An introduction to pure mathematics

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When people hear that I am a pure mathematician, the first word out of their mouths is usually “wow”, often followed by “err”, and then some pronouncement along the lines that they could never do that, or they were terrible at mathematics at school. In the more rarefied atmosphere of the College the reaction is typically more subtle, but there is still usually a lack of understanding about what modern research in mathematics is about.

A large part of it is due to the language and nomenclature that we mathematicians appear to revel in, confusing those not in its inner circle. We have tropical geometry that is far from the rainforest, forms that cannot be filled in, and circuits through which no current passes. It’s a fascinating field, but can at times be rather impenetrable.

My area of research is algebra, a subject that bears little resemblance to its namesake you might recall from school life. At university level, algebra is the study of symmetries of mathematical objects. The objects can be real-life objects, like molecules or shapes, mathematical structures like systems of equations, surfaces in 15-dimensional space, knots in pieces of string, and many other things besides.

One takes the object and looks at its symmetries, or some subset of its symmetries, for example like rotations of a cube. We can take two of these symmetries and do one then the other, like rotating front-to-back by a quarter-turn then rotating left by a quarter-turn. This produces a new rotation of some sort (question: which one?), and so we have some way of 'multiplying' symmetries, forming what is known as a group.

Although groups have myriad applications throughout mathematics and the sciences by virtue of the fact that symmetry is ubiquitous, group theory (for years if you typed “group theory” into Google it asked whether you meant “group therapy”) as a subject in its own right has grown to encompass a large part of the field of algebra.

Representation theory

Another major part of algebra is representation theory. In its most basic sense, representation theory involves taking an object and representing it in a different way, hoping that along with the new representation of it comes new insight into its structure. For groups, one would normally represent a group as matrices, so representation theory studies how a group can be written as matrices. For example, the group of symmetries of a cube has a natural representation in 3-dimensional space as rotation matrices. It has many many other ones, but some are more interesting than others. The most interesting in many cases are so-called irreducible representations. Every representation is built up out of irreducible representations, like numbers are built up out of prime numbers, and understanding the irreducible representations of finite groups is a major problem in group representation theory.

These representations are not just of mathematical importance: the dimensions of the irreducible representations of a group called SU(2) are what control how many electrons lie in each shell of the atom, and so in a sense representation theory controls how molecules behave. Recently, the representation theory of certain groups, called ‘groups of Lie type’, has been shown to vastly improve the efficiency of radar, by determining which waveform should be sent out by the control tower to most easily – hence quickly – analyze the response. More frivolously, you can use group theory, and sometimes representation theory, to solve many puzzles and games, such as Rubik’s cube.

My particular specialty in group representation theory is the local-global principle. This states that you can get information about a large structure by studying small parts of it. For example, you can determine the circumference of the earth – a global property if ever there was one – from its curvature, which can be determined in a small circle on the earth's surface, say by looking at the angle of the sun at different points on the surface. The local-global principle in representations of groups is much less clear-cut than that, and almost entirely conjectural, but due to the research of several groups of mathematicians both in the UK and abroad, its secrets are slowly becoming exposed.

Being a Fellow

I returned to Birmingham two years ago, having done my undergraduate degree here in 2000. I became a Birmingham Fellow in November of 2011, transferring from Oxford, where I completed my PhD and postdoctoral work. I was appointed on the back of an ambitious five-year research project to solve one of the deepest conjectures in the local-global approach to representation theory: Broué’s conjecture, which has been open since 1988.

So how does the Birmingham Fellowship help me in this research? It gives me the flexibility to be able to collaborate worldwide, which I have certainly taken advantage of. Without the support of this fellowship, I would not have been able to complete the research I have done over the last year. Since taking up the position I have been awarded a Royal Society University Research Fellowship, so I will be a Fellow of one form or another for the next few years!