An Innocent Question?

The well-known right triangle with sides 3, 4 and 5, has **integer** area 6 and perimeter 12.

Question: Is there another triangle with rational side lengths that has the area 6 and perimeter 12?

How many of the following criteria for a "good problem" does this question fulfil ?

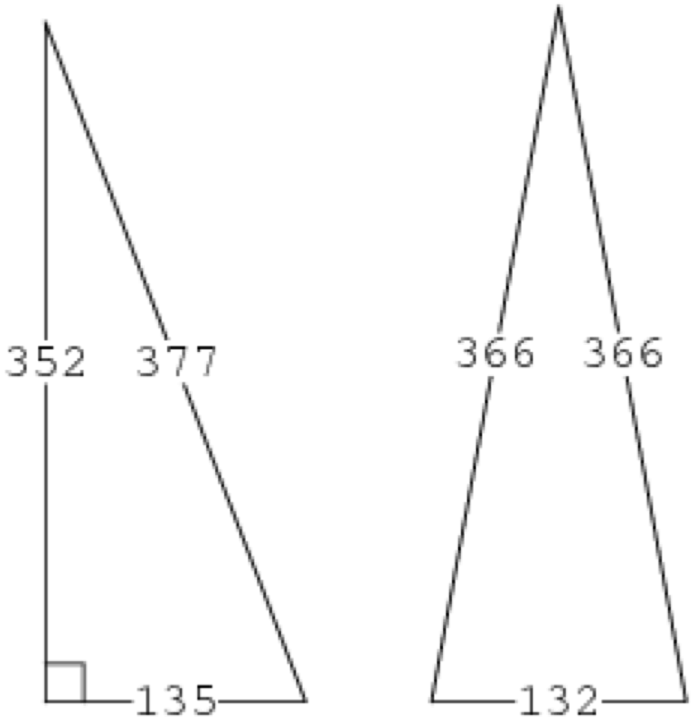
- ❑ **Novelty:** This is the one essential ingredient in the characterization of a problem. The question must be genuinely new to the learner and not just a variation on themes and arguments that are already familiar.
- ❑ **Challenge:** The solution is not obvious or routine. The learner might be expected to respond with comments like "I don't even know where to begin" or "I'm stuck!"
- ❑ **Consequence:** The result leads somewhere interesting, suggests some new questions, or sheds new light on its surroundings. Isolated puzzles with clever solutions but no wider significance won't usually count as problems.
- ❑ **Connection:** The problem's statement links disparate topics, or its solution brings to bear ideas or techniques from different areas, making connections that are not obvious or initially apparent, thereby witnessing the unity of mathematics.
- ❑ **Extension:** The questions and/or methods of solution augment or extend the learner's established knowledge.
- ❑ **Elegance and Economy:** Problems often admit many different solutions. A solution with an original approach or an incisive idea that quickly gets to the nub of the problem is preferable to one that follows an obvious but laborious routine or uses the type of case-by-case analysis known as 'proof by exhaustion'.
- ❑ **A Eureka Moment:** A flash of inspiration, a sudden insight, a surprising twist -- these moments of heightened perception give real intellectual pleasure and can yield a solution that cuts to the chase and gets to the heart of the problem.
- ❑ **General Principle:** There are well-known and often powerful methods of argument that turn up time and again in mathematical discourse. A problem that succumbs to one of these adds a valuable weapon to a learner's armoury.
- ❑ **Illumination:** The volume of mathematics covered in a lecture course (or module) is limited by its length and by convention. The lecturer rarely has time to include all the material needed to convey fully the significance and ramifications of the theory. Problems that provide additional consequences, side results, examples and counter-examples can enrich the material and improve the learner's understanding and appreciation.
- ❑ **Experimentation and Conjecture:** A problem might require the solver to experiment (by hand, with a calculator, or even with a computer program) and make conjectures based on the empirical evidence they obtain. The experience of doing this gives the learner an insight into mathematical creativity.
- ❑ **Application:** We have already mentioned the desirable property that a mathematical problem should "lead somewhere" and perhaps apply in other situations.
- ❑ **Affective Response:** A problem that evokes a pleasurable reaction, like curiosity, excitement, aesthetic satisfaction, or sheer joy, has a lot going for it. When I asked one of my sons studying mathematics what characterizes a good problem, he replied: "One that makes you smile when you come upon the solution."

References:

1. Klein Project vignettes:
http://blog.kleinproject.org/?page_id=363 and http://blog.kleinproject.org/?p=4
2. William McCallum: *Restoring and Balancing,*
http://math.arizona.edu/~wmc/Research/2010_Restoring_Balancing.doc
3. Steven Rosenberg, Michael Spillane, and Daniel B. Wulf: *Delving deeper: Heron triangles and moduli spaces*, Mathematics Teacher **101** (2008), no. 9, 656.
4. Ronald van Luijk: *An elliptic K3 surface associated to Heron triangles*, arXiv **math.AG** (2004), 35 pages

Two uniquely-interesting Heron Triangles

This special pair of triangles (*special* here means 'one right-angled and one isosceles') was discovered by Denis Boris in 2003:



They both have the same area 23760 and the same perimeter 864.

A proof***** that no other such special pairs of Heron triangles exist can be found here:

http://domino.research.ibm.com/Comm/wwwr_ponder.nsf/solutions/February2004.html/$FILE/Feb2004_dima.pdf

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*****due to Dan Dima of IBM Research