Evaluating the Accuracy of the Primary Particle Energy Estimations given by jSPARC

Charlie Hetherington

September 8, 2015

Abstract

jSPARC is an online computer model that allows high school students to work out the Primary Particle Energies of Cosmic Rays recorded by the HiSPARC Program. As part of the University of Birmingham's Summer STEM program, I spent two weeks investigating the accuracy of jSPARC's results with respect to several features of jSPARC's energy determination method: the assumptions made, the co-ordinate system employed and the values used in the NKG function that forms the basis for the estimation. Using events recorded by a group of HiSPARC detectors in the Science Park of Amsterdam, I calculated the intensity of flux at detectors using my own model, with the Particle Energy and location that gave the smallest error in jSPARC, and compared this to the recorded data values for flux intensity at these detectors. I found that over twenty events from different years and times, there was an average percentage difference of 95% between the error of jSPARC and the error given by my model. This difference can be accounted for through two main factors: the assumption that the zenith angle is 0 degrees and using the value of 77m for the Moliere radius as opposed to 92m.

1 Introduction

Cosmic Rays are high-energy particles from outer space that slam into the Earth at speeds approaching the speed of light. When measured by mass, around 79% of all Cosmic Rays are free protons and the majority of the remainder (15.8% overall) are in the form of helium nuclei – alpha particles. Curiously, the elemental proportions of the final 4.2% of particles (heavier nuclei such as oxygen, carbon and iron) do not correspond to that of our own solar system. This matches current thought about the origin of Cosmic Rays: while many low energy cosmic rays do originate in the sun as part of the solar wind, convoluted magnetic fields throughout the Milky Way Galaxy can accelerate the charged particles to high kinetic energies. This effect, known as the Fermi mechanism after Enrico Fermi who first proposed it, is limited to energies of 10^{15} eV (Electron-Volts).

This limit can be seen in Figure 1 as a break (It is often referred to as the "knee") in the curve. With energies over 10^{15} eV, particles can escape the magnetic field of our Galaxy. While this also means particles from other galaxies can enter, more rays on average will leave than arrive. The combination of these effects results in the gradient of the curve becoming steeper – the number of high energy particles drops of at an even higher rate until kinetic energies of

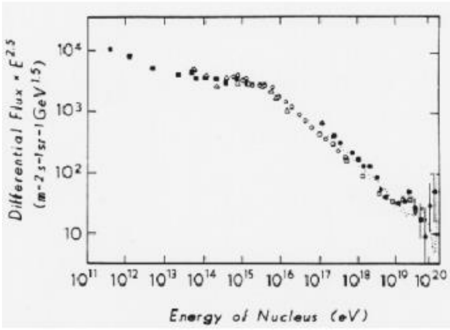

Figure 1:

10^{19} eV are reached. At this point it is theorised that the particles, travelling at speeds very close to the speed of light, would interact with a blueshifted Cosmic Microwave Background and lose energy – effectively imposing a 50 Mega Pascal range limit on these phenomenally high-energy particles known as the GZK cutoff. Nevertheless, there are still recorded instances of these energies despite there being no known sources in range!

There are many other gaps in our understanding of Cosmic Rays and as a result there are several projects across the world researching into them. HiSPARC is one of these projects. However, unlike any other, HiSPARC is run specially for High-School age teenagers.

Figure 2: High-School Students Installing a HiSPARC detector in the Netherlands. D Fokkema

Detector stations, consisting of at least two detectors (to minimise instrument noise) made from photomultipliers convert measured muons to a quantifiable electric current using scintillators. These are assembled by students and placed on the roofs of participating schools and universities. Each event is recorded by computer and matched with time, location and if possible, weather data. This is then sent to the public HiSPARC database, where anyone can use it to support investigations of their own.

For my research I investigated ways to determine the Primary Particle Energy. There are various methods to do this such as: measuring the Cerenkov light produced by secondary particles using highly sensitive telescopes; recording the amount of secondary particles and their distance travelled and measuring the fluorescent light induced in nitrogen atoms by the passing charged air shower. HiSPARC unfortunately does not have such capabilities and instead must rely on a lateral distribution function. This semi-empirical equation uses the recorded particle flux intensities (The amount of particles per meter squared per second - particle density) to quantify the intensity at an arbitrary distance from which the energy can be calculated. Put simply, it forms an estimate based on the size of the particle shower.

Upon entering our atmosphere, the cosmic rays (Named Primary Particles) collide inelastically with nuclei in the air – mainly oxygen and nitrogen – and interact via the strong force . These high-energy reactions create new particles of all types, forming air showers of pions, photons and muons. These newly born particles each carry a fraction of the initial energy and go on to create more and more particles, all the time increasing the particle density. However at a critical point, given the name x_{max}, the energy of each individual particle dips beneath the energy required to react constructively and so the particle is scattered, reducing the particle density. The depth of x_{max} is dependent on the initial energy and type of particle and is one of the many effects that must be accounted for in any model.

jSPARC is a computer simulation that that allows users to quickly and easily calculate the Primary Particle Energy for a given coincidence (A single Event recorded by three or more detector stations). By simply moving around a shower location by mouse and minimising a chi squared error, students can record the energy of events, greatly simplifying the data collation process and allowing them to spend more time analysing results. At the heart of the jSPARC is the Nishimura-Kamata-Greisen function which calculates the flux intensity at a distance r from the shower core :

$$S(r) = k \cdot \left(\frac{r}{r_0}\right)^{-\alpha} \cdot \left(1 + \frac{r}{r_0}\right)^{-(\eta-\alpha)}$$

In the function there are four constants:

- k - The factor by which the intensity is proportional to the Primary Particle Energy.
- r_0 - The Moliere Radius : a measure of how much particles scatter in the Earth's Atmosphere. The Value used in jSPARC is 92m.
- α - Empirically determined to be 1.2.
- η - Contains the influence of the zenith angle . Defined as $3.97 - 1.97(\sec\theta - 1)$ where θ is the zenith angle. As jSPARC always assumes a zenith angle of 0 degrees, η always takes the value of 3.97 in the program.

Many of the constants used were empirically calculated in the AGASA program in Japan. Using this function, jSPARC can derive the two unknowns for the event – energy and shower location. By changing the value of k (and thus the energy) such that the calculated beam intensity always matches the recorded intensity at one detector station, the user can move the beam around the map and find the place that produces the smallest deviation between recorded and calculated values for all three stations.

My aim over the ten days I spent at Birmingham was to devise my own model to check the accuracy of jSPARC having found out from Dr. Pavlidou that no investigation had been carried out into this aspect of the program.

2 Method

My initial approach was to determine the Primary Particle Energy and directly compare this with the result from jSPARC. To do this, I followed the method described by Koortland in the document 'Primary Particle Energy' . For this the zenith angle (The angle the shower makes to a line perpendicular to the ground) is needed to calcuate the value of η. The method I used to work out this angle was based on the report written last year by Lewis Anderson and the Primary Particle Angle document, also by Koortland.

To calculate the angle, a single event must be recorded by three detectors at a known time. The three detectors are then arranged with respect to their relative positions on an x-y plane (It is assumed that all three stations are at the same height – a reasonable assumption in the Amsterdam Science Park where most detectors are within 5m of the same vertical height) where Station A, the first detector to register the shower, has the position (0,0).

Each HiSPARC detector station has a GPS receiver that provides exact longitude and latitude from which, the relative x-y coordinates can be determined with each detector's bearing and distance to Station A.

To calculate the bearing in Excel the following formula is used:

```
Bearing = ATAN2(COS(Lat_a)*SIN(Lat_b)-
SIN(Lat_a)*COS(Lat_b)*COS(Lon_a-Lon_b),SIN(Lon_b-Lon_a)*COS(Lat_b))
```

Figure 3: *Veness 2002, Lewis Anderson*

To calculate the distance the Haversine function is used:

$$Haversine\left(\frac{d}{r}\right) = Haversine\left(\psi_1 - \psi_2\right) + (\cos\psi_1)(\cos\psi_2)Haversine\left(\lambda_2 - \lambda_1\right)$$

In Which:

- d - The Great Earth Distance between the two points.
- r - The Earth's Radius for which a value of 6371km was used.
- ψ_1 & ψ_2 - The Latitude of Point 1 and Point 2 respectively.
- λ_1 & λ_2 - The Longitude of Point 1 and Point 2 respectively.

This was implemented in Excel in the following form, rearranged to give a value for d :

```
cellx = SIN((Lat_b - Lat_a)/2)^2+COS(Lat_a)*COS(Lat_b)*SIN((Lon_b-Lon_a)/2)^2

celly = 2*ATAN2(SQRT(1-cellx),SQRT(cellx))

distance = celly * 6371
```

Figure 4: *Lewis Anderson*

Using these two values, the relative co-ordinates for station B can be calculated easily through simple trigonometry:

$$B_x = Distance_{a,b} \times \sin(Bearing_{a,b})$$
$$B_y = Distance_{a,b} \times \cos(Bearing_{a,b})$$

This process was then repeated for Station C with the end result that the relative positions for all three detectors were known.

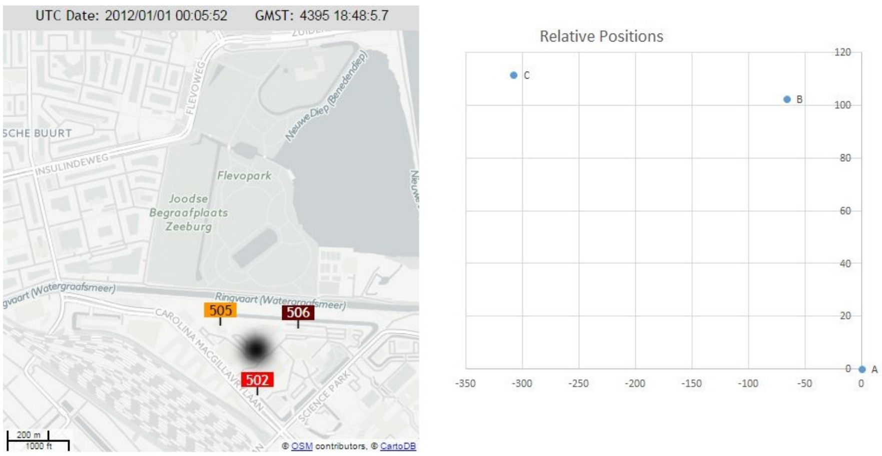

Figure 5: The Map provided by jSPARC and the graph plotted by Excel. Note that Station 506 is Station A.

jSPARC provides a map that highlights the position of all three detectors which can be checked against the positions plotted by Excel. By checking orientation and approximate distances by eye, the relative positions can be checked. This was done for all of the results. Furthermore, with the aid of internet position calculators, the distances can be exactly checked. This was done for the first four results, all of which agreed precisely.

Following the method described by Koortland, the arrival times, which were read graphically from jSPARC (The use of a Crosshair meant this could be done accurately to within 1 nanosecond), were transformed into relative arrival times (Where the time of Detection by Station A = 0) by simply subtracting the time of detection by Station A from the time of detection by the secondary detectors.

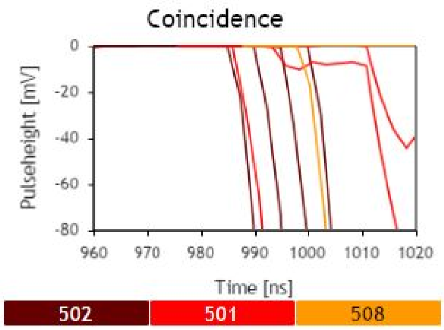

Figure 6: The Graph of Arrival times provided by jSPARC.

These times were in nanoseconds meaning it was necessary to convert them to seconds, done simply by multiplying by a factor of 10^{-9}.

Once these three pieces of data was known for each detector (Relative X Co-Ordinate, Relative Y Co-Ordinate and Relative Time), the zenith angle could be calculated.

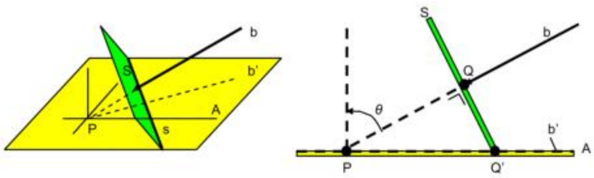

Figure 7: *Koortland*

In the left diagram, S represents the showerfront travelling perpendicular to the shower trajectory b . As the front travels toward the point P, it crosses the surface of the plane (In this case the Earth) along the line s , which moves towards the point P as the showerfront moves down its trajectory.

Taking a 2D cross-sectional view as shown on the right θ , the zenith angle is clearly visible. As the showerfront moves down the line b from point Q to point P at (Approximately) the speed of light, so too must the showerfront move from Q' to at speed v, which can easily shown to be:

$$v = \frac{c}{\sin\theta}$$

As the showerfront arrives uniformly at P, v must be greater than c – superluminal. This is just superficial. The value of v can be calculated from the detection times measured by each station allowing the value of θ to be worked out.

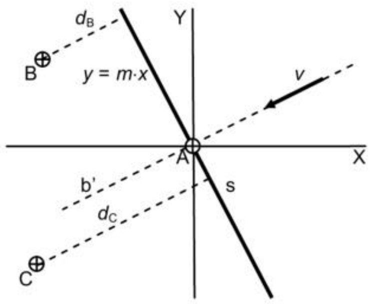

Figure 8: *Koortland*

By taking a downwards 2D view, it can be seen that the shower front makes a straight line , of gradient m, that passes across the plain crossing the origin at $t = 0$. Using the standard result for the distance between a point and a line for each station to the showerfront and combine this with our knowledge that the velocity of the shower front multiplied by the time of detection for each station also equals the distance, we can form the following simultaneous equations:

$$\frac{B_y - m \cdot B_x}{\sqrt{m^2 + 1}} = v \cdot B_t$$

$$\frac{C_y - m \cdot C_x}{\sqrt{m^2 + 1}} = v \cdot C_t$$

These can be solved for v and m.

$$m = \frac{C_y \cdot B_t - B_y \cdot C_t}{C_x \cdot B_t - B_x \cdot C_t}$$

$$v = \frac{C_y - m \cdot C_x}{C_t \cdot \sqrt{m^2 + 1}}$$

Thus allowing us to work out the zenith angle:

$$\theta = \arcsin\left(\frac{c}{v}\right)$$

With the zenith angle, we can work out the value of η from the empirical formula derived by the Agasa program.

$$\eta = 3.97 - 1.97\,(sec\theta - 1)$$

Where θ is the zenith angle.
With the value of η calculated, the remaining unknowns in the NKG function were the value of k (The Energy) and the location of the particle shower. These could only be determined through trial and improvement : comparing the data values of particle flux at each station to computed values calculated using guessed values for shower position and energy . At each iteration, an error was calculated using this formula:

$$X^2 = \sum_{i=1}^{n} \frac{(D_i - C_i)^2}{C_i}$$

Where D_i and C_i is the data and calculated value respectively for station i. This is the same error calculation used in jSPARC. Using the initial result, the estimations were refined and the model ran until the error was minimised. To do this in a feasible timeframe, I intended to use the Excel Add-On solver. However, when used without constraints, the values produced were somewhat incorrect- one estimate for the particle location had the beam in another country. To get a rough idea for the location from which constraints could be imposed, I used jSPARC's result for the shower position. It is important to emphasise that this was not the final location used, but a rough estimate. jSPARC outputs the value of the particle density for each station. To get the relative co-ordinates three circles, centered at the relative positions of each detector with radii equal to their respective distances, were drawn using the online graphing tool Desmos. Displayed graphically, the average intersection could be found easily (while all were in the same immediate vicinity, the circles never intersected in the same point). Constraints of plus and minus 350m were then set on both the x and y coordinates.

Unfortunately, even with these constraints, the solver did not provide what I judged to be reliable results. A typical X^2 error would be around 2.0 with constraints - a figure I decided too high to accurately analyse jSPARC with. At this point, I changed my approach : instead of trying to find my own value for the energy, I would instead take jSPARC's values for the energy and position and calculate, using my own model, what particle fluxes they would give at each of the detectors. These would then be compared to the recorded data values and the values given by jSPARC.

To do this, it was necessary to use the relationship between the value of particle flux at 600m from the core and the energy. The semi-empirical equation was reversed and used as the value of k in the NKG function.

$$E_0 = c \cdot S\left(600\right)^{\varepsilon}$$

$$S\left(600\right) = e^{\frac{\ln\left(\frac{E_0}{c}\right)}{\varepsilon}}$$

As jSPARC provides values for the distance between each station and the showercore, it was easy to simply input these into the function and calculate the particle intensities for each station. Both the recorded data values and the values calculated by the program are given by jSPARC meaning it was easy to calculate the errors between the recorded data and the two calculated values.These errors were then compared using a simple percentage difference calculation.

Beam Intensity (S)			
Station	A	B	C
Data Values	2.841	0.989	0.751
Jsparc Calculated	2.841	0.939	0.812
NKG Calculated	2.161036681	0.712713759	0.617152547
Jsarc Deviation (X^2)	0.007244919		
NKG Deviation (X^2)	0.350080431		
Difference in Deviation	98%		

Figure 9: Numbers in Black are inputted from jSPARC whereas numbers in Red are calculated.

3 Results

Table 1 shows four pieces of information for each event: energy predicted by jSPARC; total X^2 error given by jSPARC between its calculated values of particle flux for each detector and the recorded particle flux; the total X^2 error obtained by my Excel model, using energy and position values given by jSPARC, between its calculated values of particle flux at each detector and the recorded particle flux and the percentage difference between these two error values.

Table 1: Results

Event	jSPARC Energy (eV)	jSPARC Error	My Error	Percentage Difference
01/01/2012	9.57E+14	0.00777	0.07339	89%
01/01/2012	1.41E+15	0.01493	0.16946	91%
01/01/2012	1E+15	0.00292	0.8098	100%
01/01/2012	2.47E+15	0.00849	0.07294	88%
01/07/2013	5.17E+15	0.0138	2.77367	100%
01/07/2013	2.69E+15	0.06461	0.35806	82%
01/07/2013	1.09E+15	0.08906	1.60646	94%
01/07/2013	2.73E+16	0.00828	1.828	100%
01/07/2013	3.69E+14	0.00273	0.32636	99%
01/07/2013	4.16E+14	0.00724	0.35008	98%
01/07/2013	2.13E+15	0.00252	1.15443	100%
01/07/2013	8.94E+14	0.00595	1.68463	100%
01/12/2013	2.69E+15	0.00891	1.70337	99%
01/01/2014	1.15E+16	0.07837	2.03831	96%
01/01/2014	1.07E+15	0.06448	1.79499	96%
02/01/2014	3.68E+15	0.09259	0.51144	82%
02/01/2014	6.18E+14	0.00917	1.35239	99%
02/01/2014	3.97E+15	0.09025	0.77775	88%
02/01/2014	9.05E+16	0.00331	2.07907	100%

Table 2 gives the zenith angle for each event, calculates the value of η this corresponds to and gives the percentage difference this value and the standard result (Used by jSPARC) of 3.97. An arbitrary energy is then calculated using standard values for each constant: $\alpha = 1.2$;$r_0 = 77$m and $k = 650$. The percentage difference between these energies and the energy given by a zenith angle of 0 degrees ($\eta = 3.97$) is given in the final column. An average (Using absolute magnitude values for the zenith angle) is given. A graph of percentage error versus zenith angle is also plotted showing a linear relation for angles of

20 degrees and above.

Table 2: Change in η

Event	Zenith Angle Degrees	Difference in η	Difference in Energy
01/01/2012	-38.17843	-16%	72%
01/01/2012	32.60706	-10%	59%
01/01/2012	-31.02957	-9%	55%
01/01/2012	39.88898	-18%	76%
01/07/2013	-0.24058	0%	0%
01/07/2013	18.77723	-3%	23%
01/07/2013	-3.14002	0%	1%
01/07/2013	12.3371	-1%	11%
01/07/2013	26.4414	-6%	42%
01/07/2013	-27.43634	-7%	45%
01/07/2013	22.33278	-4%	32%
01/07/2013	27.7871	-7%	46%
01/12/2013	49.70228	-37%	92%
01/01/2014	-9.2076	-1%	6%
01/01/2014	19.67459	-3%	25%
02/01/2014	43.98086	-24%	84%
02/01/2014	6.42997	0%	3%
02/01/2014	10.3013	-1%	7%
02/01/2014	-14.29246	-2%	14%

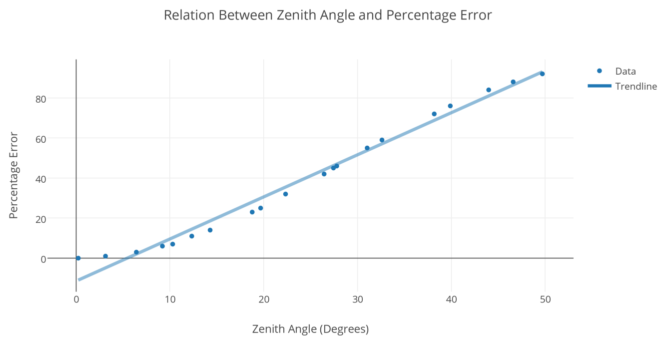

Figure 10: *Graph Showing the Relation between Zenith Angle and Percentage Error in Energy*

4 Discussion

It is clearly evident that the errors returned by jSPARC are much smaller than the errors given by my model. This indicates the values returned by jSPARC are not correct to the accuracy claimed by the program. The major differences between my calculation and jSPARC's are the value used for the Moliere radius and the use of the zenith angle in the constant η.

The Moliere radius, a constant which measures the amount of scattering a particle shower receives in a material (Formally, it is the radius of the cylinder containing 90% of a shower's deposited energy), is an integral part of the NKG function and thus, the calculated values for particle flux. jSPARC uses the value obtained by the by researchers at the AGASA Scintillation array of 92m. However, the Akeno observatory (Where the AGASA project is based) is located at a height of 900m above sea level : a stark contrast to the average 2m of height for Amsterdam. At sea level, the Moliere radius takes a value of 77m and as such , is the value used in my calculation of the NKG function.

To demonstrate the difference, the NKG function was calculated using standard values for constants ($k = 650$, $\eta = 3.97$, $\alpha = 1.2$) and with both radii. Despite there only being a 16% difference between the two numbers, this corresponds to a 48% difference in the value of particle flux at 600m from the core – S(600). As this is the value used to calculate Primary Particle Energy, the results would also differ by 48%. This substantial difference can easily be rectified by swapping the value of r_0 for a more suitable one based on the particular location.

The zenith angle determines how much atmosphere a particle shower travels through and thus plays a crucial role in determining the particle flux intensity. jSPARC does not take account of this, instead assuming an angle of 0 degrees $(3.97 - 1.97(\sec(0) - 1) = 3.97)$. As demonstrated in the results section, the error this causes grows linearly as the zenith angle increases with an average error of 36% over this particular dataset. Unlike a change to r_0, implementing this into jSPARC would be difficult. While the method used to calculate the zenith angle did make many assumptions, namely the showerfront moving at the speed of light and the x-y plane being completely flat on a curved surface , the method did seem to supply reliable results. Of particular interest is the bias towards positive values for the zenith angle: in 19 coincidences, there were only 6 negative values. My method meant that the data from only three detectors could be used meaning some events given by jSPARC could not be utilized.Also, the percentage difference method I employed to compare errors did not give a definite indication of the error in jSPARC's energy estimate. However, I did show that jSPARC was not precise to the accuracy claimed and on that basis, I feel my investigation was successful.

5 Acknowledgements

I would like to extend my thanks to Dr Maria Pavlidou for her help which I cannot understate. I would also like to thank Linda Rogers for helping arrange my placement. Additional thanks to Benjamin Barrow and Arne de Laat. I used the site plot.ly for my graph.

6 References

1. Pavlidou M. University of Birmingham. Personal Communication 2015
2. David Fokkema. 2012. The HiSPARC Cosmic Ray Experiment Data Acquisition and Reconstruction of Shower Direction.
3. Washington State University Chapter 9 Cosmic Rays. Accessed from the Washington State University Website July 2015 (I could not find the author of this work and for this I can only apologise.)
4. Koos Kortland. Primary Particle Angle. Accessed July 2015
5. Koos Kortland. Primary Particle Energy. Accessed July 2015
6. A.A Watson. Extensive Air Showers and Ultra High Energy Cosmic Rays. 2002.
7. M. Takedaa,N. Sakakia, K. Hondab, M. Chikawac, M. Fukushimad,N. Hayashidad, N. Inouee, K. Kadotaf, F. Kakimotog, K. Kamatah, S. Kawaguchii, S. Kawakamij, Y. Kawasakia, N. Kawasumik, A.M. Mahrouse, K. Mased, S. Mizobuchil, Y. Morizaned, M. Naganom,H. Ohokad, S. Osoned, M. Sasakid, M. Sasanon, H.M. Shimizua,K. Shinozakid, M. Teshimad, R. Toriid, I. Tsushimak, Y. Uchihorio,T. Yamamotod, S. Yoshidap and H. Yoshiil. Energy Determination in the Akeno Giant Air Shower Array . 2002.
8. Paul Sommers Extensive Air Showers and Measurement Techniques. 2004.
9. Lewis Anderson. Locating the Source of Cosmic Rays using HiSPARC data. 2014.
10. J Cortina, F Arqueros and E Lorenz. Methods for Determining the Primary Energy of Cosmic-Ray Showers. 1997
11. Markus Roth.The Lateral Distribution Function of Shower Signals in the Surface Detector of the Pierre Auger. 2003 Observatory.