Dr Q Wang**

Title: **Nonlinear acoustic cavitation in biomedical applications**

Description: Ultrasonically-driven cavitation can be used as a noninvasive means for drug therapy and the treatment of cancer tumours. In the former application, bubble pulsations enhance the uptake of drugs or genes into surrounding tissues, whereas in the latter case, cavitation enables real-time imaging and localized heating that causes tumour destruction. Bubbles driven by ultrasound undergo complex, even chaotic, oscillations. An understanding of the interaction between the incident sound waves and the bubble dynamics is crucial to optimizing biomedical treatments utilizing cavitation.

In this project, the student will analyse a model based on a nonlinear ordinary differential equation that describes the oscillatory behaviour of acoustically-driven spherical bubbles. The dynamics of bubbles as a function of the frequency and strength of the incident sound waves will be explored and stable and inertial cavitation regimes will be identified. Computer programming (e.g., Matlab, FORTRAN) is required along with the ability to numerically solve ordinary differential equations.

Pre-requisites: MSM 3A03 Continuum Mechanics

Desirable:
MSM 2G09 Linear Algebra: Theory and Numerical Methods
MSM 4A10 Computational Methods and Frontiers (as a co-module)
MSM 4A03 Continuum Mechanics (as a co-module)
MSM 4A06 Viscous Fluid Mechanics and High Speed Flow (as a co-module)
Matlab programming skills
An interest in biology and/or Newtonian flows

References: C. C. Coussios and R. A. Roy (2008). Applications of acoustics and cavitation to noninvasive therapy and drug delivery. *Annual Review of Fluid Mechanics* **40**, 395-420.

Supervisor: **Dr Q Wang**

Title: **Dynamic analysis of unsteady potential flow between two moving circular cylinders**

Description: The flow around two parallel circular cylinders is a classical problem in fluid mechanics. Lagally found the exact solution to the potential flow induced by two stationary circular cylinders in a uniform steam in 1929. As his MSc project, Garg (2010) obtained the exact solution for the unsteady potential flow between two moving circular cylinders. In this project, the student will calculate the pressure using the Bernoulli equation, and calculate the dynamic force on the two circular cylinders.

Pre-requisites: Fluid Mechanics, Partial Differential Equations, Conformal Mapping

References:
V. Garg 2010 Potential flow between two moving circular cylinders. MSc Thesis. School of Maths. University of Birmingham.
Q X Wang 2004 Interaction of two circular cylinders in inviscid fluid. Phys. Fluids 16 (12), 4412-4425.

Supervisor: **Dr Q Wang**

Title: **Rise and deformation of a bubble under buoyancy**

Description: The evolution of a bubble in water is a very interesting phenomenon. An initially spherical bubble in water rises due to buoyancy. The bubble expands as it rises, as the hydrostatic pressure around the bubble reduces. As the bubble becomes larger, it accelerates faster due to the increasing buoyancy. In the later stage, the bubble becomes asymmetrical, forming a kidney shape and lastly becoming a ring bubble. It rises rapidly once becoming a ring bubble. Understanding the evolution of bubbles in a liquid is a problem of both scientific and engineering importance. Applications abound, varying from underwater explosions, Linnic eruptions (eg. Lake Nyos 1986) to bubbles in medical apparatus. This phenomenon was simulated by Krishna & van Baten (1999) for 2D bubbles, and will be simulated for axisymmetric bubbles in this project.

References:
R. Krishna & J. M. van Baten 1999 Simulating the motion of gas bubble in a liquid. Nature, 398, 203.
Q X Wang 2004 Numerical modelling of violent bubble motion. Phys. Fluids 16 (5), 1610-1619.

Management Mathematics[1]

Supervisor: **Peter Butkovic**

Title: **Tropical eigenproblem for special matrices**

Description: Tropical linear algebra is a new and rapidly evolving area of idempotent mathematics, discrete optimisation and linear algebra. One of the key questions is the tropical eigenvalue-eigenvector problem which is used for instance in the modelling of multiprocessor interactive systems [2] or in that of cellular protein production [1] where the tropical eigenvalues and eigenvectors are used to describe stability of the system. Being motivated by applications in [1] this project will investigate the tropical eigenproblem for special matrices such as bivalent, trivalent and tournament. The results will be described in combinatorial terms and/or as combinatorial algorithms. Knowledge of Matlab is welcome but not a condition.

Prerequisites: MSM2D or MSM2M09, MSM3M02

References:
[1] Brackley CA et al (2011) A max-plus model of ribosome dynamics during mRNA translation, arXiv:1105.3580v1
[2] Butkovic P (2010) Max-linear Systems: Theory and Algorithms, Springer Monographs in Mathematics, Springer-Verlag, London

[1]Priority to MSci projects in Management Mathematics will be given to students taking Mathematics with Business Management

Supervisor: **Professor M Kocvara**

Title: **Optimization problems with matrix rank constraints**

Description: Many combinatorial optimization problems or global optimization problems are approximated using semidefinite programming (SDP) relaxation. The approximation consists in formulating the original problem as SDP with a constraint on the rank of the matrix variable. This constraint is then relaxed – omitted.

The goal of the project is to perform numerical study of various modern approaches to SDP problems *with* the rank constraint. This is an NP-hard problem and we cannot expect to solve it efficiently. However, recent studies suggest that we can expect these methods to be much more efficient than methods for the original combinatorial optimization problems.

Prerequisites: MSM2C Linear algebra, MSM3M12a Nonlinear programming I, MSM3M11 Integer programming

References:
[1] Anjos, M.F. and Lasserre, J.B.: Handbook on semidefinite, conic and polynomial optimization, Springer 2011
[2] Zhao, Y.B.: An approximation theory of matrix rank minimization and its application to quadratic equations, Linear Algebra and its Applications, 437 (2012), pp.77-93.

Supervisor: **Prof M Kočvara**

Title: **Approximation by non-negative splines**

Description: Cubic splines a routinely used for smooth approximation of functions. In many applications, the approximated function is known to be non-negative and it is desirable that the approximation spline has the same property. In the one dimensional case, this condition can be formulated as a condition on positive semidefiniteness of certain matrices. The approximation problem can then be formulated as an optimization problem with semidefinite constraints - a nonlinear semidefinite programming problem.
Goals: The goal of the project is to derive a rigorous formulation of this problem, solve it as nonlinear SDP and compare this approach with other approaches.

Desired skills: Calculus, fundamental techniques of linear algebra, sound knowledge of nonlinear programming, knowledge of Matlab.

References:
- Farid Alizadeh, Jonathan Eckstein, Nilay Noyan, Gabor Rudolf: Arrival Rate Approximation by Nonnegative Cubic Splines. Operations Research 56(1): 140-156 (2008)

Supervisor: **Dr Sándor Zoltán Németh**

Title: **Projection onto closed convex sets**

Description: Projection onto closed convex sets is a very important topic. Many algorithms of optimization and equilibrium problems use the projection mapping. For example the variational inequality problem defined on a closed convex set can be reduced to a fixed point problem involving the projection onto that set. Other fields where projecting onto a closed convex set is already important are statistical regression, image reconstruction, pattern recognition, various medical applications, etc. Other possible applications are non-negative solutions of linear systems of equations (this could be important in several practical problems) and portfolio optimization. A particularly interesting sub-topic is the projection onto polyhedral cones. Many of the above mentioned applications need to project onto polyhedral cones.

Unfortunately, finding the projection onto a closed convex set is usually a very difficult algorithmic task. Even for simple classes of polyhedral cones (such as simplicial cones) the existing algorithms are not too efficient and they fail for high dimensions. S. Z. Németh and his coauthors developed some algorithms which seem fast. The algorithms are based on theoretical investigations of the properties of the projection. The aim of this project is to investigate the properties of the projection mapping and the connection between these properties and the projection algorithms.

Prerequisites: Linear Algebra, Nonlinear Optimization

References:

[1] E. H. Zarantonello, Projections on convex sets in Hilbert space and spectral theory. Contributions to nonlinear functional analysis (Proc. Sympos., Math. Res. Center, Univ. Wisconsin, Madison, Wis., 1971), pp. 343–424. Academic Press, New York, 1971

[2] R. L. Dykstra, An iterative procedure for obtaining I-projections onto the intersection of convex sets, Ann. Probab., 13(3):975–984 (1985)

[3] J. P. Boyle and R. L. Dykstra, A method for finding projections onto the intersection of convex sets in Hilbert spaces, Advances in order restricted statistical inference (Iowa City, Iowa, 1985), 28–47, Lecture Notes in Statist., 37, Springer, Berlin, (1986)

[4] J. Dattorro, Convex Optimization & Euclidean Distance Geometry, Meboo, 2005, v2011.01.29 (http://meboo.convexoptimization.com/Meboo.html)

[5] A. B. Németh and S. Z. Németh, How to project onto an isotone projection cone, Linear Algebra and its Applications, 433(1):41–51 (2010)

[6] A. Ekárt, A. B. Németh and S. Z. Németh, Rapid heuristic projection on simplicial cones, http://arxiv.org/pdf/1001.1928v2

[7] http://www.convexoptimization.com/wikimization/index.php/Projection_on_Polyhedral_Cone

[8] http://www.convexoptimization.com/wikimization/index.php/Moreau%27s_decomposition_theorem

[9] http://www.convexoptimization.com/wikimization/index.php/Farkas%27_lemma#Extended_Farkas.27_lemma

[10] http://www.convexoptimization.com/wikimization/index.php/Complementarity_problem

[11] S. Z. Németh, Iterative methods for nonlinear complementarity problems on isotone projection cones, Journal of Mathematical Analysis and Applications, 350(1):340–347 (2009)

[12] S. Z. Németh, Characterization of latticial cones in Hilbert spaces by isotonicity and genralized infimum, Acta Mathematica Hungarica, 127(4):376–390 (2010)

[13] S.Z. Németh, Characterisation of subdual latticial cones in Hilbert spaces by the isotonicity of the metric projection, Optimization, 59(8):1117–1121 (2010)

Supervisor: **Dr JJ Rückmann**

Title: **Nonlinear Optimization Methods and Applications to Mathematical Finance**

Description: The use of mathematical optimization methods in finance is commonplace. They are used for pricing financial products, estimating risks or determining hedging strategies. The task is to understand how nonlinear optimization techniques can be used in the framework of mathematical finance.

Prerequisites: 3G11 Mathematical Finance, 3M12 Non-Linear Programming, 3M10 Game Theory

References: [1] G Cornuejols, R Tütüncü: Optimization Methods in Finance. Cambridge University Press, 2007.
[2] M Capinski, T Zastawniak: Mathematics for Finance. Springer, 2011.
[3] DP Bertsekas: Nonlinear Programming. Athena Scientific, 2004.

Supervisor: **Dr Yunbin Zhao**

Title: **Cardinality and rank minimization and their applications**

Description: Recently, the mathematical model for seeking sparse solutions or low-rank matrix solutions of linear systems has a significant impact across disciplines. This, however, remains an emerging new area awaiting for extensive scientific research inputs. Up to now, the NP-hard sparsity-seeking model has been investigated dominantly by probabilistic analysis and convex approximation method. While the convex method successfully solves a wide range of sparsity-seeking problems, it still fails in many situations. It is therefore imperative to develop a new rigorous theory and efficient design for the so-called weighted algorithms that, at present, lie at the research frontier of both applied mathematics and engineering. The purpose of this project is to conduct a comprehensive and systematic study of such a theory and design, and to investigate some important questions in this field, and to apply the developed algorithms to various data processing. Through this project, you will understand how mathematical optimization theory and methods can be applied to deal with important problems arising from signal and image processing, machine learning and so on. The project provides an opportunity to use linear algebra, matrix analysis, modern convex optimization, and probabilistic analysis to model and solve sparse data processing problems.

Prerequisites: Linear Algebra, Linear Programming, Matrix Analysis, Optimization

References:
1. M. Elad, Sparse and Redundant Representations: From Theory to Applications in Singal and Image Processing, Springer, New York, 2010.
2. A.M. Bruckstein, et al., From sparse solutions to systems of equations to sparse modeling of signals and images, SIAM Rev. 51 (2009), 34-81.
3. Y.B. Zhao and D. Li, Reweighted l1-minimization for sparse solutions to underdetermined linear systems, SIAM J. Optim., 22 (2012), pp. 1065-1088.

Pure Mathematics

Supervisor: **Dr. N. Bez (or Drs. J. Bennett and O. Maleva)**

Title: **Kakeya sets and Fourier analysis**

Description: A Kakeya needle set in euclidean space is a compact set of points within which a unit line segment (a "needle") can be continuously rotated through 180 degrees and returned to the starting position with reversed orientation. In two dimensions, a circular disk with diameter equal to one is an obvious example of such a set. Remarkably, given *any* $\varepsilon > 0$, it is possible to construct a Kakeya needle set with area at most ε.
On the other hand, it is reasonable to expect that Kakeya sets should be "large" in some sense. The Kakeya set conjecture, states that a Kakeya set in n dimensions is large in that it necessarily has *fractal dimension* which is equal to n (as large as it can be). This famous conjecture remains unsolved for $n \geq 3$.
Amazingly, there are strong connections between Kakeya sets and phenomena regarding the Fourier transform on L^p spaces. A project on this topic has scope to include exciting and modern aspects of Fourier analysis and geometric measure theory.

Prerequisites: MSM2B Real and Complex Variable Theory

References: [1] Tao T.; From rotating needles to stability of waves: emerging connections between combinatorics, analysis, and PDE, Notices Amer. Math. Soc. 48 (2001), 294–303.

Supervisor: **Dr. J. Bennett (or Drs. N. Bez and O. Maleva)**

Title: **The Analysis and Geometry of Distance Sets**

Description: Let $n \geq 1$ and E be a subset of $\mathbb{R}^n$. The distance set of E (which we will denote by $\Delta(E)$) is defined to be the number of distinct distances between pairs of points in E; i.e.

$$\Delta(E) := \{|x - y| : x, y \in E\}.$$

A classical problem in combinatorial geometry is to determine how the "size" of a set E manifests itself in the "size" of $\Delta(E)$. In particular, if E is a finite set consisting of N points, the celebrated Erdös Distance Set Conjecture asserts that the number of elements of $\Delta(E)$ is at least N^α, where α is a certain exponent depending on n. This discrete problem has a continuous analogue known as the Falconer Distance Set Conjecture, which asserts that if the fractal dimension of E is strictly greater than $\frac{n}{2}$ then $\Delta(E)$ must have positive Lebesgue measure. The mathematics involved in the study of problems of this type is remarkably diverse, touching on analytic number theory, fractal geometry, and the theory of the Fourier transform. The prerequisites for this project are a good grasp of real and complex analysis, metric space theory, a willingness to spend some time studying measure theory, and an interest in the Fourier transform.

Prerequisites: Real and Complex Variable Theory (MSM2B). Transform Theory (MSM3G07) or Linear Analysis (MSM3P21) would be advantageous, yet not strictly necessary.

References: [1] J. Garibaldi and A. Iosevich, "The Erdös Distance Problem", Julia Garibaldi and Alex Iosevich, Student Mathematical Library 56 (2011), American Mathematical Society. [2] P. Mattila, "Geometry of Sets and Measures in Euclidean Spaces", Cambridge Studies in Advanced Mathematics 44; Chapter 12. [3] T. H. Wolff, Lectures on Harmonic Analysis, American Mathematical Society, University Lecture Series, Vol.29, 2003; section on Stationary Phase.

Supervisor: **Dr. O. Maleva**

Title: **Fractal geometry**

Description: Sets or functions that are not sufficiently smooth are often regarded as 'pathological' or as isolated curiosities. However, in many cases these can represent classes of functions or sets to which a general theory could be applied.
Fractals, sets which are congruent to a magnification of their small parts, are an example of such a class of sets. The Cantor set is probably the best known fractal. This set already exhibits many peculiar features. For example, its *Hausdorff* and *box-counting dimensions* are strictly between 0 and 1, and arbitrarily close to any point $x \in C$ there are points $y \notin C$ such that the whole interval $(y - r/3, y + r/3)$ does not intersect C, for $r = \text{dist}(x, y)$.
The project will involve studying basic Fractal geometry with the help of a textbook such as K. Falconer, Fractal geometry. The project will go on to explore connections between Fractal geometry and questions of current research interest in Mathematical Analysis.

Prerequisites: MSM2B Real and Complex Variable Theory. MSM3P21 Linear Analysis would be advantageous, yet not necessary in terms of any specific content.

References: [1] K. Falconer, Fractal geometry.
[2] P. Mattila, Geometry of Sets and Measures in Euclidean Spaces. Fractals and rectifiability.

Supervisor: **Dr. O. Maleva**

Title: **Small and very small sets**

Description: Rademacher Theorem states that a function f defined on a finite-dimensional space $\mathbb{R}^n$ satisfying the so called Lipschitz condition,

$$|f(x) - f(y)| \leq L|x - y|$$

for some $L > 0$ and all $x, y \in \mathbb{R}^n$, is differentiable at Lebesgue measure *almost every* point x. That means that the set of points where the function is not differentiable has measure zero. An example of such a function can be $f(t) = |t|$; in this case the set of points where f is not differentiable is just one point. On the other hand, the function may be more difficult, for example the function measuring the distance to the Cantor set.
A natural question is then if we start with a set E of measure zero, can we always find or construct a Lipschitz function which is not differentiable at every point from E?
The goal of this project is to study how the geometry of small sets is related to properties of functions that behave non-smoothly on the given set, why and how the answer to the question depends on the dimension of the space and various characteristics of the set. The project will look at the porosity properties, Hausdorff and box-counting dimensions of the set.
There is scope towards the end of the project to attack some open problems in this area.

Prerequisites: MSM2B Real and Complex Variable Theory. MSM3P21 Linear Analysis would be advantageous, yet not strictly necessary.

References:
[1] M. Doré and O. Maleva, A universal differentiability set in Banach spaces with separable dual, Journal of Functional Analysis 261 (2011), 1674-1710. See also on arxiv:1103.5094
[2] P. Mattila, Geometry of Sets and Measures in Euclidean Spaces Fractals and rectifiability. Cambridge Studies in Advanced Mathematics, 44; Cambridge University Press, Cambridge, 1995

Supervisor: **Dr. O. Maleva**

Title: **Baire's category theorem and Banach-Mazur game**

Description: In 1935 the Polish mathematician Stanislaw Mazur proposed the following game. There are two players called Player I and Player II. A subset S of the interval $[0, 1]$ is fixed beforehand, and the players alternately choose subintervals $I_n \subset [0, 1]$ so that $I_{n+1} \subseteq I_n$ for each $n \geq 1$. Player I wins if the intersection of all I_n intersects S, and player II wins if he can force this intersection to be disjoint from S.
Mazur observed that if S can be covered by a countable union of sets whose closure has empty interior (S is of first category) then the second player wins while if the complement of S is of first category then the first player wins. Later Banach proved that these conditions are not only necessary for the existing winning strategies but are also sufficient. This is a very powerful method in Analysis. For example, it implies the Baire theorem which says that the intersection of any collection of open dense subsets of $\mathbb{R}$ is dense.
The game can be generalised to an arbitrary topological space X. Then in order to decide whether a certain property describes a *typical* object of X it is enough to show that there is a winning strategy for Player I with respect to the set of objects satisfying the given property. Remarkably, one can show in this way that a 'typical' continuous function is differentiable at no point!
The project will cover a range of topics connected with the Banach-Mazur game and Baire category theorem.

Prerequisites: MSM203 Metric Spaces

References:
[1] B. Bolobas, Linear Analysis. Cambridge University Press, Cambridge, 1999.
[2] J. Oxtoby, Measure and category. A survey of the analogies between topological and measure spaces. Second edition. Graduate Texts in Mathematics, 2. Springer-Verlag, New York-Berlin, 1980.

Supervisor: **Dr. J. Bennett**

Title: **Projections of small sets**

Description: Assume the set $S \subset \mathbb{R}^2$ is small in some sense. Will it have 'small' orthogonal projections on straight lines? The answer to this question will depend of course on the notion of smallness used.

For example, if we only require that the area of the set S is zero, all its 1-dimensional projections may have positive length (this is very easy to see if we take S to be the boundary of, say, a circle). However, if S is *purely unrectifiable*, that is, its intersections will *all* smooth curves have length zero and its total 1-dimensional measure is finite, then *almost all* its orthogonal projections have length zero. This is the so-called Besicovitch projection theorem. It is interesting to see that without the additional condition that the total 1-dimensional measure is finite the result is no longer true (see [3]).

The aim of this project is to study Hausdorff measures and the geometry of sets in terms of their intersections with smooth curves and their projections.

Prerequisites: MSM203 Metric Spaces and MSM3P21 Linear Analysis would be advantageous, yet not strictly necessary.

References:
[1] P. Mattila, Geometry of Sets and Measures in Euclidean Spaces, Fractals and rectifiability. Cambridge Studies in Advanced Mathematics, 44; Cambridge University Press, Cambridge, 1995

[2] F. Morgan, Geometric measure theory. A beginner's guide. Elsevier/Academic Press, Amsterdam, 2009

[3] H. R. Parks, Purely unrectifiable sets with large projections. (English summary) Aust. J. Math. Anal. Appl. 6 (2009), no. 1, Art. 17, 1–10

Supervisor: **Dr. N. Bez (or Dr. J. Bennett)**

Title: **Fourier analysis and combinatorics**

Description: A celebrated and deep theorem of Szemerédi states that given any subset of the integers with positive upper density, that is,

$$\limsup_{N\to\infty} \frac{|A \cap [-N, N]|}{|[-N, N]|} > 0,$$

then, for any $k \geq 3$, A contains infinitely many arithmetic progressions of length k. When $k = 3$, this was first proved by Roth using a delightful argument based on Fourier analysis. This argument is based on a dichotomy between "structure" and "randomness" which the Fourier transform can be used to detect.

Szemerédi's original proof was combinatorial. Since then several different proofs have appeared, including Gowers' ground-breaking argument based on Fourier analysis and arithmetic combinatorics (extending Roth's argument).

The starting point of this project will be understanding Roth's Fourier analytic proof of the $k = 3$ case of Szemerédi's Theorem. A broader goal will be to understand the fundamental dichotomy between structure and randomness which underpins all known proofs of Szemerédi's theorem.

Prerequisites: MSM2B Real and Complex Variable Theory

References:

[1] http://terrytao.wordpress.com/2010/04/08/254b-notes-2-roths-theorem

Supervisor: **Dr. J. Bennett (or Dr. N. Bez)**

Title: **Heat-flow monotonicity phenomena in mathematical analysis**

Description: One form of the classical Cauchy–Schwarz inequality states that

$$\int fg \leq \left(\int f^2\right)^{1/2} \left(\int g^2\right)^{1/2},$$

where f and g are suitable nonnegative real functions. It was discovered quite recently that this fundamental inequality has a rather striking proof based on heat flow. More specifically, if H_t denotes the heat kernel, and one forms the quantity

$$Q(t) := \int (H_t * f^2(x))^{1/2}(H_t * g^2(x))^{1/2} dx,$$

then one may prove the following.
Theorem. *The function Q is increasing,*

$$\lim_{t\to 0} Q(t) = \int fg$$

and

$$\lim_{t\to\infty} Q(t) = \left(\int f^2\right)^{1/2} \left(\int g^2\right)^{1/2}.$$

As you may have spotted, the Cauchy–Schwarz inequality is an immediate corollary to this.
This project will seek analytic, algebraic and geometric explanations for such heat-flow monotonicity phenomena in the context of a variety of inequalities in mathematical analysis.

Prerequisites: Real and Complex Variable Theory (MSM2B). Transform Theory (MSM3G07) or Linear Analysis (MSM3P21) would be advantageous, yet not strictly necessary.

References:
[1] J. Bennett, Heat-flow monotonicity related to some inequalities in euclidean analysis, Harmonic and Partial Differential Equations, Contemporary Mathematics 505, American Mathematical Society, (2010), 85–96.

Supervisor: **Dr. N. Bez (or Dr. J. Bennett)**

Title: **Inequalities in Fourier analysis**

Description: The Hausdorff–Young inequality states that

$$\|\widehat{f}\|_{p'} \leq C_p \|f\|_p$$

for $f \in L^p$ with $p \in [1,2]$ and $1/p + 1/p' = 1$, and the Young convolution inequality states that

$$\|f * g\|_r \leq C_{p,q,r} \|f\|_p \|g\|_q$$

for $f \in L^p$ and $g \in L^q$ with $p, q, r \in [1, \infty]$ and $1/p + 1/q = 1 + 1/r$. These inequalities are strongly connected and provide very useful information about the Fourier transform and the convolution operator on functions in certain Lebesgue spaces. Despite the classical and fundamental nature of these inequalities, the best (i.e. smallest) constants C_p and $C_{p,q,r}$ were fully obtained only in the mid 1970s (see [2]). One goal of the project is to understand the ground-breaking work in [2].

Recently a number of exciting and somewhat simpler proofs of the Young convolution inequality have emerged (see, for example, [1]). This project will also explore these recent developments, and the wealth of connections to other areas of mathematics.

Prerequisites: MSM2B Real and Complex Variable Theory. Transform Theory (MSM3G07) or Linear Analysis (MSM3P21) would be advantageous, yet not strictly necessary.

References:
[1] Barthe, F.; Optimal Youngs inequality and its converse: a simple proof, Geom. Funct. Anal., 8 (1998), 234–242.

[2] Beckner, W.; Inequalities in Fourier analysis, Ann. of Math., 102 (1975), 159–182.

Supervisor: **Dr. J. Bennett (or Drs. N. Bez and S. Gutiérrez)**

Title: **Oscillatory Integral Asymptotics**

Description: Many important functions in mathematics are defined in terms of integrals whose interesting properties arise as a result of the rapid oscillatory nature of their integrands. Typically, such oscillatory integrals come from applications of the Fourier transform to diverse problems from broad areas of mathematics such as partial differential equations, number theory, and even fractal geometry. This project will be largely concerned with the estimation/asymptotics of oscillatory integrals using the principle of Stationary Phase. Although the general theory is quite classical for one-dimensional integrals, it is far from satisfactory in higher dimensions. This will provide the student with an opportunity to be exposed to some challenging problems of significant current interest. In addition, it is suggested that applications of these oscillatory integral estimates/asymptotics be explored in directions that particularly interest the student. The main prerequisites are a good grasp of real and complex analysis, and an interest in the Fourier Transform.

Prerequisites: MSM2B Real and Complex Variable Theory. Transform Theory (MSM3G07) or Linear Analysis (MSM3P21) would be advantageous, yet not strictly necessary.

References:
[1] E. M. Stein, Harmonic Analysis, Princeton, 1993; Chapter VIII.
[2] Beijing Lectures in Harmonic Analysis, Annals of Math. Studies, Princeton, No.112, 1986; section by Stein (similar to his book above).
[3] T. H. Wolff, Lectures on Harmonic Analysis, American Mathematical Society, University Lecture Series, Vol. 29, 2003; section on Stationary Phase.
[4] C. Sogge, Fourier Integrals in Classical Analysis, Cambridge Tracts in Mathematics, Cambridge University Press, No. 105, 1993; Chapter 1.
[5] A. Zygmund, Trigonometric Series, 2nd Edition, Vol. I; Chapter 5.
[6] N. de Bruijn, Asymptotic Methods in Analysis, Dover, 1981.
[7] M. Taylor, Partial Differential Equations I, Basic Theory, Applied Math. Sciences, No. 115, Springer, 1996; Chapter 6, Section 7.

Supervisor: **Dr. S. Gutiérrez (or Drs. J. Bennett and N. Bez)**

Title: **Nonlinear Schrödinger equations (This is a project in analysis)**

Description: The 1-dimensional free Schrödinger equation from quantum mechanics is the partial differential equation

$$i\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} = 0,$$

where $x \in \mathbb{R}$ and t represents time. For a suitably well-behaved function $f : \mathbb{R} \to \mathbb{C}$, one may use elementary properties of the Fourier transform to explicitly find the solution of this (linear) equation subject to the initial condition $u(0, x) = f(x)$. However, for important nonlinear variants of this differential equation, such as the cubic nonlinear Schrödinger equation

$$i\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + |u|^2 u = 0,$$

this explicit analysis breaks down completely. The answers to fundamental questions concerning the existence and uniqueness of solutions to such equations (under appropriate initial conditions) are far from evident, and will form the focus of this project. The extent to which this analysis applies in higher dimensions (and to more general nonlinear partial differential equations) is of significant current interest, and will also be investigated.
The prerequisites here are a reasonable grasp of real and complex analysis, metric space theory, a willingness to study the rudiments of Lebesgue integration, and some exposure to the Fourier transform.

Prerequisites: MSM2B Real and Complex Variable Theory. Transform Theory (MSM3G07) or Linear Analysis (MSM3P21) would be advantageous, yet not strictly necessary.

References:
[1] Books on the Contraction Mapping Theorem and Picard's Theorem about the existence and uniqueness of solutions to ODE's; e.g. Metric Spaces by Victor Bryant, Cambridge University Press.
[2] E. M. Stein and R. Shakarchi, Fourier Analysis – an Introduction, Princeton, 2003 (Sections 2.2 and 3.3).

Supervisor: **Dr. J. Bennett**

Title: **The Riemann Zeta Function and the Distribution of the Primes**

Description: The Zeta Function is defined for a real numbers $s > 0$, by the absolutely convergent series

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

It has long been known that the analytic properties of ζ are intimately related with the distribution of the prime numbers. The simplest explicit manifestation of this is the celebrated Euler Factorisation, which states that

$$\zeta(s) = \prod_{p \in P} \left(1 - \frac{1}{p^s}\right)^{-1},$$

where P denotes the set of primes. From this observation alone one can easily begin to use the behaviour of ζ (such as its divergence as $s \to 1$) to deduce seemingly non-trivial information about the primes. Riemann was the first to observe that ζ may be continued to a holomorphic function on $\mathbf{C} \setminus \{1\}$, and that it is in this extension where the truly deep information about the distribution of the primes is encoded. In this project the student will be encouraged to explore the classical results in this context (and perhaps the wider context of Dirichlet Series) from a Fourier analytic perspective. The main prerequisites for this project are a good grasp of real and complex analysis, the Fourier transform, and an interest in number theory.

Prerequisites: MSM2B Real and Complex Variable Theory. Transform Theory (MSM3G07) or Linear Analysis (MSM3P21) would be advantageous, yet not strictly necessary.

References:
[1] E. M. Stein and R. Shakarchi, Complex Analysis, Princeton, 2003.
[2] G. J. O. Jameson, The Prime Number Theorem, LMS Student Texts 53, 2003.
[3] E. C. Titchmarsh, The Theory of the Riemann Zeta Function, 2nd Edition, Oxford Science Publications, 1986.

Supervisor: **Dr. N. Bez**

Title: **Bézier curves in engineering and computer graphics**

Description: Pierre Bézier (1910–1999) was a prominent engineer at Rénault and he developed Bézier curves to help in the design of automobile body parts. Interestingly, whilst working for Citroën, Paul de Casteljau (1930–) independently developed the theory of Bézier curves in 1959, slightly earlier than Bézier himself.

Bézier curves are polynomial (or rational) curves defined in terms of the Bernstein basis of polynomials

$$B_{n,\ell}(t) = \binom{n}{\ell}(1-t)^{n-\ell}t^{\ell}.$$

In terms of computer aided geometric design applicability, polynomial curves are convenient because they can be efficiently computed, and the Bernstein representation means that the curves are "intuitive" to manipulate for the designer. The latter is largely why Bézier curves have found vast applicability (including computer graphics, animation, font design).

The goal of this project will be to develop the mathematical theory of Bézier curves and discuss their applications. There is scope towards the end of the project to attack some open problems in this area.

Prerequisites: None

References: [1] Farin, G.; Curves and surfaces for CAGD: A Practical Guide, Fifth Edition.

Supervisor: **José A. Cañizo**

Title: **Stationary states for attractive-repulsive potentials**

Description: Consider N particles which interact through a potential V. This is: the force on particle i due to particle j is $\nabla V(x_j - x_i)$, where x_k is the position in $\mathbb{R}^d$ of particle k. For simplicity, assume particles are on a plane, so take $d = 2$. There are many interesting potentials which are attractive when particles are far away, but repulsive when they are close. They are called *attractive-repulsive potentials*. The forces between atoms are an example of this (though in reality you need to consider quantum effects). A typical expression of the potential is the following:

$$V(x) = \frac{|x|^a}{a} - \frac{|x|^b}{b} \quad \text{for } x \in \mathbb{R}^2$$

with $b < a$. Here, the term in $|x|^a$ is attractive and the term in $|x|^b$ is repulsive. The question is: what are the arrangements of particles in which there is no force on any of them? This is: what are the positions $(x_1, \dots, x_N)$ such that

$$\sum_{i=1}^{N} \nabla V(x_j - x_i) = 0$$

for all $j = 1, \dots, N$? These arrangements are called *stationary states* or *equilibria*, and some of them are the ones you would expect to see after particles have moved and found an equilibrium.

The problem in general in quite hard and depends strongly on the type of potential you consider. If the potential is very repulsive at short distances (such as for atoms) then particles tend to arrange themselves in a "crystalline" pattern. Proving this is hard, but has been achieved recently in some cases [4]. There has also been a lot of recent work on potentials which are not so repulsive, which turn up in the field of collective behavior, in mathematical models for the evolution of groups of animals or bacteria [1,2,3]. Then it may happen that particles reach some "limiting density" as you add more and more particles. There is a wide range of interesting behaviors exhibited by particles depending on the potential; see [2] for an illustration.

The problem has links to statistical mechanics and the calculus of variations, since some of the stationary states are ground states for the potential interaction. There are also links to partial differential equations through the models of collective behavior, which are often stated in terms of PDE when the number of particles is very large. There is also a good opportunity to run computer simulations of the models, which are technically simple to program and may give a lot of insight into the problem.

Prerequisites: A good understanding of calculus of several variables and ordinary differential equations, and a course on partial differential equations. Familiarity with C and Matlab programming will be useful.

References:
[1] Balague, D., Carrillo, J. A., Laurent, T., Raoul, G., Sep. 2011. Nonlocal interactions by repulsive-attractive potentials: radial ins/stability. URL `http://arxiv.org/abs/1109.5258`
[2] D'Orsogna, M. R., Chuang, Y. L., Bertozzi, A. L., Chayes, L. S., Mar. 2006. Self-Propelled particles with Soft-Core interactions: Patterns, stability, and collapse. Physical Review Letters 96 (10), 104302+.
[3] Fetecau, R. C., Huang, Y., Nov. 2012. Equilibria of biological aggregations with nonlocal repulsive?attractive interactions. Physica D: Nonlinear Phenomena. URL `http://dx.doi.org/10.1016/j.physd.2012.11.004`
[4] Theil, F., Feb. 2006. A proof of crystallization in two dimensions. Communications in Mathematical Physics 262 (1), 209-236.

Supervisor: **Dr. O. Maleva**

Title: **The Mathematics of Voting**

Description: We tend to feel fairly smug in the 'democratic' West about our system of government, but are elections fair? Do they really reflect the views of the electorate? Certainly many people are unhappy with the 'First-Past-the-Post' electoral system used in the UK, espousing instead some form (or other) of proportional representation. Would such a system be better? How can we make a judgement?
In fact, when we analyse voting mathematically, it becomes clear that all system of aggregating preferences (electing a parliament or a president, agreeing on who should win Pop Idols or Opportunity, deciding the winner of the 6 Nations or the Formula 1 Championship) can throw up anomalies, unfairnesses and down-right weirdness.
How weird can things get? Well, in 1972, Kenneth Arrow (Harvard, USA) and John R. Hicks (Oxford, UK) were awarded the Nobel Prize for Economics 'for their pioneering contributions to general economic equilibrium theory and welfare theory.' At the heart of Arrow's contribution to economic theory is his so-called 'Impossibility Theorem,' which (roughly speaking) says that there is no fair voting system. More precisely, once we agree what a fair voting system is, one can show that the only fair voting system is one in which there is a dictator who decides what every outcome will be. But there clearly cannot be a dictator in any fair voting system, so a fair voting system is impossible.
In this project the student will look at the mathematics of voting, in particular the proof of Arrow's Theorem.

Prerequisites: None.

References: Search on Wikipedia for 'Arrow's Theorem' and 'May's Theorem'

Supervisor: **Dr D Craven**

Title: **The symmetric group**

Description: The symmetric group is at the same time a group and a combinatorial object. The rich combinatorial structure of the symmetric group provides connections with partitions, symmetric functions, group theory and representation theory, as well as it being the archetype of a group generated by reflections, leading to Coxeter groups, Lie theory, and many more topics.
In this project the student will study some of the many aspects of this topic, from partitions to subgroup structure to the representations of the group, depending on the student's individual preferences.

Prerequisites: MSM2C Linear algebra, MSM2D Symmetry and groups, MSM203 Polynomials and rings MSM3P08 Group theory and Galois theory necessary for the more group-theoretic topics.

References: [1] James, G.; Kerber, A.; *The representation theory of the symmetric group*, Encyclopedia of Mathematics and its Applications 1,. Addison-Wesley Publishing Co., Reading, Mass., 1981. [2] Macdonald, I.; *Symmetric functions and Hall polynomials*, Oxford University Press, New York, 1995. [3] Sagan, B.; *The Symmetric Group*, Graduate Texts in Mathematics 203, Springer-Verlag, 2001.

Supervisor: **Dr D Craven**

Title: **Lie groups and Lie algebras**

Description: Lie groups are to differential equations as Galois groups are to polynomials. Originally developed to help solve differential equations, Lie theory has grown over the last century to become one of the fundamental areas of mathematics, connected to almost every branch of pure mathematics.
In this project the student will start this study by meeting Lie algebras, and then later Lie groups; there are many avenues along which one can proceed after this.

Prerequisites: MSM2C Linear algebra, MSM2D Symmetry and groups, MSM203 Polynomials and rings MSM3P08 Group theory and Galois theory.

References: [1] Erdmann, K.; Wildon, M.; *Introduction to Lie algebras*, Springer Undergraduate Mathematics Series. Springer-Verlag London, Ltd., London, 2006. [2] Hall, B.; *Lie groups, Lie algebras, and representations. An elementary introduction*, Graduate Texts in Mathematics, 222. Springer-Verlag, New York, 2003.

Supervisor: **Dr A Evseev**

Title: **Frobenius groups and representation theory**

Description: The project will involve a study of representation theory of finite groups and, more specifically, character theory. Representation theory is an important area of modern algebra and, in particular, an essential tool for investigating the structure of finite groups. The project will focus on a beautiful application of character theory to a special class of groups, known as Frobenius groups.

A finite group G acting transitively on a set X is said to be a Frobenius group if every element of G fixes at most one point in X. The student will work through a proof of the following theorem: if G is a Frobenius group, then the elements of G that fix no point in X, together with the identity element, form a normal subgroup N of G. Once this is achieved, the project may continue in one of the following directions:

1. A deeper investigation of representations and their applications to the theory of finite groups;
2. A study of purely group-theoretic methods, such as those needed for a more detailed understanding of the normal subgroup N in a Frobenius group.

Interested students are encouraged to see Anton Evseev for further information.

Prerequisites: Group Theory (MSM3P08) and a strong mastery of Linear Algebra (MSM2C).

References:
D. Gorenstein, Finite groups. Harper & Row, Publishers, 1968.
I.M. Isaacs, Character theory of finite groups. Dover Publications, 1994.

Supervisor: **Dr A Evseev**

Title: **Sporadic simple groups**

Description: The 26 sporadic simple groups occupy a special place in group theory, as they do not belong to any of the "regular" families of simple groups (such as that of alternating groups A_n, $n \geq 5$), but rather stand on their own. Many of the sporadic groups may be viewed as groups of symmetries of interesting combinatorial and geometric structures. The project will focus on a construction of several sporadic simple groups. The student will begin with the 5 Mathieu groups and will move on to other ones, such as Conway groups or the Higman–Sims group. Properties of these groups will be investigated, and related structures (such as Steiner systems, the Leech lattice, and the Higman–Sims graph) will be studied.
Interested students are encouraged to see Anton Evseev for further information.

Prerequisites: Group Theory (MSM3P08), Linear Algebra (MSM2C).

References:
J.H. Conway, R.T. Curtis, S.P. Norton, R.A. Parker, R.A. Wilson, Atlas of finite groups. Oxford University Press, 1985.
J.H. Conway and N.J.A. Sloane, Sphere packings, lattices and groups. Springer, 1999 (3rd edition).
R.L. Griess, Twelve sporadic simple groups. Springer, 1998.

Supervisor: **Dr A Evseev**

Title: **The Riemann zeta-function**

Description: The zeta-function is a function of one complex variable that encodes a lot of information about the distribution of prime numbers. The location of zeros of the zeta-function is the subject of the Riemann hypothesis, which is perhaps the most famous unsolved problem in mathematics. The project will explore the zeta-function and its applications to number theory. In particular, it will focus on a proof of the classical Prime Number Theorem, which asserts that $\pi(x) \sim \frac{x}{\ln x}$ as $x \to \infty$, where $\pi(x)$ denotes the number of primes p such that $p \leq x$. Possible further topics of investigation include:

1. The Riemann hypothesis and its consequences;
2. A proof of Bertrand's postulate: if $n > 1$ is an integer, then there exists a prime p such that $n < p < 2n$;
3. Dirichlet's L-function and his theorem on primes in arithmetic progressions.

Interested students are encouraged to see Anton Evseev for further information.

Prerequisites: Excellent knowledge of analysis, especially complex analysis (MSM2B and MSM3P22), and an interest in number theory.

References:
T.M. Apostol, Introduction to analytic number theory. Undergraduate Texts in Mathematics. Springer, 1976.
H.M. Edwards, Riemann's zeta function. Pure and Applied Mathematics, Vol. 58. Academic Press, 1974.
J.P. Serre, A course in arithmetic. Graduate Texts in Mathematics 7. Springer, 1973.

Supervisor: **Paul Flavell**

Title: **Finite Group Theory**

Description: Various topics in Finite Group Theory and the associated Representation Theory will be studied. For example, working through selected chapters and exercises of Isaacs and/or Kurzweil-Stellmacher. In the latter part of the project, it may be possible to study some contemporary papers in the area.

Prerequisites: MSM2C Linear Algebra, MSM2D Symmetry and Groups, MSM2O3 Polynomials and Rings, MSM3P05 Number Theory, MSM3P08 Group Theory and Galois Theory.

References: [1] Isaacs, I.M.; Finite Group Theory, Graduate Studies in Mathematics, 92. American Mathematical Society, Providence, RI, 2008
[2] Kurzweil, H.; Stellmacher, B.; The theory of finite groups. An introduction. Universitext. Springer-Verlag, New York, 2004.

Supervisor: **Dr N Fountoulakis**

Title: **Percolation theory**

Description: Assume that we immerse a porous stone or any other porous medium in a bucket of water. Then, what is the probability that its centre will be wetted? Simple models that try to answer this question haven given rise to a branch of probability theory, which is now known as percolation theory. The setting is as follows. Consider a large graph and declare each one of its edges *open* with some probability p (otherwise we consider it to be *closed*). Then the above question may be stated as follows: what is the probability that a certain vertex belongs to a component spanned by open edges that contains at least half of the vertices of the graph? Nowadays, percolation theory is an important branch of probability theory and a point where probability theory meets combinatorics.

Prerequisites: Probability theory is necessary, so MSM2O2 Statistics is a prerequisite; some graph theory (MSM3P15) may also be helpful.

References: [1] G. Grimmett, *Percolation*, Springer, 1999. [2] B. Bollobás and O. Riordan, *Percolation*, Cambridge University Press, 2006.

Supervisor: **Dr N Fountoulakis**

Title: **The mathematics of social networks**

Description: Social networks have become an important part of the daily life of millions of people across the world. Their rapid development during the last ten years or so has had an enormous impact on social, political and economic life. The aim of this project is the study of mathematical models that describe the structure and evolution of social networks. For example, how do we describe rigorously the observation that a friend of a friend is more likely to become our friend? We will try to give answers to questions like the above one using tools from graph theory and probability theory.

Prerequisites: The probability theory part of MSM2O2 Statistics is a prerequisite as well as MSM3P15 Graph Theory.

Supervisor: **Dr Simon Goodwin**

Title: **Algebraic combinatorics**

Description: The area of mathematics called "algebraic combinatorics" roughly refers to any topic in which there is an interplay between algebra and combinatorics. This can both involve using algebraic techniques to solve enumerative problems and using combinatorial methods to answers questions in algebra and geometry. For example, there a pleasing application of algebra in counting necklaces, and the combinatorial theory of Young tableau has striking applications in the representation theory of the symmetric groups and the geometry of flag manifolds.

For this project, the student will choose one of a variety of interesting topics in this area of sprawling current research interest. Examples of possible topics include symmetric functions, crystal bases and electrical networks. We will find a suitable book to act as the primary reference, and use this along with a selection of other sources to investigate the chosen area.

Interested students are encouraged to see Dr Goodwin for further information.

Prerequisites: MSM2C Linear algebra

The following courses will be useful, but are not necessary.

MSM2D Symmetry and groups
MSM203 Polynomials and rings, and metric spaces
MSM3P08 Group theory and Galois theory
MSM3P15 Graph theory
MSM3P16 Combinatorics and communication theory

Possible references:

William Fulton, Young Tableaux, London Mathematical Society Student Texts **35**, Cambridge University Press, 1997.

Ian G. McDonald, Symmetric Functions and Hall Polynomials, Oxford Mathematical Monographs, Oxford University Press, 1998.

Richard P. Stanley, Topics in algebraic combinatorics, `http://www-math.mit.edu/ rstan/algcomb.pdf`.

Supervisor: **Dr Simon Goodwin**

Title: **Lie algebras and finite Chevalley groups**

Description: Lie algebras play an important role in many branches of mathematics and mathematical physics. For example, semisimple Lie algebras are used for the construction of finite Chevalley groups, which form a large class of finite simple groups. The structure theory and representation theory of Lie algebras attracted a great deal of research interest throughout the 20th century and remains an active area of research today.

For this project, the student will follow the book of Humphreys; the book of Erdmann and Wildon (Introduction to Lie algebras) will supply valuable support. The highlights of the project will be: the structure theory of semisimple Lie algebras, including the beautiful theory of root systems; and the construction of Chevalley groups via Chevalley bases. An alternative direction is to study the highest weight representation theory of semisimple Lie algebras.

Interested students are encouraged to see Dr Goodwin for further information.

Prerequisites: MSM2C Linear algebra
MSM2D Symmetry and groups
MSM203 Polynomials and rings, and metric spaces
MSM3P08 Group theory and Galois theory (desirable)

References:
Karin Erdmann and Mark Wildon, Introduction to Lie algebras, Springer Undergraduate Mathematics Series. Springer-Verlag London, Ltd., London, 2006.
James E. Humphreys, Introduction to Lie algebras and representation theory, Graduate Texts in Mathematics **9**, Springer-Verlag, New York-Berlin, 1978.

Supervisor: **Dr D. Hefetz**

Title: **Topics in Positional Games**

Description: The theory of positional games is a rapidly evolving, relatively young topic that is deeply linked to several popular areas of mathematics and theoretical computer science, such as random graph theory, Ramsey theory, complexity theory and derandomisation. Results on positional games have been used to make decisive progress in these areas. The origins of the theory can be traced back to classical game theory, a branch of mathematics which has found applications in a variety of fields such as economics, management, operations research, political science, social psychology, statistics, biology and many others.
We will start by reviewing the basics of the theory and will then proceed to study certain more advanced topics. There will be some flexibility in choosing these advanced topics. This project would constitute good preparation for a student wishing to write a Ph.D. thesis in the field of Positional Games (as well as other areas of Combinatorics).

Prerequisites: MSM3P15 Graph Theory.

References: [1] N. Alon and J. H. Spencer, **The Probabilistic method**, Wiley-Interscience Series in Discrete Mathematics and Optimization, John Wiley & Sons, 3^{rd} edition, 2008. [2] J. Beck, **Combinatorial Games: Tic-Tac-Toe Theory**, Cambridge University Press, 2008. [3] V. Chvátal and P. Erdős, Biased positional games, *Annals of Discrete Math.* 2 (1978), 221–228. [4] J. A. Bondy and U. S. R. Murty, **Graph theory**, Springer Verlag, 2008. [5] S. Janson, T. Łuczak and A. Ruciński, **Random graphs**, Wiley, New York, 2000. [6] A. Lehman, A solution of the Shannon switching game, *J. Soc. Indust. Appl. Math.* 12 (1964), 687–725. [7] D. B. West, **Introduction to Graph Theory**, Prentice Hall, 2001.

Supervisor: **Dr C Hoffman**

Title: **Introduction to algebraic geometry**

Description: Algebraic geometry is the study of solutions of polynomial equations. There is a natural topology (called the zariski topology) on the space of such solutions. The study of these topological spaces is naturally equivalent to the study of the corresponding rings of rational functions. This are the basic objects of (affine) algebraic geometry. The project will introduce these objects carefully and will study their propertie. There are two possible avenues for the second part of the project, either the study of Algebraic Groups with the classification theorem as the final goal or the study of Algebraic Curves, with the final objective being the proof of the Riemann-Roch theorem.

Supervisor: **Dr RW Kaye**

Title: **An application of nonstandard analysis**

Description: Nonstandard analysis (NSA) is a technique arising from mathematical logic that allows the rigorous analysis of infinitesimal and infinite quantities, with applications to ordinary real analysis and calculus.
This project will use techniques of nonstandard analysis to investigate one area of calculus. The emphasis is on use of NSA as a 'toolkit' and its applications to a topic in analysis rather than an investigation of NSA in its own right.
A suggested topic is the calculus of variations. The calculus of variations concerns finding function $y = y(x)$ that maximize an integral

$$\int_a^b f(x, y, \dot{y})dx.$$

A numerical method to solve this problem due to Euler approximates the unknown function $y(x)$ by a step function (with n steps, where n is finite) and replaces the problem to one of maximizing a function of n unknowns. If n is allowed to be an infinite nonstandard number, this method can be used to obtain exact analytic solutions.
Other topics in analysis may be tackled in a similar way: the main idea is usually to recast a numerical method of solution into NSA with infinitely many variables or infinitessimal step size. Please discuss with the supervisor if you have other (sensible) suggestions you would be interested in investigating.

Pre-requisite: Logic

Supervisor: **Dr RW Kaye**

Title: **Machine models and undecidable problems**

Description: Computability theory and machine models for computation arise in mathematics in many different ways, but are probably of most interest since they provide techniques for proving the algorithmic insolubility of certain mathematical problems. For example Hilbert's tenth problem, asking for a general algorithm for the solution of diophantine equations, was eventually answered negatively by showing that all nondeterministic Turing machines can be represented by diophantine equations. The insolubility of the word problem for groups was proved by similar techniques.
This project will look at machine models for computation, such as diophatine equations, word problems, tiling problems and others, and investigate why they are Turing complete and what mathematical problems are shown insoluble in the process.

Pre-requisite: computability theory

Supervisor: **Dr RW Kaye**

Title: **Algorithms for Pure Mathematics in Java**

Description: Modern object-oriented programming languages provide an elegant way of representing many of the basic constructions in pure mathematics and presenting usable algorithms on them. This project concerns implementing a suite of algorithms for an aspect of graph theory and/or algebra in Java.

Doing this project will involve you in implementing and testing some algorithms concerning one area of pure mathematics as well as providing written work explaining the algorithms, how they work and their strengths and limitations.

You may choose also to investigate and discuss how well the traditional (and typically platonistic) view of pure mathematics relates to the (typically constructivistic) view of mathematics as carried out on a computer.

Pre-requisites: Strong Java programming skills (including familiarity with abstract classes, class inheritance and interfaces) are essential.

Supervisor: **Daniela Kühn**

Title: **Topics in Graph Theory**

Description: The aim of the project is to study one of the topics of last years Graph Theory course in more depth. There will be scope to study recent research developments. Possible directions include:

- *Hamilton cycles in graphs and directed graphs:* A Hamilton cycle in a graph G is a cycle which contains all vertices of G. Unfortunately, nobody knows how to decide efficiently whether a graph has a Hamilton cycle. Probably this is not possible, as otherwise $P = NP$. So researchers have been looking for natural and simple sufficient conditions which guarantee that a (directed) graph has a Hamilton cycle.
- *Graph colouring problems:* Here the question is how many colours are needed to colour the vertices of a graph in such a way that endvertices of an edge have different colours. This has connections to the 4-colour-theorem which states that every planar map can be coloured with at most 4 colours such that adjacent regions have different colours.
- *Extremal Graph Theory:* Here the general question is which conditions force the existence of a certain substructure in a graph. So this is more general than the Hamilton cycle problem mentioned above.

Prerequisites: Graph Theory (MSM3P15)

References: Diestel, Graph Theory, Springer 1997. West, Introduction to Graph Theory, Prentice Hall 2001.

Supervisor: **Paul Blain Levy (School of Computer Science)**

Title: **Coproducts and Colimits**

Description: Given sets A and B, we can form the disjoint union $A + B$. Category theory generalizes this construction to what is called the "coproduct" of objects. Several kinds of coproduct were studied by Carboni et al [1] and by Cockett [2]. The isomorphism $A \times (B + C) \cong (A \times B) + (A \times C)$ leads to the notion of distributive coproduct, while the fact that a function $f : U \longrightarrow A + B$ induces a partition of U gives rise to the notion of an extensive coproduct.
This project would involve learning some category theory and there are many textbooks that can be used, such as [4]. There are various possible directions in which you might develop this project if time allows.

- The use of these kinds of coproducts to give semantics of type theory, see e.g.˜[5]
- Looking at colimits, which are more general than coproducts; they correspond to the quotient of a disjoint union by an equivalence relation. In this setting, the extensiveness condition is given by the "van Kampen property", which can be formulated in various elegant ways that turn out to be equivalent [3].

Prerequisites: Some familiarity with abstract algebra, e.g. groups and rings, would be helpful.

References:
[1] A. Carboni, S. Lack, R. F. C. Walters (1993), Introduction to extensive and distributive categories, Journal of Pure and Applied Algebra, vol. 84, pages 145–158.
[2] J. R. B. Cockett (1993), Introduction to distributive categories, Mathematical Structures in Computer Science 3(3), pages 277–307.
[3] T. Handel, P. Sobocinski (2011). Being Van Kampen is a universal property, Logical Methods in Computer Science 7(1).
[4] B. C. Pierce (1991), Basic category theory for computer scientists, MIT Press.
[5] A. M. Pitts (1989), Notes on categorical logic, available on Andy Pitts' webpage.

Supervisor: **Paul Blain Levy (School of Computer Science)**

Title: **Dataflow Networks**

Description: A dataflow network is a simple kind of concurrent program, consisting of several processes that each input data from and output data to some channels. Kahn [2] gave a semantics for such programs using continuous functions between domains of data streams (a domain is a kind of poset) and least fixpoints. This semantics relies on the networks being deterministic. It is tempting to say that a nondeterministic network should denote a relation (rather than a function) between domains of streams, but an example due to Russell [4] shows that to be impossible. Alternative semantics for nondeterministic dataflow have been found by several researchers, in particular Jonsson [2].

One interesting kind of nondeterministic operator is ”merge”, which inputs data from two channels and outputs a merged version of the data to another channel. Panagaden et al [3] studied various ways of accomplshing this: unfair merge, infinity-fair merge, angelic merge, and fair merge. It turns out that this list constitutes a hierarchy of expressiveness: infinity-fair merge can express unfair merge but not vice versa, and so forth.

There are several parts of this story that a project could focus on:

- the correctness of Kahn's semantics for deterministic dataflow networks
- the semantics of nondeterministic networks
- the hierarchy of merge operators.

Prerequisites: None

References:
[1] B. Jonsson (1989). A fully abstract trace model for dataflow networks. In: Proceedings of the 16th ACM Symposium on Principles of Programming Languages, pages 155-165.
[2] G. Kahn, (1974). The semantics of a simple language for parallel programming. In Jack L. Rosenfeld (Ed.): Information Processing 74, Proceedings of IFIP Congress 74, Stockholm, Sweden, August 5?10, 1974. North-Holland, 1974, ISBN 0-7204-2803-3
[3] P. Panangaden (1995), The expressive power of indeterminate primitives in asynchronous computation, Lecture Notes in Computer Science 1026, pages 124-150. (Available on CiteSeerX)
[4] J. Russell (1989), Full abstraction for nondeterministic dataflow networks, Proceedings of the 30th Annual Symposium of Foundations of Computer Science, pages 170-177.

Supervisor: **Dr K Magaard**

Title: **Riemann surfaces and their automorphisms**

Description: The theory of Riemann surfaces lies at the crossroads of many mathematical subdisciplines such as analysis, algebra, number theory and topology. Compact Riemann surfaces are solution sets of homogeneous polynomials in three variables polynomials, or equivalently, via their affine model, as a solution set of two variable polynomials.
For example: The solution set of

$$x^2 + y^2 + z^2 = 1$$

is the Riemann sphere. Its affine model is obtained by making the substitutions $X := x/z_0$ and $Y := y/z_0$ where $z_0 = \sqrt{1 - z^2}$ to yield the bi-variate polynomial

$$X^2 + Y^2 = 1.$$

Other celebrated examples of Riemann surfaces are, the Fermat curve F_n which is defined to be the solution set of

$$x^n + y^n = z^n,$$

and elliptic curves, whose affine models are solution sets of

$$Y^2 = X^3 + aX^2 + bX + c.$$

Notice that F_2 is just the Riemann sphere. An automorphism of a Riemann surface R is a map from R to R which preserves the structure or R. The set of all automorphisms of R is a group. An example of an automorphism of the Fermat curve F_n is the map defined by

$$(x_0, y_0, z_0) \mapsto (\zeta_n x_0, \zeta_n y_0, \zeta_n z)$$

where $\zeta_n := e^{2\pi i/n}$ is an n-th root of unity; i.e. a solution to the equation $x^n = 1$ in the complex numbers. The automorphisms of a Riemann surface identify its symmetries and play a role similar to the one played by Galois groups of single variable polynomial.
This project would start with the study of classic texts on Riemann surfaces such as the ones listed in the references below. Time permitting the project could be developed in various different directions depending on the interests of the student. For example there are exciting connections to the inverse problem of Galois theory, see for example the book by Völklein [3].
This project is ideal preparation for students wishing to write a Ph.D. thesis with this supervisor but is also suitable for students who want to learn about the interplay between surfaces and groups.

Prerequisites: MSM2B Real and Complex Analysis, MSM2C Linear alg., MSM2D Symmetry and groups

References:
[1] Otto Forster; *Lectures on Riemann surfaces*, Graduate Texts in Math., vol. 81, Springer-Verlag, New York, 1981. ISBN 0-3879-. 0617-7
[2] Rick Miranda; *Algebraic curves and Riemann surfaces*, Graduate Studies in. Math., vol. 5, Amer. Math. Soc., Providence, RI, 1995. ISBN 0821802682
[3] Völklein, H.; *Groups as Galois Groups, An introduction*, Cambridge Studies in Advanced Mathematics 53, Cambridge University Press, 1996. ISBN 9780521065030

Supervisor: **Dr K Magaard**

Title: **Fixed point ratios of elements in permutation groups**

Description: Let G be a permutation group acting on a finite set Ω. If $g \in G$, then the set $F_\Omega(g) := \{\omega \in \Omega \mid \omega g = \omega\}$ is called the *fixed point set* of g. The *fixed point ratios* $\frac{|F_\Omega(g)|}{|\Omega|}$ as g ranges over G carry significant information about the structure of the group G. Also fixed point ratios have uses in applications of group theory to other areas of mathematics, such as the theory of Riemann surfaces and card shuffling.
We could start by going through papers [2] and [1] and some of the references contained therein.
This project is ideal preparation for students wishing to write a Ph.D. thesis with this supervisor but is also suitable for students who want to learn about the thoery of permutation groups and its applications.

Prerequisites: MSM2C Linear algebra, MSM2D Symmetry and groups, MSM203 Polynomials and rings MSM3P08 Group theory and Galois theory

References:
[1] Frohardt D.; Magaard, K. Grassmannian fixed point ratios, Geom. Ded.82: 21-104, 2000
[2]Guralnick, R.; Magaard, K. On the minimal degree of a permutation group J. Alg., 207 (1998), No.1, 127-145.

Supervisor: **Dr R. Mycroft**

Title: **Fractional graph theory**

Description: Most of the classical problems of graph theory are discrete in nature – indeed, this is a natural consequence of the discrete notion of a graph. For example, a perfect matching in a graph G is a subset of the edge set $E(G)$ (a finite set) so that every vertex is contained in precisely one edge of this subset; all the concepts of this definition are discrete. Note in particular that another way to state this is to say that every edge $e \in E(G)$ is assigned a weight $w_e \in \{0,1\}$ so that for any vertex $v \in G$ the total weight $\sum_{e:v\in e} w_e$ of all edges containing v is equal to 1. Indeed, an edge is in the matching if it has weight 1, and not if it has weight 0.

Recently, much attention has focused on *fractional* versions of classical problems in graph theory; these can be seen as continuous analogues of discrete problems. Returning to the example of a perfect matching in a graph G, we define a *fractional perfect matching* to be an assignment of a weight $w_e \in [0,1]$ to each edge $e \in E(G)$ so that for any vertex $v \in G$ we have $\sum_{e:v\in e} w_e = 1$. So edge weights are no longer required to be integers, and so, for example, an edge of weight 1/2 can be thought of as being 'half in the matching and half out'!

To put it another way, we take a classical problem in graph theory, formulate it as an integer programming problem, and then consider the linear programming relaxation (the 'fractional' version) of this problem. We can then hope to use techniques and ideas of linear programming, for instance the idea of duality, to solve the fractional version of the problem. Not only are such results interesting in their own right, but we can often make use of the fractional result to make progress towards or even solve the classical problem; for this reason fractional methods are playing a crucial role in current research in graph theory. In particular, recent work by Alon, Frankl, Huang, Rödl, Ruciński and Sudakov showed that to find the vertex degree which forces a perfect matching in a uniform hypergraph (one of the most important open problems in extremal graph theory) it is sufficient to solve the fractional version of this problem.

The first step of this project would be to understand in detail how fractional versions of a variety of classical problems can be formulated, and also the techniques which can commonly be applied to such problems. Beyond that, the project is very open-ended: most problems of discrete mathematics admit fractional versions, and there is great scope to go wherever your interest leads you!

Prerequisites: MSM3P15 Graph Theory, MSM2D Linear Programming.

References:
[1] E.R. Scheinerman, D.H. Ullman - Fractional Graph Theory: A Rational Approach to the Theory of Graphs, Dover Publications, 2011. ISBN 0486485935 (available free online at http://www.ams.jhu.edu/∼ers/fgt/fgt.pdf)

Supervisor: **Deryk Osthus**

Title: **Graph Colouring**

Description: Graph colouring has its origins in the four colour problem. Though this has been solved, many open problems remain and it is a large and active research area. Several varieties of colouring problems have been studied, which arise in different contexts (e.g. edge colourings, total colourings, fractional colourings and list colourings). The methods used to study these problems are very diverse, ranging from 'discharging' methods and algorithmic approaches to probabilistic arguments. The project would explore one or more of these areas in greater detail.

Prerequisites: MSM3P15 Graph Theory

References: Introduction to Graph Theory, by Douglas B. West, Prentice Hall, 2nd edition 2001.

Supervisor: **Professor Chris Parker**

Title: **Finite Groups of Lie type**

Description: The finite groups of Lie type make up a major part of the finite simple groups. In this project you will explore their subsgroup structure as groups with a BN pair, as classical groups and as fixed points of Frobenius automorphisms. In particular, focussing on linear groups, you will learn about the parabolic subgroups from the perspective of both BN-pairs and from the classical group point of view.

Prerequisites: MSM2C Linear algebra, MSM2D Symmetry and groups, MSM203 Polynomials and rings MSM3P08 Group theory and Galois theory.

References: [1]Carter, R.; Simple Groups of Lie Type, Willey and sons, London, 1972. [2] Carter, R.; Finite Groups of Lie Type: Conjugacy Classes and Complex Characters, Willeys and Sons, London 1985.

Supervisor: **Prof CW Parker**

Title: **Advanced topics in Pure Mathematics**

Description: Each of the following topics is suitable for an MSci Project:

(1) Topics in group theory. This project will investigate some/any area of modern group theory. The exact contents of the project can be discussed with Professor Parker in his office.

(2) Presentations of groups and group algorithms. This project will investigate group presentations. In particular, the Todd Coxeter algorithm for determining the index of a subgroup of finite index will be investigated as will the Low Index Subgroup algorithm due to Sims.

(3) Buildings. This project will use the books by Brown, Ronan and Weiss to investigate the fundamental results about the theory of buildings.

For further details please see Prof Parker.

Supervisor: **Prof S Shpectorov**

Title: **Partial Cubes**

Description: The vertex and edge graph of the n-dimensional cube is known as the Hamming cube graph Q_n. Partial cubes are isometric subgraphs of Hamming cubes, that is, a graph G is a partial cube if $G \subseteq Q_n$ for some n and for any two vertices of G the distance between them is the same when measured in G and in Q_n.
The theory of partial cubes is a lively area of Combinatorics, where new research can start literally minutes after one learns the basic definitions. Partial cubes have many applications in, as well as outside, mathematics.

Prerequisites: Graph Theory (MSM3P15)

References: "Graphs and Cubes", by Sergei Ovchinnikov.

Supervisor: **Prof S Shpectorov**

Title: **Clifford Algebras and Spinors**

Description: This might be an interesting project in particular for those who is interested in application to Physics. The focus of the project will be on the definition of the Clifford algebras over the real numbers and their action on the corresponding space of spinors. On the way there, you can learn chapters from Linear Algebra that are rarely covered in the basic university courses, but which are very useful for applications. This includes the concepts of the dual space and tensors, classification of bilinear forms, etc. The famous quaternion algebra will appear as a particular case of Clifford algebra.

Prerequisites: Group Theory (MSM3P08a) useful, but not strictly necessary.

References: "Clifford Algebras: An Introduction", by D.J.H. Garling.

Supervisor: **Prof S Shpectorov**

Title: **Groups and geometries**

Description: The concept of diagram geometries was introduced by J. Tits as a source of natural geometries on which finite simple groups act. One interesting class of geometries is called the buildings, and among buildings one finds such famous examples as the n-dimensional projective space and the polar spaces of different types. The project includes the basics of Coxeter groups, the axioms and elementary properties of buildings, and examples of buildings.

Useful: Group Theory, Linear Algebra

Supervisor: **Prof S Shpectorov**

Title: **The Omnibus Project**

Description: I would be happy to consider a project with a motivated student on any topic of her/his liking in any area of pure mathematics including, but not restricted to Algebra, Combinatorics, and Geometry.

Prerequisites: Motivation and mathematical maturity.

References: Bring your own.

Supervisor: **Dr Steve Vickers (School of Computer Science)**

Title: **Categorical logic and cartesian theories**

Description: Category theory deals with mathematical objects not through their internal, set theoretic structure, but through the "universal properties" of their interaction with other objects. This interaction is described abstractly as *morphisms* between objects. These can be very general in nature, though the prototypes are functions and homomorphisms. The categorical description allows objects to be replaced by others that are internally different but still serve the same mathematical purpose, and this is very useful when it comes to examining mathematical foundations other than set theory.

Categorical logic plays this game with interpretations of logic.

The aim of this project is to explore this in one of the simplest cases, that of *cartesian* logic. This is enough to cover algebraic theories such as groups and rings, but also, more generally, "quasi-equational" theories, and they include the theory of categories themselves.

A first objective wold be to describe the general theory with reference to particular examples where you work out quasi-equational descriptions. A more advanced objective would be to describe the initial model theorem, which is used to prove the existence of free algebras, and to work out or describe some examples concretely.

You would start with some early chapters of Mac Lane's classic [1] on category theory and then move on to categorical logic. For this you would be relying on [2] (specific to cartesian logic) with reference to [3] (an easy introduction to more general categorical logic) and possibly [4] (more advanced). You would need to find some examples of quasi-equational theories to work out in detail, and illustrate the relation between the categorical view (objects and morphisms) and the logical (operators and equations). [2] also has an account of the initial model theorem, which could form the basis of deeper study if you have time.

Prerequisites: Computability & Logic would be beneficial

References:
[1] S. Mac Lane; *Categories for the Working Mathematician*, Springer-Verlag, 1971.

[2] Erik Palmgren and Steven Vickers - Partial Horn logic and cartesian categories, Annals of Pure and Applied Logic, vol. 145, pp. 314–353, 2007. DOI 10.1016/j.apal.2006.10.001.

[3] R. Goldblatt; *Topoi: The Categorial Analysis of Logic*, Studies in logic and the foundations of mathematics vol.98, North Holland 1979. ISBN 0 444 85207 7.

[4] P.T. Johnstone; *Sketches of an Elephant: A Topos Theory Compendium*, vol. 2, Oxford Logic Guides no.44, Oxford University Press 2002.

Statistics

Supervisor: **Dr B Chakraborty**

Title: **Constructing some robust versions of multivariate mean**

Description: It is well known that means are simple yet efficient estimates of location for most of the common statistical models. However, it does not provide a reasonable estimate if there are some large observations or outliers. Outliers occur in the data due to various reasons, such as some unusual observations, or due to incorrect data entry, etc. and they are very common in large real life data sets. In such cases, trimmed means or winsorized means provide some simple alternatives to sample means, which are not perturbed much even if a certain percentage of observations are corrupt or unusual. But the definition of trimmed mean or winsorized mean for the high dimensional data is not unique. In this project, we will explore some of the ways of extending these robust estimators for multivariate data and will study their properties through simulations. We will also implement our ideas in some real high dimensional data sets with and without outlying observations.
This project could be ideal preparation for a student wishing to write a Ph.D. thesis with this supervisor.

Prerequisites: MSM2C Linear algebra, MSM2O2 Statistics II. Some background in statistical inference and knowledge of R or MATLAB would be useful but not critical.

Supervisor: **Dr B Chakraborty**

Title: **On comparison of some robust regression methods**

Description: Usual least squares estimates of linear regression parameters are quite often affected severely by the presence of outlying observations. Some suggestions to overcome this problem or to estimate the regression parameters robustly are: least median of squares regression, least trimmed squares regression, M-estimators, etc. In this project, we will consider some of them and compare their relative performance in the presence of various types of outliers. We will mainly conduct some simulation studies and also implement the methods for some real data sets. For some of the estimators mentioned above, computing is also a serious issue as there are not many good algorithms available to obtain the correct estimates. We will briefly address that issue too.

Prerequisites: MSM2C Linear algebra, MSM2O2 Statistics II. Some background in statistical inference and knowledge of R or MATLAB would be useful but not critical.

References: [1] Rousseeuw, P. J. and Leroy, A.M. (1987) Robust Regression and Outlier Detection, Wiley.

Supervisor: **Dr Hui Li**

Title: **Using meta-analysis in economics**

Description: Meta-analysis have been used in a broad range of topics in economic studies to study the nature of the internal relationship of interested topics including economic growth, openness and financial policies. Often most studies take different approaches, i.e. some look at the quantity effect, whereas other ones look at the quality effect. Moreover, some studies use firm-specific data; other studies take a cross-country approach. These different approaches also have consequences for the extent to which impact of policies can be measured: whereas firm-specific (micro-data) studies may at least potentially provide evidence on impact (i.e. when the setting is experimental or quasi-experimental), this is generally difficult for cross-country studies. Thus, finding a systematic variation through empirical evidence, for example, by using meta-regression, is very helpful.
We would start using the papers and textbooks of Sutton et al. (2000) [1] and Lipsey and Wilson (2001) [2] to study and for references contained therein, and through parts of these books.
This project could be ideal preparation for a student wishing to pursue a career in statistics or write a Ph.D. thesis with this supervisor.

Prerequisites: MSM2S01b Statistics I MSM3S02 Statistics II at least one of the following MSM3S05 (applied statistics), MSM3S08 (medical statistics), MSM3S09 (statistical methods in economics)

References: [1] Sutton, A. J., K. R. Abrams, D. R. Jones, T. A. Sheldon and F. Song (2000), Methods for Meta-Analysis in Medical Research, New York: John Wiley and Sons. [2] Lipsey, M. W., and D. B. Wilson (2001), Practical Meta-Analysis, Applied Social Research Methods Series Volume 49, London: Sage Publications Inc.

Supervisor: **P N Patil**

Title: **Goodness-of-fit Test: Density function.**

Description: Let X be a continuous r.v. with probability density function $f(x)$. Then the standard procedure to test a hypothesis that $f(x)$ is equal to any specific probability model is the standard Pearson's χ-square test. Here, the width of the class intervals are fixed, as opposed to decreasing with increasing sample size. This project will investigate a test procedure where widths of class intervals decrease as sample size increases.

Prerequisites: MSM3S04,

Co-requisite: MSM4S07

References:

Supervisor: **P N Patil**

Title: **Modal Regression.**

Description: Suppose (X, Y) is bivariate random vector, where Y is the response variable and X is a covariate, and are related by the model $Y = m(X) + \epsilon$ where m is the unknown regression function. If $m(X) = E[Y|X]$, then one gets mean regression, if $m(X) = \text{Median}(X)$ then its referred to as median regression. In this project we make a case (with a real life example) for $m(X)$ to be the mode of conditional density of Y given X and discuss estimation of $m(X)$.

Prerequisites: MSM3S04, MSM3S05;

Co-requisite: MSM4S07

References:

Supervisor: **P N Patil**

Title: **Goodness-of-fit Test: Hazard rate.**

Description: Suppose $X_1, X_2 \cdots, X_n$ are failure times with hazard rate (failure rate) $\lambda(x)$. In this project nonparametric smoothing procedures will be used to test the hypothesis that λ has a specified functional form. (If you need more information, please see Dr P N Patil.)

Prerequisites: MSM3S04,

Co-requisite: MSM4S07

References:

BSc Projects

A majority of MSci projects detailed in this booklet would also be suitable, in a scaled down form, for BSc projects. Students should discuss this directly with potential supervisors of interest.

Supervisor: **Various**

Title: **Undergraduate Ambassador Scheme**

Description: The Undergraduate Ambassadors Scheme (UAS) is targetted at helping students to appreciate how mathematics (and other science subjects) is presented in the school classroom. It is endorsed by the DTI and DfES joint project 'Science and Engineering Ambassadors Scheme' and originally received funding from the author Simon Singh. The scheme provides an opportunity for third year undergraduates to gain valuable skills and insights into mathematics education and will be of particular interest to those who may be contemplating teaching as a career.

In the first term each student will study a topic in education and then, in the second term, will spend half a day each week in a local school. Each student will be matched with a specific teacher in a school, who will act as trainer and mentor, and determine, in conjunction with the project supervisor, the tasks and responsibilities of the student. UAS have prepared a handbook and other materials to support the student.

Those interested should contact the Dr Chris Sangwin in the first instance.

Regulations for MSci projects

1. Introduction

Every final year student on the MSci programme must undertake a project worth 40 credits.
The project module gives the students an opportunity to study in depth, under supervision, some area of mathematics or statistics that particularly interests them. It is intended to give students an idea of mathematical research and teaches key mathematical skills, including writing a properly referenced project dissertation, oral presentation of advanced material and mathematical typesetting.

2. Conduct of the Project

2.1 Members of staff (as potential supervisors) will submit proposals for possible projects to be distributed to students on the MSci programme (including those studying abroad) in time for projects to be assigned before the end of the session preceding the student's final year of study.

2.2 Each project will be overseen by a supervisor and a co-assessor. Joint supervisors and/or co-assessors may be deemed appropriate.

2.3 All MSci projects will run over both terms of the session. With the agreement of the supervisor and the director, students may choose to do either 20 credits worth of work in each term or 10 credits in the Autumn term and 30 in the Spring term.

2.4 Supervisors may wish to suggest some preparatory reading or investigation over the summer vacation before the official start of the project.

2.5 Co-assessors will be appointed no later than the start of the new session and a project plan will be agreed between the supervisor, co-assessor and student at a meeting with the student held during the first 2 weeks of the Autumn term.

2.6 The co-assessor should be present for at least one additional meeting in each term. The timing of such meetings should enable the co-assessor to contribute to the progress review process in the School.

2.7 The student shall have regular meetings with the supervisor. In the Autumn term, weekly meetings are recommended.

2.8 Save in exceptional circumstances and only with the permission of the Head of School, no change of supervisor or project allocation will be permitted after the end of the second week of the final year.

2.9 Introductory training in LaTeX will be provided early in the Autumn term for those students who have not been exposed to LaTeX in their third year.

2.10 Students will produce a substantial dissertation based on the mathematical or statistical work they have carried out over the two terms.

2.11 Students will give an oral presentation based on the project work. It will take place as soon as possible after the end of the Summer Examination Period.

2.12 Members of staff will not normally be available for consultation relating to their project after the end of Spring Term. Students who do wish to consult their supervisor or other member of staff about their project after the end of Spring Term must obtain the permission of the Director of the MSci Programme.

3. Assessment

3.1 The following elements will contribute to the assessment of the project: a dissertation, two interim reports and an oral presentation.

3.2 The dissertation should include any significant work undertaken in the course of the project, though preliminary exercises may be omitted or relegated to an appendix. The length of the dissertation should not exceed 100 pages, excluding references and appendices. The font size must be 12pt.

3.3 The dissertation will be assessed by the supervisor and co-assessor.

3.4 The two interim reports will be no more than two pages in length excluding bibliography. The first will be produced by the end of the sixth week of the Autumn term, the second by the end of the second week of the Spring term. Each will provide a brief summary of the work done so far and an outline plan of future work, possibly including library search, other background research or bibliography.

3.5 The interim reports will be assessed by the Director of the MSci Programme.

3.6 The dissertation will be written for specialists in the field. They will be prepared using LaTeX (or other version of TeX) unless there are compelling reasons to the contrary and the permission of the Head of School has been obtained.

3.7 The first page of each copy of the dissertation should contain a signed and dated copy of the following declaration: 'I warrant that the content of this dissertation is the direct result of my own work and that any use made in it of published or unpublished material is fully and correctly referenced.' This page should be submitted to the Undergraduate Office by 12 noon on the Wednesday of the last week of Spring term.

3.8 An e-mail, to which a PDF file containing the dissertation is attached, should be sent to the Undergraduate Office, with copies to the Director of Year 4, the project supervisor and the project co-supervisor, no later than 12 noon on the Thursday of the first week of the Easter vacation following the commencement of the project.

3.9 Penalties will be applied for late submission. Projects submitted after the deadline will lose 1 mark. Projects submitted after 4.15 p.m. on the day of the deadline will lose an additional 4 marks. Projects submitted after noon the following Monday will lose a further additional 1 mark. For each subsequent day thereafter, an additional 1 mark will be deducted for a project not submitted before noon. No project will be accepted later than noon on the Friday two weeks after the deadline.

3.10 The oral presentation should last for about 20 minutes, with a further 10 minutes for questions and discussion about the work contained in the project. This presentation will be assessed by a small committee appointed by the Head of School and chaired by the Director of the MSci Programme. It will contain representatives from the research groups in the School.

3.11 The oral presentation should be made accessible to a general mathematical audience and should be assessed as presentations of technical material. Supervisors encouraged to offer advice in their preparation.

3.12 The School will provide 10 free OHP slides for the final presentation.

3.13 In exceptional circumstances and with the permission of the Head of School the oral presentation may be waived. In such cases, detailed assessment arrangements will be agreed by the supervisor, the Director of the MSci Programme and the Head of School. Some additional assessed element, such as a critique of an important paper in an area related to the project, may be required.

3.14 Attendance at all oral presentations is required. Penalties for non-attendance will be assessed by the Director of the MSci Programme.

4. Marking Procedure

4.1 The project will be marked according to the following categories:

- technical content of the dissertation (42 marks);
- development and execution of the dissertation (20 marks);
- presentation of the dissertation (13 marks);
- conclusions of the dissertation (9 marks);
- two interim reports (3 marks each);
- oral presentation (10 marks)

4.2 The supervisor and co-assessor should produce marks for the dissertation independently of one another according to an agreed marking scheme. After discussion as necessary, a final mark will then be agreed. If no agreement is reached between the supervisor and co-assessor, the Head of School will institute an appropriate procedure to arrive at the final mark.

4.3 The marking scheme for the dissertation should reflect the nature of the project. Suggestions of possible categories for assessment are listed below. A mark of zero will be returned for presentation if the dissertation has not been typeset using (La)TeX, unless specific permission has been given by the Head of School.

- **Technical content of dissertation**: mastery of the subject matter; historical perspective; relation to undergraduate courses; conceptual and methodological difficulty; mathematical accuracy and clarity of exposition; innovative aspects; examples cited (relevance of); originality, independently derived results and proofs, new results, proofs, simple generalizations, significant insight, original research, examples constructed.

- **Development and execution of project**: plan outline, aims and objectives; sensible development of aims; appropriate approach and methodology; evidence of work done, computer programs, experiments, field work, etc.; library research, use of references; new source materials; level of independence and of assistance; analysis and appreciation of results, comparison with experimental data or known examples.

- **Presentation of dissertation**: aims, structure and conclusions clearly laid out; suitable length; quality of exposition and organisation; diagrams; bibliography complete, appropriate, properly referenced, other source material properly referenced.

- **Conclusions of dissertation**: Conclusion and suggestions for further work; stated aims met or reasons why not; placement of work in wider context.

4.4 The supervisor and co-assessor will jointly provide a written report on the dissertation, which includes the mark scheme, and information on how their marks were arrived at. This report shall be made available to the external examiner together with a copy of the project dissertation at least two weeks before the School Board of Examiners meets to discuss examination results.

4.5 The interim reports will be marked taking into account evidence of reasonable progress and accuracy of summary of work to date, viability of proposed plan of study and other appropriate criteria. A mark of zero will be returned if a report has not been typeset using (La)TeX unless specific permission has been given by the Head of School.

4.6 Marks for the oral presentations will be awarded according to the following criteria: content and structure of talk; interest of talk; enthusiasm; use of audio-visual aids; mastery of topic; comprehensibility and quality of explanation; handling of questions.

4.7 Approximately two weeks prior to the oral presentations, there will be a meeting of project supervisors and co-assessors to discuss and moderate the marks of the written work. This meeting will be chaired by the Director of the MSci Programme.

5. Plagiarism

5.1 The School abides by the University's guidelines on plagiarism. The University regards plagiarism as a very serious form of cheating and will impose the severest penalties in all cases of cheating.

5.2 Supervision and scrutiny of project work will be sufficiently closely arranged to ensure that:

- signs of plagiarism (whether intentional or unintentional) in early drafts or pieces of work may be detected in good time and drawn to the attention of the student and then followed up by a written warning if necessary;

- students are not allowed to get so far behind with their work that they may be tempted to turn to plagiarism in an effort to catch up.

5.3 Students must make appropriate use of references and footnotes when using material from published or unpublished sources to avoid any suspicion of plagiarism. If in any doubt the student should discuss the matter with their project supervisor.

5.4 Students must include in their dissertation a signed declaration that unreferenced material is their own work as described in 3.7.

6. Health and Safety

6.1 The School abides by the University Health and Safety policy. Where a project is anything other than low risk, it is the responsibility of the project supervisor and ultimately the Head of School to ensure that appropriate Health and Safety procedures are in place. If a supervisor deems that a project is not low risk, or is in any doubt, the matter should be referred to the Director of the MSci Programme. Desk based work, standard use of proprietary electrical equipment, use of Class 1 lasers and on-campus data collection are examples of low risk activity: off-campus data collection, experiments with electrical equipment, use of robots, testing materials to breaking point are examples of possible higher risk activities.

Regulations for BSc Projects

1. Introduction

The optional 20 credit BSc project module is designed for Single Honours students on a BSc programme and gives students the opportunity to study in depth a topic in Mathematics or Statistics that particularly interests them. This may include reading mathematical or statistical works, using advanced methods to solve a given problem, explore new topics, etc.

Each project comprises a substantial task carried out under supervision. In addition, the student must write a project report, and deliver a short oral presentation of his/her work.

2. Conduct of the Project

A supervisor and co-assessor will be appointed to oversee the conduct of each project, and a project plan will be agreed between them. Joint supervisors and/or co-assessors may, on occasion, be deemed appropriate.

A 20-credit BSc project will normally occupy both semesters of a session.

Where possible, a project should be allocated before the end of the session preceding the student's final year. Save in exceptional circumstances, no change of project allocation will be permitted after the end of the second week of the final year.

The student shall have regular meetings with the supervisor. In the first semester, weekly meetings are recommended. The co-assessor should be present for at least one meeting in each semester in which the project runs. The timing of such meetings should enable the co-assessor to contribute to the progress review process in the School.

3. Assessment

Each BSc project shall be marked out of 100. Credit is given for a written report (85 marks) and an oral presentation (15 marks). The requirement for an oral presentation may be waived only in exceptional circumstances and with the permission of the Head of School.

The written report should include any significant work undertaken in the course of the project, though preliminary exercises may be omitted or relegated to an appendix. The length of the report should not exceed 50 pages, excluding references and appendices. Where possible, the report should be prepared using a word processor.

The oral presentation should last for about 15 minutes, with a further 5 minutes for questions and discussion. It will take place in the Summer term normally immediately after the examination period.

The oral presentation should be made accessible to a general mathematical audience. The supervisor is encouraged to offer advice in its preparation.

The first page of each copy of the dissertation should contain a signed and dated copy of the following declaration: 'I warrant that the content of this dissertation is the direct result of my own work and that any use made in it of published or unpublished material is fully and correctly referenced.'

Two bound copies of the written report should be submitted to the Undergraduate Office not later than 12 noon on the Thursday of the first week of the Easter vacation following the commencement of the project. Penalties will be applied for late submission. Projects submitted after the deadline will lose 1 mark. Projects submitted after 4.15 p.m. on the day of the deadline will lose an additional 4 marks. Projects submitted after noon the following Monday will lose a further additional 1 mark. For each subsequent day thereafter, an additional 1 mark will be deducted for a project not submitted before noon. No project will be accepted later than noon on the Friday two weeks after the deadline.

The School will provide 8 free OHP slides for the oral presentation.

4. Marking Procedure

The supervisor and co-assessor should produce marks independently of one another, both for the written report and for the oral presentation, according to an agreed marking scheme. After discussion as necessary, a final mark for the project will then be agreed. If no agreement is reached between the supervisor and co-assessor, the Head of School will institute an appropriate procedure to arrive at the final mark.

The supervisor and co-assessor will jointly provide a written report on the project, which includes the mark scheme, and information on how their marks were arrived at. This report shall be made available to the external examiner together with a copy of the project report.

The written report will be marked according to the following categories: technical content; development and execution; conclusions; presentation. These categories shall be worth 35, 25, 15 and 10 marks respectively. Within each category, marks shall be awarded according to a mark scheme, agreed by the supervisor and co-assessor, that reflects the nature of the project. Suggestions for possible sub-categories for assessment are listed below, though this list is not restrictive.

In the event that the oral presentation be waived, marks will be assigned for the written report alone with the following weightings: mathematical content, 40 marks; development and execution, 30 marks; conclusions, 15 marks; presentation, 15 marks.

Assessment categories:

Oral presentation: content and structure of talk; interest of talk; enthusiasm; use of audio-visual aids; mastery of topic; comprehensibility and quality of explanation; handling of questions.

Technical content: mastery of the subject matter; historical perspective; relation to undergraduate courses; conceptual and methodological difficulty; mathematical accuracy and clarity of exposition; innovative aspects; examples cited (relevance of); originality, independently derived results and proofs, new results, proofs, simple generalizations, significant insight, original research, examples constructed.

Development and execution of project: plan outline, aims and objectives; sensible development of aims; appropriate approach and methodology; evidence of work done, computer programs, experiments, field work, etc.; library research, use of references; new source materials; level of independence and of assistance; analysis and appreciation of results, comparison with experimental data or known examples.

Conclusions: conclusion and suggestions for further work; stated aims met or reasons why not; placement of work in wider context.

Presentation: aims, structure and conclusions clearly laid out; suitable length; well written and organized; word-processed; diagrams; bibliography complete, appropriate, properly referenced, other source material properly referenced.

5. Plagiarism

The School abides by the University's guidelines on plagiarism. The University regards plagiarism as a very serious form of cheating and will impose the severest penalties in all cases of cheating.

Supervision and scrutiny of project work will be sufficiently closely arranged to ensure that:

i) signs of plagiarism (whether intentional or unintentional) in early pieces of work may be detected in good time and drawn to the attention of the student and then followed up by a written warning

ii) students are not allowed to get so far behind with their work that they may be tempted to turn to plagiarism in an effort to catch up.

Students must make appropriate use of references and footnotes when using material from sources to avoid any suspicion of plagiarism. If in any doubt the student should discuss the matter with the supervisor.

6. Health and Safety

The School abides by the University Health and Safety policy. Where a project is anything other than low risk, it is the responsibility of the project supervisor and ultimately the Head of School to ensure that appropriate Health and Safety procedures are in place. If a supervisor deems that a project is not low risk, or is in any doubt, the matter should be referred to the appropriate Director of Studies. Desk based work, standard use of proprietary electrical equipment, use of Class 1 lasers and on-campus data collection are examples of low risk activity: off-campus data collection, experiments with electrical equipment, use of robots, testing materials to breaking point are examples of possible higher risk activities.