Contents

Regulations for MSci projects

1.1 Introduction

Every final year student on the MSci programme must undertake a project worth 40 credits. The project module gives the students an opportunity to study in depth, under supervision, some area of mathematics or statistics that particularly interests them. It is intended to give students an idea of mathematical research and teaches key mathematical skills, including writing a properly referenced project dissertation, oral presentation of advanced material and mathematical typesetting.

1.2 Conduct of the Project

2.1 Members of staff (as potential supervisors) will submit proposals for possible projects to be distributed to students on the MSci programme (including those studying abroad) in time for projects to be assigned before the end of the session preceding the student's final year of study.

2.2 Each project will be overseen by a supervisor and a co-assessor. Joint supervisors and/or co-assessors may be deemed appropriate.

2.3 All MSci projects will run over both terms of the session. With the agreement of the supervisor and the director, students may choose to do either 20 credits worth of work in each term or 10 credits in the Autumn term and 30 in the Spring term.

2.4 Supervisors may wish to suggest some preparatory reading or investigation over the summer vacation before the official start of the project.

2.5 Co-assessors will be appointed no later than the start of the new session and a project plan will be agreed between the supervisor, co-assessor and student at a meeting with the student held during the first 2 weeks of the Autumn term.

2.6 The co-assessor should be present for at least one additional meeting in each term. The timing of such meetings should enable the co-assessor to contribute to the progress review process in the School.

2.7 The student shall have regular meetings with the supervisor. In the Autumn term, weekly meetings are recommended.

2.8 Save in exceptional circumstances and only with the permission of the Head of School, no change of supervisor or project allocation will be permitted after the end of the second week of the final year.

2.9 Introductory training in LaTeX will be provided early in the Autumn term for those students who have not been exposed to LaTeX in their third year.

2.10 Students will produce a substantial dissertation based on the mathematical or statistical work they have carried out over the two terms.

2.11 Students will give an oral presentation based on the project work. This will be scheduled as soon as possible after the end of the Main Examination Period.

2.12 Members of staff will not normally be available for consultation relating to their project after the end of Spring Term. Students who wish to consult their supervisor or other member of staff about their project after the end of Spring Term must obtain the permission of the Director of the MSci Programme.

1.3 Assessment

3.1 The following elements will contribute to the assessment of the project: two interim reports, a dissertation and an oral presentation.

3.2 The two interim reports will be no more than two pages in length excluding bibliography. The first will be produced by the end of the sixth week of the Autumn term, the second by the end of the second week of the Spring term. Each will provide a brief summary of the work done so far and an outline plan of future work, possibly including library search, other background research or bibliography.

3.3 The interim reports will be assessed by the co-assessor.

3.4 The dissertation should include any significant work undertaken in the course of the project, though preliminary exercises may be omitted or relegated to an appendix. The length of the dissertation should not exceed 100 pages, excluding references and appendices. The font size must be 12pt.

3.5 The dissertation will be assessed by the supervisor and co-assessor.

3.6 The dissertation will be written for specialists in the field. They will be prepared using LaTeX (or other version of TeX) unless there are compelling reasons to the contrary and the permission of the Head of School has been obtained.

3.7 The first page of each copy of the dissertation should contain a signed and dated copy of the following declaration: 'I warrant that the content of this dissertation is the direct result of my own work and that any use made in it of published or unpublished material is fully and correctly referenced.' This page should be submitted to the Undergraduate Office by 12 noon on the Wednesday of the last week of Spring term.

3.8 The dissertation should be uploaded on Canvas by noon on the Thursday of the first week of the Easter vacation following the commencement of the project.

3.9 Penalties will be applied for late submission. Projects submitted after the deadline will lose 1 mark. Projects submitted after 4.15 p.m. on the day of the deadline will lose an additional 4 marks. Projects submitted after noon the following Monday will lose a further additional 1 mark. For each subsequent day thereafter, an additional 1 mark will be deducted for a project not submitted before noon. No project will be accepted later than noon on the Friday two weeks after the deadline.

3.10 The oral presentation should last for about 20 minutes, with a further 10 minutes for questions and discussion about the work contained in the project. This presentation will be assessed by a small committee appointed by the Head of School and chaired by the Director of the MSci Programme. It will contain representatives from the research groups in the School.

3.11 The oral presentations should be accessible to a general mathematical audience and should be assessed as presentations of technical material. Supervisors are encouraged to offer advice in their preparation.

3.12 The presentation slides should be submitted online the day before the presentation is scheduled.

3.13 In exceptional circumstances and with the permission of the Head of School the oral presentation may be waived. In such cases, detailed assessment arrangements will be agreed by the supervisor, the Director of the MSci Programme and the Head of School. Some additional assessed element, such as a critique of an important paper in an area related to the project, may be required.

3.14 Attendance at all oral presentations is required. Penalties for non-attendance will be assessed by the Director of the MSci Programme.

1.4 Marking Procedure

4.1 The project will be marked according to the following categories:

- technical content of the dissertation (42 marks);
- development and execution of the dissertation (20 marks);
- presentation of the dissertation (13 marks);
- conclusions of the dissertation (9 marks);
- two interim reports (3 marks each);
- oral presentation (10 marks)

4.2 The supervisor and co-assessor should produce marks for the dissertation independently of one another according to an agreed marking scheme. After discussion as necessary, a final mark will then be agreed. If no agreement is reached between the supervisor and co-assessor, the Head of School will institute an appropriate procedure to arrive at the final mark.

4.3 The marking scheme for the dissertation should reflect the nature of the project. Suggestions of possible categories for assessment are listed below. A mark of zero will be returned for presentation if the dissertation has not been typeset using (La)TeX, unless specific permission has been given by the Head of School.

- **Technical content of dissertation**: mastery of the subject matter; historical perspective; relation to undergraduate courses; conceptual and methodological difficulty; mathematical accuracy and clarity of exposition; innovative aspects; examples cited (relevance of); originality, independently derived results and proofs, new results, proofs, simple generalizations, significant insight, original research, examples constructed.

- **Development and execution of project**: plan outline, aims and objectives; sensible development of aims; appropriate approach and methodology; evidence of work done, computer programs, experiments, field work, etc.; library research, use of references; new source materials; level of independence and of assistance; analysis and appreciation of results, comparison with experimental data or known examples.

- **Presentation of dissertation**: aims, structure and conclusions clearly laid out; suitable length; quality of exposition and organisation; diagrams; bibliography complete, appropriate, properly referenced, other source material properly referenced.

- **Conclusions of dissertation**: Conclusion and suggestions for further work; stated aims met or reasons why not; placement of work in wider context.

4.4 The supervisor and co-assessor will jointly provide a written report on the dissertation, which includes the mark scheme, and information on how their marks were arrived at. This report shall be made available to the external examiner together with a copy of the project dissertation at least two weeks before the School Board of Examiners meets to discuss examination results.

4.5 The interim reports will be marked taking into account evidence of reasonable progress and accuracy of summary of work to date, viability of proposed plan of study and other appropriate criteria. A mark of zero will be returned if a report has not been typeset using (La)TeX unless specific permission has been given by the Head of School.

4.6 Marks for the oral presentations will be awarded according to the following criteria:

- delivery (audibility, use of slides, blackboard etc; style, interaction with audience);
- structure (organization, clarity of explanation, comprehensibility);
- content (level, amount of material covered, success of presentation in relation to difficulty of material);
- questions (mastery of topic and handling questions).

4.7 Approximately two weeks prior to the oral presentations, there will be a meeting of project supervisors and co-assessors to discuss and moderate the marks of the written work. This meeting will be chaired by the Director of the MSci Programme.

1.5 Plagiarism

5.1 The School abides by the University's guidelines on plagiarism. The University regards plagiarism as a very serious form of cheating and will impose the severest penalties in all cases of cheating.

5.2 Supervision and scrutiny of project work will be sufficiently closely arranged to ensure that:

• signs of plagiarism (whether intentional or unintentional) in early drafts or pieces of work may be detected in good time and drawn to the attention of the student and then followed up by a written warning if necessary;

• students are not allowed to get so far behind with their work that they may be tempted to turn to plagiarism in an effort to catch up.

5.3 Students must make appropriate use of references and footnotes when using material from published or unpublished sources to avoid any suspicion of plagiarism. If in any doubt the student should discuss the matter with their project supervisor.

5.4 Students must include in their dissertation a signed declaration that unreferenced material is their own work as described in 3.7.

1.6 Health and Safety

6.1 The School abides by the University Health and Safety policy. Where a project is anything other than low risk, it is the responsibility of the project supervisor and ultimately the Head of School to ensure that appropriate Health and Safety procedures are in place. If a supervisor deems that a project is not low risk, or is in any doubt, the matter should be referred to the Director of the MSci Programme. Desk based work, standard use of proprietary electrical equipment, use of Class 1 lasers and on-campus data collection are examples of low risk activity: off-campus data collection, experiments with electrical equipment, use of robots, testing materials to breaking point are examples of possible higher risk activities.

Projects in Pure Mathematics

2.1 Graph colouring

Supervisor: **Richard Mycroft**

Co-assessor: **Andrew Treglown**

Description: In its simplest form, the problem of colouring a graph is to give each vertex a colour from a fixed number of colours so that no edge joins two vertices of the same colour. This problem has its origins in the famous four colour problem; whilst this has been solved, many open problems remain. Graph colouring is a large and active research area, ranging from many theoretical questions to a diverse set of practical applications. Consequently, several varieties of colouring problems have been studied, which arise in different contexts (e.g. edge colourings, total colourings, fractional colourings and list colourings). The methods used to study these problems are very diverse, ranging from 'discharging' methods and algorithmic approaches to probabilistic arguments. The project would explore one or more of these areas in greater detail.

Prerequisites: MSM3P15

References: [1] Introduction to Graph Theory, by Douglas B. West, Prentice Hall, 2nd edition 2001.

2.2 The mathematics of social networks

Supervisor: **Nikolaos Fountoulakis**

Co-assessor: **Deryk Osthus**

Title: **The mathematics of social networks**

Description: Social networks have become an important part of the daily life of millions of people across the world. Their rapid development during the last ten years or so has had an enormous impact on social, political and economic life. The aim of this project is the study of mathematical models that describe the structure and evolution of social networks. For example, how do we describe rigorously the observation that a friend of a friend is more likely to become our friend? We will try to give answers to questions like the above one using tools from graph theory and probability theory.

Prerequisites: The probability theory part of MSM2O2 Statistics.

2.3 Random structures and the probabilistic method

Supervisor: **Nikolaos Fountoulakis**

Co-assessor: **Deryk Osthus**

Title: **Random structures and the probabilistic method**

Description: The theory of random graphs was founded by Erdős and Rényi during the late 1950's and it is nowadays an essential part of graph theory with numerous applications in other areas of pure and applied mathematics. The main question of this theory has to do with the typical properties that a graph has. In other words, it has to do with properties that most graphs have. For example, what is the typical chromatic number of a graph with n vertices? Are most graphs on n vertices Hamiltonian? The theory of random graphs is an essential tool of the probabilistic method. This was invented by Erdős and provides a number of elegant solutions to problems from graph theory and combinatorics, in general, as well as other areas of mathematics, that are seemingly unrelated, such as number theory.

Prerequisites: Elementary probability theory and counting from 1D is a prerequisite as well as MSM3P15 Graph Theory and MSM3P16 Combinatorics.

References:
[1] N. Alon and J. Spencer, *The probabilistic method*, Wiley, 2008.

[2] B. Bollobás, *Random graphs*, Cambridge University Press, 2001.

2.4 Topics in positional games

Supervisor: **Dan Hefetz**

Co-assessor: **Richard Mycroft**

Title: **Topics in Positional Games**

Description: The theory of positional games is a rapidly evolving, relatively young topic that is deeply linked to several popular areas of mathematics and theoretical computer science, such as random graph theory, Ramsey theory, complexity theory and derandomisation. Results on positional games have been used to make decisive progress in these areas. The origins of the theory can be traced back to classical game theory, a branch of mathematics which has found applications in a variety of fields such as economics, management, operations research, political science, social psychology, statistics, biology and many others.
We will start by reviewing the basics of the theory and will then proceed to study certain more advanced topics. There will be some flexibility in choosing these advanced topics.
This project would constitute good preparation for a student wishing to write a Ph.D. thesis in the field of Positional Games (as well as other areas of Combinatorics).

Prerequisites: MSM3P15 Graph Theory and MSM3P16 Combinatorics (taking these modules in parallel to this thesis is fine).

References: [1] N. Alon and J. H. Spencer, **The Probabilistic method**, Wiley-Interscience Series in Discrete Mathematics and Optimization, John Wiley & Sons, 3^{rd} edition, 2008. [2] J. Beck, **Combinatorial Games: Tic-Tac-Toe Theory**, Cambridge University Press, 2008. [3] V. Chvátal and P. Erdős, Biased positional games, *Annals of Discrete Math.* 2 (1978), 221–228. [4] J. A. Bondy and U. S. R. Murty, **Graph theory**, Springer Verlag, 2008. [5] S. Janson, T. Łuczak and A. Ruciński, **Random graphs**, Wiley, New York, 2000. [6] A. Lehman, A solution of the Shannon switching game, *J. Soc. Indust. Appl. Math.* 12 (1964), 687–725. [7] D. B. West, **Introduction to Graph Theory**, Prentice Hall, 2001.

2.5 Topics in graph theory

Supervisor: **Daniela Kühn**

Co-assessor: **Deryk Osthus**

Description: The aim of the project is to study one of the topics of last years Graph Theory course in more depth. There will be scope to study recent research developments. Possible directions include:

- *Hamilton cycles in graphs and directed graphs:* A Hamilton cycle in a graph G is a cycle which contains all vertices of G. Unfortunately, nobody knows how to decide efficiently whether a graph has a Hamilton cycle. Probably this is not possible, as otherwise $P = NP$. So researchers have been looking for natural and simple sufficient conditions which guarantee that a (directed) graph has a Hamilton cycle.
- *Graph colouring problems:* Here the question is how many colours are needed to colour the vertices of a graph in such a way that endvertices of an edge have different colours. This has connections to the 4-colour-theorem which states that every planar map can be coloured with at most 4 colours such that adjacent regions have different colours.
- *Extremal Graph Theory:* Here the general question is which conditions force the existence of a certain substructure in a graph. So this is more general than the Hamilton cycle problem mentioned above.

Prerequisites: MSM3P15

References: [1] R. Diestel, Graph Theory, Springer 1997.

[2] D. West, Introduction to Graph Theory, Prentice Hall 2001.

2.6 Randomness in graph theory

Supervisor: **Andrew Treglown**

Co-assessor: **Deryk Osthus**

Description: Over the last 75 years randomness has played a remarkable role in the evolution of Graph theory. Since then the topic has grown in a number of different directions. The project could focus on any of the themes below and involve studying recent advances in the topic.

- *The Probabilistic method:* The great Paul Erdős began the random revolution in the 1940s by developing the so-called 'Probabilistic method'. Roughly speaking, this approach involves one proving the existence of some desired combinatorial structure by constructing a suitable probability space and showing that our desired structure occurs with positive probability. This method has been used to solve purely Graph theoretical (and 'non-random') problems by probabilistic arguments.

- *Random graphs:* In 1959 Erdős and Rényi introduced the notion of a random graph. In this model a graph is constructed by adding an edge between a pair of vertices at random with some fixed probability. Since then the topic has flourished, both in its applications to areas such as theoretic computer science and communication networks, as well as a thriving area of pure mathematics in itself.

- *Quasi-randomness:* Another breakthrough in the field was made by Szemerédi in the 1970s; His 'regularity lemma' allows one to find 'random-like' structure in large dense graphs. So again this allows us to solve problems in Graph theory by exploiting random-like properties of such graphs.

Prerequisites: Graph Theory (MSM3P15)

References:
[1] N. Alon and J. Spencer, *The Probabilisitc Method*, Wiley, 2008.

[2] S. Janson, T. Łuczak and A. Rucinski, *Random Graphs*, Wiley, 2000.

2.7 Ramsey theory: complete disorder is impossible

Supervisor: **Andrew Treglown**

Co-assessor: **Daniela Kühn**

Description: In a party of six people there is always a group of three people who all know each other or are all strangers. However, how large do you need a party to be to ensure there is a group of six people who all know each other or are all strangers? (This question is notoriously difficult. If you solve it exactly then you would become a famous mathematician!)
Ramsey theory concerns questions of this type. Namely, how large do you require a structure to be to guarantee some desired substructure. Many problems in Ramsey theory can be stated in a graph theoretical way. However, Ramsey theory has developed into a broad area with questions arising, for example, from Number theory. The project has scope to focus on a number of topics in Ramsey theory. In particular, there will be chance to explore recent breakthroughs in the subject.

Prerequisites: Combinatorics and Communication theory (MSM3P16)

References:
[1] R. Diestel, *Graph Theory*, Springer, 1997.

2.8 Efficient algorithms for difficult graph problems

Supervisor: **Deryk Osthus**

Co-assessor: **Daniela Kühn**

Description: The aim of the project is an investigation of the computational difficulty of some optimization problems related to Graph Theory. For such problems one successful approach has been to find algorithms which work well on restricted inputs.

For instance, the travelling salesman problem asks for the shortest tour visiting a given set of cities. While difficult in general, one can solve the Travelling salesman problem almost optimally for inputs which are planar.

Often, one can also obtain efficient approximation algorithms, which guarantee a solution which is close to optimal.

The prerequisites can also be taken as co-requisites.

Prerequisites: MSM3P15, MSM3P17

References: [1] Approximation algorithms / Vijay V. Vazirani.

2.9 Heat-flow monotonicity phenomena in mathematical analysis.

Supervisor: **Jonathan Bennett**

Co-assessor: **Jose Cañizo**

Description: One form of the classical Cauchy–Schwarz inequality states that

$$\int fg \leq \left(\int f^2\right)^{1/2} \left(\int g^2\right)^{1/2},$$

where f and g are suitable nonnegative real functions. It was discovered quite recently that this fundamental inequality has a rather striking proof based on heat flow. More specifically, if H_t denotes the heat kernel, and one forms the quantity

$$Q(t) := \int (H_t * f^2(x))^{1/2}(H_t * g^2(x))^{1/2} dx,$$

then one may prove the following:

Theorem. *The function Q is increasing,*

$$\lim_{t\to 0} Q(t) = \int fg$$

and

$$\lim_{t\to\infty} Q(t) = \left(\int f^2\right)^{1/2} \left(\int g^2\right)^{1/2}.$$

As you may have spotted, the Cauchy–Schwarz inequality is an immediate corollary to this.

This project will seek analytic, algebraic and geometric explanations for such heat-flow monotonicity phenomena in the context of a variety of inequalities in mathematical analysis.

Prerequisites: Real and Complex Variable Theory (MSM2B); Linear Analysis (MSM3P21) would be advantageous, yet not strictly necessary.

References:

[1] Jonathan Bennett, "Heat-flow monotonicity related to some inequalities in euclidean analysis", Harmonic and Partial Differential Equations, Contemporary Mathematics **505**, American Mathematical Society, (2010), 85–96.

[2] Jonathan Bennett and Neal Bez, "Closure properties of solutions to heat inequalities", Journal of Geometric Analysis, **19** (2009), 584–600.

2.10 Asymptotic behavior of the Becker-Döring equations

Supervisor: **José A. Cañizo**

Co-assessor: **Jonathan Bennett**

Description: The Becker-Döring equations are a model for aggregation and clustering phenomena of particles. This model is used for the formation of crystals, the aggregation of small interstellar bodies into planets, the aggregation of lipids, and change of phase phenomena. It consists of an infinite set of ordinary differential equations which gives the size distribution of clusters, and its mathematical study was started rigorously in works by Ball, Carr and Penrose [1,3]. This project consists in writing a short review of the known mathematical properties of the equation, and conducting a numerical study to investigate some elusive conjectures on its asymptotic behaviour, expanding on our recent work [2].

Prerequisites: A good understanding of ordinary differential equations is needed for this project. Some notions in statistical mechanics would be useful but are not fundamental. Also, knowledge of a programming language (such as C) and/or a numerical analysis package (such as Octave/Matlab, the GNU Scientific Library or similar packages) is a requirement.

References:

[1] Ball, J. M., Carr, J. and Penrose, O., *The Becker-Döring cluster equations: Basic properties and asymptotic behaviour of solutions*, Communications in Mathematical Physics, vol. 104, no. 4, 657-692 (1986).

[2] Cañizo, J. A. and Lods, B., *Exponential convergence to equilibrium for subcritical solutions of the Becker-Döring equations*, to appear in Journal of Differential Equations, arXiv preprint 1211.5265

[3] Penrose, O., *The Becker-Döring equations at large times and their connection with the LSW theory of coarsening*, Journal of Statistical Physics, vol. 89, no. 1, 305-320 (1997).

2.11 Nonlinear Schrödinger equations

Supervisor: **Susana Gutierrez**

Co-assessor: **Jonathan Bennett**

Description: The 1-dimensional free Schrödinger equation from quantum mechanics is the partial diferential equation

$$i\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} = 0,$$

where $x \in \mathbb{R}$ and t represents time. For a suitably well-behaved function $f : \mathbb{R} \longrightarrow \mathbb{C}$, one may use elementary properties of the Fourier transform to explicitly find the solution of this (linear) equation subject to the initial condition $u(0, x) = f(x)$. However, for important nonlinear variants of this diferential equation, such as the cubic nonlinear Schrödinger equation

$$i\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x^2} + |u|^2 u = 0,$$

this explicit analysis breaks down completely. The answers to fundamental questions concerning the existence and uniqueness of solutions to such equations (under appropriate initial conditions) are far from evident, and will form the focus of this project. The extent to which this analysis applies in higher dimensions (and to more general nonlinear partial differential equations) is of significant current interest, and will also be investigated.

The prerequisites here are a reasonable grasp of real and complex analysis, metric space theory, Lebesgue integration theory, and some exposure to the Fourier transform.

Prerequisites: MSM2B-Real and Complex Variable Theory and MSM3P21-Linear Analysis.

References:

[1] Books on the Contraction Mapping Theorem and Picard's Theorem about the existence and uniqueness of solutions to ODE's; e.g. Metric Spaces by Victor Bryant, Cambridge University Press.

[2] E. M. Stein and R. Shakarchi, Fourier Analysis - An Introduction, Princeton, 2003 (Sections 2.2 and 3.3).

2.12 Sharp weighted estimates for singular integrals

Supervisor: **Maria C. Reguera**

Co-assessor: **Jose A. Canizo**

Description: Given the singular integral operator, known as the Hilbert transform,

$$Hf(x) := \int_{\mathbb{R}} \frac{f(y)}{x-y} dy$$

we want to study the boundedness of such operator in the weighted Lebesgue space $L^2(w)$, where w is a weight, i.e. a positive locally integrable function. It is a classical result in Harmonic Analysis that such operator is bounded in $L^2(w)$ if and only if the weight belongs to the Muckenhoupt A_2 class. A weight belongs to the Muckenhoupt A_2 class if and only if it satisfies

$$[w]_{A_2} := \sup_I \frac{\int_I w dx}{|I|} \frac{\int_I w^{-1} dx}{|I|} < \infty,$$

where the supremum is taken over all possible intervals I in $\mathbb{R}$. We will embark in studying such classical results as well as the new advances in the area related to sharp estimates. We will prove sharp weighted inequalities, namely, we will find the dependence on $[w]_{A_2}$ of the operator norm using a very powerful inequality provided by A. Lerner [3]. The mathematics of this project are fresh and remarkably powerful, the inequality provided by A. Lerner have shaken up the field of Harmonic Analysis and the study of singular integrals.

Prerequisites: MSM2B Real and Complex variable theory. MSM3P21 Linear Analysis and MSM3607 Transform theory would be advantageous, yet not strictly necessary.

References:

[1] Duoandikoetxea, J.; Fourier analysis. Translated and revised from the 1995 Spanish original by David Cruz-Uribe. Graduate Studies in Mathematics, 29. American Mathematical Society, Providence, RI, 2001. xviii+222 pp

[2] Lerner, Andrei K.; A simple proof of the A_2 conjecture. Int. Math. Res. Not. IMRN 2013, no. 14, 3159–3170.

[3] Lerner, Andrei K.; On an estimate of Caldern-Zygmund operators by dyadic positive operators. Preprint http://arxiv.org/abs/1202.1860

2.13 The mathematics of voting

Supervisor: **Olga Maleva**

Co-assessor: **Chris Good**

Description: We tend to feel fairly smug in the 'democratic' West about our system of government, but are elections fair? Do they really reflect the views of the electorate? Certainly many people are unhappy with the 'First-Past-the-Post' electoral system used in the UK, espousing instead some form (or other) of proportional representation. Would such a system be better? How can we make a judgement?

In fact, when we analyse voting mathematically, it becomes clear that all system of aggregating preferences (electing a parliament or a president, agreeing on who should win Pop Idols or Opportunity, deciding the winner of the 6 Nations or the Formula 1 Championship) can throw up anomalies, unfairnesses and down-right weirdness.

How weird can things get? Well, in 1972, Kenneth Arrow (Harvard, USA) and John R. Hicks (Oxford, UK) were awarded the Nobel Prize for Economics 'for their pioneering contributions to general economic equilibrium theory and welfare theory'. At the heart of Arrow's contribution to economic theory is his so-called 'Impossibility Theorem', which (roughly speaking) says that there is no fair voting system. More precisely, once we agree what a fair voting system is, one can show that the only fair voting system is one in which there is a dictator who decides what every outcome will be. But there clearly cannot be a dictator in any fair voting system, so a fair voting system is impossible.

In this project the student will look at the mathematics of voting, in particular the proof of Arrow's Theorem.

Prerequisites: None

References: Search on Wikipedia for 'Arrow's Theorem' and 'May's Theorem'

2.14 Baire category theorem and Banach-Mazur game

Supervisor: **Dr. O. Maleva**

Co-assessor: **Dr. J. Cañizo**

Title: **Baire's category theorem and Banach-Mazur game**

Description: In 1935 the Polish mathematician Stanislaw Mazur proposed the following game.

There are two players called Player I and Player II. A subset S of the interval $[0,1]$ is fixed beforehand, and the players alternately choose subintervals $I_n \subset [0,1]$ so that $I_{n+1} \subseteq I_n$ for each $n \geq 1$. Player I wins if the intersection of all I_n intersects S, and player II wins if he can force this intersection to be disjoint from S.

Mazur observed that if S can be covered by a countable union of sets whose closure has empty interior (S is of first category) then the second player wins while if the complement of S is of first category then the first player wins. Later Banach proved that these conditions are not only necessary for the existing winning strategies but are also sufficient.

This is a very powerful method in Analysis. For example, it implies the Baire theorem which says that the intersection of any collection of open dense subsets of $\mathbb{R}$ is dense.

The game can be generalised to an arbitrary topological space X. Then in order to decide whether a certain property describes a *typical* object of X it is enough to show that there is a winning strategy for Player I with respect to the set of objects satisfying the given property. Remarkably, one can show in this way that a 'typical' continuous function is differentiable at no point!

The project will cover a range of topics connected with the Banach-Mazur game and Baire category theorem.

Prerequisites: MSM203 Metric Spaces

References:

[1] B. Bolobas, Linear Analysis. Cambridge University Press, Cambridge, 1999.

[2] J. Oxtoby, Measure and category. A survey of the analogies between topological and measure spaces. Second edition. Graduate Texts in Mathematics, 2. Springer-Verlag, New York-Berlin, 1980.

2.15 Partial cubes

Supervisor: **Prof S Shpectorov**

Co-assessor: **Dr C Hoffman**

Description: The vertex and edge graph of the n-dimensional cube is known as the Hamming cube graph Q_n. Partial cubes are isometric subgraphs of Hamming cubes, that is, a graph G is a partial cube if $G \subseteq Q_n$ for some n and for any two vertices of G the distance between them is the same when measured in G and in Q_n.

The theory of partial cubes is a lively area of Combinatorics, where new research can start literally minutes after one learns the basic definitions. Partial cubes have many applications in, as well as outside, mathematics.

Prerequisites: MSM3P15 Graph Theory.

References: [1] "Graphs and Cubes", by Sergei Ovchinnikov.

2.16 Clifford algebras and spinors

Supervisor: **Prof S Shpectorov**

Co-assessor: **Prof CW Parker**

Description: This might be an interesting project in particular for those who are interested in application to Physics. The focus of the project will be on the definition of the Clifford algebras over the real numbers and their action on the corresponding space of spinors. On the way there, you can learn chapters from Linear Algebra that are rarely covered in the basic university courses, but which are very useful for applications. This includes the concepts of the dual space and tensors, classification of bilinear forms, etc. The famous quaternion algebra will appear as a particular case of Clifford algebra.

Prerequisites: MSM3P08a Group Theory useful, but not strictly necessary.

References: [1] "Clifford Algebras: An Introduction", by D.J.H. Garling.

2.17 Groups and geometries

Supervisor: **Prof S Shpectorov**

Co-assessor: **Dr C Hoffman**

Description: The concept of diagram geometries was introduced by J. Tits as a source of natural objects on which finite simple groups act. One interesting class of geometries is called the buildings, and among buildings one finds such famous examples as the n-dimensional projective space and the polar spaces of different types. The project includes the basics of Coxeter groups, the axioms and elementary properties of buildings, and examples of buildings.

Prerequisites: MSM3P08a Group Theory useful, but not strictly necessary.

References:

2.18 The Omnibus Project

Supervisor: **Prof S Shpectorov**

Co-assessor: **Prof CW Parker**

Description: I would be happy to consider a project with a motivated student on any topic of her/his liking in any area of pure mathematics including, but not restricted to Algebra, Combinatorics, and Geometry.

Prerequisites: Motivation and mathematical maturity.

References: Bring your own.

2.19 Riemann surfaces and their automorphisms

Supervisor: **Dr K. Magaard**

Co-assessor: **Prof. S. Shpectorov or Dr C. Hoffman**

Description: The theory of Riemann surfaces lies at the crossroads of many mathematical sub-disciplines such as analysis, algebra, number theory and topology. Compact Riemann surfaces are solution sets of homogeneous polynomials in three variables polynomials, or equivalently, via their affine model, as a solution set of two variable polynomials.
For example: The solution set of

$$x^2 + y^2 + z^2 = 1$$

is the Riemann sphere. Its affine model is obtained by making the substitutions $X := x/z_0$ and $Y := y/z_0$ where $z_0 = \sqrt{1 - z^2}$ to yield the bi-variate polynomial

$$X^2 + Y^2 = 1.$$

Other celebrated examples of Riemann surfaces are, the Fermat curve F_n which is defined to be the solution set of

$$x^n + y^n = z^n,$$

and elliptic curves, whose affine models are solution sets of

$$Y^2 = X^3 + aX^2 + bX + c.$$

Notice that F_2 is just the Riemann sphere. An automorphism of a Riemann surface R is a map from R to R which preserves the structure or R. The set of all automorphisms of R is a group. An example of an automorphism of the Fermat curve F_n is the map defined by

$$(x_0, y_0, z_0) \mapsto (\zeta_n x_0, \zeta_n y_0, \zeta_n z)$$

where $\zeta_n := e^{2\pi i/n}$ is an n-th root of unity; i.e. a solution to the equation $x^n = 1$ in the complex numbers. The automorphisms of a Riemann surface identify its symmetries and play a role similar to the one played by Galois groups of single variable polynomial.
This project would start with the study of classic texts on Riemann surfaces such as the ones listed in the references below. Time permitting the project could be developed in various different directions depending on the interests of the student. For example there are exciting connections to the inverse problem of Galois theory, see for example the book by Völklein [3].
This project is ideal preparation for students wishing to write a Ph.D. thesis with this supervisor but is also suitable for students who want to learn about the interplay between surfaces and groups.

Prerequisites: MSM2B Real and Complex Analysis, MSM2C Linear alg., MSM2D Symmetry and groups

References:

[1] Otto Forster; *Lectures on Riemann surfaces*, Graduate Texts in Math., vol. 81, Springer-Verlag, New York, 1981. ISBN 0-3879-. 0617-7

[2] Rick Miranda; *Algebraic curves and Riemann surfaces*, Graduate Studies in. Math., vol. 5, Amer. Math. Soc., Providence, RI, 1995. ISBN 0821802682

[3] Völklein, H.; *Groups as Galois Groups, An introduction*, Cambridge Studies in Advanced Mathematics 53, Cambridge University Press, 1996. ISBN 9780521065030

2.20 Fixed point ratios of permutations

Supervisor: **Dr K. Magaard**

Co-assessor: **Prof. S. Shpectorov or Dr C. Hoffman**

Description: Let G be a permutation group with permutation domain Ω. If $g \in G$, then the set $F_\Omega(g) := \{\omega \in \Omega \mid \omega g = \omega\}$ is called the *fixed point set* of g. The *fixed point ratios* $\frac{|F_\Omega(g)|}{|\Omega|}$ as g ranges over G carry significant information about the structure of the group G. Also fixed point ratios have uses in applications of group theory to other areas of mathematics, such as the theory of Riemann surfaces and card shuffling.
We could start by going through papers [2] and [1] and some of the references contained therein.
This project is ideal preparation for students wishing to write a Ph.D. thesis with this supervisor but is also suitable for students who want to learn about the thoery of permutation groups and its applications.

Prerequisites: MSM2C Linear algebra, MSM2D Symmetry and groups, MSM203 Polynomials and rings MSM3P08 Group theory and Galois theory

References:
[1] Frohardt D.; Magaard, K. Grassmannian fixed point ratios, Geom. Ded.82: 21-104, 2000

[2] Guralnick, R.; Magaard, K. On the minimal degree of a permutation group J. Alg., 207 (1998), No.1, 127-145.

2.21 Automated theorem provers

Supervisor: **Corneliu Hoffman**

Co-assessor: **S Shpectorov**

Description: The last decade computerised saw great advances in Proof assistants. In particular two extremely difficult theorems, the Four Colour theorem in Graph Theory and the Feit-Thompson theorem in Group Theory were verified with Coq. The project will deal one proof assistant (Isabelle, Coq or AGDA). Depending on the interests of the student the project will either describe the proof assistant carefully or will formalise a certain subset of a theory. Most likely the theory will be either in Algebra or Combinatorics. The student should have a reasonable understanding of Logic and should be comfortable with coding.

Prerequisites: MSM203, MSM3P17 etc

References:
[1] Tobias Nipkow, Lawrence C. Paulson, Markus Wenzel Isabelle/HOL A Proof Assistant for Higher-Order Logic

[2] Benjamin C. Pierce and all, Software Foundations

2.22 Introduction to algebraic geometry

Supervisor: **Corneliu Hoffman**

Co-assessor: **Sergey Shpectorov**

Description: Algebraic geometry is the study of solutions of polynomial equations. There is a natural topology (called the Zariski topology) on the space of such solutions. The study of these topological spaces is naturally equivalent to the study of the corresponding rings of rational functions. This are the basic objects of (affine) algebraic geometry. The project will introduce these objects carefully and will study their properties. There are two possible avenues for the second part of the project, either the study of Algebraic Groups with the classification theorem as the final goal or the study of Algebraic Curves, with the final objective being the proof of the Riemann-Roch theorem.

Prerequisites: MSM203, MSM3P08

References:
[1] Igor Shafarevich, Basic Algebraic Geometry 1: Varieties in Projective Space

[2] Frances Kirwan, Complex Algebraic Curves

2.23 Classical groups

Supervisor: **Chris Parker**

Co-assessor: **Kay Magaard**

Description: The finite classical groups $\mathrm{PSL}_n(q)$, $\mathrm{PSp}_{2n}(q)$ and $\mathrm{P}\Omega_n^{\pm}(q)$ make up the major part of the finite simple groups. In this project you will explore them via their descriptions as the stabilizers of forms and also as groups with a BN pair.

Prerequisites: MSM2C Linear algebra, MSM2D Symmetry and groups, MSM203 Polynomials and rings, MSM3P08 Group theory and Galois theory.

References: [1] R. Wilson, The Finite Simple Groups, Springer Verlag, 2009.

[2] M. Aschbacher, Finite Group Theory, Cambridge University Press, 2000.

2.24 Advanced topics in Pure Mathematics

Supervisor: **Chris Parker**

Co-assessor: **Kay Magaard**

Description: Each of the following topics is suitable for an MSci Project:

(1) Topics in group theory. This project will investigate some/any area of modern group theory. The exact contents of the project can be discussed with me in my office.

(2) Presentations of groups and group algorithms. This project will investigate group presentations. In particular, the Todd Coxeter algorithm for determining the index of a subgroup of finite index will be investigated as will the Low Index Subgroup algorithm due to Sims.

(3) Buildings. This project will use the books by Brown, Ronan and Weiss to investigate the fundamental results about the theory of buildings.

For further details please see me in my office.

Prerequisites: MSM2C Linear algebra, MSM2D Symmetry and groups, MSM203 Polynomials and rings, MSM3P08 Group theory and Galois theory.

References: These will vary according to the project.

2.25 Automorphism groups of Abelian groups

Supervisor: **Chris Parker**

Co-assessor: **Kay Magaard**

Description: Of course finite abelian groups are very well understood. This project will investigate the structure of their automorphism groups. In particular, the following question is interesting: which simple groups can be automorphism groups of homocyclic p-groups? For the project, you would first describe the full automorphism group of an abelian group and show that for $n \geq 3$ it is a non-split extension of a p-group by a subgroup $GL_n(p)$ so long as (n, p) is not $(3, 2)$. I would then like to move on to asking questions about simple subgroups of the automorphism group especially alternating groups.

Prerequisites: MSM2C Linear algebra, MSM2D Symmetry and groups, MSM203 Polynomials and rings, MSM3P08 Group theory and Galois theory.

References: [1] B. Hartley and T. O. Hawkes, Rings, Modules and Linear Algebra, Chapman & Hall, 1970.

[2] M. Aschbacher, Finite Group Theory, Cambridge University Press, 2000.

2.26 Lie algebras

Supervisor: **Dr A Evseev**

Co-assessor: **Dr S Goodwin**

Description: Lie algebras play a fundamental role in modern mathematics and physics. They are closely related to Lie groups, which are groups of symmetries with a topological structure. Lie groups are widely used in physics, including quantum physics: they are the symmetry groups of many important physical systems. For example, the group of symmetries of our 3-dimensional space is a Lie group. To each Lie group one attaches a Lie algebra, and much of the structure of the group is reflected in the properties of the corresponding Lie algebra. In some sense, analysing Lie algebras is an easier task because they are purely algebraic objects, whereas the theory of Lie groups involves topology and manifolds.

The first part of the project will be an investigation of the fundamental properties of Lie algebras and the structure of complex semisimple Lie algebras. Once this is completed, the project may continue in a number of different directions, including:

1. A study of representations of semisimple Lie algebras;
2. An investigation of Lie groups and their connection to Lie algebras.

Interested students are encouraged to see Anton Evseev for more information.

Prerequisites: Strong mastery of Linear Algebra (MSM2C), Polynomials and Rings (MSM203), and Symmetry and Groups (MSM2D).

References:

[1] K. Erdmann and M.J. Wildon, Introduction to Lie algebras. Springer Undergraduate Mathematics Series. Springer-Verlag, London, 2006.

[2] W. Fulton and J. Harris, Representation theory. A first course. Graduate Texts in Mathematics, 129. Springer-Verlag, New York, 1991.

[3] B. Hall, Lie groups, Lie algebras, and representations. An elementary introduction. Graduate Texts in Mathematics, 222. Springer-Verlag, New York, 2003.

[4] J.E. Humphreys, Introduction to Lie algebras and representation theory. Graduate Texts in Mathematics, 9. Springer-Verlag, New York–Berlin, 1978.

[5] J.-P. Serre, Complex semisimple Lie algebras. Springer-Verlag, New York, 1987.

2.27 Some sporadic simple groups

Supervisor: **Dr A Evseev**

Co-assessor: **Dr D Craven**

Description: The 26 sporadic simple groups occupy a special place in group theory, as they do not belong to any of the "regular" families of simple groups (such as that of alternating groups A_n, $n \geq 5$), but rather stand on their own. Many of the sporadic groups may be viewed as groups of symmetries of interesting combinatorial and geometric structures. The project will focus on a construction of several sporadic simple groups. The student will begin with the 5 Mathieu groups and will move on to other ones, such as Conway groups or the Higman–Sims group. Properties of these groups will be investigated, and related structures (such as Steiner systems, the Leech lattice, and the Higman–Sims graph) will be studied. Interested students are encouraged to see Anton Evseev for more information.

Prerequisites: Group Theory (MSM3P08), Linear Algebra (MSM2C).

References:

[1] J.H. Conway, R.T. Curtis, S.P. Norton, R.A. Parker, R.A. Wilson, Atlas of finite groups. Oxford University Press, 1985.

[2] J.H. Conway and N.J.A. Sloane, Sphere packings, lattices and groups. Springer, 1999 (3rd edition).

[3] R.L. Griess, Twelve sporadic simple groups. Springer, 1998.

2.28 Frobenius groups and representation theory

Supervisor: **Dr A Evseev**

Co-assessor: **Dr S Goodwin**

Description: The project will involve a study of representation theory of finite groups and, more specifically, character theory. Representation theory is an important area of modern algebra and, in particular, an essential tool for investigating the structure of finite groups. The project will focus on a beautiful application of character theory to a special class of groups, known as Frobenius groups. A finite group G acting transitively on a set X is said to be a Frobenius group if every element of G fixes at most one point in X. The student will work through a proof of the following theorem: if G is a Frobenius group, then the elements of G that fix no point in X, together with the identity element, form a normal subgroup N of G. Once this is achieved, the project may continue in one of the following directions:

1. A deeper investigation of representations and their applications to the theory of finite groups;
2. A study of purely group-theoretic methods, such as those needed for a more detailed understanding of the normal subgroup N in a Frobenius group.

Interested students are encouraged to see Anton Evseev for further information.

Prerequisites: Group Theory (MSM3P08) and a strong mastery of Linear Algebra (MSM2C).

References:
[1] J.L. Alperin and R. Bell, Groups and representations. Graduate Texts in Mathematics, 162. Springer-Verlag, New York, 1995.

[2] D. Gorenstein, Finite groups. Harper & Row, Publishers, 1968.

[3] I.M. Isaacs, Character theory of finite groups. Dover Publications, 1994.

[4] G. James and M. Liebeck, Representations and characters of groups. Cambridge University Press, Cambridge, 1993.

2.29 Voting and representation theory of the symmetric group

Supervisor: **Simon Goodwin**

Co-assessor: **Anton Evseev**

Description: This project will consider how voting can be viewed from an algebraic perspective. The algebra required is basic representation theory of the symmetric group, which will be learned by following one of the books by James or Sagan. This will be applied to the theory of voting, where voting profiles are viewed as certain elements of modules for the symmetric group. The main reference for this will be the paper by Daugherty et al.
Interested students are encouraged to see Dr Goodwin for further information.

Prerequisites: MSM2C Linear algebra, MSM2D Symmetry and groups, MSM203 Polynomials and rings, and metric spaces, MSM3P08 Group theory and Galois theory

References:
[1] Gordon James, The representation theory of the symmetric groups, Lecture Notes in Mathematics **682**, Springer-Verlag, 1978.

[2] Bruce E. Sagan, The symmetric group: representations, combinatorial algorithms, and symmetric functions, Graduate Texts in Mathematics **203**, Springer, 2nd edition, 2001.

[3] Zajj Daugherty, Alexander K. Eustis, Gregory Mintonand Michael E. Orrison, Voting, the symmetric group, and representation theory. Amer. Math. Monthly **116** (2009), no. 8, 667–687.

2.30 Algebraic groups

Supervisor: **Simon Goodwin**

Co-assessor: **Anton Evseev**

Description: Algebraic groups lie at the meeting point of group theory and algebraic geometry. Their scope covers Lie groups, which are of major importance in mathematical physics, and finite groups of Lie type, which form the majority of the finite simple groups. The book by Geck will serve an excellent introduction to this area, and will provide a platform for the student to decide which direction to take this project in.
Interested students are encouraged to see Dr Goodwin for further information.

Prerequisites: MSM2C Linear algebra, MSM2D Symmetry and groups, MSM203 Polynomials and rings, and metric spaces, MSM3P08 Group theory and Galois theory

References: [1] Meinolf Geck, An introduction to algebraic geometry and algebraic groups, Oxford Graduate Texts in Mathematics **10**, Oxford University Press, Oxford, 2003.

2.31 The symmetric group

Supervisor: **David A Craven**

Co-assessor: **Anton Evseev**

Description: The symmetric group is at the same time a group and a combinatorial object. The rich combinatorial structure of the symmetric group provides connections with partitions, symmetric functions, group theory and representation theory, as well as it being the archetype of a group generated by reflections, leading to Coxeter groups, Lie theory, and many more topics.
In this project the student will study some of the many aspects of this topic, from partitions to subgroup structure to the representations of the group, depending on the student's individual preferences.

Prerequisites: MSM2C Linear algebra, MSM2D Symmetry and groups, MSM203 Polynomials and rings, MSM3P08 Group theory and Galois theory necessary for the more group-theoretic topics

References: [1] James, G.; Kerber, A.; *The representation theory of the symmetric group*, Encyclopedia of Mathematics and its Applications 1,. Addison-Wesley Publishing Co., Reading, Mass., 1981.

[2] Macdonald, I.; *Symmetric functions and Hall polynomials*, Oxford University Press, New York, 1995.

[3] Sagan, B.; *The Symmetric Group*, Graduate Texts in Mathematics 203, Springer-Verlag, 2001.

2.32 Chaos in discrete dynamical systems

Supervisor: **Chris Good**

Co-assessor: **Jon Bennett**

Description: Suppose that $f : [0,1] \to [0,1]$ is a continuous function, then for any $x \in [0,1]$, $f^2(x) = f\big(f(x)\big)$, $f^3(x) = f\big(f^2(x)\big)$, and so on. Discrete dynamics studies the behaviour of these iterates of f. For example, a point x is periodic if $f^n(x) = x$ for some $n \geq 1$. Then Sharkovskii's Theorem tells us that if we have a point of period 3, then we have points of all other periods, and if we have a point of period 7 then we have points of all other periods except possibly 3 and 5. Li and Yorke coined the term chaos in their seminal paper 'Period three implies chaos.' It turns out that even very simple functions such as quadratic maps can exhibit chaotic behaviour and, perhaps surprisingly, one can make a detailed study of chaos using only elementary analysis of the real line.
In this project you could look at the proof of Sharkovskii's Theorm or the Li-Yorke Theorem or at other definitions of chaos such as Devaney's. An alternative would be to investigate the period doubling route to chaos.

Prerequisites: MSM2B 'Real and Complex Variable Theory' is essential. MSM203 'Metric Spaces' and MSM3P22/4P22 'Topology' would be useful. MSM3A05 'Chaos' would be a nice extra.

References: Search for 'Chaos' and 'Sharkovskii' on Wikipedia.

2.33 The Axiom of Choice

Supervisor: **Chris Good**

Co-assessor: **Richard Kaye**

Description: The Axiom of Choice says that given a collection of non-empty disjoint sets then there is a choice set containing one element of each set in the collection. The fact that this is not immediately obvious was nicely illustrated by Russell in the following example. We do not need Choice to choose one shoe from infinitely many pairs of shoes; simply choose the left shoe. However, we do need Choice to choose one sock from infinitely many pairs of socks since there is no way of distinguishing between the two socks in a pair in advance.

Choice is necessary to ensure that every vector space has a basis, or that the epsilon-delta definition of continuity coincides with the limit definition, or indeed to ensure that every infinite set has a proper infinite subset. However, historically Choice has been one of the most controversial areas in all of mathematics. Even today there are mathematicians who object strongly to the non-constructive nature of the Choice set. And the axiom has some very bizarre consequences. For example, the Banach-Tarski paradox, which is a consequence of Choice, says that one can split a solid sphere up into five pieces that can be rearranged to form two solid spheres each with the same volume as the original. There is even a proof of the existence of God from the assumption of Choice.

In this project the student could look at some the hidden uses of Choice already encountered in the undergraduate programme, study some of these ’paradoxical’ results, look at models of set theory where Choice fails or look at some of the historical and philosophical issues surrounding this problematical axiom.

Prerequisites: As many pure maths modules as possible.

References: Search for ‘Axiom of Choice’ and ‘Banach-Tarski’ on Wikipedia.

2.34 Various topics

Supervisor: **R. Kaye**

Co-assessor: **TBC**

Description: Please approach Dr Kaye for a project description.

Projects in Applied Mathematics

3.1 Models of plant root growth

Supervisor: **Dr Rosemary Dyson**

Co-assessor: **Dr Sara Jabbari**

Description: Understanding plant root growth is essential to promote healthy plant growth in normal and stressed (e.g. during a drought) environments. A root grows through the elongation of some of its cells, pushing the root forward through the soil. These cells differ from animal cells through the presence of a tough cell wall surrounding the cell, which maintains a high internal turgor pressure whilst allowing significant growth. It is the (varying) mechanical properties of this cell wall which control growth, whilst the driving force for expansion is provided by the turgor pressure.

Potential projects include investigating how bending of the root, for example as displayed under a gravity stimulus, is generated via differential wall properties across the root tissue, or modelling the movement of water through the root cross-section required to maintain the turgor pressure within the root cells as well provide a source of water for the rest of the plant.

This project will be in collaboration with colleagues at the Centre for Plant Integrative Biology, University of Nottingham.

Prerequisites: MSM3A03 Continuum mechanics, MSM3A05a Perturbation Theory and Asymptotics likely, depending on actual project.

3.2 Modelling in mathematical medicine

Supervisor: **Dr Rosemary Dyson**

Co-assessor: **Dr David Smith**

Description: The use of mathematical modelling within medical research is widespread, covering topics such as cancer, inflammation, neuroscience, physiological fluid mechanics and many others. The first two of these are the focus of the UoB Systems Science for Health (SSfH) initiative, which brings together researchers with complementary skills in modelling, bioinformatics and experimental biology from across the university to improve our understanding of health and disease. This project will involve working closely with relevant colleagues in other Schools to tackle a specific biological question of relevance to SSfH through formulating, analysing and solving (using analytical and/or numerical techniques) mathematical models. It is likely to involve models of how cells interact with each other or their environment as this is a common feature of both cancer and inflammation.

Prerequisites: MSM4A MSM3A03 Continuum mechanics, MSM3A05a Perturbation Theory and Asymptotics likely, depending on actual project.

3.3 Mathematical modelling of antibiotic resistance and novel treatment types

Supervisor: **Sara Jabbari**

Co-assessor: **Rosemary Dyson**

Description: The ability of bacteria to develop resistance to antibiotics has become a huge problem in the United Kingdom and worldwide. Antibiotics act by killing bacteria meaning that resistance is selected for by antibiotic use. Consequently, the development of novel treatments focuses on weakening the bacteria (through inhibition of their virulence) in the hope that this will minimise the likelihood of resistance to the new drugs emerging. There are several different types of targets for these drugs (e.g. inhibiting cell adhesion or toxin delivery) and this project will use mathematical modelling (ordinary differential equations) to predict the relative success of different approaches. Both numerical and asymptotic analyses will be required to investigate the resulting systems.

Prerequisites: MSM4A13 (Mathematical Biology), MSM3A05a (Perturbation Theory and Asymptotics) desirable.

References:

[1] A.E. Clatworthy, E. Pierson, D.T. Hung. Targeting virulence: a new paradigm for antimicrobial therapy. *Nature Chemical Biology* 3:541-548, 2007.

[2] S. Jabbari, J.R. King, P. Williams. A mathematical investigation of the effects of inhibitor therapy on three putative phosphorylation cascades governing the two-component system of the *agr* operon. *Mathematical Biosciences* 225:115-131, 2010.

3.4 Mathematical modelling of the acid-stress response network in *E. coli*

Supervisor: **Sara Jabbari**

Co-assessor: **David Smith**

Description: The bacterium *E. coli* can be found in the intestine of humans and animals. To survive in this environment, the bacteria must be capable of adapting to different conditions as it passes through the gut. For instance, pH varies significantly in different organs and low pH will typically hinder the bacteria's ability to function normally. To survive in these conditions, therefore, the bacteria employ a network of genes to detect the pH and adapt their behaviour accordingly. Mathematical modelling is useful to understand how this network functions and predict the consequences of perturbing the network. Ordinary differential equations will be used to represent the network and relevant parameters will be estimated using data obtained in the laboratory of Pete Lund in the School of Biosciences at Birmingham.

Prerequisites: MSM4A13 (Mathematical Biology), MSM3A05a (Perturbation Theory and Asymptotics) desirable.

References:

[1] N. A. Burton, M. D. Johnson, P. Antczak, A. Robinson, P. A. Lund. Novel aspects of the acid response network of *E. coli* K-12 are revealed by a study of transcriptional dynamics. *Journal of Molecular Biology*, 401:726-742, 2010.

[2] [1] H. De Jong. Modeling and simulation of genetic regulatory systems: a literature review. *Journal of Computational Biology* 9:67-103, 2002.

3.5 Regularised Stokeslet methods for biological flow

Supervisor: **David Smith**

Co-assessor: **Rosemary Dyson**

Description: Microscopic flows are fundamental to many processes in health and disease, from sperm swimming to the egg and left/right symmetry breaking in early development [1], to the flow of airway surface liquid that keeps your lungs healthy. There is a great need to understand these flow processes better, however the complex geometries associated with biological problems make analytic progress very difficult. One approach that has arisen in the last decade is the method of regularised Stokeslets [2], which allows the solution of the viscous flow partial differential equations through the use of point, line and surface distributions of smoothed fundamental solutions. Most existing published work however uses highly inefficient methologies. Building on the work of Smith [3] the focus will be to develop more efficient implementations exploiting representations of the underlying unknown viscous forces.

Prerequisite: MSM3A03

Corequisite: MSM4A06

References:

[1] Montenegro-Johnson, T.D., Smith, A.A., Smith, D.J., and J. R. Blake (2012) Modelling the Fluid Mechanics of Cilia and Flagella in Reproduction and Development. European Physical Journal E 35:111.

[2] Cortez, R., Fauci, L., Medovikov, A. (2005) The method of regularized stokeslets in three dimensions: analysis, validation, and application to helical swimming. Physics of Fluids 17, 031504

[3] Smith, D.J. (2009) A boundary element regularised Stokeslet method applied to cilia and flagella-driven flow. Proceedings of the Royal Society of London A, 465, 3605-3626

3.6 Finite element modelling of biological flow

Supervisor: **David Smith**

Co-assessor: **Daniel Loghin**

Description: Microscopic flows are fundamental to many processes in health and disease, from sperm swimming to the egg and left/right symmetry breaking in early development [1], to the flow of airway surface liquid that keeps your lungs healthy. There is a great need to understand these flow processes better, with time-dependent aspects such as unsteadiness of the flow and the non-Newtonian properties of the fluid being important features to address. The finite element method provides a powerful approach to these aspects, and has recently been used to show that non-Newtonian properties modify cell swimming behaviours in hitherto unpredictable ways [2]. Building on current research work, the focus will be to explore different modes of swimming (squirming and flagellar motility) and how they are modified by unsteadiness and/or non-Newtonian rheology.

Prerequisite: MSM3A03

Corequisites: MSM4A06, MSM4A10

References:

[1] Montenegro-Johnson, T.D., Smith, A.A., Smith, D.J., and J. R. Blake (2012) Modelling the Fluid Mechanics of Cilia and Flagella in Reproduction and Development. European Physical Journal E 35:111.

[2] Montenegro-Johnson, T.D., Smith, D.J. and Loghin, D. (2013) Physics of rheologically enhanced propulsion: Different strokes in generalized Stokes. Physics of Fluids, 25, 081903

3.7 An initial-value problem for the generalized Burgers' equation.

Supervisor: **John Leach**

Co-assessor: **David Needham**

Description: This project considers an initial boundary-value problem for Burgers' equation posed on the positive quarter-plane. The aim will be to develop, via the method of matched asymptotic coordinate expansions, the complete large-time asymptotic structure of the solution to this problem for selected parameter values. Burgers' equation is a canonical equation combining both nonlinearity and diffusion and as such arises in the modelling of many physical phenomena and is one of the fundamental model equations in fluid dynamics.

Prerequisites: None

References:

[1] J.A. Leach and D.J. Needham. The large-time development of the solution to an initial-value problem for the Korteweg-de Vries equation. I. Initial data has a discontinuous expansive step. *Nonlinearity* 21 (10) (2008) 2391-2408.

3.8 Squeeze flow of a viscoelastic fluid

Supervisor: **Jamal Uddin**

Co-assessor: **John Leach**

Description: Squeeze flows are commonplace in many engineering and biological systems and are often invoked in rheometry techniques in order to determine characteristics of both Newtonian and non- Newtonian fluids. In the simplest case of a parallel-plate geometry, often used for rheological measurements, where one plate is translated through a Newtonian fluid at constant velocity, exact analytical solutions of the lubrication equations can easily be derived for the radial and vertical velocity components in the film and force on the translating plate, which, neglecting slip on the solid surfaces, can be described by the Stefan-Reynolds relation.

Extensions of this case include modifications to the geometry, for example, the squeezing flow between a plate and a spherical lens, two rigid spheres or non-parallel plates. One can also consider the flow under a constant applied force rather than plate translation velocity or, for rheological measurements, a constant sample mass vs. constant contact area with the plates. The more practically relevant situation of the squeeze flow of a non-Newtonian fluid (e.g., power-law, viscoplastic, yield stress fluids, etc.) is considerably more complex. The use of a power law fluid model has been investigated in [1] with results that demonstrate the importance of such non-Newtonian behaviour in classical studies so far. In addition the occurrence of cavitation when a solid sphere is in close proximity to a wall is at present a controversial topic with [2] suggesting that cavitation occurs as the sphere approaches the wall whilst numerical and experimental studies in [3] demonstrate that cavitation may only occur when the sphere rebounds. The aim of the current project is to utilise a viscoelastic model for the squeezed fluid and also investigate the effects of compressibility and non-Newtonian behaviour on cavitation. The project will involve familiarity with fluid mechanics and perturbation techniques and would suit anyone seeking a good grounding for pursuing a PhD in Fluid Mechanics.

Prerequisite: MSM3A03, MASM3A05, MSM3A04.

Corequisite: MSM4A06

References:

[1] Uddin, J., Marston, J. O. and Thoroddsen, S. T., (2012), 'Squeeze flow of a Carreau fluid during sphere impact', Physics of Fluids, 24.

[2] Seddon, J. R. T., Kok, M. P., Linnartz, E., C. and Lohse D., (2012), 'Bubble puzzles in liquid squeeze: Cavitation during compression,' Europhysics Letters, 97, 2.

[3] Mansoor, M. M., Uddin, J., Marston, J. O. and Thoroddsen, S. T., (2013), 'Squeeze flow of a Carreau fluid during sphere impact', Experiments in Fluids.

3.9 Making holes in liquid sheets

Supervisor: **Jamal Uddin**

Co-assessor: **John Leach**

Description: Free surface flows are an important class of problems which are widespread in both nature and industry. The disintegration of a liquid sheet forms a subset of such problems and the related phenomenon of sheet retraction arises in a wide range of physical settings, ranging from fuel injectors to foams in the food industry to biological membranes. The disintegration of fluid sheets is of primary importance in the context of fluid fragmentation or atomization. Commonly, such atomization processes involve a cascade from fluid volumes to sheets to filaments to droplets, a route that depends critically on the dynamics and stability of fluid sheets and their bounding rims. Depending on the application at hand, film rupture can be either desirable, as in spray formation, or undesirable, as in curtain coating.

Our aim in this project will be to follow the paper by Savva & Bush (2009) and seek possible extensions to more practical application involving the use of surfactants and or non-Newtonian fluids. This project will use asymptotic theory with a good mixture of computational mathematics and as such will provide a good foundation for further study onto PhD.

Prerequisite: MSM3A03, MASM3A05, MSM3A04.

Corequisite: MSM4A06

References:
[1] Savva, N. and Bush J. W. M., (2009), 'Viscous sheet retraction', J. Fluid Mech, 626 pp211-240.

3.10 The propagation of fronts in time-periodic flows

Supervisor: **Alexandra Tzella**

Co-assessor: **John Leach**

Description: One of the most fundamental models in mathematical biology and ecology is the Fisher-Kolmogorv-Petrovsky-Piscunov (FKPP) equation [1]. It describes a population that evolves under the combined effects of spatial diffusion and a certain type of reaction that involves logistic growth and saturation. A particularly interesting aspect of this nonlinear partial differential equation is that it gives a simple description of the invasion of the stable saturated state into regions of space occupied by the unstable state. An invasion that, after a sufficient amount of time, proceeds via propagation of a front traveling.

In many environmental situations, the population is carried by an ambient flow. Important examples may be found in marine ecosystems e.g. interacting plankton populations [2] or in atmospheric chemistry e.g. stratospheric ozone, where chemical population dynamics may also be represented by a similar reaction [3]. A challenge is then to determine how the flow influences the characteristics of front propagation such as the front speed, its shape and location. The main objective of this project is to investigate these characteristics for the particular case of time-periodic flows (shear and/or cellular). Using a combination of perturbation theory and numerical methods, the aim is to expand on previous results that describe the front when the reactions are fast and the diffusivity is small [4].

Prerequisites: Partial Differential Equations (MSM3A04a), Perturbation Theory (MSM3A05a); some programming skills (e.g. C/C++ and/or MATLAB/Octave) are essential.

References:
[1] J. D. Murray, Mathematical Biology I: An Introduction, Springer (2002).

[2] E. R. Abraham, The generation of plankton patchiness by turbulent stirring, Nature, 391, 577 (1998).

[3] J. Ross, S. C. Müller, C. Vidal, Chemical waves. Science, 240, 4851, (1988).

[4] J. Xin, Front Propagation in Heterogeneous Media, SIAM review, 42, 2 (2000).

3.11 Large deviations and dispersion of chemicals on a Manhattan grid

Supervisor: **Alexandra Tzella**

Co-assessor: **Sara Jabbari**

Description: Consider the release of chemicals in a gridded city like Manhattan. This incident can have dramatic consequences if these chemicals are toxic. The subsequent environmental impact depends on the spatial extent of the dispersal of these chemicals. In order to assess the risks posed by such an incident we need to examine the combined action of diffusion and advection on a rectangular network where diffusion represents random motion and advection some background wind; a problem that becomes highly non-trivial as the size of the network increases [1].

Analytical progress is possible via use of the theory of large deviations which is concerned with the exponential decay of probabilities in random systems [2]. An important application of this theory is to describe the tails of the distribution of tracers carried by some ambient flow [3]. The aim of this project is to employ this theory to describe the behaviour of the concentration in a rectangular network. The factors that determine the speed and direction of spreading will depend on the solution of a family of eigenvalue problems. Analytical results will be compared to simulations of particles moving on the network which are a lot of fun to program and allow further understanding of the problem!

Prerequisites: Partial Differential Equations (MSM3A04a), Perturbation Theory (MSM3A05a); some knowledge of Probability Theory/Statistical Physics would be useful; some programming skills (e.g. C/C++ and/or MATLAB/Octave) are essential.

References:
[1] J. W. Haus, K. W. Kehr, Diffusion in regular and disordered lattices, Physics Reports, 150, 263–406 (1987).

[2] H. Touchette, The large deviation approach to statistical mechanics, Physics Reports, 478, 1–69 (2009).

[3] P.H. Haynes, J. V. Vanneste, Dispersion in the large-deviation regime. Part I: shear flows and periodic flows, to appear in Journal of Fluid Mechanics, (2014).

3.12 Distensible tube wave energy converter

Supervisor: **Warren Smith**

Co-assessor: **Yulii Shikhmurzaev**

Description: The distensible tube wave energy converter is a recognized representative of the next generation of marine energy devices, it being sufficiently different from current prototypes to achieve a substantial reduction in the cost of energy. It is a submerged tube, full of sea water, located just below the surface of the sea. The tube undergoes a complex interaction with the sea waves which run along its length. The result is a bulge wave in the tube which, providing certain criteria are met, grows in amplitude and captures the wave energy. Engineering models have been successful in guiding the development of the first prototypes of the distensible tube; however, significant open problems hinder further progress. A comprehensive mathematical model will be formulated using fundamental principles from mechanics and systematic asymptotic analysis. The open problem of the interaction between the distensible tube and the waves will be investigated.

Prerequisites: Continuum Mechanics (MSM3A03), Partial Differential Equations (MSM3A04a), Perturbation Theory (MSM3A05a)

References:
[1] F. J. M. Farley, R. C. T. Rainey and J. R. Chaplin. Rubber tubes in the sea. Phil. Trans. R. Soc. A, 370:381-402,2012

3.13 Stochastic finite element methods

Supervisor: **Alex Bespalov**

Co-assessor: **Daniel Loghin**

Description: Stochastic finite element methods (SFEMs) are modern techniques for numerical solution of partial differential equations with random data (often called stochastic PDEs). The key idea underlying these methods consists in using parametric representations of both the random data (e.g., coefficients in PDEs, boundary data, etc.) and the random solution to PDE in terms of a large (possibly infinite) number of stochastic parameters. The original stochastic PDE problem is then reformulated in a parametric deterministic form, that can be discretised with finite elements. Two main variants of the SFEMs are stochastic collocation and stochastic Galerkin finite element methods. Approximate solutions in these methods are sought in the tensor product space $X_h \otimes \mathcal{P}$, where X_h is a conventional finite element space associated with physical domain and $\mathcal{P}$ is a set of multivariate polynomials in stochastic parameters. When computing approximate solutions, stochastic collocation methods rely on samples in the parameter space to generate solutions to decoupled finite element problems, whereas stochastic Galerkin methods compute solution coefficients in the basis of $X_h \otimes \mathcal{P}$ directly via single (but very large) calculation.
Depending on the interests of the student, this project may pursue various aspects of one or both variants of the SFEM. These aspects will range from looking into advantages and drawbacks of different approaches to theoretical analysis and MATLAB-implementation of the algorithms. Interested students are encouraged to see the supervisor to discuss the details.

Prerequisites: Good knowledge of Partial Differential Equations is essential. Interest in numerical methods and ability to program in MATLAB (or strong enthusiasm to learn MATLAB) are desirable. Knowledge of basic probability theory will be advantageous.

References:

[1] R. G. Ghanem and P. D. Spanos, *Stochastic finite elements: a spectral approach*, Springer-Verlag, New York, 1991.

[2] I. Babuška, R. Tempone, and G. E. Zouraris, *Galerkin finite element approximations of stochastic elliptic partial differential equations*, SIAM J. Numer. Anal., 42 (2004), pp. 800–825.

[3] I. Babuška, F. Nobile, and R. Tempone, *A stochastic collocation method for elliptic partial differential equations with random input data*, SIAM J. Numer. Anal., 45 (2007), pp. 1005–1034.

[4] C. Schwab and C. J. Gittelson, *Sparse tensor discretizations of high-dimensional parametric and stochastic PDEs*, Acta Numer., 20 (2011), pp. 291–467.

3.14 Domain decomposition methods

Supervisor: **Daniel Loghin**

Co-assessor: **Natalia Petrovskaya**

Description: Domain decomposition methods are standard solution methods for large scale calculations arising in the area of numerical solution of PDE. The approach is to divide up the computational domain into smaller domains leading to a set of smaller, independent problems. The advantages are obvious: (i) the problems are smaller and thus easier to solve numerically and (ii) they can be solved in parallel. The downside is that this re-formulation of the problem is only possible through knowledge and suitable discretisation of boundary operators defined on the interfaces generated through the subdivision of the domain. This is a hard problem which may not have a known solution in general. The aim of the project is to review existing domain decomposition techniques for elliptic equations and to generalise and analyse some standard choices of interface operators to the case of elliptic problems with variable coefficients.

Prerequisites: MSM3A04a -Partial Differential Equations, MSM3P21 - Linear Analysis

Corequisites: MSM4A10 - Computational Methods and Research Frontiers

References:

[1] A. Quarteroni and A. Valli, Domain Decomposition Methods for Partial Differential Equations, Oxford University Press, 1999.

[2] T. F. Chan, Domain Decomposition Algorithms, Acta Numerica, 1994, pp. 61-143.

3.15 Estimating pest insect population density from trap counts

Supervisor: **Natalia Petrovskaya**

Co-assessor: **Alex Bespalov**

Description: In ecological studies, populations are usually described in terms of the population density or population size. Having these values known over a period of time, conclusions can be made about a given species, community, or ecosystem as a whole. In particular, in pest management, the information gained about pest abundance in a given field or area is then used to make a decision about pesticide application. To avoid unjustified decisions and unnecessary losses, the quality of the information about the population density is therefore a matter of primary importance. However, the population density is rarely measured straightforwardly, e.g. by direct counting of the individuals. In the case of insects, their density is often estimated based on trap counts. The problem is that, once the trap counts are collected, it is not always clear how to use them in order to obtain an estimate of the population density in the field. The aim of this project is to overcome current limitations of trapping methods used in ecological studies through developing a theoretical and computational framework that enables a direct estimate of populations from trap counts. A 'mean-field' diffusion model will be considered to investigate if it is capable of revealing the generic relationship between trap catches and population density.

Prerequisites: MSM2A Analytical Techniques, MSM2C Linear algebra and Programming, MSM3A04 PDEs & Reaction Diffusion Systems.

References:
[1] Crank, J., 1975. The mathematics of diffusion (2nd edition). Oxford University press, Oxford.

[2] Kot, M., 2001. Elements of Mathematical Ecology. Cambridge University Press, Cambridge.

[3] Okubo, A., Levin SA, 2001. Diffusion and Ecological Problems: Modern Perspecives. Springer, Berlin.

3.16 Numerical study of the predator-prey model for patchy invasions.

Supervisors: **Natalia Petrovskaya & Daniel Loghin**

Co-assessor: **David Leppinen**

Description: The goal of this project is to investigate interaction of a biological prey and predator under various environmental conditions. A mathematical model for prey-predator interaction consists of two reaction-diffusion equations written for the prey and predator population densities. Those equations require numerical solution where the solution will depend on the parameters of the system. In particular, we are interested in the set of parameters that results in so called patchy biological invasion. The student will have to check what biological invasion scenarios normally look like in response to the changes in either of the system's parameters. The student will then investigate how the patchy invasion is developed in the presence of geometric heterogeneity, i.e., in domains with curvilinear boundaries opposite to rectangular domains. There is a computational component to this project which will require numerical studies of the phenomena described above. The numerical method used for computational investigations is the finite element method; the recommended software package is Matlab.

Prerequisites: MSM2A Analytical Techniques, MSM2C Linear algebra and Programming, MSM3A04 PDEs & Reaction Diffusion Systems

References:
[1] Kot, M., 2001. Elements of Mathematical Ecology. Cambridge University Press, Cambridge.

[2] Johnson, C, 2009. Numerical Solution of Partial Differential Equations by the Finite Element Method, Dover Publications Dover ed.

3.17 How to design a calculator

Supervisor: **David Leppinen**

Co-assessor: **Daniel Loghin**

Description: The arithmatic operations of addition, subtraction and multiplication are very straight forward. How would you design a calculator to perform division? What is the most efficient way to evaluate the functions $\sin(x)$, $\exp(x)$, $\log(x)$, $\ldots$, etc?

We now take calculators (and computers, and mobile phones, and ...) for granted, but what are the underlying mathematics associated with a calculator? What is the most efficient way of designing a calculator (or a computer)? What is the most accurate?

This project will involve extensive computer programming in Maple and either C, C++, Fortran or another suitable language. One of the prime objectives will be to calculate the value of $\sqrt{2}$ to a million decimal places or more.

3.18 Symmetry in animal gaits and coupled nonlinear oscillators

Supervisor: **Dr Rachel Nicks**

Co-assessor: **Prof David Needham**

Description: Animal locomotion typically employs several distinct time-periodic patterns of leg movements, known as gaits (e.g. walk, run, gallop etc.). It has long been observed that most gaits possess a degree of symmetry. In this project you will focus on the gaits of four-legged animals. This provides an excellent introduction to spatio-temporal symmetries of coupled networks of nonlinear oscillators. Here a nonlinear oscillator is a system of nonlinear ordinary differential equations with a stable periodic solution.

You will begin by investigating the time-periodic symmetries of standard animal gaits. These gaits also have spatial symmetries corresponding to permutations of the legs. Thus the symmetries of animal gaits are spatio-temporal symmetries. You will see that symmetry constrains the gaits which can exist and allows for model-independent predictions about animal gaits.

One biological theory is that an animal's gait patterns are governed by a network of neurons called the Central Pattern Generator (CPG) that runs independently of the brain. The CPG can be thought of as a network of oscillators with symmetric coupling. You will observe that there are six assumptions that the CPG must satisfy which allow the structure of the CPG network to be deduced. You will see that these assumptions require that the CPG network has twice the number of cells as the animal has legs. You will observe why this is the case and interpret the predictions of the eight-cell model concerning quadruped locomotion. This could be extended to investigate the gaits of six legged animals.

For this project the symmetries of animal gaits will be specified in terms of a group of transformations which preserve the gait pattern. You will learn how to describe spatio-temporal symmetries and use techniques from symmetric bifurcation theory to classify gaits as primary or secondary noting that bifurcations represent transitions between different gaits.

This project can be extended to study the dynamics of rings of oscillators or other topics in coupled cell network dynamics.

Prerequisites: MSM201, MSM2D, MSM3A05

References:
[1] M. Golubitsky, I. Stewart, P.L. Buono, and J.J. Collins, Symmetry in locomotor central pattern generators and animal gaits, Nature 401 (1999) 693–695.

[2] J. J. Collins and I. N. Stewart, Coupled nonlinear oscillators and the symmetries of animal gaits, Journal of Nonlinear Science 3.1 (1993): 349-392.

[3] M. Golubitsky and I. Stewart, The Symmetry Perspective: From Equilibrium to Chaos in Phase Space and Physical Space, (2000) Birkhauser.

[4] P.L. Buono and M. Golubitsky. Models of central pattern generators for quadruped locomotion: I. primary gaits, J. Math. Biol. 42 (2001) 291–326.

3.19 Speciation and symmetry-breaking bifurcations

Supervisor: **Dr Rachel Nicks**

Co-assessor: **Prof John Leach**

Description: A classic example of speciation is ”Darwin’s finches’ in The Galapagos Islands, where over a period of about 5 million years a single species of finch has diversified into 14 species. One possible mechanism for the generation of new species is symmetry-breaking - where the single species state losses stability to a multiple species state.

In this project you will study a model in which speciation is represented as a form of spontaneous symmetry-breaking in a system of coupled nonlinear differential equations. You will learn how the model was developed, how to analyse the model and interpret the solutions to draw biological conclusions. To achieve this you will learn some of the techniques used to study symmetry-breaking bifurcations, including a little representation theory of the relevant symmetry groups and some of the results of equivariant bifurcation theory. In particular, you will see how the symmetry of a system alone can be used to determine the normal form of the bifurcation equations. You will find the primary branches of equilibria of these equations created at the bifurcation and investigate the stability of these solutions. You will then interpret the results in terms of speciation.

This project will involve mainly analytical techniques however there is scope for numerical work, including finding numerical solutions of the symmetric differential equations. Depending on your interests, you could go on to look the solutions created at secondary bifurcations (i.e. secondary speciation where a subspecies again losses stability to multiple species), or symmetry-breaking in all-to-all coupled systems in a more generic framework.

Prerequisites: MSM201, MSM2D, MSM3A05

References:

[1] I. Stewart, T. Elmhirst, J. Cohen, Symmetry-breaking as an Origin of Species. In Bifurcations, Symmetry and Patterns, 3-54 (2003) Birkhauser.

[2] M. Golubitsky and I. Stewart, The Symmetry Perspective: From Equilibrium to Chaos in Phase Space and Physical Space, (2000) Birkhauser.

[3] I. Stewart, Speciation: A case study in symmetric bifurcation theory, Univ. Iagellonicae Acta Math. 41 (2003) 67–88.

3.20 Patterns, symmetry and bifurcations

Supervisor: **Dr Rachel Nicks**

Co-assessor: **Dr Alexandra Tzella**

Description: Patterns arise in numerous situations in nature, for example animal coat markings and cloud formations. Patterns are structures that are typically periodic in space and have some kind of symmetry. Similar patterns are often seen in very different contexts, for example, stripes can be seen in zebra coats, dessert sand ripples and convection rolls, whilst squares appear in convection and layers of vibrated sand. Many patterns are universal and we can determine a lot about them from their symmetries and those of their surroundings. The possible symmetries of different patterns can be described mathematically using group theory and representation theory, and these abstract approaches can have remarkable predictive power.

Pattern formation can be modelled by partial differential equations in two spatial dimensions, and the investigation of such mathematical models is an area of much current research. Much information about the types of patterns that might be displayed in such models can be found by considering symmetries alone. In this project you will see how group theory can be used to great effect to predict the patterns which can exist. You will also use asymptotic analysis to determine the preferred patterns for particular model equations. Another option would be to study the model equations numerically.

Prerequisites: MSM201, MSM2D, MSM3A05

References:

[1] R. Hoyle, Pattern Formation, (2006) Cambridge University Press.

[2] M. Cross and H. Greenside, Pattern Formation and Dynamics in Nonequilibrium Systems, (2009) Cambridge University Press.

[3] M. I. Rabinovich, E. B. Ezersky, P. D. Weidman, The Dynamics of Patterns, (2000) World Scientific.

3.21 Instabilities in heated falling films

Supervisor: **Grigori Sisoev**
Co-assessor: **Yulii Shikhmurzaev**
Title: **Instabilities in heated falling films**
Description: Film flows are widely used in chemical engineering to provide intensive heat and mass transfer from/to a liquid. In particular, the films flowing at small or moderate values of the Reynolds number are applied to cool heated walls in microdevices. These flow regimes controlled by the viscous forces, the capillary forces and the driven forces, for instance the gravity, are accompanied by wave structures affecting the cooling process. The project deals with identifying the flow instabilities in the framework of the linear stability analysis.
Prerequisites: MSM3A04a PDE's, MSM3A03 Continnuum Mechanics.
Co-requisites: MSM4A06 Viscous Fluid Mechanics with Applications, MSM3A07 Waves.
References: [1] Acheson, D.J.; *Elementary Fluid Mechanics*, Clarendon Press, 1990.

[2] Sisoev, G.M.; Matar, O.K.; Sileri, D.; Lawrence, C.J.; Wave regimes in two-layer microchannel flow, Chemical Engineering Science, 64(2009), 3094-3102.

3.22 Rise and deformation of a bubble under buoyancy

Supervisor: **Q Wang**
Co-assessor: **D Leppinen**
Description: The evolution of a bubble in water is a very interesting phenomenon. An initially spherical bubble in water rises due to buoyancy. The bubble expands as it rises, as the hydrostatic pressure around the bubble reduces. As the bubble becomes larger, it accelerates faster due to the increasing buoyancy. In the later stage, the bubble becomes asymmetrical, forming a kidney shape and lastly becoming a ring bubble. It rises rapidly once becoming a ring bubble. Understanding the evolution of bubbles in a liquid is a problem of both scientific and engineering importance. Applications abound, varying from underwater explosions, Linnic eruptions to bubbles in medical apparatus. This phenomenon was simulated by Krishna & van Baten (1999) for 2D bubbles, and will be simulated for axisymmetric bubbles in this project.
Prerequisites: Fluid Mechanics, Partial Differential Equations
References: [1] **Q X Wang** 2014 Multi-oscillations of a bubble in a compressible liquid near a rigid boundary, *J. Fluid Mech.* (in press).

[2] G A Curtiss, D M Leppinen, **Q X Wang** & J R Blake 2013 Ultrasonic cavitation near a tissue layer. *J. Fluid Mech.* 730, 245-272.

[3] **Q X Wang** & J R Blake 2011 Non-spherical bubble dynamics in a compressible liquid. Part 2. Standing acoustic wave. *J. Fluid Mech.* 679, 559-581.

3.23 Various topics

Supervisor: **D. Needham**
Co-assessor: **TBC**
Description: Please approach Prof Needham for a project description.

3.24 Various topics

Supervisor: **Y. Shikhmurzaev**
Co-assessor: **TBC**
Description: Please approach Prof Shikhmurzaev for a project description.

Projects in Optimization

4.1 Multicriteria decision method for websites rating

Supervisor: **Sándor Zoltán Németh**

Co-assessor: **Peter Butkovič**

Description: By browsing the World Wide Web, everyone comes across many badly designed, hard-to-use websites. There are usability criteria, which should be respected by web designers in order for the users to easily find what they need, without getting frustrated. The project consists in designing a multicriteria decision method for rating websites based on usability criteria. For designing such a method, genetic programming might also be considered.

Prerequisites: Basic linear algebra and vector calculus. If genetic programming will be considered, then basic programming skills (Matlab and/or C++) are needed.

References:

[1] Thomas L. Saaty. *The analytic hierarchy process. Planning, priority setting, resource allocation*. McGraw-Hill, 1980.

[2] J. P. Brans and Ph. Vincke. A preference ranking organisation method (the PROMETHEE method for multiple criteria decision-making). *Management Science*, 31:647–656, 1985.

[3] J. P. Brans, Ph. Vincke, and B. Mareshal. How to select and how to rank projects: The PROMETHEE method. *European Journal of Operational Research*, 24:228–238, 1986.

[4] B. Roy. The outranking approach and the foundations of ELECTRE methods. *Theory and Decision*, 31(1):49–73, 1991.

[5] P. Csáki, F. Fölsz, T. Rapcsák and Z. Sági. On tender evaluations *Journal of Decision Systems*, 7:179–194, 1998.

[6] Anikó Ekárt and S. Z. Németh. Stability of tree structured decision functions. *European Journal of Operational Research*, 160:676–695, 2005.

4.2 A^* for solving Rubik's cube

Supervisor: **Sándor Zoltán Németh**

Co-assessor: **Michal Kočvara**

Description: Rubik's cube can be solved in at most 20 moves from any state. Algorithms exist which always lead to the solution, but in much more steps. A^* is an informed search method that can be used to find the optimal solution. The goal of the project is to find a suitable heuristic function and a representation of the cube.

Prerequisites: Some basic Matlab skills would be desirable for the examples.

References:

[1] Zbigniew Michalewicz and David B. Fogel, How to Solve It: Modern Heuristics, Springer (March 1, 2004)

[2] Ruhul A. Sarker, Hussein A. Abbass and Charles S. Newton, Heuristic and Optimization for Knowledge Discovery, Idea Group Publishing (February 7, 2002)

[3] S. J. Russell and P. Norviq, Artificial intelligence: a modern approach, Englewood Cliffs, N.J.: Prentice Hall, 1995.

[4] Nils J. Nilsson, Principles of Artificial Intelligence, Morgan Kaufmann Publishers; Reprint edition (June, 1986)

[5] http://www.edenwaith.com/products/pige/tutorials/a-star.php

[6] http://en.wikipedia.org/wiki/A%2A_algorithm

4.3 Rank and ℓ_0 minimization and their applications

Supervisor: **Yunbin Zhao**

Co-assessor: **Michal Kočvara**

Description: Recently, the mathematical model for seeking sparse solutions or low-rank matrix solutions of linear systems has a significant impact across disciplines. This, however, remains an emerging new area awaiting for extensive scientific research inputs. Up to now, the NP-hard sparsity-seeking model has been investigated dominantly by probabilistic analysis and convex approximation method. While the convex method successfully solves a wide range of sparsity-seeking problems, it still fails in many situations. It is therefore imperative to develop a new rigorous theory and efficient design for the so-called weighted algorithms that, at present, lie at the research frontier of both applied mathematics and engineering. The purpose of this project is to conduct a comprehensive and systematic study of such a theory and design, and to investigate some important questions in this field, and to apply the developed algorithms to various data processing. Through this project, you will understand how mathematical optimization theory and methods can be applied to deal with important problems arising from signal and image processing, machine learning and so on. The project provides an opportunity to use linear algebra, matrix analysis, modern convex optimization, and probabilistic analysis to model and solve sparse data processing problems.

Prerequisites: MSM2Da, MSM2C, MSM3M12, MSM3M02 etc

References:
[1] M. Elad, Sparse and Redundant Representations: From Theory to Applications in Singal and Image Processing, Springer, New York, 2010.

[2] A.M. Bruckstein, et al., From sparse solutions to systems of equations to sparse modeling of signals and images, SIAM Rev. 51 (2009), 34-81.

[3] Y.B. Zhao and D. Li, Reweighted l1-minimization for sparse solutions to underdetermined linear systems, SIAM J. Optim., 22 (2012), pp. 1065-1088.

4.4 Optimization problems with matrix rank constraints

Supervisor: **Professor M Kocvara**

Co-assessor: **Dr Y Zhao**

Description: Many combinatorial optimization problems or global optimization problems are approximated using semidefinite programming (SDP) relaxation. The approximation consists in formulating the original problem as SDP with a constraint on the rank of the matrix variable. This constraint is then relaxed – omitted.

The goal of the project is to perform numerical study of various modern approaches to SDP problems *with* the rank constraint. This is an NP-hard problem and we cannot expect to solve it efficiently. However, recent studies suggest that we can expect these methods to be much more efficient than methods for the original combinatorial optimization problems.

Prerequisites: MSM2C Linear algebra, MSM3M12a Nonlinear programming I, MSM3M11 Integer programming

References:
[1] Anjos, M.F. and Lasserre, J.B.: Handbook on semidefinite, conic and polynomial optimization, Springer 2011

[2] Zhao, Y.B.: An approximation theory of matrix rank minimization and its application to quadratic equations, Linear Algebra and its Applications, 437 (2012), pp.77-93.

4.5 Tropical eigenproblem

Supervisor: **Peter Butkovic**

Co-assessor: **Sándor Zoltán Németh**

Description: Tropical linear algebra is a new and rapidly evolving area of idempotent mathematics, discrete optimisation and linear algebra. One of the key questions is the tropical eigenvalue-eigenvector problem which is used for instance in the modelling of multiprocessor interactive systems [2] or in that of cellular protein production [1] where the tropical eigenvalues and eigenvectors are used to describe stability of the system. Being motivated by applications in [1] this project will investigate the tropical eigenproblem for special matrices such as bivalent, trivalent and tournament. The results will be described in combinatorial terms and/or as combinatorial algorithms. Knowledge of Matlab is welcome but not a condition.

Prerequisites: MSM2D or MSM2M09, MSM3M02

References: [1] Brackley CA et al (2011) A max-plus model of ribosome dynamics during mRNA translation, arXiv:1105.3580v1 [2] Butkovic P (2010) Max-linear Systems: Theory and Algorithms, Springer Monographs in Mathematics, Springer-Verlag, London

Projects in Statistics

5.1 Using meta-analysis in economics

Supervisor: **H Li**

Co-assessor: **B Chakraborty**

Description: Meta-analysis have been used in a broad range of topics in economic studies to study the nature of the internal relationship of interested topics including economic growth, openness and finance policies. Often most studies take different approaches, i.e. some look at the quantity effect, whereas other ones look at the quality effect. Moreover, some studies use firm-specific data; other studies take a cross-country approach. These different approaches also have consequences for the extent to which impact of policies can be measured: whereas firm-specific (micro-data) studies may at least potentially provide evidence on impact (i.e. when the setting is experimental or quasi-experimental), this is generally difficult for cross-country studies. Thus, finding a systematic variation through empirical evidence, for example, by using meta-regression, is very helpful.

Prerequisites: MSM2S01b Statistics I MSM3S02 Statistics II at least one of the following MSM3S05 (applied statistics), MSM3S08 (medical statistics), MSM3S09 (statistical methods in economics)

References:

[1] Meta-Regression Analysis in Economics and Business (Routledge Advances in Research Methods) Routledge July 2012

[2] Meta-Regression Analysis: Issues of Publication Bias in Economics (Surveys of Recent Research in Economics) [Paperback] Publisher: Wiley-Blackwell Dec 2005

[3] Meta-Analysis in Environmental Economics Series: Economy & Environment, Vol. 12 Bergh, J.C., Button, K.J., Nijkamp, P., Pepping, G. Springer; Softcover reprint of hardcover 1st ed. 1997 edition (4 Dec 2010)

5.2 Statistical process control

Supervisor: **B Chakraborty**

Co-assessor: **H Li**

Description: Controlling process parameters in any industrial process is an important aspect. It not only reduces cost and efficiency, it is a requirement in many industries to maintain the quality of their product and service. The most popular visual tool to monitor a process variable is a control chart. There are several control charting techniques available in the literature, e.g. Shewert's control chart, CUSUM and EWMA charts. Almost all of these tools depend heavily on the assumption of underlying normal distribution for the process parameter. In this project, we will investigate the performance of some of these proposed methods when the underlying distribution deviates from normality, using simulations as well as theoretical derivations. We will also consider some techniques, which does not depend on the normal distribution and hence can be applied to a larger number of situations without worrying about normality. But they may lack in efficiency when the true distribution is indeed normal. This project will involve understanding of the basic ideas and extensive simulations using either R or MATLAB. For theoretical derivations, basic knowledge of statistics and probability is good enough.

Prerequisites: MSM202

References:

[1] Montgomery, D.C. (2012) *Statistical Quality Control: A Modern Introduction*, 7th Edition, Wiley

5.3 Nonparametric classification

Supervisor: **B Chakraborty**

Co-assessor: **H Li**

Description: Statistical classification is the research area that studies the design and operation of systems that recognize and classify patterns in data. Important application domains are image analysis, computer vision, character recognition, speech analysis, man and machine diagnostics, person identification, industrial inspection, financial data analysis and forecast, genetics. The project starts with a review some basic concepts from parametric classification techniques. These techniques aim at classify data by constructing analytical functions which estimate the statistical distribution of data samples. However, it is not always possible to describe this distribution analytically. In such cases, more general, non-parametric techniques have to be applied. We will review some popular nonparametric classification techniques and if possible, some improvements on them. This project will involve understanding of the basic ideas and extensive comparison studies using simulations with either R or MATLAB. For theoretical derivations, basic knowledge of statistics and probability is good enough.

Prerequisites: MSM202

References: [1]R.O. Duda, P.E. Hart, D. Stork (2001), *Pattern Classification*, 2nd edition, Wiley

5.4 Survival analysis of time-to-event data with competing risks

Supervisor: **Dr Richard Riley**

Co-assessor: **Kym Snell (School of Health and Population Sciences)**

Description: In medical research, we are interested in observing and predicting the risk of adverse outcomes. For example, the risk of dying within one year after heart surgery, or the risk of a stroke within 5 years in those with high blood pressure. In a frail population, we may not observe an outcome of interest in some individuals, as another competing event occurs first. For example, if we are interested in the risk of a stroke, and an individual dies of another cause first, then the individual will never experience a stroke. Hence death is a competing risk. Statistically, this makes this more challenging when fitting survival models. If patients with the competing event are simply censored at their death time, then the analysis assumes an imaginary world where individuals must always have a stroke before they can die. This will lead to absolute risk predictions of having a stroke that are inappropriate. In this project, the student will have access to a large dataset with competing risks, where predictors of stroke in hypertension patients are of interest. They will start by fitting standard survival analysis methods (such as Kaplan-Meier, log rank test, Cox regression), and then apply competing risk models to ascertain the impact on model estimates and predictions.

Prerequisites: MSM3S08

References: [1] Wolbers et al (2009). Prognostic models with competing risks: methods and application to coronary risk prediction. Epidemiology 20(4):555-61.)