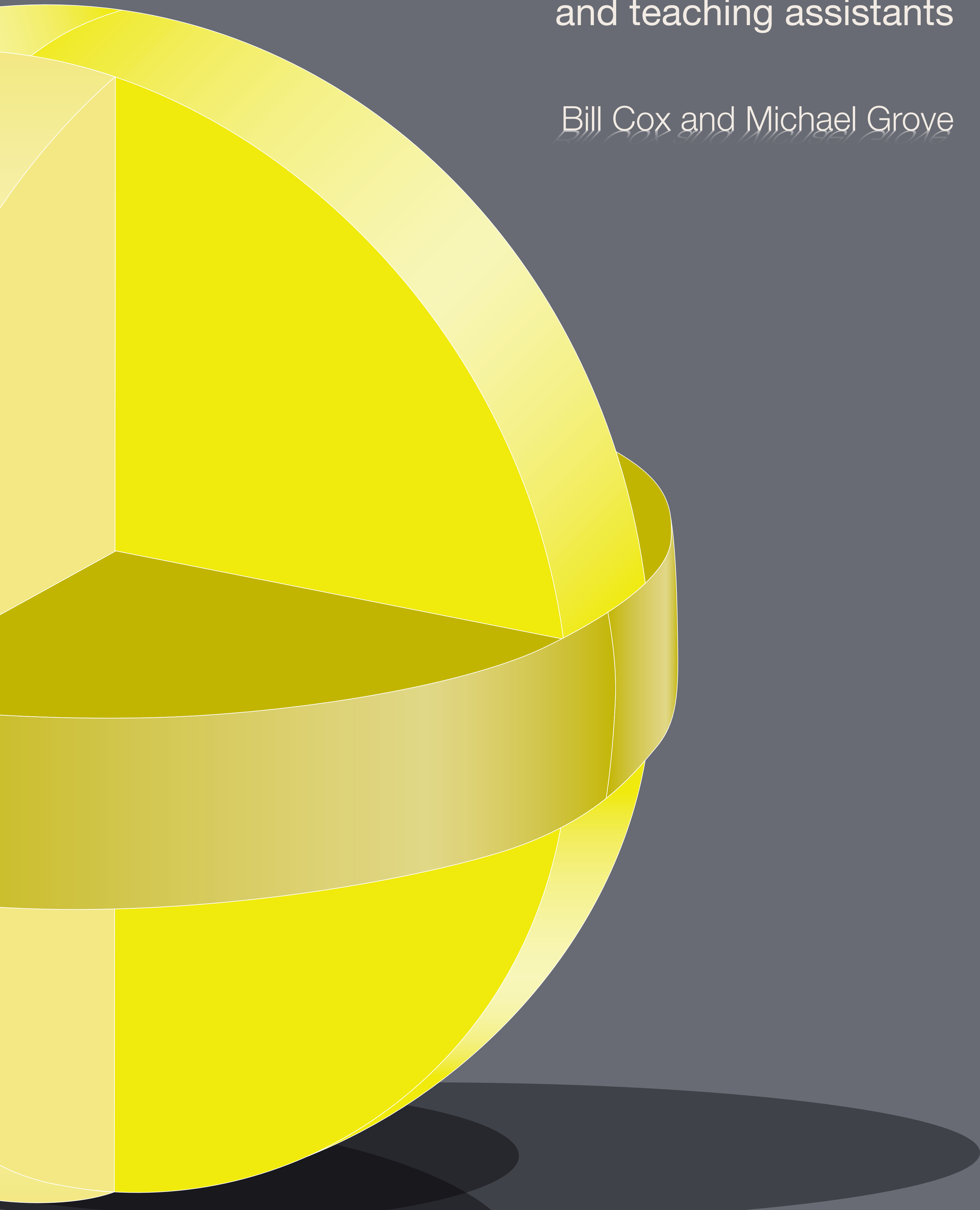
Teaching Mathematics
– a guide for postgraduates and teaching assistants

Bill Cox and Michael Grove



Teaching Mathematics

– a guide for postgraduates and teaching assistants

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Preface

This book is aimed at postgraduate students who assist in the teaching of mathematics-rich subjects - in which we include statistics and more broadly Science, Technology, Engineering and Mathematics (STEM) subjects generally. It is also suitable for others that assist to a more or less limited extent in teaching, such as research fellows, and, as they are known in the United States of America for example, graduate teaching assistants. Such assistants (which we will now refer to as postgraduates throughout the remainder of this book) have an important role in supporting undergraduate students in learning mathematics, but until recently, have rarely received training to support them in this role – certainly this is true within the United Kingdom where both authors are based. Following an initiative by one of us (MG), a series of one-day workshops were initiated across the UK, entitled *Supporting Postgraduates who Teach Mathematics*. These have been run by the Maths, Stats & OR Network of the Higher Education Academy. At these events experienced lecturers offered advice, guidance and support to postgraduates who have teaching duties. This book has evolved from our experience in running these events, along with colleagues, since 2005. Our hope is that it forms a useful guide for any postgraduate heading out for that next tutorial, or facing the next pile of marking.

This book focuses mainly upon the discipline-based aspects of the duties postgraduates might reasonably be expected to undertake. It is perhaps complemented by more generic material such as Morss and Murray (2005). There will be inevitable overlap with such material, but this will only serve to emphasize the importance of the messages we are trying to convey.

There are several key messages or themes that underpin the entire content of this book that are worth highlighting here. We cannot tell you how you should teach, there are too many variables that make each teaching session unique, but we can share our experience and suggest ideas that you might like to try. It is up to you to implement and evaluate these, if they work as intended that is ideal, if they don't, don't necessarily give up but adapt them according to needs of you and your students. Similarly, you will have your own experiences of being taught as an undergraduate student, both good and bad, build upon them; implement the ideas and approaches that you found helpful, avoid those that weren't. Talk to other colleagues, both fellow postgraduates and more experienced members of teaching staff; teaching is an activity where you can learn much from others. Finally, try to engage the students directly in each teaching activity you undertake; this is perhaps our core message. Effective teaching involves a dialogue with the students: listen to what they have to say, question them, and only then explain. Their feedback is critical to not only helping you develop as a teacher, but also for ensuring that the teaching sessions are productive and meet their needs and expectations.

Duties assigned to Postgraduates

Surveys of postgraduate students that have been undertaken through the Maths, Stats and OR Network have shown that the teaching duties usually assigned to postgraduates in mathematics fall mainly into three areas:

1. Small group teaching such as leading seminars and discussion.
2. Exercise and problem classes.
3. Marking and providing feedback on student work.

Apart from these main duties there are other activities that some postgraduates are exceptionally asked to undertake. These include actual course design and marking examinations, however such instances of postgraduates undertaking these duties are rare. Should you find yourself, however, participating in such teaching, material on these more advanced activities can be found in the book *Teaching Mathematics in Higher Education – the Basics and Beyond* (Cox, 2011, referred to as **TMHEBB** from now on).

The exact definition of what is meant by 'small group teaching' is one that varies between different institutions, however, in the majority of cases it is used to describe an interactive session in which the tutor is responsible for a small group of students, normally less than 30. In this book we will take small group teaching to be any of the following: exercise classes, small discussion groups, or examples classes What each of these teaching sessions have in common is their purpose in encouraging students to interact with both the tutor and each other, usually in the solving of mathematical problems. Such sessions are not solely intended to encourage the simple 'recall' of facts, but to develop more fundamental concepts such as application and problem solving.

The terminology in small group teaching in mathematics is also not universally agreed, but usually by an exercise class we mean one in which students work through problems together with help and support available from an expert tutor or teacher. This sort of class is dealt with in Chapter 2. By small discussion groups, covered in Chapter 3, we are considering 'real' group teaching where the objective is to develop interactions between students and tutors to facilitate learning. It is the interplay between students and tutors that provides a greater range of learning activity. By example classes we mean the situation where you work through solutions to problems, taking a more leading role in guiding students through their solutions.

Normally, postgraduates will be working for a full-time member of staff who will set the context of the teaching duties and provide supporting materials such as exercise or problem sheets. They may also moderate the marking you are asked to undertake. In general a postgraduate may appear to be directed and guided by the lecturer, supervised in their duties and therefore expected to take minimum responsibility. However, in terms of supporting students to learn, postgraduates are at the forefront and in direct contact with students when they are actively working and trying to understand the lecture and course materials. While postgraduates may receive guidance on the process of running tutorials, for example, they receive little support on the equally important skills of helping students to learn. We lay great emphasis on this aspect within this book and we might summarize the skills required as being able to:

- ENTHUSE the students about mathematics.
- ENGAGE the students in productive mathematical work.
- EXPLAIN mathematics to students with varied backgrounds.
- EXAMINE students by marking some of their work and providing feedback.
- EVALUATE your teaching.

These skills are central to teaching mathematics by any means for anyone, and they are difficult to master, especially for those new to teaching. We do not address each skill explicitly, but they pervade the material of this whole book.

Cashing in on your teaching experience

Apart from the immediate tasks highlighted above, the preparation and experience of teaching is also important as a foundation for future teaching duties since many postgraduates progress to a career in university teaching or in school, Further Education (FE) or college teaching. Additionally, and equally importantly, the interpersonal skills learnt in teaching are valuable in many walks of personal and professional life, and are sometimes a valuable asset to one's curriculum vitae.

Structure of the book

This book is not necessarily intended to be read from cover to cover, you can explore each section as required. Not all sections may be relevant to you, but we have tried to address a range of teaching activities that you may be asked to undertake, and in addition, many sections will be useful should you go on to become a teacher, whether in a school, further education or higher education.

As much as possible we want the book to be of immediate help at each stage of your development and as you progress through different teaching duties. The main duties with which you are likely to be engaged are covered in Chapters 2-5. These are split broadly between various types of classroom 'contact' teaching previously described, and assessment of student work. Typical postgraduate duties are therefore addressed as follows within this book:

Chapter 2 Running exercise classes.

Chapter 3 Supervising small discussion groups.

Chapter 4 An introduction to lecturing: presenting and communicating mathematics.

Chapter 5 Assessing student work and providing feedback.

We have devoted Chapter 4 to providing an introduction to lecturing which might be useful if you go on to an academic or other teaching career. More detail on this can be found in **TMHEBB**. In Chapter 1 we give an overview of general principles for the sort of work you might be asked to undertake as a postgraduate.

During the workshops that have informed the development of this book, interactive question and answer sessions has always been included. Within these, experienced tutors address questions posed by the delegates. Examples of common questions are collected in Chapter 6, with comments that we hope you will find helpful. You might therefore find it helpful to consider this chapter first to see if a particular question is addressed; we have tried to cross-reference to appropriate sections within this book so that you can then find further information.

Acknowledgements

We are pleased to thank many colleagues who have in various ways contributed to the production of this book, including those who shared the fun and excitement of the workshops from which it arose. We would particularly like to thank Chris Good, Joe Kyle, Duncan Lawson, Chris Sangwin, and Dagmar Waller.

We would especially like to express our gratitude to Chantal Jackson and Janet Nutall, who provided excellent administrative support for the postgraduate workshops. They cheerfully ensured that we always arrived at the right place at the right time with the right materials, and subsequently provided us with the feedback that indicated whether we had done the right thing.

Chapter 1

Getting started as a postgraduate teaching mathematics

1.1 Teaching is not easy – an analogy

At two meals a day you will probably have had more than something like 12,000 meals when you first started university, most of them typically cooked by someone else. You might be quite a connoisseur, you certainly know what you like and when it is well cooked. But how good a cook does that make you? No, eating hundreds of meals does not necessarily make you a good cook. Similarly, having attended lots of lectures and hundreds of lessons at school does not make you a good teacher. However, you can of course learn a great deal by reflecting critically on the years of teaching you have received, and adopting or adapting approaches that you found useful in helping you to learn as a student. Indeed it is a comment often made that lecturers teach how they were taught – but of course this does not invariably lead to good teaching.

As you are studying for, or already have a higher degree, your mathematical knowledge in your specialist area is no doubt very good, but we hope this book will convince you that there is much more to teaching than subject knowledge alone. You have to be able to explain your knowledge to a wide range of undergraduate students for the very first time, some of whom may not actually be that interested in what you have to say, and also you have to have the breadth of mathematical knowledge and skills to present it from different perspectives and contexts, often outside your own area of expertise. You could very well be an expert in cooking the food YOU like, but now you have to cook for lots of other people. This can give you new and exciting perspectives on your subject.

How do you gain all of the knowledge and skills needed to teach a class for the first time, and within a matter of weeks? You can't. The simple answer is that becoming an effective teacher takes experience and practice, and this is what you will gain throughout your academic career. This guide is intended to help you through your initial teaching duties, and contribute to the foundation on which to build your teaching. The authors have had extensive experience of teaching at both college and university level and offer advice and guidance that you may find useful as you embark upon your own teaching duties. But, we cannot tell you how to teach or guarantee that what we suggest will work with every group of students – you need to try the ideas and approaches we describe for yourself. If they work, continue to use them, if they don't, modify them or try new ones. Teaching involves your own continual development which you alone must direct. You will also have plenty of experience and support within your own institution; make sure you take full advantage of this.

1.2 A teaching mathematics checklist

You will soon find that even the simplest teaching duty involves a great deal to think about. Fortunately, **MATHEMATICS** itself provides a useful mnemonic for the sorts of things you need to consider when you approach any teaching situation in mathematics (Cox, 2004). Roughly, these are:

Mathematical content.

Aims and objectives of the teaching event.

Teaching and learning activities to meet these aims and objectives.

Help to be provided to the students - support and guidance.

Evaluation, management and administration of the teaching.

Materials to support the teaching and learning.

Assessment of the students.

Time considerations and scheduling.

Initial position of the students – their background in mathematics.

Coherence of the curriculum – how the different parts of a topic fit together.

Students and their needs.

This addresses the main areas one has to think about when undertaking any teaching duty.

Example

You are given a tutorial for 30 first year students who will be working through an exercise sheet on elementary integration revision. As an applied mathematician who uses calculus frequently this looks simple. Just look though the problem sheet the night before to check you can do all the problems? Afraid not! Let's do a little **MATHEMATICS**.

Mathematical content is probably fine – you have all the technical skills and can most likely cope with any question you are likely to be asked. **BUT**, all this has to be within the students' context, and at their level of understanding – be sure you know what this is and have a sufficient overview to be effective at their ability level.

Aims and objectives of the tutorial – is this simply to develop facility and speed, or deepen understanding? These require different approaches in supporting the students during the tutorial.

Teaching and learning activities to meet these aims and objectives – in the main this is prescribed by the fact that this is a tutorial in which the students' learning activities involve working though exercises, seeking help from you as and when they need it. But sometimes you may notice a common difficulty in the group, and you might then decide to focus student attention upon it by working with the whole group and writing on a black or whiteboard. In such circumstances you have to be both proactive and reactive.

Help to be provided to the students – this is the very reason why you are present in the session, to provide support and guidance to the students. But perhaps less obviously, you are there to make them think and to challenge them so as to uncover any weaknesses and provide opportunities for the students to address them. Teaching is less about providing students with the answer than it is about helping them find the answer for themselves.

Evaluation, management and administration are probably not directly applicable in this case. However, you still need to evaluate your own performance and learn from it. You may also evaluate the materials you are given – maybe the problem sheet is too hard or too easy. You need to think about how you will react to situations as they emerge, and what to change in the future. You may also need to provide feedback to the lecturer for the course, and obtain their views.

Materials to support the tutorial – as well as the problem sheet itself, you may want such materials as specimen solutions in order that you can see how the students are supposed to tackle the questions. It will be useful to have a copy of the module specification, and the copies of the course notes on the topic, as these will tell you how the students have been taught to tackle similar problems.

Assessment of the students – perhaps not directly relevant to you, but you will need to know, for example, if any of the problems are to be used for assessed coursework, because naturally the lecturer won't thank you for providing full solutions if this is the case!

Time considerations and scheduling – you must keep to the time allocated to the tutorial. But also within that time you need to think of how best the students might work though the problems. They may be tempted to work through in sequence, completing each question in order. This could mean they only work through problems they can do easily, which is little use in terms of obtaining feedback. Essentially, the students need to find their weaknesses as quickly as possible so they can take maximum advantage of your help while you there to support them. You have to identify the key 'diagnostic' steps in the problem sheet and perhaps suggest they spend a third of the tutorial on the first few questions, another third on the middling questions and the rest on the final questions – or some other such schedule.

Initial position of the students – what pre-requisites are required for this material, and how does it relate to the student background in mathematics? They may need such things as partial fractions, completing the square, or trigonometric identities, and maybe these need a little revision. What do the students actually know, how well prepared are they for the material in the tutorial?

Coherence of the curriculum – how does the topic align with the other components of their course, and how does that influence how and to what extent the students have to master this material?

Students – in a tutorial, human interaction is of the essence. Find out something about the students, and plan for building a good rapport. What courses are they studying, what are their motivations and interests? Above all, treat them as individuals who you are trying to help, respect them and they will respect you for trying to help and support them.

In this book we provide practical advice on each of these issues.

> ***Exercise***
>
> *Apply* **MATHEMATICS** *to your next tutorial or similar teaching situation*

1.3 General points in working with students

We are going to look at the technical aspects of the particular teaching activities you might undertake, whether it be tutoring or marking. Before we start, there are some general principles that apply to all aspects of your teaching and interaction with students, which we summarize here. We will keep revisiting these throughout the book as they are fundamental to teaching at any level.

Start planning and preparation as early as possible

The example given in Section 1.2 should convince you that even the most apparently straightforward teaching task involves significant thought, and you need sufficient time to gather any required information. You may have to rework your ideas a few times before the actual lesson. It is thus not only the preparation of materials that requires time up front, but thought and reflection on what you have to do. But remember you have other work to do, and your preparation should be planned in a measured way, so you spend an appropriate but not excessive amount of time on it. While it is difficult to be precise here, wide discussion with colleagues and postgraduates suggests that 1-3 hours of preparation per hour of class contact should normally be sufficient in most cases of initial delivery and this can be reduced in subsequent delivery.

Assemble the necessary teaching and learning materials well in advance

Even for an individual tutorial this can represent a significant volume of material. For example, the lecturer for the course may provide exercise sheets with outline or specimen solutions. It would also be useful to have a copy of the notes given by the lecturer to the students, so you can see how the topic aligns with the rest of the course. You will need such materials as the basis for your preparation, and may need to work on them yourself; the earlier you can assemble them the better.

Read and consult widely on the topic

We will make this point repeatedly – the teaching of a topic requires a far wider appreciation of its content than might normally be expected of a recent graduate. You may be familiar with say integration by parts, and be able to complete successfully most exercises you are likely to encounter. But are you familiar with its proof, that it is the reverse of the product rule for differentiation? Do you know why the students need it? Do you know the best way to explain it and the best method of weaning students off slavish use of the formula? For such things you need an overview and broad perspective of the topic; the easiest way to do this is to read around the topic, and discuss it with colleagues and your students. Other useful sources include books, websites, the lecturer's notes, and past examination papers.

Another way to develop breadth in a topic is to rewrite notes on the topic in your own words – reassemble and restructure the ideas and content, making sure you can prove things *ab initio* yourself, and do all the problems as the students should be able to do them. You may look into the history of the topic, or use a relevant and possibly amusing anecdote. Above all, be prepared for the constant student question 'Why do we need to do this?' Your credibility with the students will rise dramatically if you can provide convincing answers to this in an understandable way – it demonstrates that you have a deep understanding of the topic and that you are conscious of student needs. If you are unsure of real-world application of a mathematical topic, have a look online before the session. Find examples that are relevant to the discipline of the student if you are teaching non-specialists. For example, how are logarithms used in astronomy? Apparent and absolute magnitudes of stars are a natural example.

Design tasks to meet objectives

Whatever the mathematical content you have to cover in your allotted time it is likely that you will have to match what you and the students do to what you are trying to achieve; again this requires thought beforehand. Even if you are simply going to tell the students something, you have to think about **how** to tell them. What is the appropriate language, what are the main points, how will you know they genuinely understand it? But it may be more involved than this, including such things as organising the group and assigning specific learning tasks. For example, if the object is to develop skills of sustained precise and rigorous mathematical argument you might split the student group into pairs with one playing the 'prover' and the other a sceptic whose job it is to insist on precision and clarity. Or, for a complicated modelling exercise, you may split them into say groups of four or five, working together. Don't forget, all such tactics have implications for time, accommodation and possibly equipment, and you will need to provide the group with precise instructions. For a wide range of tasks aimed at various types of mathematical objective, see Mason (2002), page 105.

Ensure an appropriate amount of material

This is an area where even experienced lecturers can have difficulty, especially if they are delivering a course or module for the first time. Even if you have the 'syllabus' and know specifically what it is you have to do, there are usually still shades of interpretation that can result in too much or too little material. Returning to our earlier example of integration by parts, you can incorporate so many variations and extensions to the fundamental examples that students fail to appreciate the key ideas. By 'appropriate amount of material' we really mean in terms of the intellectual load upon students, taking into account their background and the time available (allowing for students' independent study time). There is a tendency to try and cover too much. If anything you can afford to err on the side of too little, so long as it is well taught and provides a good foundation. This is definitely an area where consulting more experienced colleagues in your institution pays dividends.

Know the students

It is likely that you will have responsibility for only a small number of students, and so you should try to learn some of their names. This not only tells them that you are interested in them as individuals, but is also useful in maintaining discipline in the group - students are less likely to disrupt the lesson if they know that you know them! If the number of students is large, then at least try to learn a few of their names. However, caution is needed here. If you manage to remember the names of a few prominent students the class may think they are favoured, or conversely, the individuals may feel 'picked on'. Strategies for learning names might include asking students to say their name as they respond to a question, having a card with their name written on it in front of them during the first few teaching sessions, or obtaining a list of names and student photographs from your departmental or faculty teaching office.

But this is not the only sense in which we mean 'know the students'. Try to learn something about their background, motivations and interests. Anything that helps you appreciate how you might best communicate your messages. This is particularly important in the case of 'service' classes, by which we mean the students are non-specialist mathematicians such as engineers and scientists. It is a two way process – let the students get to know you, open up to them a little. They will learn better if they trust you and they will trust you more as they get to know you. You might try some sort of icebreaker activity at the first meeting to generate initial discussion and engagement.

Know what the students know and HOW they know it

By this we mean how the students have learnt the material and the methods used; in effect you have to try and understand this in the way that your students do – imagine you are them. For example, when teaching the binomial theorem to first year undergraduate students you have to be aware that many will have seen it at school level only in the form of Pascal's triangle. This is of little use for more advanced applications of the theorem. If you ask the students if they have been taught the binomial theorem they will inevitably say 'yes' – but if you then ask them to write down the coefficient of x^7 in $(1 + x)^{30}$ it may take them longer than you might expect. The general point is to be wary of assuming students 'know' something in the same way that you yourself 'know' it.

Remember the students are not clones of you

In a way you were probably an exceptional student – if only because you were actually interested in mathematics! You were almost certainly good at the subject, and automatically gravitated towards the most productive learning methods and skills. Particularly if you are teaching in service classes, this will be not always be the case for the majority of your students. In fact, in higher education it is often the case that we have to teach a large number of people who we don't really **understand** too well. But this is precisely why teaching is such a difficult job – if you were teaching a class of people just like you then it would be easy, but this is very often not the case. The job is to teach the students you have got, not those you wish you had. **Respect** them, whatever their mathematical abilities. Moderate your expectations of students and view them in an objective and realistic way. Discuss this widely with more experienced colleagues and do not fall for negative value judgements and generalisations. Treat every student with due respect, even if their 'weaknesses' in mathematics sometimes exasperate you.

Generate engagement and participation

In any teaching session with which you are likely to be involved, the students need to be active participants, not passive ones. By this, we mean they are actively contributing to the delivery of the session rather than merely listening to what you have to say and taking notes. Part of this is encouraging the students to ask questions, but also critical is engaging them in the teaching process.

Consider how you might use tasks, activities that the students lead and work on themselves, either individually or in small groups. You can perhaps bring the individual groups together through a class-wide discussion; they might each be working on a different problem whose solution they then share. Tasks provide another important opportunity, the chance for you to walk around the room and speak with individual students; this will help you understand any mathematical issues they may have and provide an opportunity to identify the material upon which you should focus during the session. While students are working on a task, you should not be standing at the front of the room waiting for them to finish. You might also consider asking the more able students, or those who have completed a particular problem to explain it to others, in effect to lead a small group discussion. This is a valid teaching technique; having to explain an idea to others will help enhance their own understanding.

You might wish to invite a student to the front of the class to present a mathematical solution; some will be willing to participate, others less so. It certainly isn't advisable to force a student to participate in this way, but when students are working on tasks you can walk around the room and identify possible participants. It is important to understand why students are reluctant to present a mathematical solution to their peers; the major barrier is their fear of being 'wrong'. You can address this by identifying a student with a correct solution, telling them it is exactly what you wanted to see, and asking if they will share it. They might agree, they may ask you to comprise – "I'll write the solution, but I don't want to speak", the student can write, you can provide the commentary. In either case, you have generated engagement, and if the student is well treated while at the front, other students will see there is little to fear and will be more likely to volunteer in future.

Maintain discipline and keep order

Dealing with persistent late-comers, or those who are disruptive or lack application, is one of the most stressful and challenging aspects of teaching. Unfortunately, such things do occur. Your strongest weapon is confidence in your position, status and role. If you are placed in charge of any group of students, whether it be for a tutorial, group session, or invigilating an exam, then that is precisely what you are – in charge. You are also responsible for the session, and for ensuring that all students have a productive learning environment. Most students really do want to learn and they see it as your job to provide the best environment for learning; in fact they will often support you in achieving it. They will almost certainly support you in dealing (sensibly) with disruption and inappropriate behaviour. In fact, in small groups such things are less of a problem than in a large lecture, for example. It is relatively straightforward to control behaviour in the small group environment. What is more difficult however, and needs constant attention, is keeping students focused on the task; they can digress for a number of reasons. They might have finished early – the task was too easy, give them something else to do. The task is inherently boring and their interest wanes – discuss this with the lecturer for the course with a view to revising it. They have ground to a halt, can't see how to progress and have found something else to occupy themselves with – the task is possibly too hard, and in any case you have to decide whether to give them some help, or another task.

Understanding student behaviour

During a class, students will behave in different ways. Some will work studiously throughout, others are easily distracted and prone to talking. If students are distracted, or worse, distracting others, you should challenge such behavior, but most importantly, you should try to find out why they are behaving in this way. To do this, you need to talk to them. Use tasks to get the group working on a mathematical activity, and then speak with the student(s) individually; it may be that they have completed all the material that was set, if so, give them a challenging example or get them to present to others. They may be mathematically weak in a particular topic, if this is the case you can help them, either directly or by suggesting further reference material at the appropriate level. This is another method you can use to get to know your students.

Set expectations and standards from the beginning

It may not be the case in other subjects, but in mathematics the curriculum must be student centred, but teacher led. The students need to know what is expected of them, both in terms of the subject, their input, and the behaviour required of them both during, and outside of, the teaching sessions. These are related. Mathematics is a detailed and sequential subject that can be hard to follow at the best of times. So when you are trying to explain things in class a momentary disruption caused by a group talking, even quietly, can break the chain of thought of anyone within hearing distance. If you can hear it, then so can the students, and you have to put a stop to it. From the beginning you should emphasise to the students that there must be no undue disruption in class. The point is that the students have to know what standard of behaviour is expected. It is always better to be 'firm' with students at the start, than to start off casually. If you do this, you can always 'ease off' as the course progresses and as you get to know the students, moving in the opposite direction is almost impossible.

In terms of the curriculum, discuss with students the objectives of the session (we will say more about aims and objectives later), and the material to be covered. If possible invite questions from the students by asking if they have met any of the material before. You may wish to outline how the lessons will be structured, how students may ask questions (either during the lesson, after a particular topic, at the end of the lesson, or outside of it), or how you will approach examples and questions. Again, invite comments from the students to encourage them to engage in this learning process.

Inform the students of any relevant departmental policies, particularly with regards to attendance, assessment of problem sheets or coursework, and plagiarism. After outlining what you expect from students, and what you will cover, they may have concerns or questions. Encourage them to raise these concerns, and if necessary, create a discussion amongst the group; you can always throw open questions to the group along the lines of 'has anyone else thought about that?', 'does anyone else feel the same?', 'how should we progress?'. Be careful, however, about putting too much onus upon the students themselves. You might give students ideas as to how they may study for the course and prepare for the next class. Explain how much time students should spend on the material outside of the lesson, and suggest where they might best focus their attention (problems, examples, etc.). Tell them about support services that are available, in particular mathematics support centres, which they may wish to access for additional help, or online resources and other course material.

A good way to get students familiar with what the course will cover is to work through a problem, encouraging their input, that relates topics from the course to a practical application. For example, how Newton's laws relate to racing cars, or how trigonometry can be used to calculate distances to the stars. Try to make use of previous knowledge, and avoid quoting results from advanced material as this may scare the students. This should be looked upon as a friendly and informal way of beginning to engage the students with the material to be covered.

Some students may also like to know about various web–based resources, both for revision and for broadening their mathematical horizons. A particularly useful reference for first-year students is www.mathcentre.ac.uk.

> ***Exercise***
>
> *Think of your next encounter with students, be it a tutorial, presentation, or whatever, and think about the implications of the ideas of this section for the session and the actions these might require from you.*

1.4 Motivating and enthusing students and encouraging participation

The need to motivate and enthuse students.

This is crucial to everything we do in teaching. We all know how difficult it can be to learn something that doesn't interest us. It is no different for the students – you were one not so long ago, and we're sure you can remember courses that you disliked and perhaps struggled with because the content, lecturer, or both, were uninspiring. Whether it is in a lecture, tutorial, seminar, or demonstration class, motivating students to learn is a challenge facing all who teach. Whatever takes place in the classroom is the dominant factor for determining their future motivation, either for better or worse.

Motivating students to learn builds upon many factors: interest in the subject, its perceived relevance to them as individuals, their self-confidence and morale, their level of patience, their ambition to succeed, and peer-pressure, all being important examples. There are a number of broad, general strategies that we can use to motivate students to learn for themselves. These include providing positive feedback that supports the student's belief that they can succeed; creating a positive classroom atmosphere that is open and supportive and which makes students feel they are valued members of the group; and ensuring that every student has an opportunity for success. But first you have to make sure that you are enthusiastic.

Your (controlled) enthusiasm is essential – whatever the topic

To motivate students and sustain their interest throughout your lesson there is one absolute, invariable, and necessary requirement. Above all else: **you have to be enthusiastic – and it helps if it is genuine!**

The point is that if you perhaps find the particular topic a bit dull and you have taught it a number of times before, these students are hearing it for the first time, and they will certainly not be enthused by it if you yourself appear jaded. Your enthusiasm is necessary, but not sufficient, you still have to work on getting the students interested and engaged. But first let us concentrate on getting you enthused and inspired by a topic. It will help if, while reading this, you think of a topic in mathematics that you find less than interesting. Now, this is what you have to teach.

You first need the necessary self-awareness to recognise when your interest wanes and makes your teaching a chore, and, most importantly, when this might transfer to your students.

You need the self-discipline to modify your behaviour in such cases. You may have to act – this is part of the job. This doesn't mean pretending to be an extrovert – the students will see through this. It means exploring deeply into the topic and uncovering the components that fascinate and excite you – there are always some even in the dullest subjects. For a start, the topic cannot be utterly devoid of intellectual interest, because if it is then it is mathematically routine and you shouldn't be spending precious class contact time on it – the students can study this in their own time which is perfectly reasonable, and certainly expected, for university level mathematics.

Ask yourself such questions as:

- History of the topic, where did it come from and why?
- Where does it lead to, what areas of mathematics are built on it?
- What sort of applications does it have?
- Can I derive any of the material by different methods?
- Can it be generalized?
- How is it related to my own particular area of interest – if it appears not to be, why?
- Why do the students need this anyway?

- What concepts are at the core of the topic, and where else do these occur – can you imagine how the originators of the ideas first identified or explored them?

The purpose of these questions is not to give you ideas as to how to teach the topic (although of course it might), but rather to find some way of stimulating your own interest.

Don't let your enthusiasm run away with you, leaving the students behind. It is all too easy to become carried away by a topic you are passionate about, but you have to ensure that you continue to focus on the needs of your students, while still using your enthusiasm to good effect.

How to enthuse and excite the students

Once you have enthused yourself about the topic, you now have to motivate the students. Naturally, you need to get the technicalities of teaching right first – they have you be able to hear and see what you are doing, and to follow your 'delivery' however it takes place. That given, there are other things you can do:

Make it easy for the students to see the key points of the topic, then give them sufficient time to explore the ideas and concepts. For any given topic a student's first priority is to understand it – or at least appreciate that they will be able to understand it with further work. They can only afford the luxury of interest when they are confident that they have understood the basic ideas, that they recognise the key points, so that they are spared the chore of trying to keep up.

Talk their language Express things in the way most students are most likely to understand. We will say a great deal about this later, but when students are having difficulty in deciphering what you say their enthusiasm and interest is likely to be muted.

Examples and asides that may be of direct relevance or interest to them. This does not necessarily mean venturing into their other subject areas (for example into electricity when teaching electrical engineers). In fact, this is sometimes off-putting to students. You have to think widely about relevance. It might be something topical, completely divorced from their academic subjects. Recent examples might be a cost/benefit analysis of getting a degree, the origins of the banking crisis, or the mathematics of the internet.

Also, surprising facts from history can often evoke interest – for example it is only a few hundred years since the plus and equals signs were invented, Russell and Whitehead took 360 pages to arrive at $1 + 1 = 2$. You can get such fascinating insights and anecdotes from popular books on mathematics or by visiting appropriate websites. Fascinating questions from everyday life can also be used to enliven mundane and apparently dry mathematical results. For example, why is it colder in February than in December – or warmer in August than June? Essentially because $a \cos x + b \sin x$ can be written in the form $R \sin (x + \alpha)$ and combining two sinusoids, the earth's rotation and its orbit about the sun, produces a shift in phase.

Invoke curiosity by asking the students surprising questions (not rhetorical, and not routine) at key points throughout the session. You are trying to unsettle them slightly, so they may feel somewhat challenged and reluctant to engage, but we encourage you to persevere with this approach.

Build a good rapport with the students, so that they **want** to be interested in what you say, they expect you to say something interesting, they are used to you saying something interesting and they know you won't waste their time. Notice this is a personal skill, it has nothing to do with mathematics, but it is an important part of teaching. Once you get to know the students you can start incorporating their interests into your discussion. For example, you know Martin is a good guitarist, ask him how he became good. He will almost certainly say by lots of practice. Well, that is exactly how we will get good at integration – lots of practice. Always listen and respond positively to students' comments and questions, taking every opportunity to widen the point of discussion, draw in other students and generate interest in the topic. An occasional short amusing story or interesting anecdote related to the topic in hand can often help students remember it.

Justify sensibly, as well as logically. This is key to maintaining student interest. No matter how fascinating the subject material, a few long sentences of dry impeccable logic (of which mathematics is sometimes full!) are guaranteed to set eyes glazing over. Soften such things by simple, sensible explanation that gives the key ideas succinctly and concisely. This is actually a difficult skill and requires that you have a thorough knowledge of the topic yourself. There is no mathematical topic, especially at undergraduate level, for which such explanations are not possible (discuss!).

Tasks need to be at 'just the right level' – not too hard, not too easy. This is a matter of judgement, and your knowledge of the class as a whole. Students will vary in what they regard as the right level – a boring or perhaps trivial task for one student can be a fascinating challenge for another; you have to have a range of tasks to meet their needs. But usually the two extremes, lots of very straightforward tasks, or one or two desperately difficult tasks, are a big turn-off for everyone.

Sup with the devil. The fact is that there is one sure-fire way of getting the interest of students and that is to mention examinations. As soon as you mention that a topic often comes up on the exam you will witness furious note-taking and all ears will be yours. We like to think that students will be interested in what we are saying for its own intrinsic value and relevance, but as ex-students ourselves we know the reality and should not be afraid to use it. Make (not too) frequent reference to the exam, perhaps referring back to questions from previous examination papers. This may not score highly in the educational respectability stakes, but it works.

Avoiding things that damage student motivation

We have emphasized above some of the things you might do to interest your students. Perhaps we should also think of the things that actually damage student motivation and turn them off learning, whatever the subject. This of course includes any of the technical aspects of poor teaching, but specifically students are put off by such things as:

- ***Negative feedback*** to students, that is, anything that affects their morale to the detriment of learning, written or verbal, and tardiness in providing feedback to such things as coursework or assessed problem sheets.
- ***Too much dry theory*** without examples, inappropriate abstractions or intellectualising without concrete cases, verbosity and lack of conciseness, or arid conceptualising. The 'definition', 'theorem' and 'proof' of highly polished and finished pure mathematics should be reserved for those who are able to fully understand the meaning of such terms. While eventually necessary, it should not always be a first mode of presentation, particularly at early undergraduate level.
- ***Lack of clarity*** whether it be in the content, such as unclear notation and terminology or unclear instructions in activities.
- ***Patronising behaviour*** towards students, arrogance on the tutor's part, adopting a confrontational or challenging style, being unapproachable.
- ***Not offering choices in study methods***, or failing to adapt to student needs and progress, not allowing time for student questions.
- ***Unrealistic time pressures and expectations of students***, too much information, too high a workload, or inappropriate level.
- ***Repetitive and dull*** material and presentation.
- ***Irrelevant*** or lacking in context, or obsolete material.
- ***Poor preparation of basic knowledge*** and boring or basic pre-requisites.

- ***Poorly organised delivery and material***, lacking in structure and coherence with little overview and direction, and poor presentation.
- ***Too tutor specific***, following the teacher's interests for their own sake and paying too little attention to the interests of the students, particularly in the case of non-specialist ('service') teaching.

Most of the above represent poor or at least selfish teaching in any event, but they also have a detrimental affect on student motivation. However, you will see that some of the items are in fact unavoidable, even with the best of intentions – for example, some material has to be repeated for some students, whether they like it or not. This means that you need to work harder at minimizing the effects of such things. This can often be done by bringing the students on side, alerting them to something which they may not initially like but explaining why it is essential and how it will ultimately benefit them on their programme of study.

Examples

1. Convex sets

Some students might, for example, find it difficult to get excited about the formal definition of a convex set:

If x, y are any two points in the set then so is tx + (1 – t)y for all t between 0 and 1.

One can't argue with the precision of this definition, but what is wrong with saying 'If two points are in the set then so are all the points on the line segment joining two points'? Most students can easily appreciate this fact and then we can formalise it. It is still hardly inspiring, however, why do we need convex sets and what makes them so important and actually very interesting? Well, in a convex set you can always move between any two points by the simplest path imaginable, a straight line – that sounds quite interesting, surely? Vertices representing optimal points can therefore be located by scanning a convex set with straight lines, again, surely interesting? But how many sets do we know are convex – aren't they just a rather unrepresentative case and therefore not that interesting really? Ah, but we can always break up most sets into separate pieces that **are** convex, treat those separately then put them back together again – in other words, what we can do with a convex set you can usually stretch to any set. The point is that while the formal definition of a convex set may be pretty uninspiring for some, the teacher can make it much more interesting by providing a few minutes background information and wider explanation using language that is accessible to the students.

Often it is difficult to enliven definitions and notation, particularly in a mathematical topic like abstract algebra or analysis. But usually there is a story behind most definitions, a reason for the name or notation; occasionally mentioning some of these can break up the apparent monotony and help enliven a session. This might also help the students to understand **why** precise definitions are so important, which is something they don't always appreciate; and the sooner the definitions are then used in subsequent material, so much the better.

2. Elementary does not mean uninteresting

There is a lot of relatively low-level mathematics teaching, even in the most research-intensive of universities, for example foundation or first year service teaching. If you are a keen researcher in topology assigned to teach such a class, it may be difficult for you to become enthused about solving quadratic equations, for example. However, this is the nature of teaching – we have to try to be enthusiastic and convey that to the students, even if the material is, by our standards, elementary (although it most likely won't be elementary to many of the students!). Quadratics **are** very interesting to any keen mathematician. They lead us into complex numbers. For many geometrical purposes they may be used to approximate other continuous functions. They have a very rich history. Completing the square is an example of a magical trick used everywhere in maths – 'getting something for nothing' (for example, adding and subtracting, multiplying and dividing). Any polynomial with real coefficients .... . The list goes on.

> ***Exercise***
>
> *Think of a topic in mathematics that you find dry and unexciting. Now find something interesting about it. How would you present it in a stimulating way?*

1.5 Preparing for teaching

The importance of planning and preparation

For each type of teaching activity – tutoring, lecturing, or marking, there will be the task of preparation before you even venture into the classroom. Such things will be discussed in each relevant chapter, but for now we wish to address the general principles and issues that arise in preparing for teaching. For any teaching event you have to think about what you want the students to get from the session, and then plan and prepare accordingly. Because you will typically be working under the supervision of an experienced lecturer you will often be given prepared materials, such as a problem sheet with worked solutions. You may therefore think 'preparation' simply consists of quickly looking through the problems to check that you can do them in case you get asked by a student in a tutorial. But, as mentioned earlier, there is much more to it than this.

Your overriding priority is to ensure that all the students are given the best possible opportunity to learn during the time that you spend with them. For this you need to be able to do more than work through individual questions. You have to think about what are the really key messages you need to convey. If a student asks you a question that is peripheral to these, then you need to know not to spend a lot of time on that individual question. But if a number of students ask you questions that make it clear many are missing a key point, then you need to know when to address the issue with the entire group. This requires a mature overview of the whole topic, and an appreciation of the sometimes many different ways of approaching it.

Example

One topic in a first year calculus class might be integration by parts. No problem, you think. I know all the techniques (and indeed tricks!), the sheet looks fine, and I can do all the questions. But at the end of the session the objective should be that the bulk of the students leave with a good understanding of the technique, its limitations, variations, and its origin. You will not ensure this by answering a few individual questions – it may be that one student doesn't know which factor to integrate first, another doesn't see the trick of ln x as a product of 1 and ln x and then integrating the 1, another has to tortuously apply the formula to each example and has an inadequate facility level, another believes that every product can be integrated by parts, and the list of problems goes on. You can spend the entire lesson dealing with such individual queries and the overall level of understanding by the class will be little different to when you started. Before the lesson you have to identify what the key points really are and plan to communicate these – this will depend on the objectives of the course, the level of the students and their background, and the context within the calculus course. You may also need to revise your plans once in the classroom, in the light of student responses and feedback.

Suppose the objective is to develop student facility and speed to high levels for later use, say in differential equations. Then you might ask them to concentrate on doing lots of straightforward, routine examples, building up speed, weaning them off a formulaic approach and building up automatic facility and fluency. Then the sophisticated tricks such as the integration of ln x or $e^x \cos x$ can be left until later or for the more-able students to pursue. You need to move around the classroom checking that most students are increasing their speed. Often, students are slow with integration by parts because they don't realize that it is the inverse of the product rule. If this seems to be the case, emphasize and illustrate it to help them to speed up their differentiation of products.

You may be asked 'What is the relevance of this – will we need it later?'. Do you have a good enough understanding yourself? Can you find examples relevant and understandable to the students? If not, you should at least know where to find them and perhaps outline one or two examples at the start of the next teaching session.

The key to preparation is being clear about what you have to achieve – your objectives. In modern educational practice this is often expressed in terms of **learning outcomes** or **objectives**.

Learning objectives/outcomes – what students are expected to be able to do after the class session

We have laboured the above example a little to make the essential point that even for an apparently routine topic like integration by parts, a great deal of thought is needed before you even start. It is NOT just a matter of being able to do the questions on the sheet! Additionally, in this example we have referred to the need to think about the **learning objectives** of the session – sometimes called **learning outcomes**. These simply prescribe what the student should be able to do after completion of the course. The importance of context is also mentioned – how does the topic align with the other components of the course?

Formally, a **learning objective** (or **learning outcome**) states what a student should be able to do after learning something. The definition we give here seems to be quite appropriate in mathematics:

- A **learning objective** is a statement of what a student must DO to show that (s)he has achieved the objective – or learned the object of the statement. It will contain:
    - An action verb describing a PERFORMANCE which is both observable and measurable
    - A statement of the CONDITIONS imposed on the performance
    - A statement of the STANDARDS to be reached

While less rigorous than we are used to in a mathematical definition, it gives a reasonable idea of what we mean by a learning objective. Learning objectives are normally expressed in the form: "On successful completion of the course the student should be able to ... (learning objective)".

Example

Will be able to apply Pythagoras' theorem (PERFORMANCE), without any calculating aids or reference material (CONDITIONS), to solve any right-angled triangle, giving the results correct to three significant figures (STANDARDS).

In practice few learning outcomes would be expressed in the detailed form of this example, which is exaggerated to make an important point. Most of the learning outcomes will be explicitly or implicitly apparent in the teaching and assessment of the course. In the above example, the assessment by examination paper may not allow the use of calculators or formula sheets, for example. This would influence the sorts of questions we ask, for example we might then need to restrict ourselves to the use of a simple 3, 4, 5 triangle.

Taxonomies of learning objectives

In order to be specific about what we intend the students to know or be able to do, we need to have some sort of classification of cognitive objectives – some way of specifying the type of thinking skills we wish the students to develop. This sort of classification is called a **taxonomy** of learning objectives.

The classification of learning objectives is one of the most researched areas of teaching and learning, and few firm conclusions have been reached. Most taxonomies are actually too complicated for practical use in the classroom. One of the earliest and most influential to be developed was **Bloom's taxonomy**, and even this has been revised in recent years (Anderson and Krathwohl, 2001). In this the main thinking skills are categorised under knowledge, comprehension, application, analysis, synthesis, and evaluation. By similarly categorising teaching methods and assessment instruments one can then ensure the alignment of learning objectives, teaching and assessment methods. But this is still far removed from how most practitioner's work and to interpret it within the context of this book we have to find a workable compromise.

MathKIT is a practical three-tier categorisation (Cox, 2004) which is closer to how most higher education teachers actually work, yet gives a workable classification of the main cognitive skills and you might find it easier to remember:

- **K**nowledge/routine skills and techniques (Knowledge/remember).
- **I**nterpretation/**I**nternalisation/**I**nsight of these (Comprehension/understand, analysis, evaluation).
- **T**ransfer/**T**ranslation of these to new contexts and applications (Application, synthesis/create, evaluation).

MathKIT formalises 'I want them to *know* it, really *understand* it and be able to *use* it'. Additionally, in placing emphasis on the **K**, **I**, and **T** it enables us to discuss in a systematic way how we develop these separate skills, what teaching and learning methods we use, and how to assess whether the students have developed them. It also encapsulates what we look for when marking student work.

Example – Pythagoras' Theorem

In this case the item of **K**nowledge is the formula $a^2 + b^2 = c^2$ for the sides of a right-angled triangle with hypotenuse c. Under **K** we would, for example, expect a student to know what a right-angled triangle is, to remember the formula and to be able to use it to find the hypotenuse given the two short sides.

For **I** in this case we would expect the student to have instant recall of the theorem and thoroughly understand it and its limitations. They should be able to rearrange it to suit the problem. If they are a mathematics student we would reasonably expect them to be able to readily supply a proof, whereas if they are an engineering student then maybe a heuristic proof or understanding is required – it should at least seem 'natural' to them. They perhaps should be aware of the conditions and limitations of the theorem – restricted to a plane, for example. Perhaps the key test is that they should be able to explain it in their own words to someone new to the idea – 'I never really fully understood X until I had to teach it' is a common phrase uttered by many in higher education – including the authors!

For **T** we would expect the student to be able to apply and initiate the use of Pythagoras' theorem in new and/or unfamiliar contexts. This does not necessarily mean only applications to 'real world triangles' – diagonal of a field, for example. It may include application and transfer to other areas of mathematics – for example the distance between two points, equation of a circle, length of a curve by integration, or diagonal of a cube.

Alignment of learning objectives, teaching activities and assessment

The purpose of framing clear learning objectives is that you can then devise teaching strategies and activities, and any assessment, to meet those objectives – put simply, decide **what** you want to do, **how** you will to do it, then **confirm** that you have done it. As a postgraduate at this stage of your career, you are unlikely to be actually involved in the framing of the objectives, these will most likely be given to you and you may even be told how to address them. This may be as simple as 'help the students as they work though this problem sheet', but ideally, prior to any teaching assignment, you should receive a **module specification** for the course. As well as other things such as **content** or **material** to be covered, the teaching methods to be used, how the course is assessed, the number of **credits** that the course is 'worth' to the student, the module specification will tell you the learning objectives, sometimes in the form of a **syllabus**, which is simply an outline of the material that needs to be taught as part of the course you have been asked to deliver. There are several reasons for studying the module specification carefully:

- You may be uncertain on some topics on the syllabus, it could have been a long time since you yourself studied the material, or you may not have even met the topic before, so you need a review of this material.
- It may contain a reading list for the course or details of a recommended textbook. If a particular textbook is used ensure that you are consistent with its methods and notation. For the same reason, obtain a copy of the notes used by the lecturer for the course you are supporting. You must be able to do questions in a similar way to how the students have been taught.
- Whilst reviewing the syllabus and course material you have been given, it is useful to begin thinking about how you will approach teaching the various topics, and, in particular, the key points you will emphasize. If possible, talk with the postgraduate who delivered the course previously as they will be able to explain which sections the students understood easily, and which required more careful explanation; they will also be able to offer general advice and guidance that will undoubtedly prove helpful.
- The pre-course material will usually explain how the course is to be assessed. Common forms of assessment used within mathematics and statistics are weekly-assessed problem sheets, coursework, and examinations. Try to obtain copies of exams and problem sheets used in previous years, these will help you to prepare examples that are of a similar standard and style to those that will be used for assessment. If you don't have these it is likely the students will and they will then typically ask you for assistance – it doesn't do any harm to be prepared!
- Note that no module specification will itemise all of the detailed objectives of the module as intended by the lecturer. These will be conveyed, often verbally, by the lecturer, during the lectures – for example the lecturer might have explicitly told the students that he doesn't want Pascal's triangle used in the Binomial Theorem. This sort of level of detail can only be appreciated by frequent discussions with the lecturer and the students themselves.

Once you have a clear idea of what it is you have to achieve, you can prepare the appropriate teaching activities, whether it be for a tutorial, or setting or marking coursework. In each case, such preparation is covered in the appropriate chapter, but here we summarize the general principles.

General principles of preparation

How you approach planning and preparation for many things in life, such as an interview for a job, is largely a matter of personal preference. But in teaching we have to remember that our eventual performance, and how we prepare for it, has much more than just our own future depending upon it. If you perform badly at an interview and don't get the job, or make a bad job of your viva and don't get your PhD, then sad though this is, it only affects you. If you perform badly at a tutorial because you have not prepared well enough then you may suffer from poor feedback, but much more importantly, your students will not have benefited as much as they should. Good preparation is not an option in teaching, it is a responsibility. What follows are some principles that are important in preparation for any teaching event.

Be clear about the purpose of the event – what is it trying to achieve, what are its learning objectives? It may be for the students to practice a new technique such as inverting matrices or a method of integration.

Begin preparation as early as possible, and make sure that you have been given sufficient advance notice in order that you are able to prepare adequately. Preparation usually takes much longer than you expect; allow time for iteration and flexibility to adapt to changing circumstances.

Preparation is front-loaded in that after the first time you perform a task it becomes easier to prepare for subsequent occasions.

Preparation is always within the context of constraints, whether it be resources, workload expected of yourself and the students, time available, or background of the students. That is, in preparation you need to consider the resources available to you and any limitations upon them.

You are preparing for a form of communication, so **how** you write and say things is as important as **what** you say. The communication is comprised of **media**, the teaching and learning materials, and the **message**, the ideas you are trying to convey to the students. The real bulk of preparation time lies in the intellectual construction of what you are going to say to the students, how you will express it, what exercises you will give, how to phrase things to get the key points across, what depth of proof to use – the message.

Preparation involves reading and consulting widely on the topic, to assemble and organize your own ideas. You may be an expert in the subject, but it is unlikely that you will know everything there is to know, especially about teaching it. You will need to read a range of resources, and talk to experienced colleagues to broaden your understanding and overview.

Preparation needs to be efficient and should not occupy excessive time. This is one of the excuses often given for last minute preparation. In fact, one should start preparation as soon as possible, do it as fast as possible, then iterate to finalise it. More experienced colleagues will offer rough rules regarding preparation – for example, the number of hours suggested for preparation of a tutorial, or for marking coursework; you will develop your own such measures with experience.

Don't aim for perfection, it doesn't exist, especially in teaching. Do a good job, reflect, learn from any mistakes and improve with experience.

Preparation should be comprehensive, at least in outline, right from the start. That is, the preparation should include an overview of everything that has to be done. For example, if you are given a series of ten tutorials on a subject, plan all of them in outline before starting the first one rather than wait to prepare each one as you go along. This does not mean detailed preparation of all materials or individual lesson plans before you start, it is just to be sure that from the start you are confident that you can cover the material at the pace you have adopted, even allowing for problems on the way. In effect, make an outline of what you want to teach and how you plan to do it.

Preparation should allow for any learning and development on your part and identify things that you need to do in order to help the students. For example, this might include mathematical techniques that you may be used to doing one way, whereas the students have been taught an alternative method or approach.

Your preparation should give you an overview of the topic to be covered from a number of perspectives. As noted earlier it is often said that 'I never understood X properly until I had to teach it', and this expresses the point that you really need to 'own' any topic that you have to teach.

Remember that you are not preparing material for yourself, but for many different people who may vary widely in ability and attitude. You may have to address yourself to a mixed audience and you have to identify what they know and, more importantly, what they don't understand – this is part of the preparation. This means that you have to …

Learn as much as you can about the students beforehand, and develop this knowledge as the course progresses. What are their motivations and interests? What are the strengths and weaknesses of their mathematical backgrounds? Because they are not all like you, you need to….

Be prepared to adapt your personality – be aware of your own personality and how this might affect your teaching. Maybe by nature you are a quiet retiring person who finds it difficult to develop a rapport with people. Consciously be aware of this and try to overcome it – you may have to act a little. On the other hand you may be very outgoing and energetic. This may have the unintended effect of intimidating some of the quieter students, so try to recognise this and tone down a little to help the students feel more comfortable. If necessary mentally prepare yourself before going into the classroom (as we sometimes do when we go into an interview or viva). If you are able to show the students that you care for their interests, and you do a good job of teaching, then they will be tolerant of most of your eccentricities and foibles. Two particular personality traits that must be avoided in any teaching situation however, are impatience and sarcasm.

Exercise

Plan and prepare your next teaching event/activity, thinking of the principles described in this section.

Chapter 2

Running exercise classes

2.1 Exercise and problem classes

One of the tasks most often assigned to postgraduates is supporting students in working though exercise or problem sheets. Many departments simply refer to these as tutorials, but this term has a slightly different connotation in other subjects where they would hardly be recognised as such. One could distinguish between exercise and problem classes, even in mathematics, for such classes can serve at least two main purposes. Firstly, in mathematics it is a fundamental pre-requisite in any topic that a high level of facility is required in some basic knowledge and technique. In studying differential equations it is impossible to progress without high facility in integration. The student has to be able to choose the appropriate method and apply it quickly. This level of facility is only obtained by much practice, and this is usually provided in the form of copious exercises, starting easily and routinely, escalating to tougher and more challenging examples. Such practice is essential and unavoidable, and one of the main objectives of tutorials in mathematics is to allow students to work on this together, with help provided by the tutor. This where you come in.

As well as developing these basic skills by practice, the well-educated mathematician also needs to develop more sophisticated skills of problem solving and even problem identification, tackling extended problems of substantial difficulty, using a wide knowledge and skills base. We might say this is actually what mathematics is all about. This sort of work, although related to routine exercises, requires different teaching approaches and in general is more difficult to tutor. But tutorials are used for this purpose in mathematics and you may be called upon to deliver them. In fact, the skills and principles required in tutoring either type of class are fundamentally not that different and here we will assume we could be referring to either duty.

Ensuring that students get the best out of their exercise classes is one of the most difficult jobs in teaching. You bear the responsibility for actually encouraging the students to engage with the activity, and for assisting them in the most effective way when they experience difficulties and require support. It can also be quite intimidating entering a classroom where a, not insignificant, number of students are waiting expectantly for you to solve all of their problems. In a lecture you are out front, in control, knowing what you have to say, safe behind your notes, or the projector, it being expected that no one is going to interfere with you too much or ask you many questions. In a tutorial you are in amongst the students, have no idea what you will be called upon to do, highly visible, and the subject of high expectations, all coupled with the capability to disappoint! In reality, it really isn't like this. Provided you have prepared well it is likely that you will be able to do a good job and gain the confidence of the students; then you can relax and you will find it becomes much easier.

In Section 1.1 we gave an example of using **MATHEMATICS** to remind us of the issues we need to consider when delivering problem classes, so this might be a good time for you to revisit that example.

2.2 Preparation for exercise classes

General

The general principles of preparation outlined in Section 1.5 also apply here. The moment you receive the relevant exercise sheet you should start preparing for it. Make sure you understand the purpose of the class, what the exercises are designed to achieve, and your role within it. Precisely how is it intended to change the students' knowledge of the subject? Will you be running this class for a number of years, in which case the time you spend on preparation will be useful later. What are the constraints, how much time do you have, can you give out specimen solutions, and will the students be adequately prepared? There is little to do in preparing the **media** in this case, essentially the exercise sheet, and your engagement with the students is all there is. But there will be a number of key **messages** to convey, what are they and how do you do it in the most efficient and effective way? Read and consult widely on this, what are the different possible approaches to the problems, where do the students usually have most difficulty, what can you leave out if necessary? Try to obtain an overview of all the exercises quickly, then iterate, deepening your understanding of the subject matter and the needs of the students as you do so. Are there significant gaps in your knowledge and experience, do you need to talk to the lecturer to get some essential background? Do you really feel on top of this subject, in command and relaxed about any likely questions? Do you know the students and their background well enough, can you talk their (mathematical) language, and develop a rapport with them - that is, are you as expert in the students as you are in the subject? Do you need to adapt yourself to an unfamiliar environment – perhaps you are a pure mathematician assigned a group of business studies students on a quantitative methods course, the mathematics is straightforward, the challenges to your communications skills are extreme.

As you can see, this is about a lot more than just ensuring you can successfully complete all exercises on the sheet, this is the easy part. In order to proactively lead students in an exercise class you have to have complete mastery of the subject and related areas, and be thoroughly familiar with the students and yourself, including any limitations. We will now focus on some specific issues of preparation for exercise classes.

Doing the exercises the students' way

Probably the first thing you will do when you receive the exercise sheet is to work through each of the questions in turn. Some you will probably be able to deal with quite quickly, the harder exercises may take more time. You may be given worked solutions, and be tempted to quickly look through these checking you understand them. This is necessary, but not sufficient for preparation. You need to understand how the students will attempt the questions, and the sorts of errors they are likely to make. Then you need to know the different possible ways of tackling a particular problem and how to link them together, all in case different students take different approaches.

Examples

1. The students may be asked to minimize the function $x^2 + 2x - 3$, and your first thought may be to use differentiation. But it may be that it is intended they use completing the square – perhaps they haven't reached calculus yet. However, some of the student's have done calculus before and they may also go for differentiation. You will need to be perfectly familiar with the completing the square approach, and also be able to guide those students who differentiated, explaining why their correct answer is not actually what is required, and why it is so important that they actually fully understand the algebraic approach.

2. There may be a question on resolution into linear partial fractions. There are at least three 'different' ways to undertake a simple partial fractions decomposition – you will need to know them all and be able to explain the connections between them and their relative advantages and disadvantages. If they are required to develop high levels of facility and speed, then you may need to emphasize the cover-up rule for example.

3. There may be questions on integration by parts. Many first year students will laboriously use the formula, but again if they are required to develop high facility and speed then this is unsustainable. You will have to wean them off the formula and get them doing this automatically.

From these examples it can be seen that you actually being able to **do** a particular problem is only the first step in the task of helping the students, and a set of worked solutions is unlikely to cover all such issues; you will need to read and consult widely. Also note, in the above examples it is absolutely essential to be clear about the purpose of the class beforehand so that you can plan your approach accordingly.

Know the students

The students really are a subject of study in themselves. As we often emphasize, they are not necessarily like you, and indeed in other circumstances you might have no wish to spend any time at all with them. Even in a small class of a few students, there will be a whole range of individual personalities, and you need to be able to communicate effectively and efficiently with them all, without fear or favour. Learn as much as you can about the students beforehand, and develop this knowledge as the course progresses. What are their motivations and interests? What are the strengths and weaknesses of their mathematical backgrounds? There is no harm in a little social banter to develop rapport. Do they have any special concerns about the course? Such knowledge will help you plan and prepare your teaching, both the material and its delivery, accordingly.

Getting to know your students not only refers to mathematical knowledge, but perhaps more importantly to their attitudes and personalities. Some will be very engaging and (attempt) to take up large portions of your time. Some will verge on the obnoxious and not be a welcome experience, others will not wish to trouble you with their queries or concerns. You have to be prepared for each possible eventuality and think ahead about how you might handle the different situations. Think of some mechanism by which you will give students roughly equal amounts of time, whatever you think of them, perhaps methodically working round the class – is this feasible? Think of some way of responding equally to all requests for help, from any of the students. One way is to smile and look pleased at being asked, even if you are not! It is part of the teacher's skill to rise above any negative feelings for a particular student, and to give all students a fair deal regardless of personality traits. If a student tries to dominate your time, don't let them; spend a fair amount of time working with this student but point out that you need to work with others – you need to divide your time and attention equally amongst the entire student group.

Know yourself

You are about to go into a classroom, possibly to be inundated with questions about a topic that doesn't particularly interest you. You may be a naturally impatient person, soon becoming irritable when asked what you perceive as stupid questions. If you can't control such feelings in a teaching situation, don't take the job. Think beforehand if you need to do something about this, do you have a weakness in this respect? Then you will have to make the effort to remain controlled and rise above it all. Think nice thoughts and relax! Fear is good – if you are irritable with the students and that interferes with helping them, they will be dissatisfied, they will give bad feedback, and you may lose the job and the income. Oh, and of vital importance, although the question may seem stupid to you, to the student it is a genuine difficulty and as such you should treat it with professionalism and respect.

Perhaps by nature you are a quiet retiring person who finds it difficult to develop a rapport with others. As noted earlier, try to overcome this and develop ways in which you can be a bit more outgoing with students. Honesty is good here. As long as you care for the job and the students, they will accept you as you are and if you are a bit shy they will still find ways to get you to assist them.

So, if necessary, mentally prepare yourself before going into the classroom. Focus on the job in hand and adapt yourself to do that professionally as best you can.

Preparing Exercise Sheets

It is unlikely that you will be actually preparing exercise sheets or similar materials yourself; indeed this it is a quite sophisticated task with many factors requiring consideration. If you do have to do this then you can find more details in **TMHEBB** (Cox, 2011). Here we will give you an overview of the sorts of things involved.

A given exercise or problem sheet will have some particular objective – to develop facility in integration say. The students will have some known background, perhaps they have done no integration at all, but have a good grounding in algebra, trigonometry and differentiation. Then the exercise sheet should commence with a number of straightforward and routine problems with a low level of difficulty, building up gradually to more challenging problems requiring more sophisticated techniques or multiple applications (**stepladdering**). For a single session there should be only a small number of key ideas to convey, say facility and recall of standard integrals, simple substitution, and integration by parts. In this case, plan the session to give the students the opportunity to do something in each of these areas, with the expectation that they will complete the remainder of the exercises in their own time. If students have to complete such exercises in their own time, try to ensure these build upon material that has been covered during the session, or previously, rather than expecting them to engage with entirely new concepts; this is certainly true for first-year students or non-specialists. Note, this is different from asking them to apply the mathematical concepts to new contexts – this is entirely reasonable and essential for helping develop their problem solving skills.

You should be able to obtain examples of and ideas for such exercise sheets from colleagues, other courses, and textbooks. But wherever you obtain them, make sure that you rework them and customise them for you and your students It is not unknown for people to set problems from a book only to find later in the tutorial that they are in fact inappropriate or, worse, they can't do them themselves! It may be appropriate to set some previous examination questions, indeed you should be prepared to do this at some point otherwise the students will inevitably ask, but again get used to them yourself first.

Any exercise sheet should be **accessible** by all students in the group. This is not to say that every student will be able to answer all questions, quite the contrary, but there should be a substantial portion, say three quarters, for which every student can at least make a reasonable attempt. There should also be a range of types of questions. Some may be routine, single step questions (often called **objective questions**) to reinforce basic facts. Then there will be some longer multi-step questions requiring the assembly and use of a number of techniques within the topic. These may 'walk' the students through the solution by posing each step as a leading question, or they may provide no such clues and expect the students to navigate their own way through. Some may be synoptic questions, bringing together a wide range of ideas from across the course. A few can be very hard indeed, to keep the more-able students occupied, but let the students know that this is their purpose, and they are not all expected to master them - the idea of the exercise sheet is to develop the basic skills. In general, questions should increase in complexity; if the easier questions are at the end, then some students may not even look that far if they experience difficulties with the first question.

Exercise

Prepare your next exercise class, out-lining an initial lesson plan, with objectives, a list of the materials you will need, and some sample exercises.

2.3 Starting the session off

Take charge of the session

If you are sharing delivery of the session with other postgraduates or the lecturer, your role is still one of leadership and responsibility. If you are on your own, then you are the one in charge, and so it is you who has to run the class. Handing out the problem sheet and expecting the students to get on with the problems, asking you for help when needed, is not enough. You have to make it clear that you expect the students to work hard, that you are anxious that they learn the material, and will do your utmost to help them in this process. Make sure everyone knows who you are. As a recently graduated student yourself there may be little to physically distinguish you from the students in the class – you may be just as fresh-faced and unsure of yourself. Nevertheless, you are the teacher, they are the students and however you do it, it must be clear to everyone that you **are** the one in charge.

Commence in a business-like way

Don't be reticent about the job to be done. Take it seriously and make sure the students do as well. Ensure that the students fully understand the purpose of the particular activity of the session and that they know what they have to do. Set them a target, after a few minutes you might ask 'Has everyone done the first question yet?' Don't take yes for an answer. Some will be staring at a blank sheet. Don't allow it: talk to them, why can't they start? Move around, make clear from your body language and presence that you expect results, but you are happy to help out when anyone experiences difficulty. Your actions here are critical – help those students who experience difficulty, either on a one-one basis or in small groups; this will demonstrate your commitment to helping them learn and they will genuinely appreciate your efforts.

If there are any problems, for example someone hasn't got the exercise sheet, then you have to deal with such issues quickly – is it reasonable for you to provide copies? Is it unreasonable to expect them to bring all necessary materials to the session? You decide – but it is certainly not unreasonable to expect a student to arrive at a teaching session adequately prepared. If any students are being a nuisance in any way by engaging in disruptive behaviour, of which there are many degrees, then you have to address this quickly and effectively. We say more about this in the context of lecturing in Section 4.6, but the ideas there apply to all aspects of teaching.

Ensure that they all have the resources required

Some students may not even have brought their class notes – tell them to bring them next time, and if possible alert them before the session about what they need to bring – this can often be achieved using the communication facilities embedded within virtual learning environments; alternatively, tell them what they need for the next session at the end of the current one. Organise them to suit the particular activity – possibly in groups, or in alternate seats in a tiered lecture theatre, so that you can reach all the students.

Indicate some sort of schedule

Many students start with the first question and work progressively throughout the whole session, not getting very far in terms of overview of the topic in the time available. You may need to say something like 'spend 15 minutes on the first 3 questions, then half an hour of questions 6- 9, and finally make sure you spend a quarter of an hour on the last three questions'. This way, they are wrestling with a range of issues, can get your help on each, and can consolidate the details in their own time. Students, particularly in their early years of university need help and guidance to enable them to manage and structure their learning.

Set clear ground rules about orderly conduct of the class

You are responsible for ensuring that there is a good working environment in the class, and without being too formal about it this will normally require some simple ground rules. Particularly in first-year classes some of the students are used to different regimes and this might not always align with what you need or indeed expect. Don't be afraid to be explicit – for example, no eating and drinking in the class, no mobile telephones to be switched on, and certainly, remain focused on the mathematical task. In fact, such things are not usually much of a problem in a tutorial where the atmosphere can be quite relaxed. However, the point is that if there are any essential ground rules then these have to be highlighted at the outset. You may have to keep reinforcing them, but at least the students will have been forewarned. Your priority is to establish a good learning atmosphere, and most students will thank you for this, even if it means being a little authoritarian. You might remind the students that they are paying a lot of money for you to help them and they should make sure they get full-value for it.

2.4 Keeping things going

Keep things moving once you have started the session

Don't sit reading or doing your own work – for the entire session your students are the priority. You will find that **you** have to maintain momentum and push them to work to best effect. This is human nature. You can continue reminding the students that right now is the best time to address any mathematical difficulties they might have, while the topic is still fresh in their minds, and while you are available to assist them. It is much harder when they come to revise a few weeks before the examination – an hour spent now will save many hours later. If things go quiet, and you find you are not receiving many questions, ask the class – 'anyone stuck? Anyone finished – do you want more problems?'. Don't simply accept 'yes' or 'no' answers – use these as a means for engaging dialogue; ask how solutions might vary if certain parameters are changed – this really does test genuine understanding. Maintain the pressure in a friendly, kindly way, with the focus on the mathematics to be learned. Sometimes focused questions requiring precise answers can quite effectively break a silence – for example, 'What happens if the integral is over the range $-\pi$ to π, rather than 0 to π?'

Be everywhere

Move from student to student, seeing how they are doing. Be a pest, in the nicest possible way, continually enquiring how they are getting on. You may of course be reticent about this – you don't want to be intrusive when a student may perhaps simply want to be left alone to work. So some tact might be needed, and move on if it is clear you attentions are unwelcome. Students do tend to open up a bit more in individual conversations and then you can find out about any issues you need to address with the class. Moving around the class lets you listen to student conversations, which are hopefully about the mathematics! Don't be afraid to join in or look at what is being written – this is a very valid method of engaging the students. They will appreciate you doing it if it helps them learn and understand.

Help them with entry methods to problems – not specific hints

Starting a problem is very difficult for some students, and they just sit staring at blank paper – don't SHOW them how to start, because all that happens is the next step in the problem becomes a new starting point where they get stuck again. You have to show then how to 'enter' a problem and how to do this for themselves. Tell them to try anything at all – rewriting the question in their own words can help. Get them to write a list of everything they know that might be related to the problem. Have they seen anything like it before – look in their notes. Get them to discuss it with a fellow student(s). Get them guessing and assure them that this is a perfectly respectable tactic in mathematics; if they are unsure how to start just take a guess at it and see if that leads anywhere.

Don't answer a student's or a group's question directly

This makes it too easy. Give an indication as to the next stage in tackling the problem or exercise, but ensure that they have to work to progress. Get them to move towards the answer, and also bring in other students or open up the question for wider discussion if appropriate.

If they get really stuck go through it on the board

There will be occasions on which there is a real sticking point for most of the students, something that is conceptually and procedurally quite difficult. Most topics contain a few such instances. With your expert oversight of mathematics you may not remember or realise what these are. In fact, realising that something is actually difficult when it has become second nature to you is a valuable skill for the good teacher. A minor example we have already met is completing the square. Here the 'adding and subtracting of the coefficient of x all squared' may be a complete mystery to the student who is unable to recollect the identity $(a + b)^2 = a^2 + 2ab + b^2$. When such things prove difficult for a significant number of students then it may be the time to go though them on the board. This requires a new set of skills in itself, which we will say more about later (Section 2.6 and Chapter 4). It is often called **demonstrating**, where you are presenting some clearly defined piece of mathematics, such as a solution to a problem, or an example of a given technique. It is probably the closest you will get to formal lecturing. Here we only wish to highlight that should you need to present something to the whole class like this, then you will need to be able to **explain** it (See Section 2.5) – treat explanation as a skill that you have to learn. It is not easy, and it doesn't matter how well you think you 'know' your subject; if you can't explain it to students then you don't really know it. Be able to explain **sensibly** as well as logically. Explain by listening to student responses and talking things through with them.

A problem may be too hard

If the majority of the students are unable to make progress with a particular exercise then it is likely that it is actually too hard. In this case invent a simpler one, or go through it on the board (see below). In any event an exercise sheet should have problems gradually increasing in difficulty – 'step-laddering', which we discussed earlier. Few classes you teach will be masterclasses, most will have a fairly wide range of ability and as such you will need to help students develop their skills gradually.

Make sure that everyone is engaged

Don't concentrate on one particularly vocal group, gently bring in any shy or perhaps isolated individuals and encourage them to help each other. Perhaps you might go through the first question of a particular type at one or two points in the session (but see below). Use any particularly outgoing individuals in the class (politely and in a friendly way of course) to help develop rapport with students. Be relaxed and friendly with them, while maintaining a respectable and professional stance. This way you are always in a position to keep track of how they are progressing. Some tutors get students to present solutions to the class as a way of involving them. This should only be done if students wish – some may be embarrassed by it or be reluctant to engage. Never force a student to offer a response, but don't be afraid to encourage all students to engage.

Don't let them spend too long on a question

Many students use exercise classes very inefficiently. They will spend too much time on the first few exercises, and then spend time repeatedly doing exercises of the same type. Stress to them that the object of the session is to break as many of their mental log jams as possible, to quickly find their weaknesses and address them while you are around to help. Your students can consolidate lessons learned later in their own time. The students can quickly skim through exercises with which they are fairly confident and get onto the material they find more difficult as soon as possible. They should try these for a reasonable time, perhaps with their classmates, but then bring you in to help out. It is better to give students an idea how to proceed and then move them onto other questions because they are then experiencing a range of problems types in the class, where they have the opportunity to seek your assistance.

Encourage students to use their notes

It is amazing how many students will struggle with a question for a long time without looking at their notes, or a textbook. Often they will ask you before doing so. This is sometimes because their notes are literally a closed book to them. They have written something down from the board, but they don't really feel relaxed with the material and are almost afraid they won't be able to make sense of it. So when they ask you a question, if appropriate refer them to their notes or other material, not in a dismissive 'I can't be bothered' sense, but to get them used to using their own notes as a valuable learning resource. 'Didn't you solve a similar equation to that in the lectures?' 'Isn't that technique used in the proof of Lagrange's Theorem – have a look in your notes'. Some students, particularly first years, find this difficult because in school the problems they tackle are usually very closely related to what they have studied during class. In more advanced mathematics they may have to go back quite a long way, or interpret some of their material in a different way to how it is presented in their notes.

One reason students sometimes have difficulties using their notes is frankly that they are often a disorganised collection of material; they don't really know what is in there or how to find something in particular. When they ask a question, say on the compound angle formula, get them to revisit their notes and write a summary of all the key results and ideas, that is get them to create their own formula sheets or summaries of the material. In general, use every opportunity to encourage the students to manage their own learning efficiently. They need to know what resources they have, how and when to use them, and when to come to you for help.

2.5 Explaining to students

The art of explaining

Explaining is one of the key arts of teaching (Brown, 1978). You don't explain by telling, but by listening, learning, and engaging in dialogue with the student(s). You first have to find out what the students know so you can 'get on their wavelength'. Be sure you understand what 'know' means for your group of students. Ask the typical first year student if they 'know' the product rule in differentiation and they will most likely claim they do, but whether they have the required level of facility is less certain. Having established a common language and starting point, lead the student through the task gradually, dangling the next step just close enough to encourage them to move forward. You may have to break off and tell them to go and think about it for a while. Don't be afraid to let the students follow some unproductive paths – teach them how to backtrack and start again. When they eventually appear to understand, be pleased and congratulate them. Always be ready to praise student work and achievement when appropriate – it will mean a great deal to them and will motivate them to work harder.

In any walk of life, being able to explain things clearly and efficiently is a great gift –particularly so for a teacher. It has intellectual components (for example, knowing the topic well enough to adapt it to your listener), and emotional components (such as not becoming impatient). All of this must be appropriately marshalled when a student asks you for help. When a student asks for help they are ideally primed to learn, you should always try to capitalise on this.

Never use any sort of negative, derogatory or demeaning response to a student's question

At that particular instant your response to a student query is very important to them. If you get it wrong you can spoil the relationship with the students for the duration of the course (and other students you haven't met yet, because your reputation as someone who won't help will soon get round).

Be polite and helpful, but 'don't give away the store'

Be polite and helpful, but you don't necessarily have to give the student what they are asking for – a quick answer, or as Krantz (1991) puts it, 'don't give away the store'. 'What is the derivative of 1/x?' should not be answered by '$-1/x^2$'. **Or** by a curt 'Look it up'. Engage the students, individually or as a group, in conversation – 'What do you need it for, have you seen it before? Show me your notes/formula sheet/book. Is it in there? Is 1/x the only way to write the reciprocal?'. The object is to get the student to answer their own question. Don't think this takes up too much time on one student. Almost certainly students around them will pick up and learn from it (you might notice how suddenly students nearby start to listen to you). Also, the students will see that you are on their side, that you will spend time with them, that you genuinely want to help – this will encourage them to raise their own questions with you. It is about building a supportive atmosphere within the classroom, with which students of all abilities can engage.

Examples

1. This example might resonate as an illustration of how important is the manner by which you explain things to someone. Many of us have had to learn to use a new piece of computer software in recent years. In this we are in a similar position to, say, an engineering student learning mathematics. We want to be able to use it reasonably well, and will invest some effort in learning it, but it is really a small part of what we do and we have so many other priorities. If we get stuck we may ask 'techie1', a computing specialist, for help. He talks way above your head, berates your lack of expertise then wearily sorts it out himself. However good he is as a techie, he is no good as a teacher.

You call in techie2. He listens patiently to your problem, reflects a moment, then leans across, taps a few keys, and it's done. He then lets you follow suit to check you can do it yourself – job done. But is it? He has just got you to imitate him – what do you do if a similar but different thing happens again? This individual is very helpful and has solved your problem by doing it for you, but he has not helped you to learn, he is not necessarily a good teacher.

You call in techie3. She listens to your problem, sees the solution, but doesn't let on immediately. Instead, she asks questions to find out what you already know, how motivated you are, how much time you can devote to this task, what your precise needs are. Then she explains, using language that she now thinks you will understand, and in a depth that she thinks will benefit you most. Your problem is solved – only now your understanding will be much more permanent and portable. She is a good teacher. She appreciates that although you are willing to learn, this topic does not have a high priority, so she has to explain in the most efficient and effective way in the context of what you already know and your possible future needs. You don't want to be an expert, and you are not interested in the finer details – you are not lazy or stupid, you are just busy and in an unfamiliar environment. Techie3 has had to work harder to get onto your wavelength, but she has done the job required. She has also reduced the probability that you will have to seek her assistance again with a similar query.

The analogy with teaching mathematics, especially to non-specialists, for example, is obvious. Being able to explain things clearly, at the level of the student, is one of the most important skills of a teacher.

2. As a group exercise the delegates at one postgraduate workshop were asked to choose a particularly difficult piece of mathematics or statistics, at any level, and prepare a short presentation (5-10 minutes) explaining this topic to a typical first year student in terms they are likely to understand. One postgraduate proposed a presentation on homology groups, which we thought might be a bit ambitious, but in fact he did an excellent job and demonstrated perfectly how even the most difficult ideas can be explained simply by: paraphrasing technical terminology, use of notation, careful use of diagrams; appropriate analogues; providing a range of viewpoints, converging and diverging. At a more mundane, but no less challenging level, another workshop looked at explaining completing the square to a group of engineering students. In this case the explanation is usually made difficult because students are not given a clear view of the key ideas involved (expansion of $(a + b)^2$ and $A - A = 0$) and lack sufficient skills and fluency in the use of these, therefore another key pre-requisite of good explanation is ensuring that the basic key components are fully understood beforehand.

Exercise

Choose a particularly difficult piece of mathematics, at any level, perhaps something you are soon to teach. Prepare a short presentation (5-10 minutes) explaining this topic to a typical first year student so that they would develop a reasonable interpretation of it in their own terms. List the methods you used to construct the presentation

2.6 Working through problems on the board

We will discuss this further within Chapter 4, which is devoted to presenting and communicating mathematics. Here we simply focus on the instances where you might work through a solution on the board, by which we mean a physical space or location where you can write mathematical expressions so that they are visible to the entire group at once; this might be a blackboard, whiteboard, or even an overhead projector (OHP) transparency. Such an approach may be the most efficient way of moving the entire class past some challenging sticking point.

You are not really there to go through the problems

In an exercise class the key point is for students to actually do mathematics as opposed to watching it being done, as they perhaps might in a lecture. You are there to help, not to do the problems for them. However, you might sometimes have to explain a particular question or key point on the board, because most students seem to be experiencing difficulty in exactly the same place in a given problem. Using the board effectively is an important skill that you need to develop and we will say more about this later. Krantz (1991) gives a lot of useful advice on board work in the context of mathematics. Here, we are not so much interested in the technicalities of using the board, but in how you work through mathematical solutions in front of the entire class. The emphasis is not on actually presenting the material, but more on enticing the students to think and work though the material with you, to an extent, telling you what to write.

You are simply the scribe

When working through problems on the board you should hardly do anything other than write down what the students tell you to. They do the work, maybe with a few leading questions from you. There is no harm in following suggestions that you know are not necessarily leading in the right direction – a lot can be learned from understanding why something doesn't work. After all, this is precisely how real mathematicians work. But make sure they understand this is what you are doing, and explain to them why the suggestion didn't lead anywhere, and that you are going to backtrack to try another route – which they have to again suggest. While it is a valid learning approach to allow students to explore an unproductive path towards obtaining a solution, you should never write down anything that is mathematically incorrect. If a student makes a point that is mathematically wrong, and other students don't correct it, it is your responsibility to do so. Do this quickly, and effectively before moving on – don't dwell on individual mistakes or focus attention upon the student who suggested it, but if this is a common misunderstanding you can generate an interesting discussion to explore why this is so.

One of the reasons you might hesitate to allow students to explore unproductive paths is that it appears to take up considerable, and precious, class time; shouldn't we show them the right way as quickly as possible? Not always. You are teaching them how to think mathematically, not how to imitate. The lessons they learn by their mistakes will be far more valuable than what they can gain from watching you do the thinking. Encourage the students to develop the solution by asking them pertinent leading questions and giving careful hints that require them to think. When you have done it in this rough way, get them to write up the final solution tidily and carefully. When completed, leave the solution on display and as they work through a similar problem keep asking them to refer to it should they become stuck again.

An important feature of mathematics is being able to structure mathematical arguments correctly. To your students, this will mean writing the solutions to mathematical problems or exercises, and is an area where many experience great difficulty. You should ensure that when you write mathematics for students to see, it is done in a mathematically correct manner. By this we mean writing it logically, including any definitions, with an appropriate level of detail so that the evolution of the solution can be readily followed. This is important as it shows students how mathematical arguments should be structured and presented – you will be particularly grateful for this if you are responsible for marking any of their work!

Asking students to come up and work through problems on the board

This is sometimes worthwhile, but only if they volunteer. No student should have to do this if they don't want to. Remember that for many people speaking in public is a great fear, and they are there to learn, not be embarrassed. It might be argued that asking students to work through problems in front of the class is a way of developing their communication skills, but this is only valid if it is one of the learning objectives, the students know and expect it, have signed up for it, and are prepared to do it. If the objective is simply to learn some mathematics, then some students may object – they may not want to learn by presenting to others. They might even point out that they are paying you to teach them, not to do your job for you!

However, there are ways of encouraging this if you understand why students are reluctant. While it is true that some students may readily volunteer to present, and if this is the case, don't be afraid to utilise this enthusiasm, others are reluctant. For many students, the fear of being 'wrong', particularly in front of their peers, is the critical factor in preventing engagement. Why not walk around the room, identify a student with a correct solution, praise them and ask if they will present to the others? They might just agree even if they only write their solution and you need to explain it to the group. If that doesn't work, invite a student to the board to act as a scribe for others – you then moderate the discussion. If you handle enabling such engagement in a non-threatening way, other students will be willing to participate in future sessions.

2.7 Maintaining a productive working atmosphere

Section 1.4 deals with encouraging student participation, and keeping students interested. This applies equally to all aspects of teaching. A problem-solving class is not as difficult to run smoothly as a lecture – for a start there are usually fewer students. However, it does have some special features that we will discuss here. Largely these amount to keeping the whole class on task, and curbing any disrupting or distracting behaviour.

Be clear about your duties, responsibilities and status

We have emphasized this previously, but in an exercise class it is particularly important to be clear about why you are there. It is all too easy to let the hour drift by with the odd question now and again, while you (and many of the students!) focus on other things. Not only do you need to remember why you are there, but also ensure you remind the students of this as well. You may only be a year or two older than some of the students, but don't be afraid to exert authority, if necessary, to get them working. If some students are 'messing around' and not working on the problems, get in amongst them and encourage them to work. This will not only help them, whether they appreciate it or not, but also the rest of the class.

Set ground rules early on and stick to them

It is much better to be firm with the students to begin with and ease up if necessary as the course progresses; it is much more difficult, if not impossible, to tighten up after a relaxed start. You might negotiate some of the ground rules with the students. Maybe the start time is not convenient for all students and a change is helpful to everyone (discuss this with the lecturer first), but then stick to the new start time agreed. Maybe the work of the day is easy and not something they experience difficulty with - perhaps you can provide some exercises in an alternative area where they would appreciate help. Or perhaps the student lack the necessary preparation for the exercise sheet and time is better spent reviewing some previous work. Although we say 'stick to ground rules', there will be exceptional circumstances where you have to be adaptable, but again, you are the one responsible for what takes place during the teaching session. Naturally, if you see the need to depart significantly from the original objective of the class, then let the lecturer with responsibility for the course know in advance before doing so.

Keep order

One of your key responsibilities is to keep order in the proceedings – when students (or indeed anyone) are engaged in free discussion they can easily stray from the task in hand and the situation can become unruly unless some form of order is imposed (you might have been an unruly student yourself not long ago!). Usually humour and an appeal to the students' good sense and courtesy will settle things down – after all they are there to learn and instances of extreme unruly behaviour are fortunately very rare. But if it doesn't, remain in control and politely but firmly insist that they keep to the task set. If this doesn't resolve the problem then consult the lecturer responsible for the course, or a member of academic staff.

Never be rude, sarcastic or derogatory

No matter what the provocation, remain firmly objective and in control. If you are rude or otherwise unpleasant then this will alienate most of the class, and in any case it is bad manners. It is not setting a good professional example. As a young lecturer, in a moment of frustration, one of us sarcastically announced 'You lot are supposed to be the brightest 10% of the population – I dread to think what becomes of the bottom 10%!'. Quick as a flash, from the back of the class, in a beautiful Irish lilt came the retort 'They become university lecturers, sure they do!'. Fortunately this lightened the tone considerably but sarcasm was never again used with students.

Allow leave only for essential purposes

Anyone leaving usually creates distractions. The class has been timetabled and announced well in advance and usually lasts no longer than an hour. An adult should be able to manage their time to accommodate such things – make clear this is what you expect.

Keep on task

As has been emphasized, a typical session will have just a small number of essential and key ideas to convey. You and the students have to keep focused on these and ensure that progress continues to be made towards them. It is your responsibility to keep the students on task, that is, solving problems, or talking about solving problems. However, there is no need to drive students relentlessly for the entire session. A few minutes light banter can refresh everyone and help build up a rapport, but in the end there is an important job to be done, so make sure you know how to bring a group back on task. Stand at the front, sometimes your mere presence will act as a focus. If that doesn't work, speak, or shout loudly enough to attract the attention of the group – or make a loud tapping noise on the desk using a board eraser or other such item. Try mentioning (loudly) the exam or a key mathematical idea they need to know – if students think you are making a point that will help them learn, their attention will naturally be yours.

Be aware of any diversity issues

This might include disabilities, multicultural and language issues. In mathematics we are relatively lucky since it is a universal language – however, it is well known that 'word problems' in mathematics can present special difficulties for some overseas students. You need to be sensitive to such issues and ensure that they are appropriately addressed. This might mean explaining the nature of a particular question again to an individual or small group of students – it could be that they fully understand the mathematical ideas, but not some of the terminology used to describe them. It may be they need support with English language skills – if this is the case, it isn't your role to suggest it to the student, make the suggestion to the lecturer for the course or their personal tutor; they can then follow this up with the student themselves.

In such instances, be careful about stepping outside of your responsibilities and expertise, some of which can be difficult even for the experienced lecturer. It may be that you have a student in your class with a physical (such as a visual or hearing impairment) or learning (such as dyslexia or dyscalculia) disability. Universities have an obligation and responsibility to support such learners and so reasonable adjustments might need to be made to aid their learning. However, the exact nature of these adjustments can vary, and at this stage of your career a member of university staff should not only determine what these are but also advise you on how to implement them for any students within your class.

Chapter 3

Supervising small discussion groups

3. 1 Discussion groups in mathematics

Discussion groups are not often used in mathematics teaching; they are more common in arts or social science based disciplines. In mathematics such a group, normally small, comes together to discuss an extended problem or application, mini-project, or perhaps a particularly extended and important proof. The object is to deepen understanding by debate and interaction, and to sharpen analytical and logical skills. In fact, it is some of the most sophisticated and technical teaching you might be asked to undertake. It is important to realize that of all the tasks a teacher engages in, this is probably the activity during which students can potentially learn the most. It is therefore key, but also very difficult.

3.2 Benefits of small discussion groups

Enhanced motivation

In a discussion group the objectives are usually broader, deeper and often less precise than in a typical exercise class; as such there is less pressure to produce detailed correct work. There is more room for personal opinion (although not too much in mathematics!), and debate can range over a wider area. The students have more control over the proceedings, more personal input and opportunities for direct feedback; they can also follow their own interests much more easily.

A wider range of learning activities

In a properly led discussion, a wide variety of skills are being exercised, including learning by doing, learning by trial and error in a safe environment, learning through interaction, the need to explain and justify your ideas, and more general transferable skills such as communication and teamwork. In mathematics, students will also be constantly justifying their arguments, since many ideas raised can be put to the test. This helps students develop skills of constructing logically based arguments and precise reasoning skills, since ideas raised must be tested promptly and arguments mathematically justified.

Learning through interaction

This is perhaps the prime strength of discussion. It involves all of the key aspects of a learning activity: initiating ideas and lines of thought; explaining them; listening to new ideas; questioning with a genuine desire to know; and, responding in a disciplined, not to mention socially-acceptable, and organized way. Each of these is a skill we should aim to develop to high levels amongst students throughout a mathematical sciences undergraduate programme.

The role of discussion in mathematics

The use of discussion is restricted profitably to specific areas of mathematics. For example, it would not be used for routine topics such as matrix algebra calculations, or for problem solving for which conventional exercise classes are more appropriate. It would best be used for topics that benefit from discursive and argumentative treatment. For example, a particularly long and involved proof; a tough mathematical modeling exercise where you are seeking to model a real-world scenario; a sizeable practical statistics exercise; identifying a wide range of applications of a topic such as quadratic equations or complex numbers. All of these require a broad range of ideas and skills that an individual student might lack, but the group, as a whole, possesses. Members of the group will have to use all their interactive skills to produce the final product. The specific objectives of a discussion group session to which you may be assigned will be provided by the lecturer responsible for the course; they will also provide guidelines on the sorts of activities that will be appropriate for use with the students.

3.3 The MATHEMATICS of small group teaching

We can again do a little **MATHEMATICS** before we start:

> **Mathematical content** is particularly important in small discussion groups – this is not the place for developing routine skills and facility. Rather, discussion is the means by which we delve deeper into conceptual issues and tackle the most difficult ideas of the course. The emphasis is on the students themselves addressing those topics that require really careful and sustained thought and developing their own understanding; debate with peers is one of the best ways of achieving this.
>
> **A**ims and objectives of the session – this will most likely be to deepen understanding of a challenging topic. In a discussion group it is especially important to be clear about the objectives, and remain on track towards them; it is all too easy for a discussion to diverge from its primary purpose or topic. Perhaps write the main objectives of the session on the board as an ongoing reminder to keep the group on task. Note that this is a key distinction between a mathematics discussion group and one in an arts-based subject. In the latter the ability to broaden, extend and explore unanticipated directions is often prized above all else – the quality of the discussion being more highly regarded than the conclusions reached. This is not usually the case in mathematics. Usually (although not always) the idea or concept that is to be understood is present from the outset, but is difficult of access and the purpose of the discussion is to help others towards achieving understanding. There is little point in prolonged discussion about finding the derivative of x^2, but there is much useful to discuss about the circumstances under which the derivative of an arbitrary function does or does not exist.
>
> **T**eaching and learning activities to meet these aims and objectives – the structure of the discussion will depend on the precise objectives – for example, if the objective is to understand a lengthy proof then different people in the group may be assigned different parts with the remit of trying to understanding each and then explaining them clearly and concisely to the others; you, as tutor, would then bring these individual parts together through a closing plenary. If the objective is to understand a difficult concept, such as convex sets (Section 2.5), then an open debate about what the definition means, how it works, why it is useful, might be the optimal approach. You will still, most likely, need to steer the discussion towards the outcomes you are trying to achieve.
>
> **H**elp to be provided to the students – in a discussion your best way of helping might be to do nothing. You are certainly not there only to provide answers to the queries students might have – this removes their reason to think. You may need to gently bring them back on track if you think they are going in the wrong direction, and you may need to bring order to the proceedings. You are

not providing the mathematical ideas directly, but assisting the students to learn and think about the mathematics for themselves. This is one of the most difficult things to do in teaching and one reason why you shouldn't be leading such groups until you have had more teaching experience.

Evaluation, management and administration of the curriculum and its delivery is largely a matter of you evaluating your own performance and learning from the informal feedback you gather during the class and from the students. You may evaluate the materials you are given – maybe the topic and instructions are not ideally suited to the discussion format, or are not sufficiently precise to keep the students focused. Either way, feedback your views to the lecturer responsible. Think about how the session went: what did, and didn't, work well? What would you change next time? How could you better help focus student thinking?

Materials to support the discussion group might include instructions on the roles of different students, the objectives of the session, or any handouts to be used. It might be useful to have a copy of the module specification, and the lecturer's notes on the topic. You might also find it helpful to have an outline plan of the points you would ultimately like students to discuss, so you can steer discussion in productive directions.

Assessment of the student learning may not be directly relevant in this case, but the session could be developing understanding of a particularly difficult topic that might be assessed through coursework or a problem solving exercise. It may equally be a debriefing session on a coursework recently returned by the lecturer – a discussion of student attempts and what might be learned from them.

Time considerations and scheduling are particularly important in a small group discussion because of the open-ended nature of the task. The students will need to work through a number of stages – understanding what the issues are, developing strategies to tackle them, brainstorming and evaluating approaches, drafting and testing conclusions and finally bringing all components together in some form of overall conclusion. If everyone contributes as they should, then this represents a significant work and time commitment, and by its very nature there will need to be time for mistakes, dead ends and frank, if not heated, discussion. As the leader of the group (whether within the group or as an observer) you will need to keep a very tight rein on the schedule, seeing that each stage receives adequate time and attention, all the required skills are given appropriate practise, and things brought to a conclusion in a timely fashion. Either you or the students will therefore need to map out the schedule in advance and keep to it.

Initial position of the students. A group discussion will only really be beneficial if all of the students in the group have a similar background preparation – preferably the topic should be new to the majority of them. Sometimes students are assigned to groups precisely to this end.

Coherence of the curriculum – discussion groups are ideal for achieving the sort of broader overview that establishes coherence in the curriculum. Indeed this might be one of the objectives of the session. It provides time and opportunity for students to take a global view of the subject and the group might survey a range of methods for tackling a particular problem, evaluating their relative merits in turn. It might seek to establish links between different parts of a course. It might probe more deeply than a problem or exercise class can allow into the implications of a particular definition and its mathematical significance.

Students – in a discussion group they are really able to demonstrate their individual personalities, and you have to capitalise on this in a sensitive way. Try to find ways of engaging all the students equally, whether they be introvert or extrovert. Maintain a lively, stimulating and productive friendly atmosphere. It is inevitable that some students will be wrong some of the time, and lose arguments, but you need to ensure that they don't take it personally and that it doesn't ultimately undermine their confidence.

3.4 Preparation for small group discussion

Apart from the general principles outlined in Section 1.5 there are a few specific things we need to consider in preparing to run a discussion group.

Get an overview of the topic of the group session

Naturally, you would do this for any form of teaching, but in a discussion group it is particularly important because the debate may range over many aspects – in fact, this is exactly the point. You need a breadth of knowledge of the topic, so that if a student suggests a particular direction, you can assess how productive it will be. What are the core ideas of the topic? Is it to prove something, what then is the shape of the proof? Is it to explore applications of a particular technique, say integration by parts, and if so, what then are the underlying steps in the technique that can be transferred to other contexts? What sort of questions does this pose for the group as a whole?

What are the key points?

Particularly important in a discussion, because by its nature students are prone to stray off the point, is the need to ensure they remain on task. In the average discussion topic there would normally be at most three or four key ideas, and the session built around ensuring that the students understand each. Unless so directed, students will naturally think everything discussed is of equal importance. Note that this is one other way in which a mathematics discussion group differs markedly from that of a more arts-based subject. In the latter the emphasis is more on the quality of the debate and the outcomes may be wide ranging, potentially controversial and possibly inconclusive. In mathematics on the other hand, there will typically be some well-defined conclusion and the focus is upon the efficiency with which the debate reaches that conclusion. Indeed one valuable group exercise would be to ask students to study a body of work for themselves and identify the key messages, mathematical ideas and concepts.

Identify intellectual sticking points

One valuable use of a discussion is to tackle difficult conceptual issues and to do this you need to have a good idea of such sticking points so that you can steer students through them if need be (only as a last resort, they should be able to sort most things out between themselves, after all this is an essential part of the problem solving process). You may need to design specific activities to deal with these challenging concepts. There shouldn't be too many, but you need to allocate sufficient time for the students to adequately consider each.

Example

In solving a linear differential equation by integrating factor, the step when, having multiplied by the factor, you convert the left hand side to the derivative of a product often causes difficulty for students. This is typically because they are not sufficiently fluent with use of the product rule, and in reversing it. Give each group ten minutes working though some examples of the product rule, gradually working up to the sort of techniques you are using in the solution of the differential equation. This is time well spent. Like learning to ride a bike - when they haven't quite understood it, it seems impossible, but when it suddenly 'clicks', they'll not forget it easily.

Find an acorn for the topic

A powerful tool in developing any area of mathematics is the use of a seminal example, an 'acorn' from which a whole tree grows. Such a starting point is ideal for a mathematical discussion group. It is something students can explore, and the right 'acorn' encapsulates the essence of the topic and is something to which they can refer throughout the session. This provides the students with a familiar 'home page' on which to try out their ideas.

> **Example**
>
> In teaching first order differential equations the simple initial value problem $y' = y$, $y(0) = 1$, provides a suitable 'acorn' on which everything else can be built (Cox, 1996). Find hooks or aide memoires for key ideas – make them easy to remember, and in fact a useful group activity might be to search for such simple devices for remembering particularly complicated mathematical results and ideas.

Ensure accommodation and resources are appropriate,

Once you have decided how you will use the session you need to consider such practicalities as seating arrangements, any instructions needed for the activity, and any materials the students will need to have both at the start of the session or possibly in advance. This is quite a technical matter and may be out of your hands, but don't be afraid to raise the issue if, for example, the room is not suitable. If you don't already know the room then you should always check it out in advance of the session – indeed this is good practice before commencing the teaching of any class. Perhaps the equipment provided is not be ideal for the purpose – maybe the board is too small. In such cases, unless you can get an immediate change, you need to be flexible and inventive about making do with what you have – this requires thought in advance.

3.5 Keeping things moving and engaging students

Set objectives and rules from the beginning

Tell the students what the objectives of the discussion are, the ground rules, and consider assigning duties or roles, including your own, particularly if it is the first occasion you have run such a class - ensure you explain the individual roles clearly to the students in advance. This may range from you being a formal chair leading the discussion, to being an observer or referee, or providing expert insight as a 'witness'. Make sure everyone understands the requirements for a meaningful discussion. This includes timekeeping, disciplined working, and the behaviour expected in a civilised discussion.

Set the background and motivation for the discussion

You may have to set the discussion topic in context. How is it related to the rest of the course? How will the outcomes move them forward? Perhaps a bit of relevant history, an example, reference to the exam or coursework, something directly relevant to the students that acts as a motivating factor. The topic should in any case be interesting or important, otherwise you wouldn't spend time discussing it.

Clear instructions

Time constraints are very important in a discussion. Students need to understand the issue, all have a share in discussing it, make mistakes, backtrack and iterate their ideas – this all takes time. The students themselves then need to bring the discussion together, formulate, and agree, useful conclusions – such a learning experience may be quite new to them. For this, they need clear instructions, possibly on a handout, making clear the outcomes expected and their contribution to the learning process.

Start with easier tasks

This is to get students used to working together to build confidence through readily achieved success. Such tasks can also acts as icebreakers early on in a course to generate discussion and interaction amongst the students.

Provide 'entry methods' to topics

Not 'hints'. That is, general principles students can adopt to get started on any discussion or project. Have they seen a similar thing before? How does it differ from other examples they have seen, and how can they generalise what they already know to suit this new case? Can anyone guess an approach that the rest of the group can explore and apply? If you find it is essential to provide a hint to actually get a group started, then perhaps this implies the task is not expressed well in the first place and needs to be revised. The key point about successful group work is that within any reasonably sized group there should typically be someone within it who can make a start on the task, and make it accessible in order that other students can subsequently engage.

Make sure everyone is involved and engaged

General approaches for encouraging participation were discussed in Section 1.4. In the case of group work the key point is that the group should regulate itself in maintaining a disciplined, productive working environment in which everyone contributes. However, in overseeing the groups you should be alert to such matters ensuring you enable, and indeed encourage, more bashful members to contribute while maintaining control of the more vocal ones in order that they don't dominate the group.

Be only a supportive, positive and inspirational presence

Don't intimidate, intrude unnecessarily, embarrass or threaten anyone. Learn to stand back and let go when necessary. Groups should only interact with you to tell you about progress, or to ask for general advice. Sometimes you might be asked to adjudicate on a disagreement between group members. Then you have to ask whether this is something you should be doing – isn't the point of the group to resolve such issues themselves?

Keep them guessing – literally!

Encourage students to use trial and error and (educated) guessing as a technique for tackling problems. Let them see that making mistakes is fine – just do it quickly, and learn from it. Students can sometimes learn as much from mistakes as they can from the correct approach.

Get students to help each other

This is the key purpose of a small group session, but in the early stages students might be a little inhibited by this. Emphasise that this is exactly what they should be doing. One important role you can play in the groups is as an impartial channel, helping the members of the group to talk to each other initially through you; you can then withdraw and leave them to it. With practice they should eventually develop into a team, sharing duties and ideas seamlessly, practice which will stand them in good stead when they enter the workplace or pursue further study.

Ending the session

Finish appropriately with some form of plenary session to allow groups to feedback on their progress which you can then bring together to summarise the lessons that have been learned. If the exercise was properly structured then similar messages may emerge from the individual groups, and the final result will be all the more convincing. Give a summary for the students to take away, and suggest follow-up work to consolidate and build upon the outcomes, possibly to be handed in later. Describe briefly what you will do next session, and ask the students to bring necessary materials and undertake any essential preliminary work. Make notes for yourself on how the session has gone (this is a form of reflective practice and essential for improving teaching), how you might do better, and any key items you need to follow up with, or for, the students. This is particularly useful if you have promised the students you will follow up the session in some way. It is important to be reliable in such things, it sets a good example to the students.

Chapter 4

An introduction to lecturing: presenting and communicating mathematics

4.1 Introduction

At this stage of your teaching career you are unlikely to be undertaking formal lecturing or be in control of a full course, but you may be working through examples in an examples class, or working through a topic on the board. We will usually refer to this simply as a presentation although sometimes you may hear it called **demonstrating**; think of it as a little less formal than a lecture. You are likely to be presenting something that the students have already been told about in the main course lectures. You may have been given material to use for the notes of the presentation, but it is best if you convey the key messages in your own words. It might require a deeper treatment of a particular topic, a technique, or working though examples. Even so, you should still aim to develop all the skills that a good lecturer needs. Your presentation should be clear, concise, engaging, stimulating and responsive to student needs. If you need further material on delivering a complete course or mathematical lectures, then see **TMHEBB** (Cox, 2011).

In any form of communication we can split it into the media and the message. In your case the media is likely to be 'chalk and talk' – writing mathematics on a board and addressing the group to explain the key ideas. The message is the overall mathematical idea you are trying to convey. This can be more difficult, but also much more interesting. Two lecturers can present exactly the same topic, in the same time, via the same medium, yet one might be completely clear and exciting while the other is unintelligible and dull – let's aim for the former! Remember, the key purpose of a presentation is to convey the most important messages of a topic to the maximum number of students as efficiently and effectively as possible. It is 'prime time' teaching – we therefore need to make it quality time.

4.2 The MATHEMATICS of presentations

Suppose you have to give a presentation on a specific mathematical topic – say completing the square, integration by parts, the ratio test of convergence, Lagrange's Theorem in groups, or the Central Limit Theorem. What do you need to think about?

> **M**athematical content – what exactly do you have to cover and at what level of rigour? Is the emphasis to be on the rigorous proofs involved, or more upon the use of a technique or theorem?
>
> **A**ims and objectives of the presentation – what is the precise purpose of the content? Is it to develop deep conceptual skills with an understanding of the basics of a proof and the ability to design similar proofs for ones self? Or is it to develop facility and speed and the ability to adapt a technique to novel circumstances? Maybe it is simply to motivate and prepare students for a forthcoming topic of unusual difficulty, or a difficult problem solving exercise.

Teaching and learning activities to meet these aims and objectives – different objectives will have different implications for the presentation. If the purpose is to develop facility and speed then there will need to numerous examples, gradually increasing in length and depth. If the emphasis is upon rigorous proof then the presentation has to provide a structure for the proof; include a few examples illustrating and motivating the key steps, a preliminary outline of its structure and a final summary outlining how key conclusions have been reached. Different strategies will have differing forms of interaction with the students, sometimes a dialogue, sometimes setting them group or individual tasks.

Help to be provided to the students - support and guidance. Particularly in your case, the purpose of a presentation **is** to **help** the students, rather than to just **inform** them. The emphasis has to be much more upon participation and interaction than is perhaps usually experienced in a typical lecture. This needs to be incorporated into your delivery. You might actually be presenting for only half the time, the rest is dialogue and interaction with the students, and encouraging them to undertake specific activities for themselves.

Evaluation, management and administration of the curriculum and its delivery. Even if only inwardly, you should be regularly asking yourself 'are the students understanding this, am I making it sufficiently clear?' You can find this out by regularly asking them – not by simply saying 'do you understand?', they will invariably answer 'yes' – pose questions that test genuine understanding. You can explore different ways of expressing ideas, experiment with lines of argument, reordering sequences of development. Suddenly, after giving the presentation a few times, an idea occurs to you on how to improve your treatment of a topic. This is what we mean by real evaluation and development of your teaching.

Materials to support the presentation – You might need a handout for the students or a problem sheet. Perhaps more importantly, make sure the students have the relevant materials for the session. Some students do not like to overburden themselves carrying notes from previous sessions. Emphasize that you expect them to bring such materials to the class in order that you can refer them to something that might have been studied previously. Ideally they should bring all materials related to the course – this will also help you see exactly what prior preparation or information they have received or have access to.

Assessment of the students – perhaps an assessed coursework is due. You'll know this because the students may actually attend the class. The challenge is to help them as much as possible, without simply providing solutions. Or perhaps it is providing feedback on some coursework that has just been marked and returned. You'll know this because some students may not turn up to the class - they just want the mark, yet it is the feedback that is the most important component.

Time considerations and scheduling – always important in any teaching activity. Make the presentation a well paced, self-contained package with a clear beginning, middle and end. Allow time for questions and activities, and for impromptu diversions if anything looks interesting or worthy of further follow-up or exploration. Your actual presentation may take only half of the allotted time, interspersed with intervals of participation from the students. Naturally, always start and finish on time – the students may have another class across campus that they have to attend immediately after yours.

Initial position of the students – their background in mathematics. When you design your presentation your starting point should be one that is comfortable for the students. You might already have a good idea of what the students know that is relevant to the topic, but if not then you have to explore this as you go along. Almost invariably, on average, they know less than you might reasonably expect. Additionally, your students have to know it sufficiently well to adapt to the new situation you might be presenting. For example most first year students will 'know' $(a + b)^2 = a^2 + 2ab + b^2$, but how many will have this knowledge immediately accessible so that they can identify:

$$a^2 + 2ab = (a + b)^2 - b^2$$

when you write it down during completing the square?

Coherence of the curriculum – how the different components fit together. Part of your presentation might be devoted to setting the topic in context – how it relates to previous work, work to follow, or to the course as a whole. For non-specialist mathematics students, try to give an idea of how the topic relates to their 'home' subject(s) and in particular why it is important in the context of their course.

Students – in a demonstration class you will probably have a relatively small group and the atmosphere can be quite intimate and relaxed – an ideal time to develop rapport and good relations with the students.

The above provides a quick checklist for the sorts of things you need to consider in preparing and delivering your presentation. The remainder of this chapter looks at each aspect in more detail.

4.3 Planning and preparing the presentation

For general points about preparation refer back to Section 1.5. Here we cover specific issues arising in preparation for a presentation, assuming you are starting from the beginning.

Whether lecturing a complete course or simply giving a one-off presentation, we need to plan as much as possible in advance as a single entity, rather than prepare each session as we go along. The point is that you need to have an overview of the whole subject before you start. You need to map out roughly what needs to be covered and how the different topics fit together (coherence), and identify what sort of sequence is appropriate – this is not always the same. You need to at least identify the indicative content of each presentation in advance, but you can always modify and reorganise as you go along in response to how things progress during the session. Even though you may think you know the subject well, you will still need time to collect together the many ideas you will have about teaching a topic, to organise them, evaluate their effectiveness and then package them together in a coherent way. It is not only whether you are satisfied with the results of your preparation, but whether your students are. You might be assigned the course at short notice, with little time to prepare, but even then the first step in planning is obtaining a sufficiently detailed overview of the whole area you have to present.

An important point – giving a lecture or presentation is NOT the same as presenting a research seminar or a conference paper. In the latter you are giving a talk to peers who will know as much, if not more, than you do, will readily ask questions if they feel the need, and expect a fast pace of delivery with little embellishment. The purpose of a presentation to students is the opposite – the emphasis is on getting as many as possible to understand and follow you through the mathematical material, and to continually monitor their understanding by eliciting questions, or repeating or reinforcing key points in differing ways. You may also need to present it at multiple levels of increasing complexity to cater for the individual needs and abilities of students within the group.

Aims and objectives

The first thing to do is broadly decide what you have to cover to meet the aims and objectives of the session, that is, precisely what are the students supposed to obtain from your presentation? The aims and objectives of a presentation should: make clear to the students what will be expected of them; fit coherently within the context of the course so the students can fit the new material into what they already know and can appreciate its later role or purpose; progress at a reasonable pace in terms of both timescale and intellectual demand;

take account of the mathematical background of the students, that is your starting point should be at a level accessible to the majority of the students.

Example

For example, if you have to teach integration by partial fractions, are you sure beforehand that they are sufficiently skilled in resolving into partial fractions? Do they have the right level of facility and speed? If not then you might need to provide some revision in this area before commencing the primary task (integration by partial fractions).

In planning the teaching and learning activities for a given presentation we might need to think about:

- The resources available.
- The abilities of the students and their mathematical backgrounds.
- Alignment with the learning objectives.
- Methods of engaging the students in learning for themselves.

The objectives of the session will comprise some essential ideas and techniques that it is vital to convey. You could probably present these to a colleague in five minutes with little trouble, but here you have to not only explain to the students what they need to know, you also need to help them assimilate the ideas and understand the key points. This is where having a teaching strategy comes in; each of the basic points may need a substantial amount of explanation and activity which you have to build into the delivery of the presentation.

Example

Having stated a key theorem, you may need to spend some time motivating it, explaining its implications, outlining a 'common sense' approach to its proof, and making it seem reasonable. You might then need to give the students a few minutes to discuss it amongst themselves, or maybe think about the proof. You might then move on to the proof itself; if it is technically detailed and complicated you might present a specific example alongside it. You have to think about the best means of conveying your key messages to your students in the allocated time available. For this you need to plan your lesson carefully – you need a **lesson plan** which, starting from the aims and objectives, maps out the different teaching tasks you will use and the learning tasks the students need to undertake to achieve these. This may range from a few succinct reminder notes for yourself to a more detailed schedule of the lesson – the level of detail is entirely up to you.

Possible learning activities during a presentation depend upon the type of lesson you are delivering but include: presenting new material; recapping or summarising previous material; and, demonstrating examples. The activities that work best will be determined by the aims and objectives of the session. However, you also need to make use of activities that work well with your group - obtaining direct feedback from students is the best way of determining if an approach works effectively for them.

A good approach is to structure your lesson so that the following elements are incorporated:

- A concise and attention grabbing introduction preferably providing motivation for the topic.
- An overview of the key points to be covered and their direct relevance.
- A detailed but well paced and succinct explanation of several specific key points using clear examples.
- Opportunities for student engagement.

- A closing summary of the key points, with appropriate references or directions for follow-up, including further work the students might undertake in their own time.

If the presentation is of significant length (over say twenty-minutes) then remember that the attention of students varies as the session progresses. You may need to convey the key points when they are most attentive, at the start of the session for example or right at the end, and provide short breaks or changes in activity; additionally, ensure you factor in sufficient time for student questions.

Preparation of materials for a presentation - media

In a typical presentation the main materials that might be needed could be:

- Student notes, either transcribed from your presentation, in a handout, or electronic file.
- Problem/exercise sheets.
- Occasional administrative materials such as timetables or course handbooks.
- Ongoing work such as coursework, solutions and any feedback.

There are two aspects to such materials – the **media** (paper-based, electronic, etc.) and the **message** (actual mathematical and intellectual content), which we consider separately. The advice on media is mainly routine technical points, while the message is by far the most difficult, being the crux of good teaching.

The list below summarizes the sorts of things we have to think about when preparing teaching materials for a presentation session:

- What the students actually take away from the session – notes from your board-work, handouts or problem sheets, and how these will help them to better understand or master the topic.
- The clarity and conciseness of the materials and the style of presentation, consistent with efficient explanation and ease of understanding.
- If you use handouts make sure you do not overload the students, and remember that if you issue them before the presentation then students might not attend the session itself!
- The volume of learning materials. Too little and students may not have enough to follow the ideas. Too much and they may feel there is too much to learn. In a presentation it is usually the case that 'less is more', certainly in terms of materials that the students take away.

Design of learning materials for a presentation – message

In preparing learning materials we always need to be think about how they will look to the students, how they will help and support them in better understanding the key ideas. This is difficult. When we write something such as course notes, a handout, or a problem sheet for someone else we have to view these from the student perspective; think about what they will make of it. When the audience is a class of students this means assessing their current state of knowledge on which you have to build. What sort of language will they understand? You will need some mental image of your audience that enables you to address yourself to them in an accessible way. Below we give some ideas on how to do this. The learning materials should:

- be appropriate to the needs of the student and their level of understanding of the topic. This requires research into their background and discussion with colleagues, as well as with the lecturer for the course. Remember that what the students actually know, and can do, will vary widely and you have to judge the appropriate starting level. Err on the low side initially, and be prepared to adapt as the course develops if need be.
- support the achievement of the objectives of the course, which should be clearly emphasized and continually referenced.

- highlight or reinforce the key ideas, of which there should only be a very small number for any single session.
- provide 'hooks' for students to develop ideas – aide memoires or mnemonics, to make it as easy as possible for them to remember and internalize key results and ideas.
- anticipate the difficulties students might encounter and support them in dealing with these. Whenever you have to deal with a difficult piece of work, think about the best way to present it and express it in this form in the notes and associated materials. This is usually the appropriate place for incisive examples, and maybe a short exercise for the students to attempt during the presentation.
- highlight the essence of complicated ideas, theorems or methods, and support the development of students' intuitive views of formal arguments. As we approach a substantial topic or proof, the materials should build-up to this, preparing the students and identifying prior results they might need. There is no harm in warning the students that they are approaching a difficult topic and outlining roughly, in straightforward terms, what it is all about, and why it is mathematically important to them.
- provide 'roadmaps' and overviews of difficult sequences of arguments. Demonstrate how two topics might coalesce, and provide examples that enable students to work through multiple aspects of a topic. Having a key example to which students can continually refer is particularly valuable for the most challenging material.
- include devices and techniques for developing a rapport with the students, and proactively involving them in the learning activities. Successful learning is a two-way process, the students themselves have a critical role in managing and progressing their own learning.

Example

Suppose you are giving an introductory session on Group Theory that the students are seeing for the first time. The key ideas in this case are the four group axioms – you will be repeatedly referring back to these when you undertake straightforward proofs such as uniqueness of the identity. You might have a separate overhead transparency, slide, or part of the board where you display these axioms, so that you can continually refer to them. But there is also another key idea here, the actual motivation for the group in the first place, a holistic view of the axioms. You may use the application of symmetry, or you may emphasize that a group is the simplest non-trivial structure in which we can solve equations. Either way, this is a key idea that we can continually highlight and we can use the learning materials to help us. Aim to identify the key ideas and ensure the learning materials are built around those.

4.4 Giving the presentation

Introduction

For many of us, particularly early on in our careers, this can be something of a trial. The following might help you prepare, but it may be a long time, if ever, before you feel entirely relaxed when you go into the classroom. It will help in your early presentations if you can discuss the material below with a more experienced colleague or lecturer, or your fellow postgraduate tutors (why not schedule informal meetings to discuss teaching issues, much as you might do with your research?), who may be feeling just like you. Your department may provide a mentor for this purpose, but if not, try to find your own.

In delivering a presentation there are a wide range of factors to consider, even when we have prepared all the materials we need. First there is the basic classroom technique including establishing a proper professional demeanour, running the session to ensure objectives are met, structuring it for maximum impact, and starting and finishing appropriately. There are the technical issues of using the various resources in ways that will help students to learn and the very important task of producing the right atmosphere in the classroom – creating a warm, relaxed, but industrious environment in which students want to learn, enjoy the activity and feel free to get involved. Last, but undoubtedly the most important, there is the task of ensuring that students are engaged in effective learning. As with all such things, the first presentation is usually the most nerve-racking – not just your very first presentation, but the first one with a new class. These factors are so important that we will treat them separately, at the beginning. Following this, the section is arranged as follows:

- Basic classroom teaching and learning technique,
- Use of resources,
- Maintaining a conducive learning environment during the lecture,
- Supporting student learning,

The first session

The first presentation is often the most important, not necessarily in terms of content, but since it sets the standard for those that follow. Students will often feel some apprehension at meeting you for the first time, and you might be apprehensive about meeting them; it is important to create a relaxed classroom environment that makes you all feel at ease. Visit the classroom you are using before the first class and familiarise yourself with the room layout, and how lights, blinds, and any audiovisual equipment works. See what teaching resources are to hand – do you have a board on which to write mathematics? Is there an overhead projector? If not, you need to either think about how you will approach teaching your class, or try to identify a more suitable room. You can also take this opportunity to check the acoustics and practice writing on the board to ensure your writing can be read easily from the very back.

On the first day of class, arrive at the classroom early and finish a little early; you can greet the students as they arrive, and deal with any questions after the session. Introduce yourself – students at least need to know your name! Since this session sets the tone for the remainder of the course, you should start and finish on time, encourage questions, and provide students with the opportunity to speak and engage. Don't rush off at the end, stay behind a few minutes for any students that may wish to speak with you privately.

There may be some first day 'housekeeping' tasks to do, such as writing up details of the course, key dates, room changes, or your contact details for example – here decide how you wish the students to contact you outside of the class. This may be to visit your office at a particular time during the week, or to contact you via email. You might tell them something about your background and let them know that you are approachable. Do not fear offering an open door policy, because very few students will take advantage of it anyway. Hand out any course documentation, such as the course syllabus or a reading list. If appropriate bring along a copy of the recommended text and tell them where it is available. Make any health and safety announcements that are appropriate, ensuring you yourself know what to do in case of a medical emergency or a fire alarm. Set the ground rules for your sessions explaining them clearly. These may relate, for example, to lateness, talking in class, or eating and drinking. Be sure you know departmental policies on such matters, but if none exist adopt those you think are reasonable to make the class work well. Note once again, here it is not so much whether you are satisfied, but whether the students are happy with the arrangements. You may for example be able to ignore a couple of students chatting to each other at the back of the room, but the students close to them will probably be very irritated and look to you to control their behaviour. You may have to take an attendance list at the start of the class, find out what your departmental policy is and ensure you follow it.

Basic classroom technique

Here we will consider the technical aspects of delivery – the media. In fact, you are probably the most important part of the media, so included here is consideration of your behaviour and your control over it. Teaching is a matter of human interaction. As well as the professional responsibility to do a good job technically, the teacher also has to cope with the extra demands of the human side of teaching – remaining detached in difficult situations, setting an inspiring example, and treating all students equally despite any personal feelings towards them.

Time and again, students rank enthusiasm highly on the list of qualities they like to see in their teachers along with providing a good set of notes. We have covered enthusiasm in Section 1.4 as it applies to all aspects of teaching. In the context of a presentation however, it is particularly important and sometimes difficult. You have to allow time and plan specifically for maintaining enthusiasm and interest. Continually monitor yourself to check that you don't slip into monotone delivery to the board or your notes. Having already identified the key messages during your preparation emphasize these by your verbal and body language, keep revisiting them from different directions (quite literally – move around the room!) and motivate them.

Teaching can be quite stressful and difficult. You need to be aware of the problems this may bring and have the confidence to deal with any issues or challenges. Good preparation will help your confidence levels, but it is likely that problems will arise, you will make mistakes, you will be uncomfortable with the behavior of some of the students, and you may have to revise your plans as the teaching session progresses. Such things may get to you – but usually things are not as bad as they may feel. Try to maintain a sense of humour, and never lose your temper or control. Learn to not let errors fluster you. Being nervous is acceptable, but learn to control it, and certainly don't show it. Talk to colleagues about this, you will find they will most likely have had similar concerns and will probably have anecdotes that will put your problems in perspective; they will certainly be able to offer advice.

Remember that everyone makes mistakes. The real question is how you deal with any mistakes, and how you try to minimize them. Some common difficulties that can arise include:

- losing your train of thought during a long sequence of mathematical arguments.
- minor errors such as missing a sign or writing the wrong symbol.
- saying something different to what you actually write.
- omitting a crucial step because of your high (or maybe low!) familiarity with the material.
- going off in the wrong direction or being unable to solve a particular problem yourself.

Usually the students will identify such issues for you and so be attentive to this – rather than seeing this as threatening or embarrassing, it is extremely positive as it means they are paying attention to what you are doing. Be alert to advance warning signs, such as students suddenly starting to talk to their neighbours – sometimes they are just checking if you really have made a mistake before raising it with you. Sometimes a number of students may shout out with queries or concerns at the same time, and you can't understand what they are saying. Take charge, quieten them down, and ask one of them to make their point scanning your work as you do so; if this doesn't locate your error (it nearly always does), ask for one student to point it out, thanking them for their contribution. If you haven't made a mistake and they have misunderstood, explore where the confusion has arisen and re-explain the mathematical point you originally made in an alternative way. When you re-read your writing on the board you usually read what you think is there, so if students have pointed out an error, try to read more carefully A few errors, corrected, at least demonstrates to the students that you are human – but you need to ensure there are no errors in key ideas and concepts. If you cannot address the error there and then, and sometimes this happens, leave it, address it out of class and let the students know the result at the next opportunity. If you have prepared for your session appropriately, mistakes should be restricted to typographical errors which are usually straightforward to correct.

To ensure that your sessions run smoothly and encourage productive learning, you will need to insist on good, responsible behaviour from the students. The best way to achieve this is to set a good example yourself, being punctual and proper in your approach, in particular how you address and engage with the students as individuals; treat all students with respect and courtesy. Students are people first, mathematicians second (or third, or ...) and so don't let your feelings for them as mathematicians affect your treatment of them as individuals. Building a rapport with students can help make the class enjoyable for both you and for them; treat them as you wish to be treated in return.

A common description applied to the good teacher is 'hard in the head, soft in the heart'. You have to set high standards of behaviour, thinking and debate, particularly in mathematics, but you also sometimes need to be generous and give the benefit of the doubt. This does not mean compromising on quality or standards, and it is perhaps not so much about being 'soft' on the students, but being hard on yourself. There may be many occasions when you get frustrated, irritated or impatient with some of the students and it is then you have to guard against inappropriate behaviour – you should certainly never actually display any of these emotions to the students, as a professional you have to rise above such things and maintain a professional demeanor at all times, regardless of what else is taking place within your life.

As Krantz (1999, page 42) argues, you are the most important thing in the lecture room – and you are a person interacting with other people. But you are not interacting in the ordinary sense of people interacting with others say in a social or work context. Here interacting to produce the best environment for learning is actually part of the job – in some sense you are actually acting. You have to adapt your personality and disposition to the needs of the job and continually monitor yourself to remain fresh, self-possessed and alert. You can, as much as possible be yourself, so you can relax, and help your students to get to know you. Avoid waffle and don't be patronizing. Take charge of ensuring that the students have maximum freedom within an enlightened educational environment. The most important attitude a lecturer can have is to be on the side of the students, to help them achieve their goals, and to keep encouraging them.

Starting the presentation

The way you start the session can set the mood for the class. You might settle the group down with light small talk, give time for the (slightly) late arrivals, or get administrative matters out of the way. When you do actually start do it clearly and obviously – that is, command their attention and concentration. Then start with a brief outline of the purpose of the session, perhaps recapping previous work.

Example

When discussing the Factor Theorem, you can remind the students of previous examples of factorizing simple polynomials such as quadratics and some cubics by inspection, and note the limitations of these methods. Then emphasize the fact that the factors indicate precisely where these polynomials vanish, and indeed to find such points is one of the purposes of factorization. One can then point out that by reversing this process we can devise a way of factorizing more complicated polynomials, and this leads naturally into the subject of this particular demonstration. In doing this we have reinforced previous methods of factorization, and established a context for the new work on the Factor Theorem.

Note that there is a slight danger when announcing up front what you are going to do in the lecture. Some of the students might think they have seen it before and mentally switch off which is the last thing you want.

Example

Suppose the lecture is to first year mathematics students with good A-level grades and is devoted to integration using partial fractions. Now many of the students will have seen this before, will tell you this, and may be impatient at you 'going over A-level stuff'. They might not see the purpose of the session. But the real purpose of the presentation is actually to consolidate and extend their skills and speed up their performance, possibly introducing them to quicker or more efficient approaches to techniques they have met before. This is what should be emphasized to the students at the outset of the session.

They may have met partial fractions before, but it is unlikely that their skills will be as highly developed as they need to be.

Ensure satisfactory delivery

This includes such things as how you speak, how you write, and how generally you present both yourself and the mathematical message to your students. We have to ensure that what the students take away from the session will help them to learn the topic. This usually translates into providing them with a good set of notes and instructions for future tasks. This doesn't mean you must provide formal handouts, but it is a well known fact that most students will invariably copy whatever you write on the board; use this as the basis for good notes that they can take away and use in their own time to follow-up on the ideas and concepts. Use sectioning, headings, and other forms of formatting skillfully. Link what you say and what you write considerately to improve the quality of the notes you provide – identify key ideas and use appropriate notation.

You need to ensure that the students can see and hear whatever you do. If you have concerns over your voice projection, use a microphone. You can enliven your delivery with intonation, emphasis, and body language. Face and talk to the students, establishing eye contact with them. This will help you to see what is happening during a session – if you glance at a student, they may give you some indication, such as a nod or a shake of the head, that indicates whether or not they understand what you are saying. It can also indicate whether they are becoming tired or distracted and you need to vary the pace or engage them in an interactive exercise. To emphasize a key point pause, say and write it out clearly, tell the students it is important (and why), repeat it, ask the students if they have any questions – ensuring these are appropriately addressed before moving on. You may want to reference where the mathematical ideas will be needed later, either in the module, their course, or in everyday life.

Talking in front of a class of students is completely different to an informal conversation with friends, and so it really is a good idea to think about this before you enter the classroom. Some important points to consider are:

- Go to the classroom before the lesson starts and chat informally with students. This helps to build a rapport and also enables you to gather feedback on how the sessions are progressing. It also provides the opportunity for the students to advise you on areas where they would like guidance – you can then tailor the session to address these and students will then thank you for taking the time and trouble to respond to their mathematical needs.
- Keep your delivery style as conversational as possible and avoid what might be perceived as a patronising tone. Vary the pitch and tone of your voice to emphasise what you are saying. Try not to use a constant or monotonous tone – your voice should indicate the really key or fascinating concepts.
- Vary the pace at which you speak. Students need time to make notes and absorb the information. Deliver key points and principles slowly and deliberately. If you find you are talking too fast, recap any key points you have made – it never hurts to make these twice. If you think you have talked for too long and students are losing concentration, take a short 'refreshment break', or initiate a change of activity. If you do take a break – make it clear as to when students should return and ensure you restart on time.
- Ideally you should keep the volume of your voice constant throughout the lesson. You need to ensure your voice carries all the way to the back of the classroom, check this with your students to ensure you can be heard by all.
- Avoid talking to the board or to your notes. It is acceptable to talk while you are writing, but when finished stand back, face the students, and recap the key points you have just written. If the

only dialogue is while you are talking into the board, your voice may not carry sufficiently for all students to hear.

- Sometimes, pauses can add real effect to what you are saying and provide opportunities to emphasise a key point or idea. They allow students to focus their thoughts on what you have said, explore within themselves whether they have any questions, and importantly, allow a natural opportunity for them to ask those questions. Pauses can also be used to control any unwanted talking during the class – if you pause mid-sentence, it won't be long until all students are focusing their attention on you.
- Always display enthusiasm for the material you are delivering, emphasizing that you really want the students to learn, not only because they need to, but because it is intrinsically interesting.
- Don't simply 'read out loud' your notes to a class *ad verbatim*. Reading from a script shows a lack of command of what of what you are saying, does little to establish confidence amongst students that you genuinely understand the material yourself, impedes voice propagation, and reduces eye contact with students.
- Aim for a smooth, focused and succinct presentation of the key points, with only occasional references to your notes. An effective approach is often to have written down in your lesson notes everything that you will write onto the board, and then simply 'talk around' this core material.
- Don't think that just because students have a full and accurate copy of the lecture notes it means they have learned during the lesson. Many people do in fact learn effectively by copying material down (in this case from the board or slides) coupled with a suitable audio commentary (from you), but others simply imitate, intending to interpret their notes later on – if you want to check this, have a look at how some students interpret mathematical symbols. One compromise is perhaps to use 'white space handouts', in which students complete particular sections as the session progresses.
- Avoid using phrases such as 'um', 'well', and 'you know'. It is better to have a silent pause. In the same way, avoid using phrases such as 'well this should be obvious' and 'it should be clear to you all that…'. Perhaps it should, but it doesn't mean that it is, and any such comments act as a barrier towards ensuring student engagement – they may be reluctant to ask what they perceive is an important question if they are concerned how you will react to it. In the same way, sarcasm should always be avoided, even if it is intended purely as a joke. It also goes without saying that sexist or racist comments have no place within civilised society let alone within the classroom.

The key to effective communication is not just speaking, but also listening to students and observing their actions and facial expressions. This will tell you whether you are talking too quickly or too slowly, whether they are becoming bored or restless, whether you need to vary the activity, or whether they need another example to emphasise the key points covered. Keeping the attention of the students during a class is a challenge for any tutor, experienced or otherwise. To maintain student interest you can ask questions, possibly recapping earlier material, at strategic points throughout the lesson, or alternatively ask students how they themselves would solve a particular problem. Keep the students engaged throughout, if they think they are likely to be asked a question or are to be asked to participate in some way, they will be more likely to be attentive throughout.

Try to engage the students in eye contact as this will give them the impression that you are talking to them as individuals. To ensure the students remain engaged with the lesson try moving around the classroom to provide a different focus. Try walking up and down between rows while talking, there is no reason why a tutor should always remain at the front of the room. Use facial expressions and arm movements to emphasise key aspects of the material. Again, if students feel you are enthusiastic about the topic, they are more likely to engage.

Of course, even if the delivery is perfect few of us can concentrate for very long without needing a change of activity or a break. Particularly in mathematics, many of us learn best if we are actively engaged in the process. When appropriate, provide some change of activity to give the students a chance to 'recharge their batteries'. This is very straightforward in mathematics – provide them with a short problem that either emphasizes a point you have made, or primes them for something you are about to say. Such activities can be directed at the most challenging components of the course material, the intellectual hurdles, or at some subtly that might otherwise be missed. A caveat about this issue is that we should perhaps question the view that a proficient, able student should not be expected to maintain attention for an extended period on an important topic. A presentation therefore forms a continual balance between maintaining the attention of the students, trying to stretch their powers of concentration, and maximizing the effectiveness of delivery.

One of the most important aspects of good delivery is the management of the pace of the presentation. Inappropriate pace is a frequent student complaint, so maintain a pace that the students can handle; do this by asking them; just because you think it is appropriate pace doesn't mean that they do! Try to estimate how long each part of your session will take, leaving a small amount of contingency time for questions and unplanned activities, and then try to maintain an unhurried even pace. If things don't go according to plan, maybe because a topic took longer than you estimated, don't accelerate to cover the material; readjust later by moving content to your next session or consider whether that additional example is absolutely necessary – could it form a question for the students to undertake in their own time while you make the solution available on the virtual learning environment? If you finish your plan early improvise some additional material, continue on to the topic planned for the next session, have an exam or other question ready for the students to tackle, or failing that, finish early –students rarely complain if you finish a few minutes early.

Another component of satisfactory delivery is the balance between theory and examples. Few of us can concentrate for long on a presentation of theory without soon needing some sort of concrete example or application – something we can play with to check we have the right idea. On the other hand too many examples and applications may slow down the delivery so a careful balance needs to be struck.

Examples

1. Immediately following the presentation of any major new technique such as inverting a matrix or integrating by parts, we might present a range of worked examples showing the main features of the technique. For example, for matrix inversion of several non-singular matrices, we can then illustrate what happens for a singular matrix. Or for integration by parts, illustrate by a straightforward example such as xe^x and then one that has to be repeated such as x^2e^x or circulated, like $e^x\sin x$.

2. Another use of theory and example in conjunction is the parallel treatment where a theory is presented on one side of the board, and the relevant stages of a specific example on the other. Thus, one may work through the proof of a theorem in the general case, and in parallel present a particular example. Or, one might list the general stages in some technique such as inverting a matrix, and in parallel undertake a particular example that demonstrates how these are applied.

Closing the presentation

Close the session with a summary of the main points using appropriate mathematical language – this provides students with key words or phrases that they can follow-up in their own time, either through further reading or using the internet. It is important to end the presentation on a high, reinforcing to the (hopefully) exciting and valuable aspects of the lecture. Always end appropriately – not in mid-sentence or halfway through a point, better to finish a few minutes early. Don't leave the classroom quickly at the end of the session, make yourself available for any students that want to see you – after all, this is a positive sign that the students view you as approachable. When you close the presentation, you can mention the material that you will aim to cover next session, perhaps highlighting any recommended or essential reading that the students can undertake in advance.

4.5 Use of resources and media

This section addresses issues of the use of the various available resources – board technique, use of overhead projectors (OHP), and electronic resources - which we can summarize in terms of **media** and **message**. The media issues relate to actually using the resources to ensure that the communication of the message is efficient and effective. This includes writing clearly on the board, large enough writing on the OHP transparencies, not standing in front of the board or projector, talking sufficiently loudly or using a microphone if necessary, and the correct writing and pronunciation of mathematical symbols and terms.

Using resources to convey the required message in the most effective and memorable way is as important as the technical aspects, and more difficult. When working through calculations with a colleague on a board you each have the opportunity to query what is written, to explore different interpretations of what you see, and to link closely with what is said. Most of this is denied to the student in a presentation. Often their main priority is to write something down that they can decipher later, so they at least have some notes from the session.

Your use of resources might be developed in two stages: first the technical use of the equipment or the environment, then the application of those skills in communicating the key mathematical ideas you are teaching. Here, we will focus on black (or white) boards, overhead transparencies and handouts.

The lecture room is also important. For large classes certainly, the standard arrangement for the lecture is a tiered lecture theatre. This is not always satisfactory, because you cannot easily mingle with the students (unless there is room for them to use alternate rows). It is useful if you can get close enough to assist any individual. But even failing this, you can still walk up and down the aisles, which helps in giving them the impression of being 'amongst them' rather than in front of them. Try to use the space in the room constructively. If you can, try to get a flat room, then you can turn the presentation into a tutorial when you set the students a task.

Black(white)board technique

Good board work is important. Write neatly in large letters in plain longhand or print. Work horizontally across the board, proceeding linearly, with not too much material. Label key equations. Draw sketches neatly and persist in getting students to do so as well. Use sliding boards effectively. Do not erase until you have to and then do it properly, checking with the students that they have finished with the material. Don't reuse material already written, write it out again. If your writing is poor, slow down and take particular care.

When communicating the mathematical message by writing material on the board we have to think about the actual purpose of displaying it on the board. If it is fairly routine, then all students have to do is copy it as a record. Note that this is not the waste of time and resources that some might think. The mere act of copying something down can help the students to absorb it. This is particularly so with mathematics, because even when copying mathematics down verbatim it is usually necessary to think through the symbols and manipulations as you write.

On the other hand if the purpose of displaying material on the board is to develop deeper understanding (which most of it should be) then we need to do more than just write the material on the board. We have to actually use it in some way. Once the material, say a proof of a theorem, or a worked example, has been written down and the students have had time to transcribe it, then we need to study it in more detail. You can walk up to the back of the class and look at it with the students, discussing what you all see. Get them talking about it with you, see it and discuss it from their perspective. Discuss together with them what the important features are – would it have been a good idea to highlight or underline some things? If so, do it. Point out important features, or get the students to. Indicate to them the logical structure, the steps made and the reasons for them. Can we improve the presentation in some way, is it clear enough? Interact with it, pointing things out, drawing connections. Repeat the key phrases or points and continue to revisit them. Say things in different ways, and remind them of the meaning of key words and concepts. In all this you are using the board as a tool in developing understanding, rather than simply transmitting information. If this serves no other purpose than providing a short break it can still be very worthwhile.

Writing out mathematical equations and calculations on the board requires consideration. So, for example don't skip steps – write each one out along with a few brief words of explanation. It is easy to simply write the equations, and not write down the intermediate words and explanation you are saying at the same time. This is actually storing up trouble, because what the students then get as a set of notes is a list of equations, and this is then how they tend to write out solutions of mathematical problems. We all know this is not always good mathematical presentation, and prose interjections are sometimes very helpful. They articulate what the symbols and equations really mean. Also, writing out the prose can slow you down, which might be a good thing for the students!

There are a number of tips for using the board:

- Always use the board in an orderly way. The writing on the board should be clear, of the correct size (so that it can be read from the back of the room, but not taking up too much space), and written level.
- Don't erase immediately, leave material up long enough for students to at least copy it down.
- You can save space by using abbreviations, but make sure you tell students what these abbreviations mean as you are using them. Also define mathematical terms and notation before or shortly after you use them, certainly on the first occasion and perhaps on subsequent occasions as needed.
- Highlight key points or formulae using an outline box or other means.
- While writing on the board, try to stand slightly to one side so that students can see exactly what is being written. Also, once you have finished writing, stand to one side, leaving the board clearly visible, to explain it. Avoid 'talking into the whiteboard': a good approach is to explain a point, then write the notes onto the board, adding any additional information as you go along. You can also reemphasise the point after you have finished writing.
- Note that most people write, think and talk at different speeds, so it is not uncommon to make mistakes by inadvertently writing what you happen to be currently thinking or saying. Try to discipline yourself to keep everything synchronised, perhaps reading out verbally what you are actually writing.
- Make sure there is no glare from either the lights or windows on the whiteboard.
- For the same reason, ensure the board is clean before you write upon it, and that you erase it thoroughly – particularly so at the end of the session in order that it is ready for the next user.
- Practice drawing any complicated diagrams or graphs ahead of the lesson, alternatively, arrive early and draw them on the whiteboard in advance (although this is not always possible as the teaching room may not be free, and additionally the students should ideally see you do the drawing – they will have to do it). If you use a prepared slide for a complicated diagram, give the students a copy, but they will normally learn a lot more by copying from a free-hand sketch that you actually do in front of them. Alternatively, and perhaps as a compromise, provide a partial diagram on a handout and then complete this with the students during the session.
- Have a supply of chalk or (the appropriate) pens and eraser available for the lesson – they are not always available in the teaching room.

- If you are using a blackboard, try to avoid chalk that squeaks or is too faint. Use thick chalk and hold it at an angle to the board.
- If you use colours on the board, be considerate of contrasts, and check with students that what you write is sufficiently clear. Be wary of using reds and greens – some students may suffer from colour blindness. Also, make sure the students have the same facilities – different colour pens for example - if they have to copy down what you are writing.
- Finally, if in doubt, **ask the students**, they will be more than happy to provide advice and suggestions if they think you are doing something that could be improved!

Use of overhead projectors, transparencies and other visual aids

Using such resources takes particular practice, and your staff development department might have courses that provide useful support in this area. Some advice is obvious, for example, don't stand between audience and screen, don't obscure projector with your shoulder while writing on a slide. In other words make sure every student can see everything they need to see when they need to see it. The best place for the screen is in the corner of the room at a slight angle, if possible. See Mason (2002, Page 44) for advice on this topic.

Once projected, the slide on the screen is no different to the board in terms of display (except of course that transparencies can be prepared in advance with greater quality, for example it is easier to use colours or different fonts), and so everything said previously for boards applies equally for overhead transparencies or PowerPoint slides. However, slides are better for presenting summaries and overviews of what you are saying, and they can be shown or removed at any time. You can give students copies of the slides, whereas you can't provide them with copies of the material written on the board (unless you have access to a new generation of electronic whiteboards that are becoming increasingly common within schools), but don't allow this to overload the students or effectively increase the pace of the lecture beyond the 'assimilation threshold'. Multiple slides can be used, one showing a summary, the other, further detail. Overhead transparencies or slides and boards can also be combined to good effect (Krantz 1991, Page 40, Mason, 2002). For example, with a set of standard equations, such as Maxwell's displayed on a slide, you can work through examples of their application in real time, using the board.

Other visual communication media available to you include PowerPoint presentations, transparencies, slides, demonstrations, videos, and mathematical software such as GeoGebra, all of which require further training in the context of presentations to students as well as the pedagogy of their use in the classroom. They can sometimes be used effectively to convey key points when appropriate, but should not distract from the mathematical message intended. If you have difficulty drawing complicated diagrams and graphs upon the whiteboard, then a prepared transparency or PowerPoint slide might be useful or a piece of software such as GeoGebra.

There are a few points to consider when using transparencies or PowerPoint:

- Transparencies and PowerPoint slides typically convey less information than 'chalk and talk' presentations, and so they need to be changed more frequently. Make sure students have time to take notes, or provide copies of handouts. A solution is to provide copies of the presentation and transparencies, however, these are usually somewhat substantial documents due to the layout and size of the fonts necessary to make them visible on the screen, and might be expensive to produce.
- If using transparencies, it is best to prepare them in advance, using different colours and layouts to emphasise key points. Prepare them tidily so that they are easily readable, and perhaps consider using a computer to do this – the advantage is that they will be readily available the next time you have to teach the class.

- You should face the students, not the screen, when using transparencies or PowerPoint. Use a pointer to indicate key information on the screen as you are talking about it. An alternative with a transparency is to point at it with a pen, thereby casting a shadow onto the screen.
- PowerPoint is not always the best means of displaying complex mathematics, and so one option is to produce the slide using LaTeX. Once you have done this, you can then convert it to a .pdf file, which can then be run as a presentation. Again, you should bear in mind how much material can realistically be conveyed on a single slide and also how easily the students can reproduce it during the sessions.
- Some tutors use 'gapped' handouts which are given to students; both you, and the students, then complete these during the teaching session by 'filling in the blanks'.
- Ensure the projector does not block the view for some students. Also, be careful not to stand in front of the screen yourself.
- If using PowerPoint, make sure you use an appropriate colour scheme that provides good contrast. You do not necessarily have to use 'black on white', but you do need to ensure it is accessible to all students. Again, if in doubt, ask the students.
- While running a presentation, or using transparencies, lower lighting improves visibility, but ensure it is not so low as to make note taking difficult.

Note that useful though they may be in some contexts, such things as PowerPoint and prepared overhead transparencies are rather sterile when used as the sole means of presenting material. Students often testify to the fact that they benefit greatly from seeing mathematics developed as written down on a board, with an ongoing commentary from the presenter – i.e. 'chalk and talk'. Additionally, the presenter is forced to concentrate more on what they are actually writing, rather than simply tracing lines on a slide, and this also helps to moderate the overall pace of the session. While we would encourage you to experiment using different forms of visual and electronic media to emphasize key points, computer software is particularly suited for this purpose, we would strongly advocate the benefits of the 'chalk and talk' approach for the majority of your teaching – certainly at this stage of your career.

Use of handouts

As far as mathematics is concerned you can learn a lot by looking at the various examples used by your colleagues. Don't use handouts or slides to rush the course or overload students and, especially for large classes, think of the costs of replicating large volumes of paper. These days, online virtual learning environments are available for storing course materials so that they are accessible to the students.

Everything that has been said about boards, transparencies and slides applies equally here, but in addition the students can take handouts away and therefore have the opportunity to customize them to create their own reference materials or revision aids. See Mason (2002, page 64) for different types of handouts and their use. Handouts may vary in form from level to level to assist students in the development of independent learning skills. Thus, in the first-year, students often require significant support with materials such as handouts – these are typically self-explanatory, perhaps stand-alone documents, and contain numerous different examples. The level of guidance may be relatively high and demand little from the student in terms of their own original input. By the second-year, students need to be encouraged to become more independent and take more responsibility for their own learning. The handouts can then be quite concise, brief and have considerable 'white space' that they are expected to complete with their own work and notes away from the session. In the third-year their independent learning skills should be almost fully developed, and students can be expected to make much more use of books, and other reference materials during their own time. One option here is to either use a dedicated textbook for the course, or provide book-quality handouts, a little more advanced than they actually need. You can help the students understand the material within your presentation by covering the seminal parts, and leaving the students to follow-up on the detailed or wider elements for themselves.. Your role in this instance is less about providing the material as it is about helping the students to assimilate it for themselves.

Putting it all together

We have discussed the main learning resources separately, but by combining boards, slides or transparencies and handouts (and possibly demonstration models) you can engage the students in many imaginative ways to help them learn. This is rather a technical matter however and certainly in the early days of your teaching career you should perhaps keep things simple, although it certainly never hurts to try out new approaches and evaluate how these work. See **TMHEBB** (Cox, 2011) for further ideas.

4.6 Maintaining a good learning environment in the session

Some environments are better than others for learning – and these can vary between individuals. In a presentation we have to ensure that the environment is optimized for learning for most of the students. This has many meanings, for example understanding how your students are likely to learn best, getting them relaxed and able to ask questions, and keeping appropriate discipline so that the class runs smoothly. We have touched on such things earlier, but will be more explicit here because in giving a presentation you have to deal with the class as a whole rather than one or two individual students at a time as might typically occur in a tutorial session.

Establishing good staff-student relations

Easier said than done. It may require many qualities that you feel you do not possess to a sufficient degree – a friendly nature, sense of humour, or infinite patience. But there is one overriding quality that will take you a very long way – you must really **care** for your students. Failing that, be able to pretend you care for them. Students will forgive a lot if you if have their interests at heart. Krantz (1999) holds as one of his key maxims for relations with students the need to respect them. This is absolutely right. You have to treat them as people, not only as mathematicians – and as in most environments, until you are given cause to behave otherwise, it pays to be nice to people. While you are the one in charge, extend courtesy, consideration and respect to your students. Give value and do your best for them. Be reliable, do what you say you are going to do. Be firm and fair and ultra-tolerant. Never lose your temper or become impatient with your students, be utterly objective and detached when dealing with them, especially in difficult situations. Find an ultimate deterrent or escape route for those situations when things do start to get out of hand.

You may sometimes find that the students are not all that interested in the topic you are covering – maybe they are engineering students who regard the mathematics more as a 'tool'. It is very difficult to avoid letting this affect your relationship with the students, but it is an important part of the job to do this – keep good relationships with the students even though you may have little in common. You will be constantly asking the students to do things they don't really want to do – so if you have to also do some things you don't want to do, then you at least have this in common!

Another important aspect of building good staff-student relations is to treat them as individuals. Never categorize students unjustly. It is tempting to fall for such generalizations as 'these are engineers, they are not really interested in mathematics', 'these students are not interested in learning, they just want to pass the exam'. But as Baumslag (2000, page 140) notes, their motives might not be as you would like, but as individuals they are entitled to their motives.

Classroom management

By this we mean maintaining an orderly learning environment, ensuring all the teaching and learning activities proceed expeditiously in a disciplined and productive manner. Circumstances may sometimes arise where it is necessary to deal with some kind of troublesome behaviour, such as noisiness during lectures or disruptive late arrivals. If this becomes a serious concern then refer it to the lecturer responsible for the course, although you should perhaps try to deal with it yourself in the first instance. The advice that follows may be useful, as it is distilled from a number of experienced mathematics lecturers who have to deal with the issues on a daily basis, across a wide range of students. Also, there are actually some specifically mathematical aspects to these issues, but what we must stress is that extreme cases of bad behaviour are fortunately very rare – most of what you will encounter may best be described as 'irritating', to both you and other students.

The first issue to address is the question of when a problem actually becomes a problem. It may be tempting to ignore unwelcome behaviour. You may not think it is worth making a fuss, or you simply may not know what to do about it anyway. The way out of this is to ask yourself not whether it bothers you, but whether it interferes with the learning of other students, or the running of the class. If it does, then you must deal with it. The students will, quite rightly, expect you to do so, and will often support you in dealing with it.

Let us assume you have got to do something about it. Of course, prevention is better than cure; set the ground rules in the very first session, clearly and concisely. Try to get the students on side by involving them in setting the rules, perhaps have a discussion as to what are reasonable ground rules during the first teaching session. Students are less inclined to misbehave if they feel you know something about them, so learn some names and get to know as much about them as you can reasonably manage. Keep seeking their feedback, informally and by frequent measures of progress. Keep them active. Be everywhere – wandering about the class rather than standing only at the front. Any sanction imposed must be systematic and fair – a first warning in pleasant mode, a second warning including a statement of consequences, at the third offence impose the consequences without embarrassment or compromise. If the circumstances are extreme and it cannot be resolved by removal of some students, seek advice from the lecturer for the course.

Perhaps the most common behavioural problem students present is talking and inattention when they are supposed to be listening to you. Deal with this as soon as it occurs, rather than letting it build up, focusing upon the culprits. While everyone must see that this behaviour is unacceptable, the class as a whole must not be chastised if disruptive behaviour is taking place, but the culprits must be identified and warned. If students are talking, make direct eye contact with them so that they know you see them. Sometimes stopping the presentation, looking directly at the offenders and only resuming the lecture when talking stops is enough to resolve the problem. However, this is allowing students to dictate the progress of the class and should not be allowed to delay proceedings for too long. Physically move toward that part of the room, again making eye contact with the students. If you cannot identify the source of the noise simply ask who is talking. If there is no answer, their embarrassment may be a sufficient lesson.

Having given fair warning, if you finally do identify someone talking, make an example of them and impose the threatened sanction firmly, immediately and without malice – this might involve asking them to leave, or asking them to explain to the group exactly what their problem is. If they wish to talk, ask them a mathematical question about the material that requires a response or bring them to the front of the room to work through a mathematical problem on the board – this is not unreasonable if they have been warned that this is a possible consequence of their actions. The authors have never had a student refuse to leave, but if the situation progresses that far you should refer the matter to the lecturer of the course, phone security, or maybe even leave yourself if the situation really is that untenable. If appropriate, speak to the student(s) privately out of class, or refer the matter to their personal tutor. Explain that their behaviour distracts the other students. Above all, make it clear that you bear no grudges, and that once the issue with them is resolved it is forgotten completely and will have no further consequences. It is important to realize that normally students are not being deliberately troublesome, but have possibly simply not learnt the rules of reasonable behaviour, or have previously been set a bad example by being allowed to talk in classes. Additionally there are times when you want them to talk – during exercises, or to tell you that you have made a mistake for example. Getting the balance right between keeping the students quiet and allowing reasonable discussion is something that you will develop with practice.

Another common problem is lateness and absence. Students shouldn't miss classes, be late, or leave early, particularly if this is disruptive to others. It is best to be firm on this matter, particularly in the first few classes. If you feel that a student's absences are excessive and are jeopardizing their academic performance, inform the student's personal tutor – this isn't a matter for you to address at this stage of your teaching career. If a large percentage of students don't attend the session, consider the possibility that they do not find the sessions useful. On the other hand, students are adults and whether or not they attend is really up to them. If absence is problematic, seek advice from the lecturer responsible for the course to explore what might be done. As for lateness, this is different because it can disrupt the class, and should not be tolerated. If a student arrives late and without apology, challenge them and don't be afraid to do so in front of other students. It may be that there is an alterative entrance to the classroom that is less disruptive; make it clear that it is unacceptable for students to be late, but if there are exceptional instances where this occurs they should use this entrance. If you find a number of students are regularly late on a particular day, maybe this is because they need to travel from another location across campus following previous session – here, you might wish to delay the start by five minutes to allow for this but ensure the lecturer with responsibility for the course knows this is what you are doing and is in agreement.

There also seems to be a recent and growing phenomenon of students eating and drinking in lectures and resultant litter in classrooms. Often, there are university regulations about this, be clear as to what these are and ensure they are rigorously enforced from the outset. In any case, it should be made clear that litter in the lecture room is unacceptable.

At some point, albeit exceptionally, you may have to face a student who is resentful, hostile, or challenging during a session. The following are a few suggestions for dealing with such behaviour:

- Don't become defensive and take the confrontation personally. Respond honestly to challenges, explaining – not defending – your objectives in the class.
- At all costs, avoid arguments with individual students – do not participate in such challenges, and no matter how hard, maintain a calm, composed and professional manner.
- When talking to a disruptive student, be objective about expressing your concerns and explain your reasons for doing so.
- Be honest when something doesn't work as you had planned.
- As a last resort, leave the class and report the matter to the lecturer concerned.

Despite the perhaps daunting nature of some of the issues discussed in this section, take comfort in the fact that they are generally of an exceptional nature. If you genuinely care for your students and do your very best to help them learn, then most of the time you will be able to create a lively but disciplined environment where they will work with you to learn mathematics and help challenge any irritating behavior. On the rare occasion when things do go wrong, the majority of the students will be behind you in helping address the issue – never underestimate the power of peer pressure.

Encouraging student engagement and interaction in a presentation

In a presentation, as opposed to an exercise class, you are limited in what you can do to get the students directly involved, but there are still techniques for achieving this. To encourage student engagement we need to respond sympathetically to any queries, and welcome student questions. We also need to 'be everywhere' within the teaching room, and demonstrate to the students that we expect them to interact and contribute at appropriate times during the session. Concentrate not only on the academic content, but devices for developing rapport with the students, getting them (not too) relaxed so they feel able to ask questions and make a contribution to the delivery of the session. It is not easy to do this – some people exude empathy, warmth, and helpfulness without even trying, some simply leave others cold no matter how helpful and sympathetic they are. Methods suggested elsewhere such as helping students over the difficult concepts naturally help, but other approaches are needed, such as being relaxed yourself or a little light-heartedness. Perhaps it is the things you shouldn't do that really make the difference – no sarcasm, no impatience, and no responding to a student's question with a dismissive 'go and look it up'.

When students do summon the courage to ask questions, and indeed for many, it does take genuine courage to ask a question in front of their peers, respond in such a way as to help them understand as much as possible. One student may have asked the question, but it is almost certain others will be interested in the response or wishing they had the courage to ask it for themselves. When a student asks a question they are at their most receptive for learning and we need to capitalize on this and take the opportunity to help them learn. Build an environment for interaction, and build student confidence – respond positively to any question, and thank the student for asking it. If the question, or perhaps an answer, isn't what you expect, don't dwell upon it, move quickly on – never laugh or mock a student question and do not allow the opportunity for others to do so either. Ask the students questions throughout the session, but insist you get answers before moving on; if you want students to answer and ask questions you need to:

- Explain the question clearly, possibly displaying it visually so that students are certain of what you asking.
- Give students time to collect their thoughts, possibly even to discuss their possible responses with others.
- Provide privacy so that asking and answering does not feel so threatening – allow students to submit questions post-class and address these at the start of next teaching session to the entire group.
- Allow students the opportunity to prepare questions and answers before 'going public' – let them do this in groups.
- Never criticize or ridicule an answer – instead try to build upon it. 'That wasn't quite what I was looking for, but how about if we consider….?'
- Persist. Don't give up, if you do, you will find your voice is perhaps the only one that is heard throughout the entire session! If you quietly count to twenty while waiting for a response, usually someone will answer.

Explanation during your presentation

Contrary to what some might think, students do not necessarily benefit most from a pristine, perfectly delivered presentation with impeccable, meticulous logical development of the material, simply vocalizing the board work or the slides. In such instances, all the students obtain is essentially a copy of your notes and what you say. Instead, what is most important is how you actually **explain** ideas, either in writing or through dialogue with the group. You should try to develop the arguments in real-time, noting down the main points and actions on the board or slides, showing how you might reason your way through the material as if your had never seen it before. 'Now, I need something which when I differentiate it is going to give me [this result] on the LHS, any ideas? Didn't we see something like this last week?' 'How about if we try….?', 'How about you try…..?'. This way, you not only present the material in a more honest and understandable way, but you are demonstrating to the students how a mathematician actually thinks (as opposed to how most textbook writers write). As well as noting down only what you say, students will learn to copy what you do and how you think, they will get a feel for the traditions and ethos of the subject – this will benefit them far more in the longer-term as they begin to think like mathematicians.

This is really what teaching is all about – constructing explanations that help the students to understand ideas and techniques as efficiently and effectively as possible. Naturally, careful explanation of a point takes more time than simply telling the students. Explanation is not only a matter of what you say, but also about how efficiently you say it. You should certainly have a good insight into the background knowledge of your students, their 'facility level', and be able to talk to them in language they will understand, but there is more to it than this. You need to empathize with your students and 'get on their wavelength'. A presentation is not **talking** to quick thinking stenographers. It is about **explaining** to widely varying human beings. Being able to explain things clearly, at the level of the student, or indeed at a number of differing levels, is one of the most important skills of a teacher, and it is during a presentation or lecture where this skill is most essential. In a tutorial you can speak with and question an individual student at a time, and shape what you say in the light of their responses. In a presentation, you have to rely more on what you say, often at differing levels of complexity to cater for the natural variation in student ability that exists within any group, and how they students are likely to receive it, with little opportunity to directly obtain feedback from them. It is less easy interrogate students to find out what they know or whether they have understood you. For this reason it is best to work near to the lowest common denominator of what you think they know, and you must continually be on the lookout for signs that they are failing to understand, ready to react and respond accordingly. How well or easily students learn is a direct function of how well or effectively we explain what we are trying to convey. There are good explainers, and there are bad; your skills in this area can be improved by education and training, but more typically by practice and experience. Below we suggest some ideas for developing careful explanation, going into some detail. For further information or guidance you can consult **TMHEBB** (Cox, 2011). This also contains a number of exercises which might interest you if you wish to explore this aspect of teaching more deeply.

Assist students in 'chunking' information
The 'chunking' of information, ideas and processes is familiar in mathematics (after all, that is what theorems, lemmas, and propositions, for example, are for!). To be any use, the chunks of mathematical material have to be internalized and put together to give a coherent and total picture of the topic. In the presentation the 'bite-sized chunks' translate into breaking the topic into smaller, more easily memorized results, continually reinforcing them and from time to time checking that the students have genuinely understood them.

Example

In teaching introductory calculus one might regard the derivatives of standard functions (such as x^2 or ln x) as bite-sized chunks. The important cases of such results can be thoroughly learned and understood independently as separate entities. These bite-sized chunks are then readily available to the students as they move on to more complicated functions such as products, where they can concentrate on the structural aspects without being distracted by the need to think hard about the standard derivatives involved. Equally, the same is true for standard integrals.

Provide 'hooks' for students to hang ideas on – aide memoires or mnemonics, to make it as easy as possible to remember key things
Such techniques are familiar in mathematics, for example **BODMAS** (for describing the order of operations in a calculation), or **CAST** (for describing the quadrants in which the three basic trigonometric functions are positive), and indeed we ourselves have used **MATHEMATICS** as an aide memoire for teaching itself. But emphasize that the student should eventually internalize the idea and dispense with the hook; no-one can progress far in algebra by having to use **FOIL** (a standard method for multiplying two brackets) each time for the multiplication of binomials.

Provide appropriate step-laddering and lubricate arguments
In sequences of arguments, as for example involved in putting together the bite-sized chunks, one important factor is the 'height' of the steps in the arguments. If the logical or intellectual jump from one step to another is too large, then many students will simply not be able to follow it. Try to obtain the right balance, which takes practice, and give the students a suitable starting point with analogies, by noting a quick example or by asking an appropriate question of the class.

Example

The product rule in differentiation. Having ensured that the students know their standard derivatives thoroughly (see above), you can move on to the product rule. You might prove it for mathematics students and even for engineering students because it is so very important. But for engineers try and lubricate the argument by giving the analogy to the expansion in area of a heated rectangular plate with sides u, v. When it comes to actually applying the product rule you will need to think carefully about the steps you include. Eventually, you don't really want the students to use the formula, but rather to be able to differentiate the product without the need to introduce u and v. But for complete beginners this is probably a step too far initially, so we include all the steps, 'Put u =..', write down and substitute into the formula, collect and simplify terms. Eventually we have to wean them off this approach; this can best be achieved through lots of practice, much of which they will need to undertake in their own time.

Anticipate the difficulties students might encounter and support them in dealing with these
Some of the mathematical ideas that you are going to introduce and discuss will be quite subtle and challenging for some students. There is nothing wrong with letting the students know this in advance, and then modifying your pace and delivery accordingly. Let them see that you are anxious to help them understand the topic.

Examples

1. When teaching the integrating factor method in linear differential equations students usually find difficulty in the reversal of the product rule used to convert the left-hand side of the equation to the derivative of a product after multiplying by the integrating factor. You can help considerably by revisiting the product rule and presenting a few examples of converting expressions to derivatives – in effect using the derivative rule backwards.

2. Concepts such as convexity, linear dependence, or equivalence relations, always cause problems – why? Because they **are** difficult! Be considerate in presenting such topics, and include in your explanation the reasons why these concepts are used.

3. Sometimes you may have to rethink how you will phrase things to make them more accessible to the student. For example, in partial fractions we are often tempted to say something like 'equating coefficients ...'. Even the more able students would be hard pressed to really understand this move as they are not likely to see immediately the connection with linear independence (Mason 2002, p 27). It is probably kinder, and perhaps more truthful, to emphasize the fact that since the result is an identity, it must therefore be true for all values of x, which can only be the case for a polynomial if the corresponding coefficients are equal. Even if this is presented in a somewhat vague manner, it will probably be better received by the students than saying 'equating coefficients...'.

Provide regular overviews
Even when the bite-sized chunks have been put together and students understand the connections, they still require an overview of the material to see what the general message is, and to develop an intuitive feel for what is going on. This can be developed by ensuring from the start that the students understand what the main objective is, and that they are fully aware of the purpose of the presentation. You might, for example, state this at the beginning, repeat it at appropriate intervals in the session, and at the end in a summary. This aspect of teaching, giving an overview, may not be easy for you. Even though you graduated with a good degree and may now be doing quite advanced research, you may be overspecialized and may not yet have developed the experience to form such overviews yourself.

Example

Having developed a lot of techniques of integration – by parts, substitution, partial fractions, or trigonometric identities, you can point out to the students that this plethora of methods can broadly be summarized under three main approaches:

- Manipulating the integrand into a more helpful form (partial fractions, trigonometric identities).
- Reversing the product rule (by parts).
- Reversing the chain rule (substitution).

Then ask the students to notice one obvious omission (reversing the quotient rule) and thus discover for themselves a new and unusual method of integration. This gives them a compact oversight of what integration is about. It packages it neatly and (hopefully) makes it look more manageable.

Continually highlight/reinforce the (small number of) key ideas conveyed in the presentation
Mathematics is particularly concept efficient, which can either make it easy or difficult to learn depending upon your approach. Any presentation should contain only a relatively small number of basic ideas. But by the time these have been recycled, applied, and tweaked, the resulting set of notes can run to many pages of complicated looking mathematics that can be intimidating to the novice. The job of the teacher is to reverse this process, to emphasize the important core ideas, help the student to absorb these and to recognize them in their many different guises. You can do this by listing the key points at the beginning of the session, clearly identifying them when encountered, and summarizing them at the end of each session.

Examples

1. In a short presentation on partial fractions to first year engineering students for example, the key points may be i) the reason they are needed, ii) what a typical resolution into partial fractions looks like, iii) what can be done with the resolution that cannot with the combined fraction, iv) the use and properties of an identity, v) the process of decomposition, vi) checking by returning to a common denominator. A single lesson may consist of many examples of these key points.

2. Completing the square is conceptually difficult for many students at an elementary level for a number of reasons – they don't always realize how important it is and so perhaps do not take it seriously. However, they also have difficulties because amongst all the manipulations involved, it is not always clear to them what the key ideas actually are. These are simply: thorough facility with the expansion $(x + a)^2$ and use of the $0 = A - A$ ploy. These are the two building blocks for completing the square. If the students are thoroughly familiar with them, then they should have little trouble with completing the square. If either is shaky or unfamiliar, then they are likely to struggle. So, before embarking on the method, continually reinforce these ideas, keep referring to them throughout, and list them as the key ideas at the end.

Bring out and highlight the essence of a complicated idea, theorem, or method, and support the development of students' intuitive views of formal arguments
Mathematics is full of complicated ideas which can be overwhelming to the novice, but many of them, certainly at undergraduate level, can be explained in relatively simple ways, almost accessible to the educated layman. Try to give the student an intuitive feel for an idea – what are the core points? One way to do this is to treat it as a challenge for yourself to explain a difficult piece of mathematics to a non-mathematical friend. In fact you sometimes find that when you attempt such explanations it enhances your own understanding of the topic.

Example

Students are often confused by the large number of 'different' methods used for some types of integration – partial fractions, completing the square, trigonometric and hyperbolic identities. Emphasize to them that these are not really 'methods' of integration at all, but rather simply rearrangements of the integrand which puts it into a form in which the integration is more transparent. It may not be the integration they are having trouble with, but their knowledge of essential algebraic and trigonometric identities.

Emphasize the respectability of guessing and the necessity of checking

Often the formal, finished, polished presentation of mathematics gives an impression that it always moves with inexorable logic from A to B with no intervention from the imagination. We know this is not the case – but students may not. In mathematics we have phrases for making guesses and leaps of faith seem respectable – 'conjecture', 'by inspection', 'trial and error', 'method of undetermined coefficients'. All are nothing other than guesswork – educated though it may be. Share this secret with the students. Encourage, or indeed insist, that they get used to guessing and help them to lose the fear of being wrong - 'make mistakes – but quickly!'. In such guessing, get the students used to asking questions such as 'what would be the simplest thing to do in these circumstances?'. Most importantly, they must realize the imperative of checking their guesses.

Example

The method of undetermined coefficients in finding particular integrals for inhomogeneous linear differential equations is a classic example of 'guessing' made respectable. It may be so familiar to us now that it hardly seems like guessing – but this is what it actually is, and should be emphasized to the students. By inviting them to make guesses themselves, before showing them the 'correct' choice, you can give students a feel for the whole idea of progressing in mathematics by exploring promising avenues. In the method of undetermined coefficients we are forced to 'check' in order to find the coefficients.

Alert the students to patterns that may assist their understanding and learning

Some define mathematics as the science of patterns, and certainly patterns pervade the subject. Students should always be alerted to patterns in whatever they are doing and they should be encouraged to search for them themselves. The patterns can be of all sorts of different types – algebraic, geometric, logical, conceptual or numerical.

Examples

1. Baumslag (2000, page 155) notes how the right picture can illustrate the structure of an idea, giving the well-known example of the triangular proof of the countability of the rational numbers.

2. Another type of pattern in mathematics is the generalization – 'perpetuating the present pattern'. The simplest examples are perhaps sequences – like 1, 4, 9, ..., but there are others that are much more powerful. For example generalizing the Pythagorean length $\sqrt{(x^2 + y^2)}$ from the plane to R^n , $\sqrt{(x_1^2 + ... + x_n^2)}$. All these are **generalizations** of a pattern. In the latter case one can carry the pattern further into **abstractions** such as metric spaces.

Help students to build up connections by providing roadmaps and overviews of difficult sequences of arguments – try to express proofs 'sensibly', as much as 'logically'

This is related to the last principle, but particularly relevant with a long proof. You can write down a summary of the steps to begin with, or provide a handout. Try to make each step 'sensible' (not necessarily the same as logical) – why would you do that, why did the originator do that? Sometimes the standard proof is presented in a sanitized, 'perfect', logically most efficient way that often prompts the question 'why?!' from any curious student. Sometimes it may be best to use a different form of the proof in which each step follows naturally.

Examples

1. The proof of a major theorem such as Cauchy's theorem cries out for a form of 'road-map' linking together all the individual steps and showing how they integrate into the final picture. This can be treated as a group exercise within the session, only going through it when the students have had a good try themselves first. Here, the basic idea is to build up the proof by first deriving it for triangles, using a reduction process on smaller and smaller triangles to provide estimations for the integrals required. This can then be used to derive the anti-derivative theorem and thence Cauchy's theorem.

2. A similar example is the proof of Lagrange's theorem in Group Theory. In this case the structure is one of continual searching for cosets by a repetitive process until the group is exhausted.

Alert students to the fact that some teaching and learning is provisional, and may have to be refined at a later date
As higher levels of mathematical knowledge or application are reached it becomes increasingly necessary to compromise between exactitude and economy of explanation. Due to time or other limitations, we sometimes have to be a little economical with the truth in the interests of progress through the material. We perhaps avoid a subtlety that the students will revisit in a later course. On such occasions at least warn them, as a matter of intellectual honesty, that this is what you are doing, and that you cannot at the moment provide the entire perspective. The students don't have to believe everything they learn in a course is the absolute last word – indeed, informed skepticism is an essential asset for any educated individual. The examples below are cases where at some stage in a child's mathematical education they are taught firmly established 'facts' usually without qualification, which are later overturned to reveal whole new realms of mathematics.

Examples

1. Students do not always realize that such things as Pythagoras' theorem or '180 degrees in a triangle' are restricted to plane geometry and do not, for example, hold on a sphere. But then, when alerted to this fact, they may protest that one would not hesitate to calculate the diagonal of a field using Pythagoras, despite the fact that it is on a 'spherical' earth. This is an example where their previous teachers have had to be 'incomplete' in their explanation to avoid confusion. Tell students that this will often happen, because we are continually adapting to new circumstances in mathematics. We cannot at every stage explore all of the detailed restrictions upon our statements.

2. The same is true for the often quoted early school assertion that we can't take the square root of a negative number.

Getting feedback from and to students

In a presentation we should also be continually interacting with students to gather and provide continuous feedback. This is discussed elsewhere within this book: in maintaining a conducive learning atmosphere, and engaging students during the session, but it is as well to think of it explicitly as a tool or technique to enhance student learning.

A good way to elicit feedback from students and provide them with feedback is to occasionally set a 'must do' task which focuses on a particularly important or difficult point. In this you tell the students something that they must absolutely do before the next session – only taking 15 minutes to an hour say. But emphasize that this will be something that will greatly ease their subsequent learning. You don't necessarily need to ask the students to submit the work, but you do need to follow it up in some way, for example, by using a quick quiz in the next session.

Another useful way of obtaining feedback is to use a few minutes in a lecture to explore an individual student's notes and discuss these with the class – did others make the same point, what does it really mean, can we generalize or particularize it, can someone give an example? It is merely another method of exploring what the students actually think, and in particular, whether they have understood the point being made.

Example

We have mentioned elsewhere how insecure first year students are about methods such as the product rule. This greatly impedes subsequent progress. If they have to use u = …, v = …, for the product rule, then students will struggle to follow such things as the integrating factor method in differential equations. Tell the students that one 'must do' exercise in the next week is to wean themselves off the u, v formula. Tell them to practice the product rule for as long as it takes to be able to use it, and reverse it, quickly and accurately – this means working through many examples. This will speed up their subsequent assimilation of material.

Evaluating the presentation

Even the most experienced lecturer often finds something to improve in a presentation they may have given a number of times. Certainly, in your first presentations, each is a learning experience as much for you as for the students, and so each one needs to be evaluated in order that you can continually improve your teaching.

This area is important, because it is really how we measure the effectiveness of the impact of any of the advice given and the principles adopted. Baumslag (2000, page 142) gives a delightful analogy to illustrate this – the difference between a good and an indifferent carpenter. The good carpenter sharpens his tools before putting them away after finishing a job. So the good lecturer, on leaving a lecture, spends a few minutes noting the main topics of the lecture, good and bad points, and ideas for doing things differently next time. This is also necessary for your presentations. Occasionally perhaps, give the students a very quick feedback form, your department will have a version that it uses for lecturing staff, to see how things are going, but this can never replace a good rapport with the students that enables you to speak to them directly about any issue and thereby ensuring it is addressed quickly and effectively.

There is also the minute-by-minute 'micro-evaluation' we do throughout a session, in which we watch the students to get a feel for how the session is progressing. It helps if you can cultivate a couple of students who you can rely upon to give you honest feedback on progress. If students make comments, good or bad, don't take it personally. The classroom is by far the easiest place for a student to address their mathematical problems and misconceptions, and almost certainly other students will benefit from the issues addressed. This is one reason why you should identify in advance the challenging components of the material, where you think many students may experience difficulty, then you can anticipate such issues arising and be ready to address them before moving on.

Chapter 5

Assessing student work and providing feedback

5.1 Assessing student knowledge and understanding

Introduction

As a postgraduate you are unlikely to be setting or marking examination papers. You are more likely to be marking work set by the lecturer, working to a ***marking scheme***. However, as background we will say a little about the general principles and terminology of assessment. This may be useful if you later move to an academic position where you actually set exam papers. Some of the readers to whom this book is addressed (Graduate Teaching Assistants in the United States for example) may already be setting papers. If this is the case you can consult **TMHEBB** (Cox, 2011) for more information. But most of what we say will be aimed at marking and providing feedback on student work. We will go into some detail on this, not to expect you to imitate our approach, but to simply illustrate the detailed aspects of marking – the need for fast and fair decisions, and the difficulties of deciphering what a student actually knows.

There is one key point that we must emphasize at the outset. We have many times said how important it is for you to consult widely on aspects of teaching. There is no aspect in which this is more important than when marking student work. You will almost certainly have someone checking (or to use the correct terminology, **moderating**) your marking, most likely the lecturer for the course, who should be the first person you contact if you experience any problems or difficulties. But don't confine yourself to only this, consult as widely as possible, accessing the experience from across all of your colleagues, including your fellow postgraduates. Even amongst the most experienced academic staff you will still hear heated arguments over marking – it is by no means a rigorous science!

Sum marking

A pile of student homework scripts lands on your desk. No problem, mathematics is easy to mark – the answer is either right or wrong isn't it? Not like essays where they can be so subjective. In mathematics we can see precisely what the student has done, and there can be no argument whether or not they have earned the marks. OK, try the exercise overleaf….

As mathematics goes, certainly at university-level, it doesn't get more basic than this. Don't be offended by the simplicity, similar examples are used in generic teacher training (Morss and Murphy, 2005), because they illustrate a key point in assessment. You will quickly identify where the error is and award an appropriate mark. But it is likely you will feel uncomfortable with whatever you award as you might imagine two extremes:

Either

- the student has made a small slip in a carry-forward and clearly knows how to multiply two numbers, dock a mark, call it 9/10.

Exercise – Sum marking

Students have been asked to evaluate the product 1234×34 without a calculator. Mark out of ten this attempt by one student:

```
  1234x
     34
   4936
  37120
  42056
```

Compare your mark with those of a few colleagues. Do you all agree? Discuss any differences. Why are there differences? What is the 'correct' mark?

Or

- the answer is wrong, this could be critical in, say, the calculation of a drug dose and lead to serious consequences – some might give a zero mark.

If you are hesitant about your mark and such questions make you think, then you are in good company. If you are quite sure about your allocated mark and think there is no problem with it, then you have a lot to learn. This exercise has been used in many of the postgraduate workshops we have run and the spread of marks is always very wide – indeed, we have regularly had both 0 to 10 as marks awarded at the same workshop.

The fact is, without specifying in advance exactly what it is we are trying to assess, there is no way of fairly allocating any sort of mark. If the intention was to test whether the students could multiply two numbers, including carries, and the students were aware of this, then this is the basis upon which their solution must be assessed. This student has demonstrated mastery of all the necessary processes, and has made a single mistake in a carry forward. This error could be put down as a minor slip and a mark of 9/10 would be difficult to argue with. On the other hand, if it is accepted that the students can multiply and this is simply a test of their accuracy, and the student knew this in advance, then a much lower mark might be in order. Indeed, one may even adopt the policy that if the answer is not exactly correct, no marks.

If you think further about this exercise, you will notice a number of key points:

- It is very difficult to phrase our intentions to such a degree of accuracy that any two people would agree on a specific mark in the range 1-10. No matter how carefully you specify your objectives, there will still be room for argument within a few marks.
- You may have different intentions or objectives that need to be assessed, each demanding different cognitive skills from the student – simply **knowing** the method of multiplication, an **insight** into the idea of multiplication with an understanding of place value, and the ability to **apply** the method accurately in different contexts.
- It is essential that the students know what is expected of them – in this case, method or accuracy, or both?
- Your decision on the matter is a mixture of rigorous objective logic, your attitude and generosity.
- If you mark the question, with the same objectives, on two separate occasions then it is quite possible you will return two different results.

As seen from the exercise, in valid assessment we have to be clear that what we are asking the student to do will (hopefully) measure what we expect them to have learned. Furthermore, the student should have been told what we expect them to be able to do. The common vehicle for articulating this is the **learning objective** (Section 1.5). Learning objectives must be defined and published for all to see for a given module, and then the assessment must measure the attainment of these learning objectives. We say **the assessment must be linked (or aligned) to the learning objectives**. Fair assessment is difficult and is a continual compromise between rigour and practical time constraints. We will be quite precise and rigorous in our coverage of the basic principles, but even then this only narrows the range of possible marks that might result in the above exercise, and does not entirely remove the variability in marking.

Definition and purpose of assessment

- **To assess** (Chambers): to estimate, judge, evaluate e.g. person's work, performance, and character. Here by **assessment** we mean **the measurement of the extent to which students have met the learning objectives of a course of study**.

The form assessment usually takes is a sampling of student learning across a wide range of material, and is as much a measure of their ability to learn a subject as it is evidence that they have actually achieved what we wanted them to. In effect, we only assess a small sub-set of what the students have studied. Because the students do now know in advance what this sub-set is, to maximize their chances they must study as much material as possible, and it is in this sense that assessment drives learning.

It is worth reminding ourselves of some of the purposes of assessment, because this can influence the sort of feedback we provide. Some of its most important purposes are listed below:

- To judge the extent to which knowledge and skills have been mastered.
- To monitor improvements over time.
- To diagnose student difficulties, misconceptions, or misunderstandings (this is sometime called **diagnostic testing**).
- To evaluate the effectiveness of the teaching methods used.
- To evaluate the effectiveness of the course.
- To motivate students to study (in effect, to scare them – you probably remember this well!).
- To predict future behaviour and performance.
- To qualify students to progress or to award a degree classification.

The purpose of the assessment will influence its form and conduct, but whatever its purpose, it is primarily a measure of student learning. It is not necessarily a good measure, and poor assessment can adversely affect student learning. Assessment has to be taken in context – for example, the sort of coursework assessment used to encourage student learning should be used carefully in predicting future performance, if only because there is no certainty that it is solely the student's own individual work.

Some key definitions in student assessment

Listed below are definitions of some commonly used terms in the study of student assessment. To be frank these are rarely used in everyday teaching as most academics appreciate their importance in a more informal way. However, they **are** important:

- **Formative assessment** is designed for developmental purposes and does not contribute to student marks or grades, allowing them to make mistakes without penalty – this includes such things as diagnostic tests or non-assessed homework.

- **Summative assessment** is designed to establish student achievement at stages throughout a programme and normally contributes to the overall mark or grade of the student.
- **Criterion-referenced assessment** is an assessment system in which the performance of a student is marked and graded according to specified criteria and standards. In theory, all students could fail outright or achieve the highest grade.
- **Norm-referenced assessment** is an assessment system in which students are compared with each other and placed in rank order on a (normally!) normal distribution curve. Only a proportion of students will obtain a particular grade or class of degree.
- **Validity of assessment**: the assessment measures what it is supposed to – by this we mean the learning objectives set.
- **Reliability of assessment**: the outcome is consistent for students with the same ability, whenever the assessment is used, whoever is being assessed, and whoever conducts the assessment.

5.2 The MATHEMATICS of assessment

Assessment requires continuous thought throughout the teaching of a course. Assessment is a prime motivator for learning, and, as coursework, a tool for learning. We can use **MATHEMATICS** to remind us of the things we need to think about in relation to assessment:

Mathematical content should be clear by the time any assessment is set, and in your case will probably be clear from the problem sheets you have been using. You should therefore have a good idea what you will be marking.

Aims and objectives of the course have to be aligned with the assessment – you can only assess what the students are expected to have learned. Additionally, the aims and objectives can affect the form of the assessment. Deep understanding might be assessed through an essay, a challenging problem or application, or a mini-project. Students can be helped to learn a particularly difficult topic by appropriately set coursework. Since you will probably be marking material in which you have been supporting the students, and also using a marking scheme, you should have a good idea of what the students are expected to be able to do.

Teaching and learning activities to meet these aims and objectives – the assessment needs to reflect and be aligned with the way the topics are taught. If a high level of mathematical rigour has been exercised in the teaching then this must be reflected in the way the work is set and marked. You will be familiar with this from your tutorial work with the students.

Help to be provided to the students – includes ensuring that they know how they will be assessed, along with important information such as coursework submission dates. You might hold revision classes before the work is due, or give examples of a (not too) similar type to those they have been set.

Evaluation, management and administration of the curriculum and its delivery is, in the case of assessment, particularly formalised in most departments. For the actual examinations the system will be quite involved and regulated, and the setting and marking of end of year examinations will normally be moderated. In the case of marking problem sheets, for example, which is where you are most likely to be involved, things may be less well regulated, the marks will only contribute a small amount to the overall assessment and any second marking is likely to be as much to support you as to fulfil university requirements.

Materials to support the assessment might include coursework, or solutions with an appropriate marking scheme.

Assessment – well this is it!

Time considerations and scheduling are particularly important in the context of assessment. You have to plan ahead as deadlines can be quite tight - there are usually departmental policies on how quickly marked work and feedback should be returned to students. Tardiness in this respect is one of the most frequent student complaints.

Initial position of the students – Such forms of assessment as problem sheets will normally be tied quite closely to the backgrounds of the students, and setting assessment that matches this is unlikely to be an issue for you. However, be careful not to make unjustified assumptions about prior student learning.

Coherence of the curriculum may be reflected in synoptic questions in the assessment, calling upon knowledge and skills from across the whole course.

Students – In fact, your experience of assessment is likely to be more immediately linked with the students than the lecturer who sets the final examination. You might possibly be returning work directly to them, and receiving directly their feedback on your marking – they may even try to negotiate the mark with you, or query your comments on the script! Be personable, but professional, and remember that the marks probably count a very small amount to their overall assessment. You simply don't have the time to revisit every student request.

5.3 Marking student work

Your responsibilities

You are unlikely to be actually setting exams, and your marking will probably be confined to non-assessed student work such as problem or exercise sheets. At most, you might be asked to mark assessed coursework, although as a student yourself, this should not really be your responsibility. However, in any event, you will most likely be working to a marking scheme, or at least a set of worked solutions, even if they are ones you have designed yourself.

However, if any of your marking is at all **summative** – that is, assessed and contributes to the student's overall mark - THEN YOU MUST MARK TO A MARKING SCHEME PROVIDED BY THE 'EXAMINER'. Since you might be using a marking scheme, we say something about them below.

There are two situations in which you might need to be particularly careful when marking. One is if you are an overseas postgraduate student, not familiar with the UK system and the standards expected of the students you will be tutoring. The other is the situation when you are tutoring non-specialist mathematicians, such as engineers, scientists or business studies students. In the former case you must consult widely to find out as much as you can about students at UK universities. This will be a familiar issue to people in your department and there should be plenty of experience to help you out. Have a look at past exam papers in your course to get an idea of the standards, and if possible have a look at previous A-level papers to obtain an idea of what UK students study at school and the standards expected. In the second case, of teaching mathematics to non-specialists, remember that mathematics is usually just a 'tool' to such students. They are not always interested in the finer points, and will be more satisfied with heuristic arguments than formal proofs. Additionally, proofs and details will generally come lower down their priorities – possibly like you yourself when you take your car into a garage for repair – the mechanic may be highly enthusiastic about the fuel injection system within your engine, but all you really want is to get it to start properly. You must respect such differences and try to help the students as engineers learning mathematics, not as mathematics students. You have to learn to 'get into their world' and judge them in that environment when marking their work.

Marking schemes

Marking schemes are the normal means of conveying the criteria of assessment in mathematics. They allocate marks to different portions of a solution to illustrate how much credit is awarded to each part. The essentials of good criteria are that they:

- match the assessment task and learning outcomes.
- enable consistency of marking.
- can pinpoint areas of disagreement between assessors.
- help students to achieve the learning outcomes.
- can be used to provide useful feedback to students.

A marking scheme should present the solution as the students are likely to do it in the light of what, and how, they have been taught. This helps focus your attention on just how difficult it really is for them and exactly what the assessment tests.

The marking scheme you are provided with might be quite detailed, breaking the marks down into a number of levels, or it might be a rough outline leaving a lot to your own interpretation and judgement. In any event, it should leave you feeling confident in awarding marks to students in such a way as to reflect their performance. If you have any doubts at all then discuss them with the lecturer concerned. Be aware that things will improve dramatically with experience – as you mark more and more student scripts you gradually build up 'rules of thumb', intuition and insight that makes marking easier. Also be aware that it is not an exact science – don't aim for perfection or meticulous detail – in marking we must be 'sensible'.

Marking to a mark scheme

Marking requires numerous fast judgements on individual scripts and most caring teachers find this quite stressful, because after all it can mean a great deal to the student. You have to be fair without being too harsh or too generous. In reality, this actually depends on whether the marking really matters – for example you can afford to be a bit harsh in coursework, but maybe less so in an exam. You need to be sensible – time spent on agonizing over a precise mark is rarely fruitful, student assessment is simply not that accurate a process and it is better to take a step back and look at a question globally to get an overall feel for what the solution is worth. One specific point – if the student has made a poor attempt that you consider makes very little contribution to the solution, don't necessarily award zero marks. The student may well have anguished over the question and tried to tackle it. A flat zero mark without comment can then be very demoralising, so try to find something encouraging to say and if there is sign of real effort on the part of the student, then give a token small mark – this makes very little difference in the grand scheme of things.

At least in the UK (other countries have different traditions), when marking mathematics in higher education we try to identify exactly what the student knows, in the fairest possible way. We all know what a sequential subject mathematics is, the solution of a typical question proceeding through a number of stages. Often a student will make an error at one stage, but proceed subsequently perfectly correctly, carrying through the original error to the final answer. This is one reason why it is sometimes best not to provide the answer in the question, for example, 'show that the integral of cos x.sin x is…..'. The student may worry unduly and waste time trying to locate their error if they don't obtain the given answer. If the student makes an error early in the question which makes it difficult to continue as intended, then try to find clues that the student did know how to proceed but got bogged down in the detail of the calculation. Don't penalise the same error twice. Carry consequential errors through the solution, unless of course the error makes the subsequent part of the solution much easier than was intended. However, you should stick to the marking scheme as much as possible, unless there is a compelling reason to deviate from it.

Avoid the halo effect in marking (allocating excessively high or low marks for reasons other than objective interpretation of the marking scheme). An example here is to avoid being unduly influenced by untidy work or poor handwriting. But, if you are really unable to read a student's handwriting, you simply cannot award marks for things you don't know are there. Also, as you mark, your mood can change – don't let it influence your marking – try marking one question at a time and then shuffling or reversing student scripts so that the second question attempted by students is marked in a different order.

It is worth making a practical point here. If you are marking a test or exam type question, set under examination conditions in which there is no possibility of collusion between students, then it is not unreasonable to simply use the student's answer, provided it is correct and not given in the question, as the basis for allocating marks. If you are sure that the student couldn't have got the answer by guesswork or by avoiding the necessary working, then you can sometimes take the working for granted. This speeds up the marking considerably. What you can't do however is award zero marks if the answer is incorrect – then you have to look more deeply into what the student has done and try to identify the extent of student understanding.

While we always try to be consistent and totally objective in our marking it is worth remembering that this is an imperfect world and occasionally we might need to exercise generosity or severity in the wider interests of the students and the teaching. Your baseline in such decisions should be 'What is in the best interests of the student'. This doesn't mean giving them marks they don't deserve, and may even mean being a bit harsh. On the other hand there is no point in being gratuitously hard on them, and sometimes it doesn't do any harm to give a student the benefit of the doubt – in fact, if anything, we would encourage this. In any event you should always have a considered educational reason for your decisions.

An important issue that sometimes causes postgraduates problems is the degree of resolution of the marking scheme. For example, you might have a short question for which four marks can be awarded. The student makes a minor error so you don't think it is worth full marks, but dropping it to three seems a bit severe; you feel like awarding a half-mark. Of course, this is ultimately down to you, after all, you are doing the necessary arithmetic to collate the marks at the end. But you might wish to consider whether it is worth anguishing over such a decision in the first place. The issue about the half mark is really one of not having a sufficiently fine grading scale to resolve decisions on what to award and being forced to perhaps penalise something too heavily, or to give too much benefit of the doubt. The first thing to do in such situations is clarify how important such decisions are. If you were marking a final year exam then they are very important. On the other hand if it is a piece of coursework that only contributes a few percent to overall assessment then whether it is worth a '3 or 4' is hardly relevant – although to a student, it will often have far more significance. Some academic members of staff argue they will **never** award full marks. Why not?! If the student has achieved all that has been asked of them what hidden requirements are these penalties being based on? On the 1-4 scale discussed here that means students could never achieve more than 75% - one hopes that the reason for this is explained to them very carefully! On the other hand, if you are doubtful about giving 3 even though they have made a small mistake, remember that this is still a first class mark. Another ploy is to compensate for what you consider might be a harsh judgement on one question by being lenient on another. The main thing is to be sensible, practical and fair throughout. Often, as we will see later, the feedback on the work itself is more important than the actual mark awarded.

Sometimes a student's solution departs from the marking scheme and they produce a completely different but also correct solution that has not been allowed for. This may mean the student has shown 'flair' and initiative and deserves good marks. It may also mean that they have not learned what has been taught them, but have relied upon previous knowledge from another source – that is, their solution does not constitute evidence that they have achieved the learning objectives for the course. Here, the mark you award should be firmly referenced against the learning objectives; we will say a little more about this shortly (Section 6.3).

5.4 A detailed example

Here we will look in detail at the marking of a simple question set for engineering students. This is simply to illustrate a number of key points that frequently arise during the course of marking student work. It illustrates the sort of detailed thinking one has to go through even in the simplest of questions, and this has to be done quickly. Sometimes you might skim quickly through a student's solution and jump to the wrong conclusion at first glance. In the question no aids, such as calculators or formulae sheets are allowed, and there are no explicit presentation marks:

Expand $(2 + 3x)^5$ *by the Binomial Theorem.*

This would seem to be a perfectly innocuous question, as it was intended to be, either the student knows the Binomial Theorem or they do not. There are 6 marks available for this question, and it has been extensively used in the workshops that form the foundation for this book with around a thousand postgraduates to date.

The first key point about this question is that most students will have seen the Binomial Theorem before but almost invariably by Pascal's Triangle. However, it was made clear during the particular course in which this was used that they would be expected to learn the general binomial expansion with the coefficients n(n-1) ... (n-r+1)/r! because this is easier to use for large powers and applies without modification when the power is not a positive integer. Predictably many students simply use what they already know and stick with Pascal's triangle. Strictly speaking such students have given no evidence that they have actually learned what was expected. On the other hand they may simply have chosen what is the simplest option if they can actually remember the coefficients from Pascal's triangle. This is a typical situation where the student might depart from the mark scheme and you are left with the problem about what marks to award. In this case you might need to seek further guidance from the lecturer for the course.

Some students might evaluate the expansion by longhand expansion of the brackets without using the Binomial Theorem, and then clearly they have not demonstrated that they have learned this – here, you would be entitled to give zero marks. But if they for some obscure reason they expanded it in the form $(2 + 3x)^2(2 + 3x)^3$ using the Binomial Theorem for the two factors and then expanded longhand, then you might show some generosity and award say half-marks. As noted earlier, the mark awarded in such situations also depends on the outcome of any marks for the student and the purpose of the assessment.

Another key point about this question is that it is possible to get the answer almost completely wrong by making the single small error of forgetting to include the 3 in the expansion – that is writing x^r or $3x^r$ instead of $(3x)^r$. This is a very simple oversight for the student to make if it were an exam question and otherwise the student may understand the Binomial Theorem perfectly well. In such a case, it is not overly generous to penalise this with the deduction of just one mark out of the six, even though the final result might have four out of six of the coefficients wrong. Possibly, this error was not allowed for in the mark scheme. Few schemes can be so detailed that they highlight every possible error; then experience and judgement is called for. Ask around to get a range of opinions. Give the benefit of the doubt where appropriate. Ask yourself whether the student has really demonstrated that they have learned what the question was intended to test. If they have not, have they still demonstrated some ability that is worthy of merit? Is the question explicit enough to render the mistake important (cf: the multiplication question in the Sum Marking exercise of Section 5.1 - how serious is the error for the learning objective being tested?). Does the student 'really know what they are doing'?

If a student makes an up-front mistake with a coefficient, this will affect their final answer – how can such an initial mistake be taken into account and how can you work out what the answer would be in this circumstance so that you do not penalize the student further? You might need to work out the subsequent solution yourself with this different starting point, but this can be time intensive for multiple cases. Instead, you might wish to utilize a computer algebra package to calculate the result – such systems are now available readily online and can prove invaluable to markers.

Now, let's turn to the actual marking of this question, assuming it to be an exam question, so we are being careful about the mark. Appendix 1 gives a mark scheme for the question and Appendix 2 gives the attempts by three students, A, B, C. You may receive mark schemes in a wide variety of forms and degrees of helpfulness. If you feel the one you receive is not sufficient for you to feel confident in your marking then ask the lecturer for more information. The attempt of Student A has been used in a number of workshops to illustrate the variation in marks between different markers. Since the students have no formula sheet they are expected to know the Binomial Theorem for $(a + b)^n$ for a positive integer n and be able to use it for any a and b. Note that the question doesn't ask for the general form of the Binomial Theorem, and Student A has shown great facility in using it directly without solely relying upon inputting the information available into a formula; this has to be allowed for in marking the solution.

Student A has indeed learned the form required and the first line has the correct factorial coefficients. However, they have made the common error referred to above of forgetting that in this case b = 3x, instead taking b = x. This is the only error in the solution, which is subsequently continued correctly. So for simply using x instead of 3x we might deduct one or at most two marks, and this student deserves at least 4 marks out of the six – one wouldn't quibble over 5. In fact, the marks given for this by 25 delegates in one workshop, without using the marking scheme, were:

0	1	2	3	4	5	6
1	3	3	9	6	3	0

with an average of three. Even without a marking scheme, three out of six is harsh. This student clearly **knows** the Binomial Theorem and has made a small mistake in its application in this case. To award only half of the marks is simply unfair. To award less, as done by over a quarter of the markers here is unacceptable. Certainly the zero, implying that the student has learned nothing at all, is evidence of a failure of the marker, not the student! If a marking scheme such as that in Appendix 1 is used then we find the spread of marks reduces somewhat and the average mark increases, but it is still not unusual to obtain a distribution that spans a range of degree classifications.

The previous exercise has been used in many postgraduate workshops and you may be interested in the spread of marks awarded at one workshop (out of 6) for the three attempts. Perhaps quickly give your own marks before looking at these results. In the actual exam all students made different types of errors but obtained the same overall mark of 4/6:

Student	0	1	2	3	4	5	6
A	0	0	2	12	8	0	0
B	0	0	1	3	5	7	3
C	0	1	5	2	6	1	0

If nothing else, such exercises taught the delegates at the workshops that there was plenty of room for debate about marking even the simplest of questions. Remember that for the purpose of the exercise it has been assumed these are student attempts under exam conditions. Of course if it is set as a coursework exercise (for which such a question would really be too routine), where students have time to check and improve, then the marking would be less generous. For example, student A should have spotted their error with the missing 3 and corrected it – then awarding only half marks would send a strong message to check more carefully next time.

5.5 Feedback on students' work

Normally one does not have the opportunity to give feedback on exam solutions, so now let us assume the marking considered above is for the question tackled as coursework. One of the primary purposes for coursework is for students to benefit from the feedback you provide on their work. This is a key tool for helping students to learn, and it is important that feedback on coursework is returned as promptly as possible, while it is still fresh in their minds. By feedback on student work we mean comments about the work that will enable the students to learn from what they have done and improve their understanding.

The human aspect is particularly important in providing feedback. This is probably the most sensitive area of teaching in terms of human interaction. In all other situations you have sufficient distance, physically and officially, between the students and yourself to control interactions. But with coursework you are directly involved with the students and you are interacting on matters that directly affect their future. Some students take it very seriously and will want their marks explained and justified. Some regard your mark as nothing more than an initial offer on which negotiations can commence – usually in the upwards direction! It doesn't help to be formal in such situations or to 'pull rank'. If you cannot easily justify your marks then you perhaps ought to consider whether the student is correct. In our experience, if you are considerate to your students and respect them, are honest and rigorous with yourself then such things are invariably resolved amicably.

Additionally, coursework is wide open to the possibility of plagiarism, or students copying from each other. While this is perhaps an issue in essay-based assessment, say in the arts or social sciences, where plagiarism is almost impossible to conceal, it is really a non-issue in subjects like mathematics, where it is almost impossible to detect, investigate and prove. Particularly in techniques-based topics, two completely independent solutions to the same problem can look virtually identical. The only real indicator is when a number of scripts are submitted with identical 'silly' mistakes; even then it is still difficult to assign responsibilities. If the assessment regime is such that significant gain can be made through cheating or plagiarism, then it is not really reliable or valid. If in any event you believe there is an issue with plagiarism then report it to the responsible authority, typically the lecturer for the course in the first instance. Under no circumstances should you accuse or challenge a student of plagiarism yourself. Not only is plagiarism hard to prove conclusively, your university will have a clear policy on how suspect cases are investigated.

On providing feedback on student work, there are a wide range of mechanisms possible, including full specimen solutions and notes, annotations on student scripts, oral feedback in class, classroom demonstrations, personal summary sheets, feedback forms, group sheets identifying common mistakes or errors, posting on notice boards or the virtual learning environment, workbooks, follow-up teaching sessions, an effective open-door policy, and individual discussions with students. The important thing is to make every effort to ensure that all students receive the feedback in one form or another. Below we summarise some key points when providing oral and written feedback:

Oral feedback

- **Mix positive and negative feedback** in a constructive way, don't be too critical, and try to direct the student towards improvement.
- **Ask students what they think** first, before giving your feedback.
- **Give them the opportunity to respond** with solutions and identify improvements for themselves.
- **Be descriptive**, not evaluative.
- **Specific** rather than general feedback, so that students can easily focus on what is needed to improve.
- **Direct feedback towards actions** that can be taken to improve.
- **Check the students have understood** the feedback, because they will sometimes say they have, even if they don't really understand what you are talking about!

- **Take into account the situation** – confidentiality or painful embarrassment are two examples. Be sensitive to the feelings of students when providing feedback.
- **Make appropriate allowances** for students and don't have excessive expectations of them. Sometimes we say things like 'you should know that already' which immediately discomforts them. There are many reasons why students might not know something we think they should know, not all of them the fault of the student - it might be our poor teaching, after all!
- **Only feedback with an amount of information the student can use**. It is no good giving so much feedback that the student can't see which part to use or how to use it. Structure your feedback as efficiently as possible. For example, in solutions of a large number of questions a student may consistently repeat similar mistakes (for example, applies initial conditions to the complementary function before adding the particular integral in solving inhomogeneous differential equations). Bring these mistakes together and summarise the basic underlying error succinctly, leaving it for the student themselves to correct individual instances.
- **Be a good listener** when a student is responding to your feedback. It is amazing how often we think students are saying something we understand when they really mean something different. They literally might not be speaking the same language as you, or they may have completely different view of matters.
- **Agree on outcomes of feedback** and actions to be taken, and then follow-up on these.

Feedback on written work

- **Prompt**. Provide feedback as soon as possible, before the topic moves on too much.
- **Don't just note errors**, provide constructive responses leading the student towards improvement.
- **Give suggestions** for further reading or work indicating where the student needs to concentrate their efforts.
- **Balance** encouraging and critical feedback, and try to end on an optimistic high note.
- **Focus on a few points** that will result in greatest improvement rather than highlighting every minor error.
- **Relate to marking criteria**, distinguish between different skills that the student needs to improve.
- **Write detailed comments** on a separate sheet if necessary and provide a summary of the most common comments to all students.
- **Follow up with oral feedback** when possible.
- **Make sure your feedback aligns with the mark given** and try to indicate how individual marks have been lost.
- **Give an overview** or summary statement of the feedback.
- **Feedback efficiently** – if you note the same mistake being made by a number of students, give selective feedback to the whole class.
- **Don't agonise over feedback** – a few short words will usually have more effect than a long, detailed critique – the students can always discuss things further with you during the class.

A further key point about feedback is that there is such a thing as being **too** helpful to students. It is often argued, for example, that one should not issue worked solutions to coursework, because students just file them away and never look at them again (until too late!). Instead it is advised that one picks up any error in a student's script and suggests appropriate ways forward from there, addressing the particular difficulty of the individual student. While this seems to be very commendable and helpful to the student, it is time consuming, especially with large classes, and it is also not quite so clear cut as it might at first appear.

Firstly, if a student loses marks then that should immediately tell them they have an issue that requires attention, and if their response is to file away the solution then so be it – we cannot regulate the learning methods of every student. They could come to you to help address the problem, but they also need to develop the skills of criticizing their own work, comparing their solution with the specimen solution and identifying where and why they have gone wrong. These are important skills for the students to acquire in order to develop as independent learners – this is a critical skill.

Perhaps the best advice here is to decide an upper limit on the time you can commit to providing individual feedback, and within this identify the really essential messages you must convey in the most efficient way. Don't worry too much with details – perhaps simply put a circle around minor mistakes and let the student sort the details for themselves. If there is a really important point that does need careful and full explanation, then take some care over this – maybe it reflects a misunderstanding that you could have anticipated and avoided in your presentations or tutorials. Perhaps go through it in class next time, or consider making a summary sheet available highlighting common mistakes so that you don't have to write these on every individual student answer script. Some people recommend making comments even when a student has not made an error – simply to extend on what they have done or give them a different viewpoint. In practice you will probably find your time fully employed covering those issues where students **have** made mistakes. In general, you shouldn't take 'a long time' over anything in teaching – time should be used efficiently but effectively for all concerned.

Example

Consider feedback on the Binomial Theorem question marked in Section 5.4 with the solutions given in Appendix 2. If we pretend these are **coursework** questions (in fact, as noted above, it is not a good coursework question as it is too routine and easily copied) on which we have to provide feedback then we might adopt a strategy as follows.

All students have got the Binomial Theorem mostly right, but there are some minor errors. In general feedback to all we might re-emphasize the need to get the details of the theorem exactly right – remind the students they won't have a formula sheet during the examination. Two of the students may have forgotten the 3, so in the general feedback note that the rth term should be $2^{n-r}(3x)^r$.

Having summarized the common errors, there is not much to say on each individual script. For Student A you might say nothing at all, or simply '3x not x'. For Student B, something like 'Your coefficients are out of synch' will typically suffice:

$$\frac{5}{1!} \times 2^4 \times (3) \text{ not } \frac{5(5-1)}{1!} \times 2^4 \times (3x).$$

For Student C, we might simply write on the script '$(3x)^r$ not $3x^r$' and '$na^{n-1}b$ not $a^{n-1}b$'. These individual comments, along with issue of the full solutions and an analysis of common errors should suffice for these students to at least begin to identify their problems or misconceptions. It is then up to the students themselves to address these.

A final word of caution, particularly if you are considering providing full solutions or a mark scheme for a previous examination paper. It is the policy of some universities not to allow this; make sure you yourself fully understand the local rules and regulations regarding assessment and marking. The lecturer for your course or other colleagues will be able to advise you.

Chapter 6

Frequently asked questions

6.1 Your questions answered?

More than a thousand postgraduates have so far attended the workshops for which this book was written. At each workshop, postgraduates were asked to pose questions that particularly concerned them regarding any aspect of their teaching duties, from exercise classes and presentations to marking student work. The wide and stimulating range of questions has strongly influenced the content of this book. However, such strong themes emerged in the questions and these were so directly relevant to the teaching duties of a typical postgraduate that we decided to devote this chapter to those questions that were most frequently asked despite the fact that answers can usually be found in the previous chapters of this book. We have divided the questions under the following headings:

- Small group teaching (Exercise classes, discussion groups and presentations).
- Marking and feedback on student work.
- Anything else related to typical postgraduate teaching duties.

We have organized and rephrased the questions to encompass the wide variety of forms they took in the actual workshops.

6.2 Small group teaching (exercise classes, discussion groups and presentations)

How to deal with a group of students who are at very different levels?

This is a common problem, especially in classes where you are teaching non-specialist mathematics students. Some postgraduates questioned the focus of the workshops on the teaching of 'elementary' mathematics. There were a number of reasons for this including the wide range of subject areas represented – from pure mathematics to statistics, from engineering to physics. Postgraduates are often employed to work at first-year level with non-specialist mathematics students, where the main challenge is communicating straightforward mathematical ideas to students who are not always highly motivated or qualified in mathematics. Such teaching is some of the most difficult you will face, and it is not difficult to translate the lessons learned on such material to the teaching of more advanced mathematical topics.

When we have a wide range of ability levels in a group we must resist the temptation to focus on the two extremes – the less able or more able student; both are entitled to equal 'added value'. We need a range of problems, interactive activities, and work to support and challenge all levels of ability. All students should typically have the necessary mathematical pre-requisites for the course, and as such, there is a limit to the variation in ability that might be expected. All students will have the opportunity to consolidate the material discussed during the class in their own time (for example, worked solutions may be issued following completion of an exercise or problem sheet), and so we can perhaps afford to let the more able students start to address some of their mathematical difficulties themselves, or set them more challenging problems to work through outside of the class.

When presenting material to the group as a whole, aim for a 'middle level', but ensure you clearly emphasize and highlight any particularly key points that the less able students might miss. On the other hand, you can occasionally present even the most elementary material from an advanced standpoint that the more-able students might not have seen, and that the less-able students will also be able to understand. Be very specific about what all students will need to know when it comes to assessment. For example, the students will be keen to know whether the proof of a particular result is necessary. Here you can reassure the less-able students about what you will be looking for, but you can also challenge the more-able students to fully understand the finer details of the proof, regardless of whether it will be examined or not.

Often, students are at different mathematical levels because they have different degrees of motivation and interest, and so try to find presentations and explanations that will appeal to a wide range of individuals. This is the subject of Section 1.4. A point perhaps not sufficiently emphasized there is that there is one very strong motivating stimulus for many students in higher education – their self-esteem, which is generally high. When challenged with a problem, such students often feel compelled to solve it, even if for no other reason than they feel that they should be able to. You can capitalize upon this by demonstrating (reasonably) high expectations of them and indicating that you expect them to be able to make some form of progress, even with more challenging problems.

Some postgraduates expressed a form of this question by asking how one deals with perhaps a student who is very much more able than the others within the group. Firstly, find out why their ability is so different – it may be that they are in the wrong class or stream. You can set such students more challenging problems, obtained from a book or previous examination papers, to work on during, or outside of the session. If you do this, you need to ensure you too are familiar with the material as they may ask you to help them at any point. Alternatively, can you utilize this student to help the others learn? Could you get them to lead a group, or explain the material to others students? Certainly, but is such a technique reasonable? All students need to gain benefit from the session, and the benefit of such an approach to this student is that it helps develop or enhance their own understanding of a topic if they need to explain it to others – it also helps develop their wider skills such as presentation, teamwork and communication.

Supporting less able students is more of a challenge, and your role is more one of guiding the student to develop their own learning. You have only a limited amount of time inside, or outside of the class, to spend with all students, and as such you need to encourage less-able students to focus upon knowledge and techniques that are absolutely essential. Does your university have a mathematics support centre where they can go to get further help and guidance? If so, direct such students towards it. There is also a wealth of online material that is appropriate to the student and available for them to use, encourage them to engage with it. If you feel a student is really struggling, you may need to highlight this to the lecturer for the course or their personal tutor so that they can follow this up. Is it acceptable to suggest such a student visits you outside of the class for further help? Absolutely, but remember that you have to balance the demands of your own work and study and there is a limit to any additional support you can provide.

With less-able students, you may need to moderate the level of the mathematical knowledge and techniques upon which they focus. For example, if you are teaching rules of differentiation to first-year engineering students, concentrate primarily on the product rule and function of a function rule with those who are less-able. They are less likely to need the quotient rule anyway, and if they have a thorough understanding of the product rule and function of a function then they can work their way through a quotient. If you are proving a difficult theorem in, say, complex analysis, encourage the less-able students to concentrate more on the actual effect of the theorem rather than upon the conditions under which the theorem holds. For example, if proving Cauchy's theorem and the various Cauchy formulae, for less-able students you might not overly emphasize the conditions imposed on the functions involved, whereas you might expect the more-able students to examine carefully how these incorporate into the proofs. This might sound like heresy to the pure mathematician, but teaching is a continuum of such compromises.

Another variation on this issue is the widely varying levels of motivation displayed by the students. The extreme case arises when a student is resolutely refusing to participate in class activities. There is a limit to how much time you can devote to such students, and it may be that you simply have to give up on them in order not to unduly affect the learning of others. Try to find out why they won't participate; key to dealing with any particular forms of student behaviour is understanding why they are behaving in that way in the first place – you can only do this by talking to them. Your students are young adults and provided you have done your best to try to help them, they are capable of making their own mistakes and dealing with any consequences. You are not a counsellor and certainly shouldn't act as such – if any personal issues arise, ensure you refer the student to their personal tutor or the lecturer for the course who will know what referrals to make if necessary.

How do you deal with disruptive students?

This is not a matter of being 'strict', but of being firm and confident about exercising appropriate control so that all students within the group get the best out of the activity. It is important to emphasize there are differing degrees of disruptive behaviour, and most of what you will experience is relatively low-level, typically students talking when you don't want them to.

If one section of the class is talking or making unreasonable noise then that will affect the working conditions of others and must be controlled, but before you do, try to find out what they are talking about. If they are discussing mathematics, this is exactly what you want from the students but maybe they need to do this at a more appropriate time. If someone is talking while you are trying to say something to the whole class, you need to stop it; for your sake and for the sake of the students around them. Some teachers find it difficult to maintain control in this way and can find it quite stressful. You may feel you are being unnecessarily authoritarian, or worry that you might upset and alienate some of the students. However, provided you are fair and sensible and make it clear that you have the best interests of all the students in mind then this should not be a problem. Politely mention to the students that talking when you are is not acceptable and that there will be opportunities during the session for them to engage in (mathematical!) discussion with others. Students are very reasonable and will typically respond well to appropriate direction. Never lose your temper. Always say please when you ask them to be quiet. Don't hold grudges or allow your actions to go beyond the classroom when dealing with students. The majority of students will invariably support a teacher who controls the class and exercises discipline, and are often contemptuous of those that don't.

If students continue to talk after being asked not to, either split them up or ask them to leave altogether, on no account change what you are doing in order to accommodate them. Sometimes you can deal with such students in a productive way by asking them a mathematical question that requires a response in front of others, or by bringing them to the board to work on a problem; These are perfectly valid techniques if a student has been warned that this is a potential consequence of their behavior. Often it pays to have a quiet word with the student individually when the others are working on a task; find out why they are talking – is it because they find the material easy or too challenging? If you know the reasons for their behavior, you can adapt and react accordingly.

If a student continues to be disruptive, ask them to leave. They are potentially disrupting the learning of other students, who will invariably be on your side in taking a strong line. If the situation gets this far, inform the lecturer responsible for the course or another member of university staff. Don't express anger, malice or any other strong emotion. Stay in control. Don't use sarcasm. If they refuse to leave, you yourself leave and report to the lecturer. You are certainly not paid to handle personal confrontations.

We looked at the issues around maintaining a good atmosphere during the class in Section 4.6. Applying the principles highlighted in this section you should be alert to any possible problems before they arise; there are almost always prior indications of potentially disruptive behaviour long before it manifests itself . Normally, during the first few classes most students are on their best behaviour but some may start to test you to see exactly what you will tolerate. For example, there will inevitably be some students who will start talking amongst themselves when you are trying to talk. As we advise in Section 4.6 deal with such issues quickly and effectively, not with rancour, sarcasm, or anger, simply make it clear that it will not be tolerated. Monitor the future behaviour of these students and keep them in line; you may need to keep reinforcing the point but it is one well worth making. It is also worth seeking advice from the lecturer or other colleagues; if students are disrupting your session it is likely they are doing it in others classes as well.

How to get everybody involved, motivated and engaged.

Many postgraduates at the workshops asked a variation of this question. In the sort of work you are undertaking, this is critical. We have said a great deal about it within Section 1.4, and it is a large area.

Many postgraduates expressed a desire to know how to deal with the long pause and silence in a class that often occurs when they ask students a question during an explanation of a solution, for example. 'Count to twenty' is popular advice here, persist! Eventually a student will contribute a starting point to break the silence. However, there are things you can do at the outset in order to encourage a response. Make sure the question is carefully posed, and present it in a number of different forms. Vague, open, questions are not necessarily the best means of generating debate, similarly it might work better to go from specific cases to more general ones, certainly within mathematics – try to broaden the nature of question, for example, rather than discussing a particular integral, discuss how you might approach integration of certain functions in general. Try to make your questions stimulating, possibly provocative, and maybe intriguing. If you have developed the right rapport with the students, you may be able to ask the question to a specific student (but don't do this if it embarrasses them), sometimes simply by making eye contact with them. If that doesn't work, break the class into smaller groups and get them to discuss the question, ask one member to report back, or get them to submit a response on a piece of paper that you yourself can then discuss with the group..

Perhaps the most useful advice is to remember the students are not you; they may not be as motivated or as confident as you are; indeed you may not always have been as confident and motivated as you are now and so think back to how you felt when asked a question as a student – what would encourage you to respond? Throughout, remember that the primary function in a problem class, for example, is to help students learn and so there should be an of exchange of questions, discussion and debate. There is therefore nothing wrong in moving around the classroom, engaging students in 'spontaneous' discussion, either individually or in small groups. You will almost certainly find that after a short while you identify a problem, challenge, or concern - something you can help them with, something upon which you can build, either with the individual student or the wider group.

If no-one at all is asking questions, remind them that you are there to help them and now really is the best time to obtain your help and assistance. Make it clear that you really do want to help them, at that point someone will usually ask you over. This is an important aspect of taking tutorials or problem classes. If you develop the right rapport with students, earn their trust and give good value you will find increasingly more students will begin to ask you questions. Asking a question in a classroom situation is quite a daunting prospect for any student you might be teaching – there will be some who are always willing to ask questions or offer answers, but try not to let them dominate the group; equally, be wary of such students shouting out answers, particularly when you want other students to think for a while about the question or statement that has been posed. Consider why many students don't ask questions or volunteer answers, it is the fear of not only being incorrect but also the fear that others will laugh and ridicule them, This is where you need to build a safe and friendly learning atmosphere, never laugh at, or ridicule a student yourself, regardless of their question or answer, and do not tolerate it from other students.

Some postgraduates were worried that if they encourage interaction too much, some students will stop coming to classes. Yes, this can happen and it is sometimes a fine balance. The fact is, many students simply do not like problem classes, or classes where they have to have a significant input. Many prefer to simply work on their own, at home, or in the library with a friend. There is nothing wrong with this, we all like to choose the environment in which we work and learn best. Your students are adults, entitled to make such choices, so although we emphasize the need to actively engage them in a classroom environment, this really only applies to those that want to – you cannot force them to participate. In one sense it is acceptable for those that don't want to be there to stay away from the session, then you can concentrate on those that do. But, if they all stay away then there is something fundamentally wrong with the support you are providing. As in any 'public' event such as tutorials, problems classes, or lectures, if you want people to attend you have to have something that is of value, benefit or interest to them. The session does have to be valuable to the students – addresses the challenging topics and exemplifies the key techniques. More prosaically, work through past examination papers, exercise sheets, or previews of coursework. One point where you may need to intervene is if you feel that the students, by their lack of involvement, are being disruptive, or adversely affecting the atmosphere of the class for others. Here you must challenge such behaviour, and if necessary, ask them to leave.

It is possible to exert gentle pressure on students to engage, but this is a subtle skill, needing a mix of humour, firmness and earned respect and authority. It also requires a good sense of when this should be left well alone, either with the group as a whole or individual students. The ability to balance the cost benefits of spending time on recalcitrant students is probably a key quality of the good teacher. Remember that in such a learning environment you are actually doing more than just simply mathematics. Part of the purpose of such classes is to teach the students to be able to work with others in solving problems. They therefore need to learn how to contribute positively, to be able to take risks with ideas, to criticize (appropriately) the views of others, and (when) to ask the right questions. You are therefore perfectly entitled to encourage them to engage – be up front about this and consider building it into your 'ground rules' at the outset that you expect the students to actively participate during the sessions. Sometimes the students might not know how to engage, they may not want to interrupt you or appear rude; make it clear at the outset of the class how they can best engage – if you can encourage this, it will make your teaching experience a lot easier and more enjoyable.

If you start a tutorial and everyone has already done the sheet correctly, what would be the best thing to do? It could be difficult to ad-lib.

Preparation is key to teaching, and you should therefore plan ahead with this possibility in mind; if you do so, adapting and reacting won't seem such a challenge. If you have worked through the problem or exercise fully yourself, in depth and breadth, then you should be able to suggest some harder variations for the students to work through – this might involve modifying an existing question or having a similar, but more challenging example, that you can give to the students. If the students have completed all questions on the problem or exercise sheet without challenge, then you may need to feed this back to the lecturer with responsibility for the course – the students may be (unusually) diligent or more likely the tasks were too easy. More likely, you will find it is a small number of students who have completed all of the tasks – in this scenario, you may wish to bring them together to work on a more challenging example as a group while you work through the existing sheet with the other members of the class.

Another ploy is to look ahead to the work scheduled for the next session, or expand upon the current topic, relating it to other areas. For example, if the work relates to integration by parts and comprises a number of standard examples, perhaps show the students applications to reduction formulae. The key point – you have to know the subject sufficiently well to be able to ad-lib, this requires preparation in advance.

Students lose concentration about halfway through an hour's class. Any ideas?

Most people lose concentration after half an hour working on a task. Ideally you need to introduce some change of activity after about 20-30 minutes, even if it is only a two-minute break. Some short story, joke, anecdote, piece of history, or recent application relevant to the topic can be discussed with the students and you can find many examples on the internet; there are some fascinating historical and personal stories behind many areas of mathematics. Alternatively, you may spend a few minutes discussing something that isn't mathematical – if you do, you need to think about how you will restart the session afterwards and ensure that this doesn't go on for too long – the students are in the class to learn mathematics. The key thing to remember is that not only might **some** students lose concentration after a while, but it is **inevitable that all** students will lose concentration eventually; part of the role of a teacher is to anticipate when this occurs and deal with it. Look round the room for signs that students are losing focus, there might be discussions taking place, or even yawning(!). If you think a change of pace is needed, do it, engage the students in a different activity.

How to explain clearly to students?

The importance of good explanation is a central theme of this book, and Section 4.6 treats it at length. The sorts of questions postgraduates raised about this issue were often quite specific, and related to individuals issues they had faced. For example, some wanted to know how to put long solutions into perspective when working through them on the blackboard. This is a key component of good explanation, setting the sometimes complex mathematical ideas and details into context. We mentioned this in Section 4.6, providing regular overviews, highlighting the essence of an idea, a proof, or technique and communicating these clearly to the students; all are key aspects of explanation. Break the topic or idea into 'bite-sized chunks', but be sure you also give what might be termed a 'road map overview' that shows the students how to bring the individual 'chunks' together. Identify and emphasize key points – for example $(a+b)^2$ and A-A in completing the square. Be economical with the truth – sometimes we have to skip quickly over finer detail or more challenging cases during a first treatment or at a certain level – in either case, ensure that the students understand that this is what you are doing; some might have encountered these aspects before and could ask about them, alternatively, they may need to know such details for a future course. Use memorable phrases that encapsulate the topic. For example one of the authors uses 'rings and sings' in contour integration, which reminds students they are always having to think about the contour (ring) and the singularities. When they meet a contour integral, 'rings and sings' (hopefully) springs to mind!

In pure mathematics there are lemmas and propositions which are in effect, ways of breaking up long pieces of mathematical work. But in fact, this is not always helpful and can impede fluidity. Explain 'sensibly' as well as logically. Keep asking the class what they think the next sensible step might be. Imagine how the person who first tackled this topic might have approached it. Ask why each step is taken. Sometimes in mathematics there is a tendency to 'pull rabbits out of a hat', for example we say things like 'Consider the function ...', without saying why and then proceed through extensive logical steps until the result we are looking for suddenly pops out. This may be elegant in the final polished presentation of the topic, but it is not the way most people think and is a particularly unfair way to introduce difficult ideas and concepts to those who are still relatively new to the subject. Admit the *ansatz* (an educated guess that is verified later by its results) for what it is, a device to give us the result we want – the proof won't work unless we take that particular type of function, or make a particular assumption. The person who first did this quite possibly started from what they wanted to prove and worked backwards to get the function they then 'start' with.

When it comes to actually explaining a mathematical argument to a student face-to-face, during a tutorial or a mathematics session for example, it is best to gently interrogate the student as you go along. Question the student until you are clear about their current level of understanding, and while doing so try to break down the logical steps needed to reach the necessary result that enables the student to genuinely understand – these will be different for different students. Most students, if you get the pace correct, while be genuinely trying to follow your explanation and understand the steps, but may deviate from the correct path because they are not thinking along the same lines as you. By repeatedly quizzing them as you go along you are hopefully bringing them back onto the correct path. Encourage them to ask you questions so that they themselves can check if they are progressing in the correct direction towards the end result. It is more difficult to do this sort of thing in a class of twenty students, but you can get an approximation by trying it within a small sub-group – the other students will then hopefully be able to follow the discussion and see the argument more clearly.

What do we do if they ask something you don't know, so you can't answer their question?

This is a very common concern, and the answer is simple – whatever you do, don't blag it! Be open to yourself; if this is a question you are **supposed** to be able to answer, for example, one of the tutorial problems, then apologize and try to rectify the situation as soon as possible, either during the class when the students are working on another task, or outside of the session; you can then communicate the result to the students either electronically (virtual learning environment) or at the start of the next session. If it is a question that is peripheral to the object of the class, or for which there is no reason for you to have prepared, then consider trying it for a while either during the class or with a small group of students afterwards. If you can't do it relatively quickly (you mustn't allow it to distract you from the main purpose of the class) admit it, and if you think it appropriate say you will deal with it later and make the solution available. However, for such questions, there is no obligation that you should try to tackle them; if it isn't appropriate, explain this to the students – you need to exercise care with such questions to ensure that some students don't misunderstand and feel this is the kind of mathematical content that they are expected to address, for example, during the exam.

We must emphasize that there is nothing wrong with occasionally not being able to answer a question, particularly if it is one that is somewhat unexpected. The students will benefit greatly from watching you tackle it, even if you don't succeed – this is, after all, part of the problem solving process. They will see how a mathematician works, see that it is OK to cross out, make mistakes and educated guesses. They will feel better about themselves and realize 'it isn't just them'. It might even spur them on to try harder. If you think the question is interesting and appropriate, open it up to others in the class to see if they can tackle it. Be careful to ensure though you are not solving someone's coursework for another course! The key point is not to be embarrassed if you can't do a problem - we all have off days and none of us are perfect. It is the way in which you subsequently handle the situation that is critical, but for questions that are on a student problem sheet, preparing adequately in advance should greatly minimize the risk of not being able to solve a particular mathematical problem.

One variant of this question is worth considering separately, that is when the question from the student appears so obvious that you feel you cannot say anything without revealing the answer. Remember that while the question may appear short, and most likely, 'trivial,' to you, it is unlikely to be so for the student that raises it with you. Take something like "What is the integral of 1/x?". It may be that the student is generally fine with calculus but has just forgotten this standard result and wants you to remind them. You are there to help students learn, but not to be a resource of standard mathematical results. Suggest that they try to tackle it, and then ask whether they think this might be a standard result – if so, where can they find it? Suggest they try a few standard derivatives, or ask why the formula $\frac{x^{n+1}}{n+1}$ doesn't work – anything to get them thinking. On the other hand the student may be very weak at calculus and the question reveals a much deeper issue – they may not have been taught calculus from first principles for example, but just as a set of rules to apply. In this case, where do you start? You have to talk to the student to establish precisely what the difficulty is. You could start with "do you know what a derivative is – how it is defined, what does it represent?". Note that since the student has found the courage to ask you the question, they are most likely already **engaged** and **enthused** - you are two parts there and now you have to **explain**!

How can I distribute my attention to all students in a fair way?

A specific example was cited here. Suppose you have just a minute or two at the end of a tutorial and three students ask for help. One student you know to be very weak and struggling with the most straightforward of questions, one at the level reasonably expected but with some quite common difficulties, and one who is very able and has a mastery of the material and who has a subtle issue unlikely to concern other students. Which question do you address? There many options for dealing with this. You might answer the question of the least-able student since their need is the greatest. You might answer the question of what we might term the 'average' student because their question is likely to be of interest and relevant to the majority of the other students. You might answer the more-able student because they have asked an interesting and incisive question; it might be one that particular interests you. However, the simple fact is that you must try to respond to **all** the students in one form or another. For example, consider if the queries are related in some way, you can answer all three together (another reason for having a broad view of the topic), stretching your explanation a little. Or spend the majority of the time on the most important question (which may be that of the weaker student or that of the average student since this will relevant to many others), make quick comments on the others and ask any student still in doubt to remain behind for a few minutes. This is a somewhat exaggerated situation, but the key point is that you often need to think laterally and quickly if you are to try and accommodate all of the student needs sympathetically; however, we fully recognize that this is not always possible, even with the best of intentions.

How to avoid just telling them the answer to a problem when they really seem to have no idea how to start?

This question often occurs. It really comes down to how to help students when they are struggling, say when working through a problem in an exercise class, but key is not to help them **too much**! It is, as Kranz says, being helpful without giving away the store, we mentioned this in Section 4.6. By definition, many mathematical problems will contain a number of steps, and to tackle the problem the student needs to not only understand each of the individual steps but also have an overview of how they all fit together. Ask the students questions that tease out these different aspects, leading them to hopefully to fill in the gaps themselves – this is an ideal opportunity to generate discussion amongst the group with different students contributing ideas.

Ask the student to write out the question in their own words, or highlight key words within the question upon which they should focus; do they understand what the question requires? This may be the issue. Get them to write down some key facts about the particular topic – using their notes if necessary. Encourage them to guess. Think about yourself – how do you begin a difficult problem, like within your own research? One of the most common reasons why students can't begin a particular problem is because they try to begin with steps that are too large. Suggest they look for a similar example within their notes, or try to reduce the problem to a more straightforward one that could act as a starting point (this is ideal for mathematical modeling). Even with such hints, sometimes a student is still not able to solve the problem and there may be a temptation to go straight to the solution. 'Giving hints' is something one does with people who have 'nearly' got the idea, but need a bit of a prompt, and usually it is tantamount to filling a gap within the solution that the student hasn't yet bridged. This may be appropriate if the gap isn't too large or too serious. But, if it doesn't enable the student to proceed then you need to explore deeper within the understanding of the student and identify the mathematical 'tools' they need to tackle this problem for themselves – this could involve 'backtracking', if a student cannot solve a first-order differential equation by separation of variables, do they understand what a differential equation is? Are they sufficiently happy with re-arranging equations? If not, these are your new starting points for helping the student. Remember the objective is not so much to help them solve a particular problem, but to help them develop the necessary skills for solving any such problem themselves.

In essence, the overall approach is about questioning the students constructively in a way that leads them to better understanding – usually you don't actually answer their questions or tell them anything mathematical at all, you simply ask the right questions and they identify the solutions. Another ploy is to work through a similar question, but if this is too close to the one they are struggling with then it doesn't really help them to learn – they just imitate you. Help students to develop the skills of identifying their own 'sticking points' and finding ways to address them – this includes encouraging them to look things up in their own time. Often students don't know where to start because they are frightened to start, a gentle nudge or a little pressure can often be the motivating factor they need.

How can I be sure, at the end of the class, that the students have learned something?

The honest answer is that you can never be sure what any student has learned, although you can try. Provide a short quiz based upon the material covered, ask questions at the end of the session or at the beginning of the next based upon the material in their notes. They might be specific mathematical questions, or more general - something like 'What are the three key points of this session?'. Watch their reactions and any contributions (or lack of) that they make; this is much easier in a small class of the sort we are considering than it is in a large lecture. You need to think carefully about the sort of questions you ask, to ensure they are specific and clear, and your ability to obtain responses from across the group, not just one or two individual students. For example, if you have just covered integration by parts give the group a few minutes at the end of the session to write down the methods they would use to integrate each of x exp(x), x exp(x^2), x^2 exp(x), x^2 exp(x2). You might provide a list of multiple choices on a slide that they can choose from, and then encourage them to vote, or you might collect their responses; either way, you will soon see who has learned what. Don't regard such an activity as a waste of precious class time – it will help them learn and help you understand how they learn. Additionally, beware of taking the student's word for it if they say they have understood something – sometimes they will answer 'yes' simply because this is the response that is least likely to lead to further follow-up on their part. If you really want to know, persist by asking questions that require mathematical responses.

How to make best use of the blackboard and other media?

We have dealt with this at length in Section 4.5. Briefly, the blackboard is not just a place to write notes for the students to transcribe. Naturally this is one function, and when you do you have to observe all the rules of good communication – an adequate writing size, concise, tidy, properly sequenced and ordered, and all written at a steady pace. But the real use of the blackboard comes in as a discussion object. Having written down some item, an example or proof maybe, stand back with the students and discuss it, highlighting salient features, debating key points or ideas with them, do they understand each step? Can they identify intermediate steps or sections that have not yet been completed? You can sometimes use different parts of the board for different things at the same time. For example, if you are working through a proof on one half of the board, you might undertake a numerical example in parallel on another part of the board, illustrating the general argument. Also, as you write on the board, sometimes regard yourself as the scribe, getting the students to tell you what to write next.

Some postgraduates expressed specific concern about the quality of their handwriting and how to improve it. Writing on a board is a challenge, and the first time you do it, it is somewhat 'unnatural'. Firstly, slow down and write as little as possible by making your notes succinct and to the point. Print rather than producing cursive writing – this will also, most likely, be easier for the students to read. Occasionally walk to the back of the class to see how your writing looks, if you find it hard to read, this can often be the stimulus that motivates (perhaps temporary) improvement. If necessary, try to practice in advance on your own, or failing that, consider producing slides or transparencies in advance of key points, but these should ideally never wholly replace work at a board. If the students cannot read your writing, they will usually be quite vocal in telling you.

There were a number of questions regarding the use of PowerPoint or other electronic media. The determining factor in the use of any kind of media is its effectiveness in communicating your message. Many mathematical topics are best conveyed by 'chalk and talk' or its variants. Prepared OHP slides can be useful for adding emphasis (such as summaries or diagrams), but rarely in isolation – it is difficult to imagine any effective mathematical presentation in which the lecturer is not in some way active at the board, developing the mathematical ideas and arguments as the session progresses. Slides can be used as objects for discussion, perhaps to support handouts that the students can annotate. Mathematics really isn't a 'spectator sport', you learn mathematics by doing it, and the second best thing to tackling a problem yourself, is watching someone else tackle it.

There may be occasions when PowerPoint can be useful, say to play a video clip, display an image or diagram, however, its role is to enhance delivery rather than enable delivery. Similarly for computer software, resources such as GeoGebra can provide real-time manipulation or graphing of mathematical functions – this makes them ideal as a basis for showing ideas and generating discussion.

How do you avoid feeling intimidated if you are in front of 60/70 students?

When you find out, tell us! For most of us walking into a classroom is one of the most nerve racking parts of our day. It is hard to avoid **feeling** intimidated, but key is not **showing** it. This is where acting comes into teaching – exude confidence and composure, appear totally relaxed, smile and get down to the mathematical business of the session. Focus on the task, learn to deal with distractions, expect and encourage student cooperation, and maintain order sensibly. All are easy to say, but not necessarily easy in practice. By far the most effective strategy is to have confidence in your mastery of the topic and to be entirely clear about how you will approach the teaching session. The material should be second nature to you and at your fingertips so that you simply don't have to think about it. Prepare material meticulously, have confidence in your status and responsibilities, and treat the students as you wish to be treated; if you do this, all will be fine. But one of us, with decades of teaching experience will shortly after writing this be going into a class of 150 engineers for a two-hour session – I am less than relaxed about it!

How to structure lessons?

This large topic is the subject of Sections 1.5 and 2.2. Identify the (few) absolutely key points, ideas, concepts, and techniques that you need to communicate during the lesson. Then, design the most efficient and effective way of communicating these to the students during the session. Try to use a range of different methods to vary the pace: explain and motivate key points, give the students examples, ask them questions, provide problems for them to tackle individually or as a group, identify topics for which you can have a discussion or sharing of ideas. Try to balance the time available appropriately, you shouldn't spend the entire session presenting material yourself – a significant component of a tutorial or problem class should be student led activity. Remember, contact time with the students is prime teaching time and so don't waste it on minor details or routine matters – address the fundamental, difficult ideas that really unlock the topic, and provide the students with the opportunity to apply these ideas for themselves.

6.3 Marking and feedback on student work

What is a good way of marking? How should I evaluate different kinds of mistakes?

This is the purpose of a good mark scheme. However, few are (or even should be) so detailed that they will cover every possible mistake that a student can make; then experience and judgment is called for. In such instances, give the benefit of the doubt to the student where appropriate. Has the student really demonstrated that they have learned what the question was intended to test? If they have not, have they still demonstrated some ability that is still worthy of merit? Is the question explicit enough to render the mistake important (see the multiplication question in Section 5.1 - how serious is the error for the learning outcome being tested?). Does the student 'really know what they are doing'?

It may be that you do not agree with the mark scheme, or with departmental suggestions on marking and feedback. It might be that that the feedback is too 'numerical' (i.e. a mark) and not helpful enough, that feedback should be communicated verbally rather than in writing. Both are good points, but a departmental policy is there for a reason and you should ensure you follow it. There is some research that suggests that when students receive their coursework back they only look at the marks achieved and don't read the feedback itself. Possibly they already have some preconceived idea of what a reasonable mark for the work is and if the two align, they are happy to accept it. The problem is that if you don't provide marks for work then, sadly, few students will actually hand it in. Naturally, providing copious individual feedback on each student's work may simply take up too much time, so a balance is needed. See Section 5.5 for ways of providing feedback in general.

A number of postgraduates expressed concerns regarding what might be termed 'the minutiae of marking', for example if they are not allowed to award half marks what is better to give, the higher or lower mark, for example, a 3 as opposed to a 4. One could boost a student's confidence with 4 but a 3 might demonstrate more effectively that the student needs to exercise greater care; which is best? We discuss this situation in some detail in Section 5.3, but briefly, all marking is a blunt instrument, and calls for common sense and compromise. It is a continual argument with yourself that, as marker, you need to resolve quickly. Usually the mark you are worried over awarding is not that significant in reality, and perhaps the only mark we should think particularly carefully over awarding is zero. Awarding zero ends a significant message to the student, that there was nothing in their solution of mathematical merit – if this is genuinely the case, so be it, but make absolutely certain. If the student has made a genuine attempt (on a question that is worth say 4 marks or more), it is worth perhaps considering a 'token' mark.

How much is it worth writing on a student's script?

This is a balance between the time you have available, your other priorities and your commitment to teaching. As noted above, research suggests that if students are reasonably satisfied by the mark provided on their script then they won't necessarily be interested in the feedback. Place an upper limit on the time you can commit per script and stick to it; ensure you only address points that are key for that student, and you do not have to address all mistakes the student might have made.

Don't worry too much with the minor details when providing feedback –highlight (circle) these and allow the students to identify what this means for themselves; they could always ask you in the session. Commonly occurring errors can be collected on a handout for all students. Take the greatest amount of time over the really important points, which may suggest ways in which you can modify your delivery to ensure they are addressed, or at least highlighted, during the teaching session. For example, if you get a number of cases where students repeatedly confuse integration by parts and integration by substitution, then next time you cover this during a class, clearly highlight the issues and provide a number of examples of both methods. If you don't mark the student work for your class, speak with the postgraduate who does to obtain feedback that you can use to plan and structure your teaching.

As we mention in Section 5.5, there is such a thing as being too helpful in feedback to students. The very fact that they have lost marks should immediately tell then they have an issue that requires attention; if they don't follow this up themselves in order to resolve it, it isn't necessarily our job to remind them or ensure it happens. Students need to develop the essential skill of being able to criticize their own work, comparing their solution with the specimen solution and identifying where and why they have gone wrong.

When marking work, if the mark scheme doesn't seem to fit the work well can we suggest changing the distribution of marks?

This depends on what you (or the lecturer you are assisting) is trying to achieve. If you think the marking scheme can be improved to better reflect better the achievement of the objectives of the work being assessed, then discuss it with the lecturer. But, this is not always as clear-cut a decision as it might at first seem, and ultimately it is a decision that is the responsibility of the lecturer for the course. For example, take the case of the binomial theorem which we discussed in Section 5.4. When you teach this to first year students, the likelihood is that they have already seen it, but many have only seen it in the form of Pascal's triangle. The task is to wean them off this approach and apply the binomial formula, which extends to negative or fractional powers. This in itself can be a difficult task, students are sometimes reluctant to learn new approaches to the same thing and will tend to remain with their familiar method. To discourage this, you will need a marking scheme heavily weighted towards the correct choice of method – indeed all the marks might be awarded for this, with errors in the details of calculation (to an extent) ignored. On the other hand, if you are happy for the students to continue with the method they prefer, then the marks would be more evenly spread across method, implementation and accuracy. We demonstrated in Section 5.4 how great the range of marks can be without a precise mark scheme (and even with one in fact), so before changing the distribution of marks, ensure you are entirely clear of what the objectives of the exercise really are.

A related issue here is the case of the student who has attempted a particular question using a different method to that on the mark scheme. If their solution is perfectly correct do they get all the marks? What if the solution is incorrect but still has some merit, possibly attracting part marks? The answer to such questions really depends on what is meant by 'different'. Provided they have demonstrated that they have learned what it is they are supposed to, one can be generous in awarding marks for 'different' approaches. For this reason, one should normally stick to the marking scheme. The example relating to the binomial theorem exercise in Section 5.4 illustrates this point well. If a student evaluates $(2 + 3x)^5$ by the binomial theorem as instructed in the question, and gets it correct, then this should attract full marks. But if they evaluate it by longhand expansion of the brackets without using the binomial theorem then they have not demonstrated that they have learned the binomial theorem and you would be entitled to award zero marks. But if they for some obscure reason they expanded it in the form $(2 + 3x)^2(2 + 3x)^3$, using the binomial theorem for the two factors and then expanding longhand, then you might show some generosity and award say half marks. It also depends on the outcomes you are seeking to assess and the purpose of the assessment (for example, is it formative or summative? See Section 5.1).

There is another aspect to this particular question that is also relevant here. Many students undertaking the course from which this example was derived, have seen the binomial theorem previously using Pascal's triangle. They are, however, emphatically told during the course that they are not to use Pascal's triangle, but have to learn the general expansion result (so that it can later be generalized to negative powers, for example). Some students do not accept, appreciate or even understand this message and would answer the question using Pascal's triangle. That is, they rely on their previous knowledge, and have given no evidence that they have learned what was actually taught on the course. In this case it is very difficult to decide on a mark that is fair and reflects attainment of the learning outcomes. Unless learning outcomes and questions are phrased in a rigorously precise manner – for example, "Expand by the binomial theorem, but not using Pascal's Triangle", then the student has completed the task set within the question, but on the other hand, has not demonstrated that they have learned an important mathematical technique. In general, re-teaching a topic by a different method is very difficult, you need to explain to the students this is what you are doing, and explain the reasons why very clearly.

When should I be generous/harsh with marks?

The official answer would have to be – 'you should never be generous and you should never be harsh – only fair to all students'. You should be scrupulously fair and objective in all your marking. But, like all generalisations, it isn't always as clear-cut as it might first seem. As discussed in Section 5.3 your benchmark should be 'what is in the best interests of the student?'. This doesn't mean awarding marks they simply don't deserve, and may even mean being a little harsh to act as an incentive to improve, but you must always have a considered educational reason for your decisions, and importantly, ensure this is applied equally to all students.

How can we make marking consistent across different markers?

In theory, this is exactly what the marking scheme is for; in practice, we will still get variation. The Open University, for example, has long had very detailed marking schemes and yet still utilizes moderators to check peoples' marking and ensure consistency across the cohort. The reasons people award different marks can be as varied as their reasons for voting for a particular political party, or liking a particular type of music. Strategies such as double marking (where two people mark the same paper or question, compare marks and subsequently reach agreement), and marking schemes, can mitigate against too much variation in this respect, but won't totally eliminate it.

In reality, the marking you undertake will most likely be formative, or contribute very little towards the overall mark or grade of the student – in this sense you are fortunate. If you are marking summative work, then it may need to be moderated across individual markers and this will typically be done by the lecturer of the course. In some universities, departments have policies that mean two individuals cannot mark the same question on an examination paper.

Should I worry about the way students write and present mathematics?

Yes, but getting student to change how they write mathematics is a challenge. It is time consuming and difficult to get students to write mathematics properly, and requires concerted efforts by all members of staff across a department to ensure its importance is clearly communicated. All you can realistically do is set a good example yourself when writing mathematics on the board – one of the reasons students write brief mathematical explanations is because that is what they have been previously taught. As students sometimes tend to imitate your presentation, ensure that you present mathematical arguments on the board in the appropriate way – this is your contribution towards influencing students in this regard.

Another reason students present mathematical work poorly is that they think they are writing either for themselves, so it doesn't need to be thorough, or they are writing for the teacher, who will 'know what they mean'. Particularly for coursework, emphasize to them that they are actually writing for, say, someone who will be taking the course next year, who may not be able to fill in the gaps, or easily understand what they mean. A further reason they write mathematics poorly is because they don't always understand what they are doing. Explain that the actual process of writing out the mathematics clearly, precisely and correctly actually helps them to learn. Alternatively, explain to them that they need to present mathematics in an appropriate way when submitting coursework or during examinations, if it is presented with an inappropriate structure it not only makes it harder for you to mark it but harder to ensure the students get a fair mark for their efforts; this can sometime be the motivating factor they need!

Related to this issue, some postgraduates asked how you should mark a student's work if you cannot read their writing very clearly. Of course, this depends on the degree of severity. Some work is now so badly presented that a number of lecturers have taken to awarding presentation marks to encourage greater care. Ultimately, you cannot award marks for things you don't know are there; if you cannot decipher what a student has written, then you have no grounds on which to award marks – although there are times when you might wish to 'give the benefit of the doubt' to the student. You might look for evidence elsewhere that the student knows what they are doing, but frankly if you need to work too hard to compensate for poor handwriting, then they really need to be pulled up on it – mention it to their lecturer, or their personal tutor; there will most likely be places within the university where they can receive help.

What should I do if a student disputes the way in which I've marked their work?

This, to an extent, depends upon the degree of disagreement. But, if it cannot be settled amicably refer to the lecturer in charge. Don't be bullied or intimidated! There might be cultural factors – some nationalities consider it reasonable to negotiate their marks – others believe the marker's word is final and wouldn't dream of questioning the decision. You should always be prepared, and able, to justify any marking decisions you have made, but remember, the type of marking that you will most likely be undertaking is likely to contribute a very small component of the student's total assessment, and the effect of you being one or two marks 'out' is likely to be miniscule. Students rarely find this a convincing argument however, but ask them to consider the converse situation, there may be occasions when they receive more marks than they are perhaps entitled to – do they really want you to revisit all cases?!

How to deal with students copying work?

We need to establish what we are talking about here as this too is rarely simple. Are we talking about 'working together', which we should be keen to encourage, or outright plagiarism? The latter is something you shouldn't address yourself at this stage in your career – if you suspect cheating or copying, refer this to the lecturer or other appropriate member of staff, this really is one for them to address. Never accuse a student of plagiarism and never raise this with an individual student. You may wish to say something at the outset, highlighting that you expect students to submit their own individual solutions, and indeed, many universities are now asking students to sign forms to this effect.

Emphasize to the students that working together is fine, even encouraged, but it is in their interests that any work submitted is their own effort, not a reproduction of someone else's. You will reduce the chance of copying not by excessive vigilance of what the students are doing, but by creating an open and secure atmosphere in which students are not afraid to expose their weaknesses and problems.

6.4 Anything else related to your teaching duties

If English is not your first language, how should you behave and act in order that students understand you?

If you are teaching in a university where English is the language that is used for teaching then you should firstly try to improve your spoken and written English; your institution may have courses that can help in this regard.

However, for your immediate purposes if you do experience difficulties communicating with students, there are some things you can do yourself. In mathematics we are perhaps more fortunate than other disciplines in that mathematics is a universal language; you can always write down symbolically what you are trying to say in words – and you can asks students to do the same (note, the reverse is true if you have overseas students within your class and find yourself sometimes unable to understand what they are saying). There may, however, be occasions when this is not always possible, for example, when explaining some subtle geometrical ideas, or using an analogy to get across a mathematical point, in such situations, you might try using a fellow student to explain the ideas or set this as a task for the other students.

As well as language issues, you should also consider the cultural aspects of the differences between yourself and the students. For example some overseas tutors are not typically used to providing the same levels of support to students as might be expected in UK universities, and may have different marking practices and attitudes. On the other hand, in other countries, some practices are considered as a mark of respect that might be considered as plagiarism within the UK. It is therefore vital that you familiarize yourself with the cultural environment in which you teach and adapt your own beliefs and approaches to teaching mathematics accordingly; try to learn as much as you can about the students and their backgrounds. This is an area where talking to other colleagues can really help.

How much planning and preparation?

As long as it takes to make you feel confident that you can do a good job for the students. If this turns out excessively long to meet your other commitments then you have to ask whether you should be undertaking this duty or whether your preparation is as efficient as it perhaps might be. While planning may take a long time for the first occasion you teach a class, if you repeat the duty in subsequent years, it will be much quicker. We have said a great deal about planning and preparation in Chapters 1-4, reflecting its importance. The essential point is that you **begin** planning as early as possible. It is not the actual time spent on planning that is so important, but more the period of time over which planning takes place – the longer you have to think about it, the more your ideas and approaches can naturally develop. As a postgraduate, preparation is particularly important. This may be one of your first experiences of teaching and you may be feeling apprehensive or insecure. As noted previously, the best antidote is good preparation so that you can relax a little about the **content** of the teaching, and concentrate your energy on the more difficult **context** of the teaching.

While preparation is a compromise between the time that you have available and your commitment to teaching, it is so crucial to good teaching that this is perhaps an occasion where it won't hurt to err on the side of being more generous and allow significant (but certainly not excessive) time for preparation. Experienced teachers have various rules of thumb such as two hours preparation per one-hour lecture, so ask around to find if there is a general rule within your institution. Additionally, a specific period of preparation time might be explicitly specified in your contract of employment.

In general, you can get some idea of the preparation required by asking yourself the following questions:

- What are the really key points I want to convey to the students?
- Who am I talking to, what is their background?
- How long have I got with the students? Make sure to include within this any self-study time that is expected of the students.

This can actually be done quite quickly. You can assess what you actually need to do, including any materials you might need to produce, but often such resources will already be available to you – if not have a look to see what is available in books or online (but make sure you check these carefully before using them!). Don't aim for perfection - students will be quite content with someone who delivers a reasonable session provided it addresses the areas where they are experiencing difficulty.

Allocating time to preparation for teaching is also a very good method of developing (your) time management skills generally. You need to balance this task with other priorities you might have such as your research for your PhD, which is ultimately your first priority. The students themselves will be able to provide very quick feedback on how you have coped, and indeed you will know this yourself post-session; you will soon be found out if your preparation is below par, and will soon see where less effort could have been applied without compromising the session. Your teaching duties will often be very well defined, with quite precise tasks. For example, you might simply be assisting in an exercise class with a set problem sheet. Here, your preparation consists of ensuring that you can do all of the problems, perhaps in a number of different ways, and that you have an overview of the topic as a whole. This is the absolute minimum that you must do, no matter how long it takes. Don't be afraid to ask for help from others. You might find you are not able to do all of the problems immediately – you will certainly not be alone!

After four years as an undergraduate, most people have a good idea of what to do.

This is a common point made by some who think teaching is 'common sense' and there is nothing further to learn. To an extent, teaching is about applying common sense; think back to the good and bad classes you received as an undergraduate student – try to implement the good approaches and ideas and avoid the bad. However, there is always something new to learn or experience in teaching, even if you have been teaching the same course for a while. Your students will not necessarily be the same, there may be some new cutting edge research or application that has emerged that could form the basis of a fascinating discussion, or the course may be assessed in a different way in response to a change of approach within the department or university.

The implication of the original question is that only minimal training is necessary for teaching and then one learns through experience. Certainly, you learn from experience, but 'being on the receiving end of an activity' does not make necessarily make you an expert in that activity. You may be a pure mathematician asked to teach mechanics to engineers. There are also variations according to location, some European countries apply an approach to marking whereby tutors stop marking a question as soon as they identify a mistake, within the UK, we carry the student's mistake forward so they are not penalised more than once: this is fundamentally different and perhaps quite uncomfortable for a new tutor starting out.

A lack of training for teaching in higher education has never been justified, but particularly now is indefensible when students are directly paying tuition fees and when global competition demands that we provide the best possible teaching and learning experience for our students. The skills required for such high quality teaching are not innate, not easily developed and it is unfair to expect postgraduates to develop them by trial and error when teaching for the first time. Our experience is that postgraduate students are keen to learn and find out more about teaching, and this book, along with the workshops we have run over the years, is our contribution towards helping you with this process, We have tried to share our knowledge and things we wish we had known at the time – we hope it proves useful to you.

Appendix 1

Marking scheme for binomial expansion question

Notes:

- The question does not ask for the binomial theorem to be quoted
- No formula sheets or calculators are allowed
- Pascal's triangle is not to be used

$$(2+3x)^5 = 2^5 + 5 \times 2^4(3x) + \frac{5\times 4}{2!} \times 2^3(3x)^2 + \frac{5\times 4\times 3}{3!} \times 2^2(3x)^3 + \frac{5\times 4\times 3\times 2}{4!} \times 2(3x)^4 + (3x)^5$$

(4 marks)

$$= 32 + 80(3x) + 80(3x)^2 + 40(3x)^3 + 10(3x)^4 + (3x)^5$$

$$= 32 + 240x + 720x^2 + 1080x^3 + 810x^4 + 243x^5$$

(2 marks)

Total 6 marks.

Appendix 2

Student solutions to binomial expansion question

Student A

$$(2+3x)^5 = 5(2^4)x + \frac{5(4)}{2!}(2^3)x^2 + \frac{5(4)(3)}{3!}(2^2)x^3 + \frac{5(4)(3)(2)}{4!}(2)x^4 + x^5$$

$$= 32 + 5(16)x + \left(\frac{20\times 8}{2}\right)x^2 + \frac{60\times 4}{6}x^3 + \frac{5(4!)}{4!}(2)x^4 + x^5$$

$$\Rightarrow (2+3x)^5 = 32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$$

Student B

~~$(a+x)^n = a^n + x^6$~~ $(a+x)^n = a^n + \frac{n(n-1)}{1!}a^{n-1}x + \cdots$

$$(2+3x)^5 = 2^5 + \frac{5(5-1)}{1!}2^4(3x) + \frac{5(5-1)(5-2)}{2!}2^3(3x)^2 \cdots$$

$$+ \frac{5(5-1)(5-2)(5-3)}{3!}2^2(3x)^3 + \frac{5(5-1)(5-2)(5-3)(5-4)}{4!}2(3x)^4$$

$+$ ~~$5(5-1)(5-2)(5-3)(5-4)(5-5)$~~ $(3x)^5$

$$(2+3x)^5 = 32 + 20\times 16\times 16(3x) + \frac{(20\times 3)}{2}(8)\,9x^2 + \frac{(60)(2)}{6}(4)\,27x^3$$

$$+ \frac{120}{24}(2)(8)x^4 + 163x^5$$

$$(2+3x)^5 = 32 + 960x + 2160x^2 + 2160x^3 + 810x^4 + 163x^5$$

Student C

$$\underset{\uparrow\, b}{(2} + \underset{\uparrow\, a}{3x)}^5 = a^n + a^{n-1}b + \frac{a_n\, n(n-1)}{2!}\times a^{n-2}b^2 + \cdots$$

$$= 2^6\, 3x^5 + 3x^4 * 2 + \frac{5(4)}{2!}\times 3x^3 * (2)^2 + \frac{5(4)(3)}{3!}3x^2\times(2)^3$$

$$+ \frac{5(4)(3)(2)}{4\times 3\times 2\times 1}3x(2)^4 + \frac{5(4)(3)(2)(1)}{5\times 4\times 3\times 2\times 1}(3x)^0(2)^5$$

$$= 3x^5 + 6x^4 + 120x^3 + 240x^2 + 720x + 32$$

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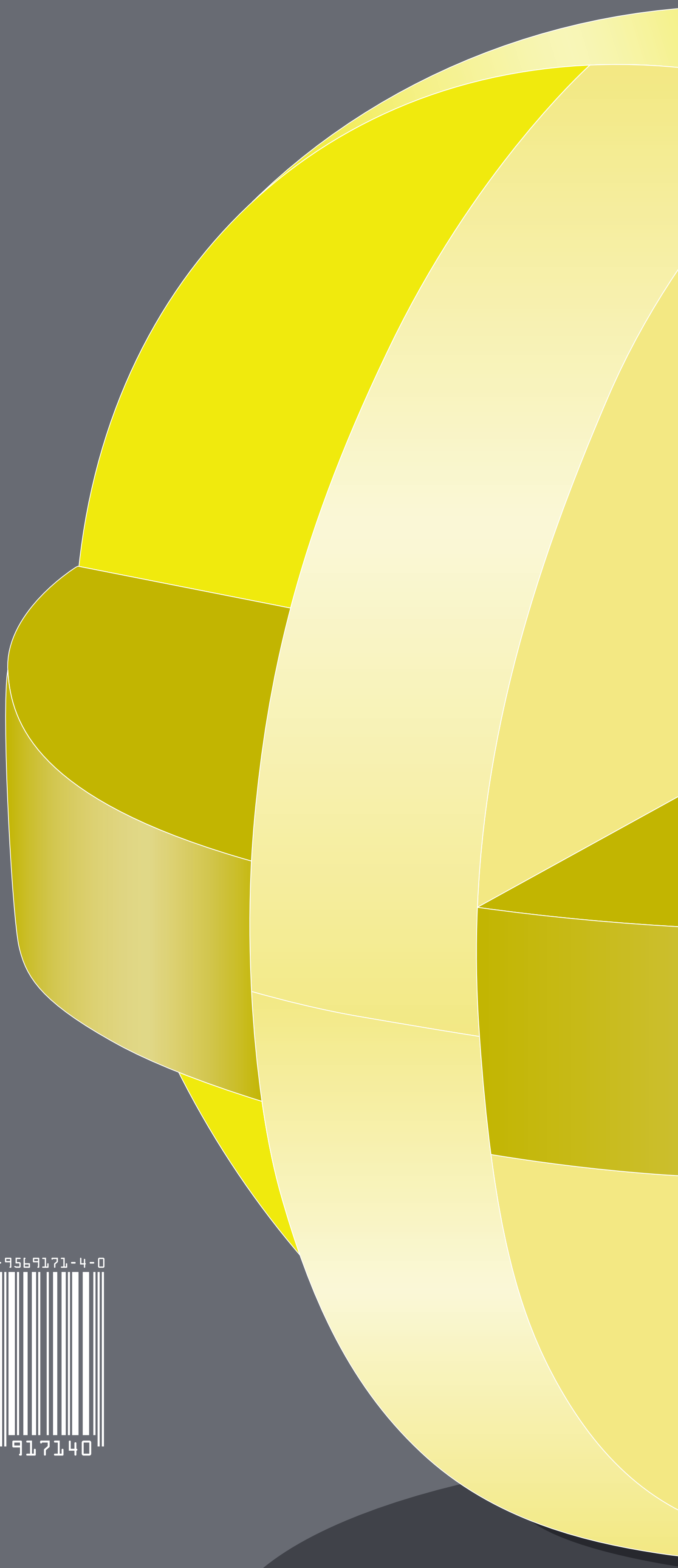

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