Loughborough University

How Round is your Circle?

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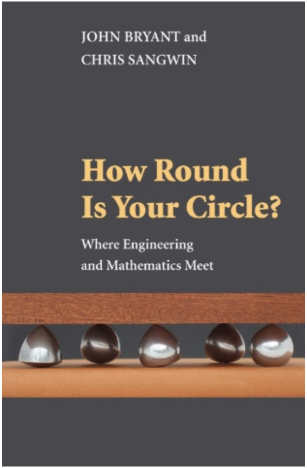
How round is your circle?

Chris Sangwin and John Bryant

Princeton University Press, 2008



The book

1. Hard Lines
2. How to Draw a Straight Line
3. Four-Bar Variations
4. Building the World's First Ruler
5. Dividing the Circle
6. Falling Apart
7. Follow My Leader
8. In Pursuit of Coat-Hangers
9. All Approximations Are Rational
10. How Round Is Your Circle?
11. Plenty of Slide Rule
12. All a Matter of Balance
13. Finding Some Equilibrium

Introduction

1. An unexpected puzzle!
2. Some interesting geometry.
3. Applications of this geometry.
4. Implications to engineering.
5. A new shape!

A puzzle.....

How do you know if something is "round"?

How would you judge a freehand circle drawing competition?

Round = circle (2D) and sphere (3D).

Width

Width is the distance between parallel tangents.

If we have a circle then the *width* is constant.

If we have *constant width* then do we have a circle?

Shapes of constant width

Families of shapes: circle $\leftrightarrows$ Reuleaux rotor.

- Symmetries not necessary
- Circular arcs not necessary
- Circumference? (Barbier's Theorem)

3 dimensions

See also *Meissner's Tetrahedron*.

Franz Reuleaux (1829–1905)

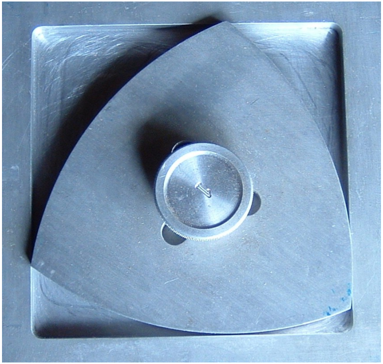
- Kinematics of Machinery, (1876)
- The Constructor, (1904)

Applications: Coins

UK coins all have constant width.

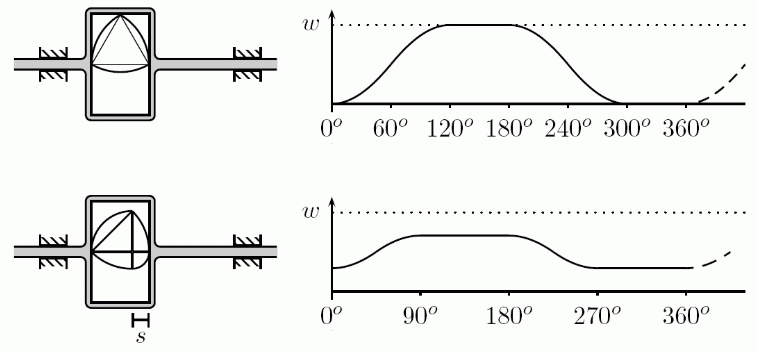
"Rotor of a square"

Shapes of constant width rotate in a square touching all four sides.



Cams

Cams are important in machinery.



(§§ 72 and 76), its formula runs $(C_3'' P^{\perp})^d - b$. In Hornblower's train the curve-triangle $\tilde{C}$, (which we have already examined in

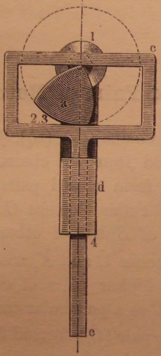


FIG. 407.

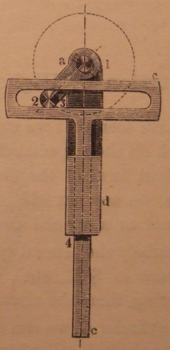
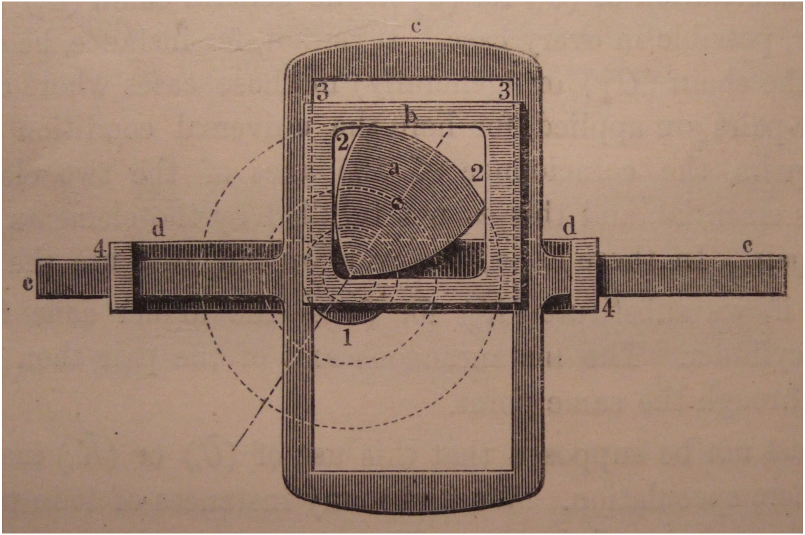


FIG. 408.


c
3
3
b
2
a
2
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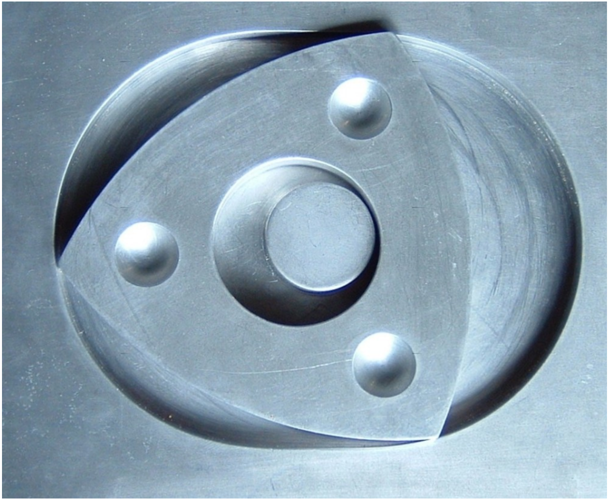
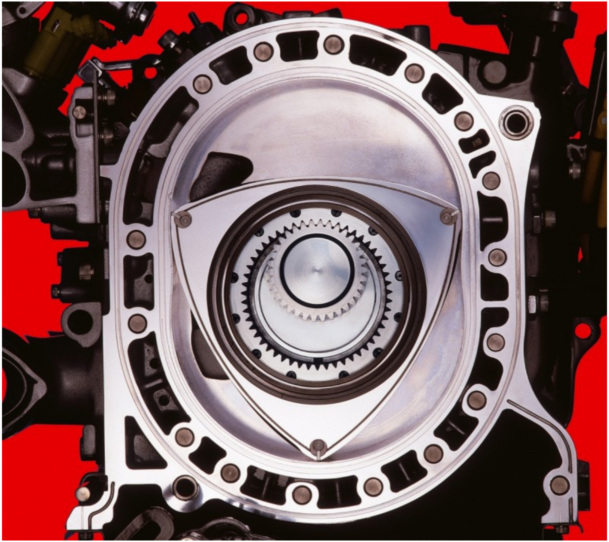
4
c
c
4
1

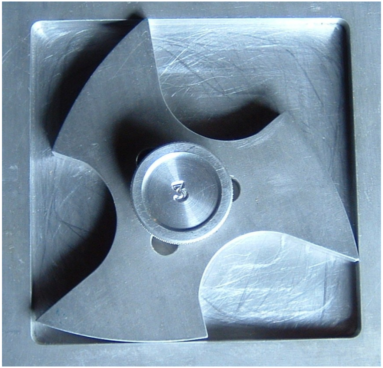
Rotary engine



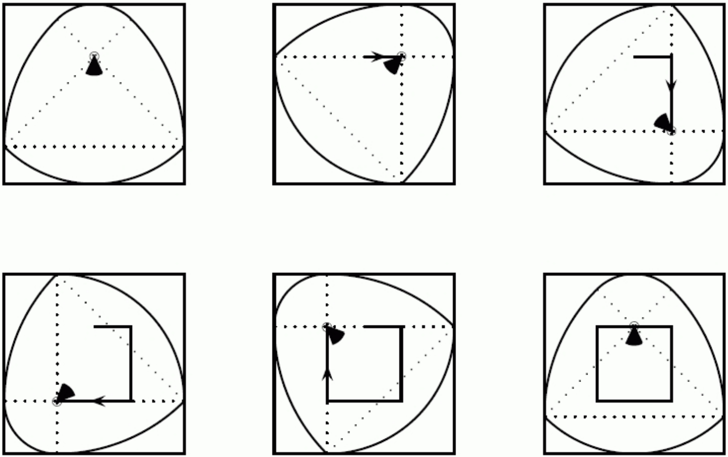


Drilling a square hole

Harry James Watts, 1914



... not a perfect square.

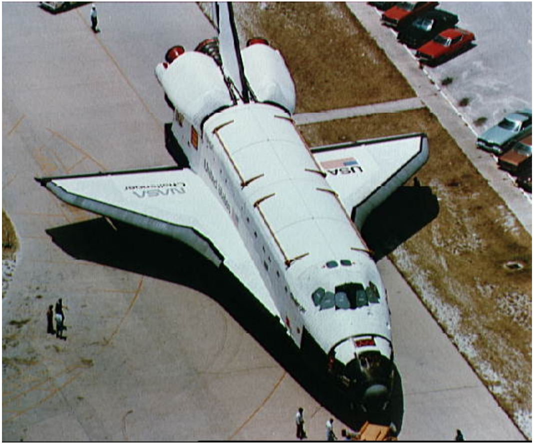


Fun geometry .... but so what?!

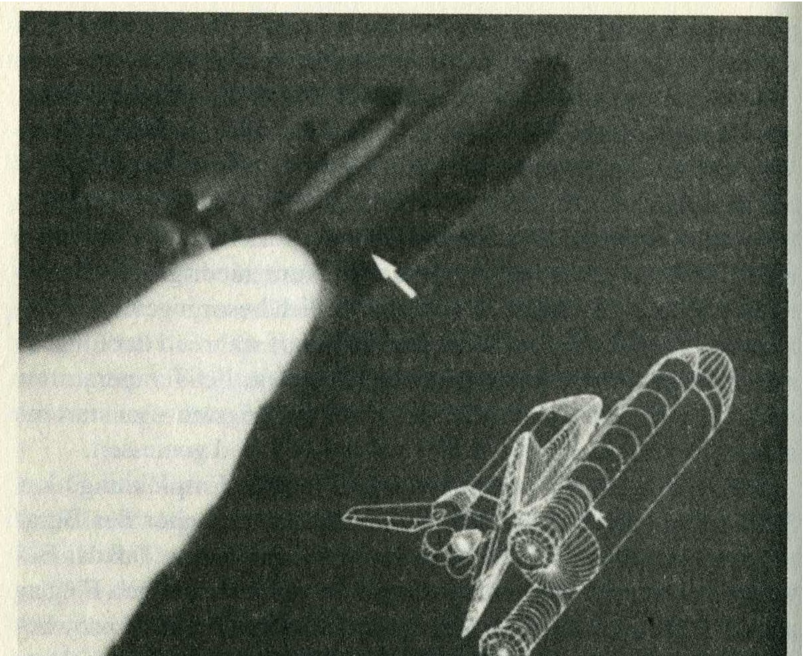
In engineering many applications rely on roundness.

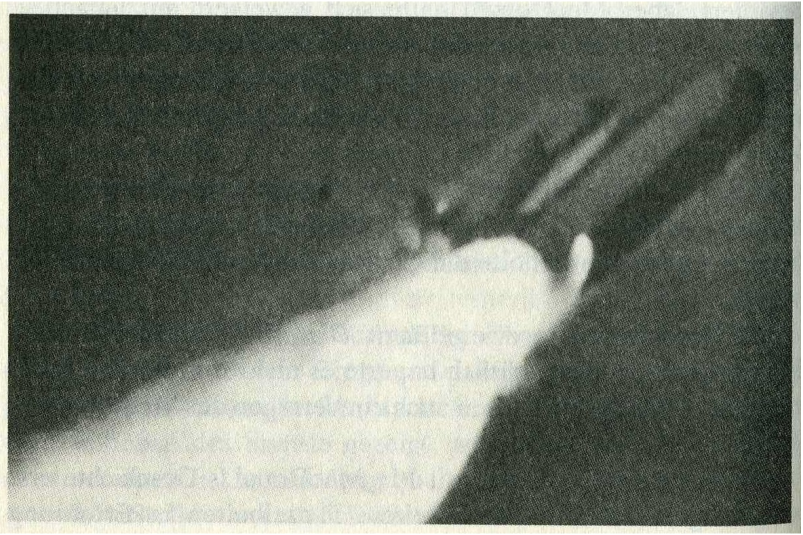
No manufacturing process achieves perfection.

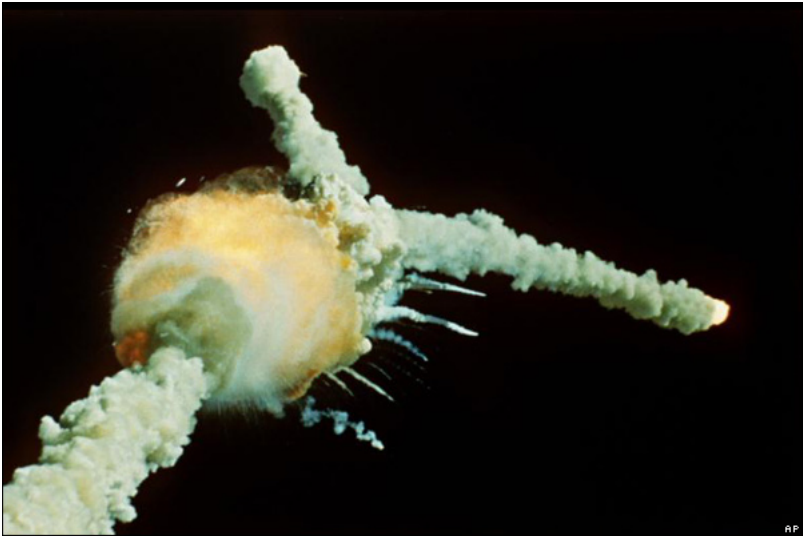
28 January 1986, Shuttle Challenger



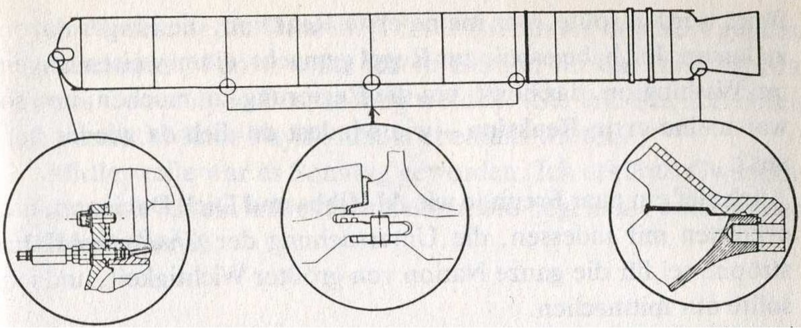
Broke apart 73 seconds into its flight





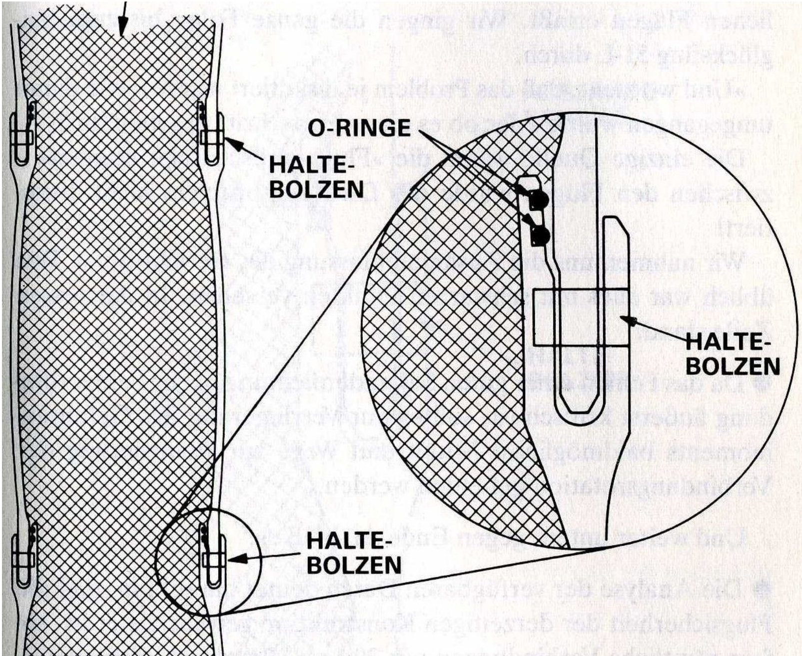

AP

What went wrong?



The “out of roundness” was a significant factor!

See Feynman, R. P., *What to* you *care what other people think?*, (1988)


O-RINGE
HALTE-
BOLZEN
HALTE-
BOLZEN
HALTE-
BOLZEN

Measures of roundness

Engineers measure *departure from roundness*.

What can we do?

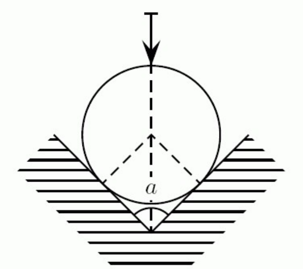
- Theoretical measures.
- Practical tests.

Smallest outside circle....

.... or largest inside circle ...!

Formal: *Maximum deviation from the minimum circumscribed circle.*

A simpler method

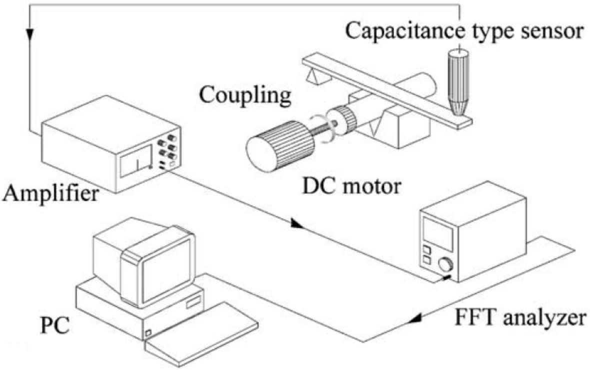


If a circle is rotated then the position stays constant.

Can we conclude the converse?

If, when rotated, the position stays constant then is the shape a circle?

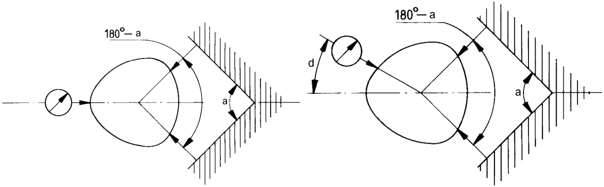
Goho (1999)



Goho, K. and Kimiyuki, M. and Hayashi, A. (1999) Development of a Roundness Profile Measurement System for Parallel Rollers Based on a V-Block Method *International Journal of the Japanese Society of Mechanical Engineers* C42 (2) 410–415.

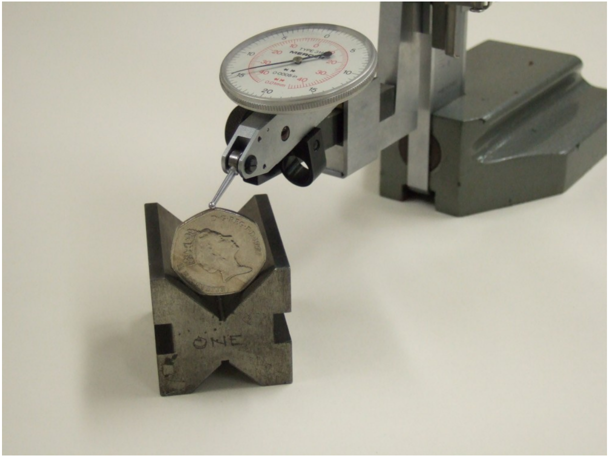
British Standard 3730 - part 3

"*Methods for determining departures from roundness using two- and three-point measurement*"



...take one two-point measurement and two three-point measurements at different angles between fixed anvils.

Our experiments



A 50p piece (7 undulations) in a 90^o vee-block.

Rotors of a general triangle

A shape of constant width is a rotor of the square.

Can we construct a *rotor of a triangle?*

How

$y = mx + c$ can be written as

$$y\cos(t) - x\sin(t) = p(t). \tag{1}$$

1. t is angle to the horizontal,
2. $p(t)$ is the perpendicular distance to origin.

Given a function for $p(t)$ we generate a family of lines.

$$p(t) = \alpha + \beta \cos(nt), \tag{2}$$

where n is an odd integer > 1.

eg, if $\beta = 0$ then a circle.

The distance between parallel lines is $p(t) + p(t + \pi)$.

$$p(t) + p(t + \pi) = 2\alpha + \beta \cos(nt) + \beta \cos(nt + n\pi) = 2\alpha$$

So (2) automatically generates a shape of constant width.

Further details

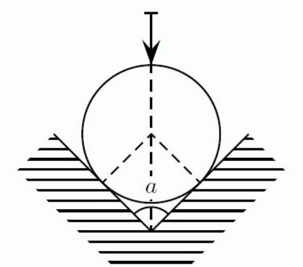
- Convexity,
- Choose right parameters

Angle as fraction of a rotation

- Degrees
- Radians
- Grads
- “Rotations”

Is BS 3730 “wrong”

No.... we specified three *tangent lines*



BS3730 specifies two tangent lines and a point on the angle bisector.

These are quite different.

[I’m not convinced everyone appreciates this!]

Science projects

> The main use of a model is the pleasure derived from making it.
> Cundy and Rollett, *Mathematical Models* (1951)

3D printing....

Conclusion

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