



School of Mathematics Newsletter

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Tropical linear algebra and the generalised max-plus eigenproblem

by PETER BUTKOVIČ

Congratulations go to Professor Peter Butkovič and his PhD student Daniel Jones for winning the EPS paper of the month prize for August. Their paper 'On special cases of the generalized max-plus eigenproblem' was recently published in a leading mathematics journal, the SIAM Journal on Matrix Analysis and Applications. Below, Peter describes their work.

Tropical linear algebra (also called max-algebra or path algebra) is an analogue of linear algebra where the operation of addition is replaced by maximisation and multiplication by conventional addition. The interest in tropical linear algebra was originally motivated by the possibility of dealing with a class of non-linear problems in pure and applied mathematics, operational research, science and engineering as if they were linear due to the fact that the obtained algebraic structure is a commutative and idempotent semifield. Besides the main advantage of using linear rather than non-linear techniques, tropical linear algebra enables us to efficiently describe and deal with complex sets, reveal combinatorial aspects of problems and view a class of problems in a new, unconventional way. The first pioneering papers appeared in the 1960s, followed by substantial contributions in the 1970s and 1980s. Since 1995 we have seen a remarkable expansion of this research field following a number of findings and applications in areas as diverse as algebraic geometry, geometry, control theory and optimization, phylogenetics, modelling of the cellular protein pro-

duction and railway scheduling. A number of research monographs have been published.

The generalised max-plus eigenproblem $Ax = \lambda Bx$ (GEP) is a generalisation of both the basic version of the (polynomially solvable) tropical eigenproblem and two-sided systems of linear equations $Ax = Bx$ (TSS - of undecided computational complexity). The latter have been studied since 1978. It has been proved that the solution set to TSS is finitely generated and that these systems are equivalent to mean payoff games. A number of solution methods exist, however none of them is polynomial and this problem is known to be in the intersection of the classes NP and co-NP. It is therefore expected that eventually a polynomial solution method will be found. Consequently, it is now one of the most intensively studied problems in tropical mathematics.

The present paper analyses GEP where the main interest is to study the spectrum, since for a fixed eigenvalue the problem reduces to a TSS. In contrast to the basic (one-sided) tropical eigenproblem where the complete spectrum can be found in a relatively easy way, in low order polynomial time, the spectrum for GEP is much more complex, possibly empty or a continuum, and no polynomial method is known for finding even a single eigenvalue or deciding whether such a value exists. It has been known for some time that for symmetric matrices GEP admits at most one eigenvalue. However, no polynomial method is known to determine this unique candidate. The paper presents a number of special cases, all polynomially solvable, and links the problem with the theory of matrix games.

If you would like to find out more about this novel research, the paper is available on Peter's homepage <http://web.mat.bham.ac.uk/P.Butkovic>.

EPSRC funded project in Harmonic Analysis

Dr Maria Reguera was recently awarded an EPSRC First Grant to support the project entitled 'Harmonic Analysis in rough environments'. Below Maria describes the project's main goals and potential applications.

The project will focus on the study of boundedness of singular integrals in two specific rough contexts. The study of boundedness of singular integrals is one of the fundamental questions in Harmonic Analysis. Classically, the question of boundedness has been considered and extensively studied in particularly smooth environments. The current project is concerned with the development of a theory for singular integrals in rougher environments, namely in Lebesgue spaces where the underlying measure can be quite irregular.

It is expected that progress in the direction of the project will shed light onto important questions pertaining to other areas of mathematics such as the David–Semmes Conjecture in Geometric Measure Theory, the characterization of removable singularities for bounded Lipschitz harmonic functions in Complex Analysis, the study of properties of model spaces in Operator Theory and the geometric properties of harmonic measure in any domain in PDEs.

Prizes galore!

Over the last couple of months several of the young mathematicians in the School have been recognised for their research and teaching successes. Third year PhD student Stefan Glock was the College of Engineering and Physical Sciences' winner of the Michael K. O'Rourke Best Publication Award 2016. This award 'recognises outstanding achievement in the publication of a postgraduate researcher's research'. His paper (joint with Daniela Kühn and Deryk Osthus), entitled 'Optimal path and cycle decompositions of dense quasirandom graphs' considers decompositions of graphs: Given any graph, one can repeatedly remove (the edges of) a cycle until the remaining graph has no more cycles, that is, it is a forest. But what is the least possible number of cycles needed? In the 1960s, Erdős and Gallai conjectured that this number should be linear in the number of vertices of the graph. This conjecture is still wide open. The Birmingham Combinatorialists were able to determine the exact number

in the case of dense random-looking graphs. They also obtained results concerning related problems, e.g. path decompositions. More information about Stefan's research can be found on his homepage <http://web.mat.bham.ac.uk/S.Glock/>.

Research Fellow Paul Roberts was recently awarded a Universitas21 fellowship to visit Australia for the month of February next year. Whilst there, he will speak at the ANZIAM (Australia and New Zealand Industrial and Applied Mathematics) conference as well spending 3 weeks visiting the maths department at the University of Melbourne. In Melbourne Paul will work with Professors Kerry Landman and Barry Hughes, and will apply discrete-continuum approaches to the modelling of bacterial infections. Paul was also recently awarded the 'WIREs Systems Biology and Medicine Best Poster Award' at the 10th European Conference on Mathematical and Theoretical Biology for his poster entitled 'Mathematical Models of Hyperoxia-Induced Retinal Degeneration'.

Congratulations also go to 3rd year PhD student Ryan Davies for winning a Teaching Academy Award. Ryan was one of only 5 PGRs at the University to win an award for 'excellence in their contribution to teaching'. At the recent EPS Research Conference, PhD student Tássio Naia received an honorable mention for his poster on trees in tournaments.

MathSoc update

by HEATHER COLLIS AND LUCY MORRIS

Freshers' week was all about the family scheme for us. On the Monday we had Parents' Welcome Drinks in local bar, Urban Village. Parents had to propose to each other throughout the night, this created a great atmosphere and was a good bonding experience for the older year groups before meeting the freshers later in the week. Following this, on Wednesday the parents led a tour of campus for their new families, introducing the first year students to MathSoc and the family scheme. Finally on Friday we held a family quiz with drinks and pizza. The winning family got a 'student hamper' as top prize, and it was a great opportunity for the students to socialise within family groups.

Next up was our biggest event for this term – The MathSoc Annual Bar Crawl. We sold all our tickets with 200 members joining us for the event. The bar

crawl saw students visit the Lounge in Selly Oak before heading off to various bars and clubs on Broad Street. Following this we held our first movie night for those who aren't so keen on drinking, watching $X + Y$ with freshly made popcorn. We have also kick started recruitment for our pilot year of the Teaching Opportunities in Dame Elizabeth Cadbury School, which should see the scheme beginning in the upcoming weeks following DBS checks and a training session.

We have had a great start to the term, but there is lots more to come! We are collaborating with Poynting Physics Society and AstroSoc to host a Games Night with both board games and a competitive Mario Kart tournament. Our annual Careers Networking Evening isn't far away and we are expanding on our career focused events by holding a PwC event with ChemSoc and CSS.

At the start of next term we will be going to Budapest. Our first release tickets sold out in half an hour, so it looks like it's going to be big! We are also planning a trip to the science museum, our Annual Spring Ball, and, keeping to tradition, we will be running the Coventry Half Marathon to raise money for our charity of the year, Mind.

Mathematical Biology Study Group

The 2016 Multi-Scale Biology Study Group (MBSG 2016) will take place here in Birmingham from 12th-15th December 2016, hosted by the UoB Mathematical Biology Group. The event is jointly funded by POEMS (Predictive modelling for healthcare technology through maths) and MSB-Net (UK Multi-Scale Biology Network). MBSG 2016 will bring together biologists and clinicians with mathematicians and computer scientists to tackle research questions posed by the experimentalists, following the successful blueprint of the Mathematics in Medicine and Mathematics in the Plant Sciences Study Groups. More details of the event can be found here <http://web.mat.bham.ac.uk/D.Smith/StudyGroup.htm>.

LMS workshop on Interactions of Harmonic Analysis and Operator Theory

by ALESSIO MARTINI AND ANDREW MORRIS

The LMS Midlands Regional Meeting and Work-

shop on Interactions of Harmonic Analysis and Operator Theory was hosted by the School of Mathematics at the University of Birmingham during 13-16 September 2016. The event examined recent exciting interactions between harmonic analysis and operator theory at the forefront of applications to partial differential equations across a variety of geometric contexts. More than 65 mathematicians took part, ranging from UK based undergraduates with a burgeoning aspiration for research, to internationally renowned research leaders. The event had a distinct international flavour with almost half of participants based outside the UK; many arrived from a number of European countries as well as Australia and Japan.

The Workshop featured two mini-courses by Kaj Nyström (Uppsala, Sweden) and Javier Parcet (ICMAT, Spain). These distinguished speakers provided incredibly well-motivated, enthusiastic and detailed accounts of recent developments in the theory of “parabolic equations with complex bounded measurable coefficients” and “Fourier L_p summability with frequencies in nonabelian groups” respectively. The mini-courses were aimed at a level accessible by doctoral students and provided an effective introduction to recent interactions between harmonic analysis and operator theory for early career researchers whilst stimulating the interest and insight of analysts more generally. Indeed, more than one third of participants were research students, with most based in UK receiving financial support from the LMS.

The Workshop included a further ten expert plenary talks covering a broad selection of topics at the interface between harmonic analysis and operator theory. These included functional calculi, analysis on Lie groups, Euclidean and noncommutative harmonic analysis, first-order methods in elliptic and parabolic PDE, multilinear inequalities, optimal constants and weighted estimates. The talks allowed researchers to delve deeper into exciting recent developments with many commenting on the novelty and effectiveness of bringing together such a diverse range of topics that would foster potential future interactions.

The Regional Meeting took place on the afternoon of 15 September 2016. Pascal Auscher (Paris-Sud, France) delivered a moving tribute to the mathematical legacy of Alan McIntosh, Charles Batty (Oxford) articulated a comprehensive overview of the

preservation of holomorphic semigroups, and Tony Carbery (Edinburgh) presented an intriguing multi-linear analogue of duality. The Meeting was opened by LMS Vice President John Greenlees with LMS representatives Elizabeth Fisher, Iain Gordon, Fiona Nixon, and Chris Parker also in attendance. It was followed by a wine reception and the society dinner at Browns Birmingham restaurant in the city centre,

which was favoured with delightful autumn weather as participants enjoyed views of St. Martin's in the Bull Ring.

More information about the workshop, including abstracts and slides of the talks, is available from the website <http://web.mat.bham.ac.uk/lmsmidlands2016/>.



The LMS workshop photo

News in Brief

- At the same time as Dr Maria Reguera's recent grant success, Dr Will Perkins was also awarded an EPSRC First Grant for his project on 'New approaches to Gibbs measures at the interface of probability and computational complexity'. More on this in the next newsletter!
- Alessandro Paolini completed his PhD earlier this year, and has now moved to Kaiserslautern for a postdoc position with Professor Gunter Malle to work on representation theory of finite groups of Lie type.
- Congratulations go to Craig Holloway for his marriage to Sarah back in September! Craig recently commenced a teaching and research fellowship in the school having previously been a PhD student here.



Sarah and Craig Holloway