Optimal two stage committee voting rules

Ian Ayres\textsuperscript{2}  Colin Rowat\textsuperscript{3}  Nasser Zakariya\textsuperscript{4}

January 26, 2006

\textsuperscript{1}The authors thank Toke Aidt, Steve Bellenot, Aaron Benjamin, Mayling Birney, Tilman Börgers, Gautam Chinta, Jayasri Dutta, Marco Mariotti, Herakles Polemarchakis, Debraj Ray, Indra Ray, Thomas Richard, Daniel Seidmann, Demetri Sevastopulo, Philippa Townsend, Ben Zissimos, and seminar participants at Brown University, ESEM 2004, ESRC Game Theory workshop in Essex, the University of Warwick and the Midlands Theory workshop. Alejandro Caceres’ and Serge Uogintas’ generous assistance with the Maple code is particularly appreciated.

\textsuperscript{2}Yale Law School

\textsuperscript{3}Department of Economics, University of Birmingham

\textsuperscript{4}FAS, Harvard University
Abstract

We study option management by committee. Analysis is illustrated by tenure decisions. Our innovations are two-fold: we treat the committee’s problem as one of social choice, not of information aggregation; and we endogenise the outside option: rejecting a candidate at either the probationary or tenure stage return the committee to a candidate pool. For committees with \( N \) members, we find three key results: (1) a candidate’s fate depends only on the behaviour of two ‘weather-vane’ committee members - generalised median voters; (2) enthusiastic assessments by one of these weather-vanes may harm a candidate’s chances by increasing others’ thresholds for hiring him; and (3) sunk time costs may lead voters who opposed hiring a candidate to favour tenuring him, even after a poor probationary performance. We also characterise the optimal voting rule when \( N = 2 \). A patient or perceptive committee does best with a (weak) majority at the hiring stage and unanimity at the tenure stage. An impatient or imperceptive committee does best under a double (weak) majority rule. If particularly impatient or imperceptive, this rule implies that any hire is automatically tenured. Perversely, the performance of a patient, imperceptive committee improves as its perceptiveness further declines.

Key words: intertemporal strategic voting, real options, social choice, heterogenous priors, tenure

JEL classification numbers: C73, D71, D72, D80, G12

This file: 060126twostage.tex
1 Introduction

Many decisions involve more than one stage of decision making. When a committee is responsible for making such decisions, it is natural to ask what voting rules it should follow. This paper does so in a real options setting: the first decision purchases the right to make a later, final decision. Rejection at either stage presents the committee with a new option to consider. As the committee will then re-apply its voting rule, the value of the outside option is endogenous. When committees are comprised of two members, we determine the voting rule that maximises the value of this option.

For expositional purposes, the real option that we consider is drawn from labour economics: the right to offer an employee a permanent position. Tenure in academia and ‘up or out’ rules in law, accountancy and the military are all examples of employment decisions that fall within the scope of our analysis (Meyer, 1992).\(^1\)

Within finance or corporate governance, portfolios of both real and financial options may be managed by teams, particularly in the non-investment community (e.g. pension and insurance companies). In political economics, bills in parliamentary democracies must survive votes after their second and third readings before being enacted into law; bicameral systems often require that bills be passed by both houses in sequence; international treaties tend to be signed by States and then ratified. In law and economics, judicial decisions motivated Condorcet’s interest in committee decisions. The possibility of appeal and penalty phases introduces a second decision making stage in both cases.\(^2\) In the economics of the family, a period of engagement often precedes marriage which, if unsuccessful, typically ends candidacy.

While all of these situations differ in details, we believe that our model captures the central feature of all of them, namely the management of an option by committee.

To focus on the intertemporal issues associated with options management

\(^1\)McPherson and Schapiro (1999) provides a recent introduction to the tenure literature, which includes Chatterjee and Marshall (2001); Chen and Ferris (1999); Ehrenberg, Pieper, and Willis (1998); Carmichael (1988) and Ito and Kahn (1986). Another academic example is the practice of refereeing articles, which are often accepted for publication only after surviving an initial round of refereeing.

\(^2\)The most notorious penalty phase decision dates to Socrates’ trial. Athenian juries first voted on the defendant’s innocence or guilt; a guilty finding then led to a second vote over the proposed penalties. Thus, Socrates was found guilty by 280 jurists, but 360 voted for his execution.
we abstract from many intratemporal ones, including incomplete information between committee members. Our hiring committee considers a single candidate. Their opinions as to whether or not the candidate will be a good hire are common knowledge. As their opinions may differ, we model ‘diverse beliefs’ (Kurz, 1996) or ‘open disagreement’ (van den Steen, 2004). In contrast, then, to much of the committee literature (e.g. Feddersen and Pesendorfer, 1997), our committee’s problem is one of social choice, not information aggregation.

By assuming heterogenous priors, we do not follow the ‘Harsanyi doctrine’ that “differences in subjective probabilities should be traced exclusively to differences in information” (Aumann, 1976). If subjective probabilities are seen as beliefs held before receipt of any informative signals, then their heterogeneity may be as well founded as, for example, ability heterogeneity (Morris, 1995).³ We find this argument compelling. Further, heterogenous priors seemed a very natural way to model the present environment.

Having formed priors, committee members vote either in favour of or against hiring the candidate.⁴ If hired, the candidate emits a publicly observed performance signal. All members use this signal to update their beliefs about the candidate. They then vote either in favour of or against granting the candidate tenure. Rejecting the candidate at either stage returns the committee to the candidate pool. Before drawing a new candidate, each committee member expects to receive $V$, the value of the expected game.

Framing the committee’s problem recursively is the main technical contribution of this paper: the value of the outside option, $V$, is determined endogenously, as a function of the voting rule. This structure embeds a fixed point problem in the analysis: the value of the tenure process depends on committee members’ voting strategies. Those voting strategies, in turn, depend on the value that committee members ascribe to rejecting a candidate and returning to the market.

³This seems to be the basis of Harsanyi’s own arguments against his eponymous doctrine: “by the very nature of subjective probabilities, even if two individuals have exactly the same information and are at exactly the same high level of intelligence, they may very well assign different subjective probabilities to the very same events” (Harsanyi, 1968). In addition to the papers already cited, recent work with heterogenous priors includes Morris (1996); Allen and Gale (1999); Duffie, Gärleanu, and Pedersen (2002); Yildiz (2004). Theoretically the heterogenous priors assumption is a weaker one than that of common priors (Heifetz, 2001).

⁴See Ghosal and Lockwood (2003) and the references therein for an analysis of abstention.
We first analyse agents’ optimal voting strategies for general committees of size $N$ taking $V$ as given. Then, to solve the fixed point problem and assess optimal voting rules, we specialise to the cases of $N = 1$ (to develop intuition) and $N = 2$. Technically, each additional committee member adds a dimension of integration to the problem. While our methods apply to larger $N$, we as yet can only apply them on a case-by-case basis rather than for general $N$.

We initially analyse the $N$ member committee, instead of specialising to $N = 2$, for three reasons. First, it provides results for two stage committee problems when the value of the outside option, $V$, is exogenous. These are novel and interesting in their own right. Second, the generality facilitates extensions of the optimality analysis to larger committees. Finally, there are almost no costs of this generality.

For general committees, we find rich intertemporal patterns of strategic voting. Some of these are specifically intrapersonal: sunk time costs can create situations in which a committee member who opposes hiring a candidate goes on to support tenure in spite of a bad probationary performance.

Others are interpersonal. Once a voting rule is fixed, a tenure ‘weather-vane’ voter - the member who always votes with the winning side in the tenure vote - may be identified. Other committee members then condition their votes at the hiring stage on two factors: their own priors about a candidate, and the tenure weather-vane’s.

They therefore vote against hiring a candidate that they personally favour if the tenure weather-vane is sufficiently opposed to ensure his defeat at the tenure stage. At the other extreme, a sufficiently enthusiastic tenure weather-vane can cause committee members to apply higher standards in the hiring vote. We term this effect ‘inverse enthusiasm’. Otherwise, enthusiasm by the tenure weather-vane lowers other committee members’ thresholds. Insight into the conditions governing these opposing effects is gained by explicit appeal to an options interpretation.

Analogously to the tenure weather-vane, a probationary weather-vane may also be identified. A sufficient statistic for a candidate’s fate is the priors of these two committee members. Increases in the mean priors about a candidate may harm him; increases in the variance of those priors may help him.

Finally, optimal voting rules for two person committees are derived. The rising threshold rule - a (weak) majority at hiring rising to unanimity at tenure - is optimal for patient or perceptive committees. This seems consis-
tent with the practice in many departments: whatever the formal rule, one wants to go to the dean with unanimity. It is also intuitively appealing: the committee hires more readily than it grants tenure as hiring entails either little cost or gains good information on the candidate.

One comparative static result is peculiar: patient but imperceptive committees’ performance improves slightly as their perceptiveness falls. This owes to the tendency of the more optimistic committee member to ‘take a chance’ on a candidate under the rising threshold rule. As the probationary signal’s informativeness declines, the optimist loses the ability to believe that the pessimist will be swayed by a good probationary performance. The optimist thus takes fewer such chances.

The rising threshold result has the flavour of that in Meyer (1991, Section 7), in which it may be optimal to bias against the winner of an earlier contest. Here, discounting and aggregation via the committee work in opposite directions: an individual committee member lowers her standards, perhaps approving of a candidate once the time costs are sunk and he has become eligible; the committee, as a whole, however, has a higher standard for the second vote.

The double (weak) majority rule - a single vote to pass at each stage - is optimal for committee that are both impatient and imperceptive. This less demanding rule helps the committee by tenuring candidates more rapidly than does the rising threshold. When the committee is particularly impatient or imperceptive, the rule never yields genuine probationary periods: if hired, a candidate is certainly granted tenure, removing a further impediment to rapid tenure. In these cases, the committee’s problem is endogenously reduced to a one stage problem.

Two voting rules - double unanimity and falling thresholds (unanimity followed by weak majority) - are dominated for all parameter values. As there is no private information to pool, the former is dominated for reasons other than those developed in Feddersen and Pesendorfer (1998). For reviews of the literature on jury decisions and information aggregation, we recommend Persico (2004) and Ghosal and Lockwood (2003).

The social choice literature is also large. Much of its concern with decision

\[^5\] Their jurists, who share common interests, may convict more innocent defendants when required to reach unanimity than under majority votes: a pivotal jurist who initially believes in a defendant’s innocence will allow guilty votes from the remaining jurists to overrule her private signal. Under weaker rules, the jurist does not have this incentive to vote against her private signal.
rules has, however, been positively motivated. Caplin and Nalebuff (1988), for example, show that the Condorcet paradox can always be overcome with a super-majority rule of 64% when individual preferences and their distribution are appropriately restricted. We avoid problems of cycles by restricting committee members’ choice sets to two elements, allowing us to concentrate on normative questions.

To our knowledge, ours is the first recursive analysis of a committee decision problem. Similarly, there have been few analyses of two stage committee decisions. One exception is Manzini and Mariotti (2005), which takes each committee’s (rational) preference ordering as given to focus on the rationality properties of their composition. Polborn (2000) analyses an environment in which voters decide on tax reform voting rules when young (and poor); after a stochastic shock, when they are older (and wealthier), they vote on tax reform. He shows that median voters will support supermajority rules.

Section 2 formally presents our model. Section 3 then analyses the committee members’ decisions during their two decision stages, taking the voting rule and $V$ as given. Section 4 solves the fixed point problem for $V$ when $N = 1$; Section 5 addresses $N = 2$. Section 6 concludes and compares our model to a standard options model.

2 The model

Consider a pool of *ex ante* identical candidates. In the game’s prehistory, one of them is shortlisted and presented to a committee of $N$ risk neutral members, indexed by $i$. For expositional clarity, assume the committee members to be female and the candidate male.

The candidate will either be good for the department ($\tau = 1$) or bad ($\tau = 0$) if tenured. While committee members thus share common interests, their assessments of the candidate vary. The subjective probability that each assigns to the candidate being a good hire is $p_0i \sim [0, 1]$ with $p_{01} \geq \cdots \geq p_{0N}$. Generically, these can be written as strict inequalities; we assume this in what follows. Finally, these priors are common knowledge.

At time 0, the committee members vote on hiring. Voting is costless and mandatory: abstentions are not allowed. If strictly more than $0.60N$ of committee members approve, the candidate is given probation.

---

6This model differs from those that allow an agent to specify the order in which candidates are tested; q.v. Weitzman (1979).
If the candidate is hired, his probationary period emits a public signal, $\theta \in \{0, 1\}$. It is common knowledge that all committee members view the signal as accurately reflecting the candidate’s type with probability $\sigma$: $P(\theta = 1|\tau = 1) = P(\theta = 0|\tau = 0) = \sigma$. Without loss of generality, $\sigma \in (\frac{1}{2}, 1]$; otherwise, the signal’s complement could be used as the informative signal.

Thus,

$$E_{0i}(\theta) = \sigma p_{0i} + (1 - \sigma)(1 - p_{0i});$$

where $E_{0i}$ are member $i$’s expectations at time 0.

After the signal’s receipt, committee members calculate $p_{T1}$, their posterior beliefs:

$$p_{T1}(\theta, p_{0i}) = \frac{P(\theta|\tau = 1)p_{0i}}{P(\theta|\tau = 1)p_{0i} + [1 - P(\theta|\tau = 1)](1 - p_{0i})}.$$  

(2)

Thus $p_{T1} \geq \cdots \geq p_{TN}$.

At time $T > 1$, a tenure vote is taken. If strictly more than $\delta T N$ committee members approve, the candidate is tenured.7 As this date cannot be brought forward, the real option is European rather than American.

If, at either vote, the candidate fails, he is dismissed and the department returns to the pool of candidates.8

This pattern of play is summarised in Figure 1. Payoffs are discounted to time 0. Thus:

1. if a candidate is not hired, the department returns to the pool the following year. The value of restarting the hiring process is $\beta V$, where $\beta \in [0, 1]$ is the discount factor.

2. if a probationary candidate is denied tenure, the department immediately returns to the pool. It neither suffers costs nor gains benefits from having granted probation to a candidate.

---

7 We address the fixed point problem, below, by setting $T = 2$. Until then, we develop the exposition for the fully general $T > 1$ as this allows the effect of the probationary period to be explicitly seen.

8 It is therefore assumed that the department cannot hire directly at the tenured level. Established and recognised lawyers are occasionally hired directly as lateral partners. Some departments, such as Yale’s Comparative Literature department, only hire at the tenure level directly.
3. finally, if a candidate is granted tenure, committee member $i$ anticipates an expected payoff of $\beta^T p_T$: the actual payoff is equal to the candidate’s type.

That the probationary period does not impose costs or deliver benefits in its own right may be interpreted as the probate’s teaching load financing the probationary period. In options parlance, no dividends are paid. Granting tenure to a $\tau = 0$ candidate may be similarly interpreted. (As tenure does not convey salary guarantees, this may be thought of as adjusting a tenured, non-research productive faculty member’s salary.) This structure may fit the legislative process more closely: a bill does not impose costs or benefits until it is enacted, but information about these is gained during its consideration.

Under some parameter values, the decision tree in Figure 1 degenerates: some terminal nodes are dominated for committee members holding any priors. For example, were it possible that $\sigma = \frac{1}{2}$, refusing tenure to a candidate granted probation would always be dominated by not hiring the candidate. Similarly, $T = \infty$ would leave tenure refusal dominated by initial rejection. Finally, if $\beta = 1$, all candidates are hired: hiring allows no worse a payoff.
to be obtained than that possible by returning to the pool; a positive signal, however, convinces any committee member that the candidate is worth tenuring. In what follows, only generic cases are considered, unless otherwise specified.

It follows that $\beta < 1$ implies that $V(\cdot) \in [0, 1)$: even if a $\tau = 1$ candidate is discovered immediately, it take some time to tenure him. The lower bound is achieved whenever the terminal node is reached with a $\tau = 0$ candidate tenured. If a perfectly patient committee never granted tenure, the indeterminate $V = V$ would result.

At this point $V$ is treated as constant. Fixed point arguments will later be used to compute $V$ as a function of its parameters.

Now define strategies:

**Definition 1.** A strategy for committee member $i$ is a pair of functions

$$v_{0i} : [0, 1]^N \rightarrow \{0, 1\} \text{ and } v_{Ti} : [0, 1]^N \times \{0, 1\}^N \rightarrow \{0, 1\}.$$

Strategies map to votes: action 1 corresponds to a ‘yes’ vote, and action 0 to a ‘no’. The hiring vote maps from priors, while the tenure vote maps from posteriors and past votes.

The expected utility of member $i$, as assessed at time $T$ and discounted to time 0, is:

$$u_{Ti}(v_T) \equiv \begin{cases} \beta^T p_{Ti} \text{ if } \sum_{j=1}^N v_{Tj} > \delta_T N \\ \beta^T V \text{ otherwise} \end{cases}.$$

Her expected utility at time 0 is

$$u_{0i}(v_T, v_0) \equiv \begin{cases} u_{Ti}(v_T) \text{ if } \sum_{j=1}^N v_{0j} > \delta_0 N \\ \beta V \text{ otherwise} \end{cases}.$$

**Definition 2.** A strategy profile $v^* = \{v_{0i}^*, v_{Ti}^*\}_{i=1}^N$ and posterior beliefs $p_T(p_0, \theta)$ constitute a voting equilibrium iff:

- $v^*$ contains no weakly dominated strategies;
- $v_{Ti}^* \in \arg\max_{0,1} \left\{ P \left( \sum_{j \neq i} v_{Tj}^* + v_{Ti} > \delta_T N \right) \beta^T p_{Ti} \\
+ \left[ 1 - P \left( \sum_{j \neq i} v_{Tj}^* + v_{Ti} > \delta_T N \right) \right] \beta^T V \right\};$
- $v_{0i}^* \in \arg\max_{0,1} \left\{ P \left( \sum_{j \neq i} v_{0j}^* + v_{0i} > \delta_0 N \right) u_{Ti}(v_T) \\
+ \left[ 1 - P \left( \sum_{j \neq i} v_{0j}^* + v_{0i} > \delta_0 N \right) \right] \beta V \right\};$
\( p_{T_i}(p_{0i}, \theta) \) is defined by equation 2;

\[ \forall i = 1, \ldots, N. \]

Thus, a voting equilibrium is a restricted Perfect Bayesian equilibrium with trivial learning. Eliminating weakly dominated strategies reduces the effective domain of \( v_{T_i} \) to the support of \( p_{T_i}, [0, 1] \): committee members vote ‘sincerely’ at time \( T \). As dominated strategies cannot be present in the support of mixed strategies, this allows concentration on pure strategies at time \( T \). Finally, it ensures a unique equilibrium in the time \( T \) stage game (see Theorem 1), simplifying time 0 analysis. It will be seen later that this effectively makes the domain of \( v_{0i} \) two dimensional.

3 The meetings

3.1 The tenure committee meeting

Perfection allows \( v_{T_i} \) to be considered independently of \( v_{0i} \):

**Theorem 1.**

\[ v^*_{T_i}(p_{T_i}) = \begin{cases} 1 & \text{if } p_{T_i} \geq V \\ 0 & \text{otherwise} \end{cases} \forall i \in N. \]

**Proof.** When committee member \( i \) is pivotal, she chooses the terminal payoff that she judges higher. When she is not, her choice is without consequence. Voting against her preferred terminal payoff is weakly dominated. \( \square \)

Therefore:

**Definition 3.** Member \( k \) is a weather-vane voter at time \( T \) for beliefs \( p_T \) and voting rule \( \delta_T \) when

\[ k = \begin{cases} \lceil \delta_T N \rceil & \text{if } \delta_T N \text{ is not an integer;} \\ \lceil \delta_T N \rceil + 1 & \text{otherwise; (3)} \end{cases} \]

where \( \lceil \cdot \rceil \) is the least integer function.

A weather-vane voter is therefore an \( \delta_T \)-percentile voter, a generalised median voter. If \( \delta_T N \) is not an integer, then \( \lceil \delta_T N \rceil > \delta_T N \) and there are enough votes for passage. If \( \delta_T N \) is an integer, then \( \lceil \delta_T N \rceil = \delta_T N \) and,
since we use strict inequality, this is not enough for passage; \( \delta T N + 1 \) will be enough.

Thus, the committee’s decision coincides with the weather-vane’s vote. A weather-vane differs from a dictator in that the aggregation rule does not privilege her \textit{ex ante} or independently of the beliefs of others. To see how a weather-vane and a pivotal voter differ, consider:

\textbf{Example 1.} Let \( N = 5, \delta T = \frac{1}{2} \). Therefore, \( k = 3 \). Suppose further that the priors are such that even committee member 5, the most skeptical, is willing to vote for tenure. In this case, \( k \) is not pivotal: withholding her vote would leave a majority in place. Now suppose that the priors are such that members 4 and 5 will vote against tenure. In this case, the remaining members are all pivotal.

The weather-vane exists as a consequence of mandatory binary voting and the elimination of weakly dominated strategies. Generically, the weather-vane is unique.

\section{3.2 The hiring committee meeting}

Now consider behaviour at the hiring committee meeting. Again, each committee member compares her priors to threshold levels. Now, as each member seeks to predict the fate of the candidate if he is brought to tenure, the thresholds depend on the tenure weather-vane’s priors as well.

Define

\[
\bar{p} \equiv \frac{\sigma V}{\sigma V + (1 - \sigma)(1 - V)}; \quad (4) \\
p \equiv \frac{(1 - \sigma) V}{(1 - \sigma) V + \sigma (1 - V)}. \quad (5)
\]

Thus, \( \bar{p} \) (resp. \( p \)) leaves committee member \( i \) indifferent between returning to the candidate pool next year and granting the existing candidate tenure if he produces a bad (resp. good) performance signal. If \( p_{0i} \geq \bar{p} \) (resp. \( p_{0i} \leq \bar{p} \)) then committee member \( i \) will vote for (resp. against) tenure even when the candidate emits the bad (resp. good) signal during probation:

\textbf{Lemma 1.}

\[
\begin{align*}
p_{0i} \leq \bar{p} & \iff p_{Ti} (1, p_{0i}) \leq V; \\
p_{0i} \geq \bar{p} & \iff p_{Ti} (0, p_{0i}) \geq V.
\end{align*}
\]
Proof. By equation 2 and definitions 4 and 5,

\[ p_{Tk}(1, \bar{p}) = p_{Tk}(0, \bar{p}) = V. \]  \hspace{1cm} (6)

The result follows from the monotonicity of \( p_{Ti} \) in \( p_{0i} \).

As we would expect, a committee member must have a higher prior if she is to support tenure for a candidate who has performed badly than she must have for one who has performed well:

**Lemma 2.** \( 1 > \bar{p} > p \).

**Proof.** The former inequality follows from equation 4.

Assume that the latter inequality does not hold. Thus, there exists a \( p_{0i} \in [\bar{p}, \tilde{p}] \) such that, by Lemma 1, \( p_{Ti}(1, p_{0i}) \leq V \leq p_{Ti}(0, p_{0i}) \). As the posterior increases in the signal, this is a contradiction.

As priors are common knowledge, all committee members know at \( t = 0 \) that \( p_{0k} \geq \bar{p} \) ensures that the tenure weather-vane \( k \) will vote in favour of the candidate if he reaches the tenure meeting. Refer to candidates so hired as permanent hires. Similarly, \( p_{0k} < p \) assures them that \( k \) will oppose the candidate. Finally, when \( p_{0k} \in [p, \tilde{p}] \) the candidate, if hired, is a probationary hire, whose probability of winning tenure is assessed by committee members to be \( E_{0i}(v^*_{Tk}) = \sigma p_{0i} + (1 - \sigma)(1 - p_{0i}) \).

Now define

\[ \tilde{p} \equiv \beta^{1-T}V; \]

the initial belief that leaves \( i \) indifferent between returning to the pool and hiring the candidate when she knows that the weather-vane will support the candidate’s tenure (\( p_{0k} \geq \tilde{p} \)) regardless of his performance. Thus, \( \tilde{p} \) is independent of \( \sigma \). The definition is derived from

\[ E_{0k}(\beta^T p_{Ti}(\theta, \tilde{p})) = \beta V. \]

As the committee members are rational and \( p_{0} \) is common knowledge, \( E_{0k}(\cdot) = E_0(\cdot) \forall i \in N \).

Now define the expected payoff to hiring a candidate if \( p_{0k} \in [p, \tilde{p}] \), so that the weather-vane only votes for tenure if \( \theta = 1 \):

\[ f(p_{0}) \equiv P(\theta = 1) \beta^T p_{T}(1, p_{0}) + P(\theta = 0) \beta^T V \\
= \{ \sigma p_{0} + [(1 - \sigma) p_{0} + \sigma (1 - p_{0})] V \} \beta^T. \]  \hspace{1cm} (7)
Lemma 3. The function $f(p_0)$ is strictly increasing over $(0, 1)$.

Proof.

$$f' = [\sigma + (1 - 2\sigma)V]\beta^T;$$

so that $f' > 0 \iff V < \frac{\sigma}{2\sigma - 1}$. This last term monotonically decreases from infinity at $\sigma = \frac{1}{2}$ to $1 \geq V$ at $\sigma = 1$.

Finally, let $\hat{p}$ be the initial belief that leaves $i$ indifferent between returning to the pool and probationarily hiring the candidate, so that $f(\hat{p}) = \beta V$. As the performance signal is now used, $\hat{p}$ is a function of $\sigma$:

$$\hat{p} = \frac{\beta^{1-T} - \sigma}{(1 - \sigma)V + \sigma(1 - V)}.$$  

The analysis of $\frac{\sigma}{2\sigma - 1}$ in the proof of Lemma 3 shows that $\hat{p}$ is finite.

We expect $\hat{p} \geq \underline{p}$ as there would otherwise be a range of priors in which members were willing to hire even if the candidate stands no chance of obtaining tenure. This is easily confirmed: $f(p) = \beta^T V \leq \beta V = f(\hat{p})$. Lemma 3 establishes the inequality. The inequality reduces to an equality as $\beta \rightarrow 1$.

These results allow us to express the optimal behaviour of voter $i$ as a function of $p_{0i}$ and $p_{0k}$:

Theorem 2.

$$v^*_i(p_{0i}, p_{0k}) = \begin{cases} 1 & \text{if } (p_{0i}, p_{0k}) \geq (\hat{p}, \bar{p}) \text{ or } (p_{0i}, p_{0k}) \in [\hat{p}, 1] \times [\underline{p}, \bar{p}] \end{cases} \forall i \in N.$$  

Figure 2 illustrates and explains the results:

I Member $i$ knows that, if hired, the candidate will fail tenure. As this delays the department’s return to the pool, she opposes hiring. When $p_{0i} > \bar{p}$, she votes against a candidate, even though he has already convinced her that he should be granted tenure.

II Member $i$ knows that, if hired, the candidate will gain tenure, regardless of his performance. Since she is sufficiently skeptical about the candidate, she opposes the hire. When $p_{0i} \in (\hat{p}, \bar{p})$ she votes against the candidate even though she would vote for him were the tenure weather-vane less enthusiastic. We term this effect, indicated by the arrow, ‘inverse enthusiasm’.
III Again, member $i$ knows that the candidate will gain tenure if hired. Now, however, she is sufficiently confident in the candidate to take the chance of hiring him.

IV Member $i$ is skeptical of the candidate. She therefore votes against hiring the candidate, rather than allowing $k$ the opportunity to resolve her uncertainty.

V Member $i$ votes in favour of the candidate as she is willing to allow the weather-vane to resolve her uncertainty.

We now present two lemmata on the relative magnitudes of $\hat{p}, \tilde{p}$ and $\bar{p}$. These depend, in part, on the following object:

$$L(\beta, \sigma, T) \equiv \frac{\sigma\beta^{T-1} - (1 - \sigma)}{(2\sigma - 1)} \leq 1.$$
Lemma 4. $\hat{p} \leq \tilde{p} \iff V \leq L(\beta, \sigma, T)$.

Proof. By definition of $\hat{p}$ and $\tilde{p}$, the lemma’s first inequality may be expressed as
\[
\frac{\beta^{1-T} - \sigma}{\sigma + (1 - 2\sigma)} V \leq \beta^{1-T} V.
\]
As, by Lemma 3, the left hand side of this is positive for $\sigma \in \left(\frac{1}{2}, 1\right]$, this may be manipulated for the result.

Similarly, then:

Lemma 5. $\bar{p} \geq \tilde{p} \iff V \leq L(\beta, \sigma, T)$.

Thus,
\[
V \leq L(\beta, \sigma, T) \iff \hat{p} \leq \tilde{p} \leq \bar{p}.
\] (8)

The sign of the inequality in $V$ and $L$ will be seen to divide the committee’s parameter space into two regions.

Lemma 4 examines how the standard that committee member $i$ applies depends on her perception of the weather-vane’s support for a candidate. When $V \leq L$, an enthusiastic tenure weather-vane ($p_{0k} \geq \tilde{p}$) discourages committee members from hiring a candidate ($\tilde{p} \geq \hat{p}$). When $V \geq L$, the opposite holds: an enthusiastic tenure weather-vane ($p_{0k} \geq \tilde{p}$) now encourages committee members to hire a candidate ($\tilde{p} \leq \hat{p}$). The first, inverse enthusiasm, has already been introduced. Analogously, we term the second ‘pro-enthusiasm’.

To understand these effects, think of $L(\beta, \sigma, T)$ as akin to the strike price of the option and $V$ as the return to buying the market.\(^9\) Thus, when the strike price is high relative to the return to the market ($V \leq L$), $i$ requires a higher standard to favour hiring if she knows that $k$ will grant tenure ($p_{0k} \geq \tilde{p}$): hiring exercises the option at the relatively high price. If, however, the weather-vane is not convinced, then hiring will only exercise the option in the good state of nature, a proposal more appealing to $i$.

On the other hand, when the return to the market is high relative to the strike price ($V \geq L$), $i$ is more easily convinced to favour hiring if she knows that $k$ will grant tenure ($p_{0k} \geq \tilde{p}$): hiring exercises the option at the relatively low price. If $k$ is not convinced, $i$ is more easily convinced to return to the market, with its relatively high rate.

\(^9\)This interpretation is developed further in Section 6.
Thus, a committee member will demand a higher prior to support initial hiring if she wants to hire on a different basis (e.g. probationary or not) than does the tenure weather-vane voter.

Now consider Lemma 5. When $V \leq L$, the prior at which $k$ is indifferent between probationary and permanent hire is larger than that which leaves $i$ indifferent between rejection and permanent hire. In the special case in which $i$ and $k$ are the same committee member, her priors lie along the dotted diagonal line in Figure 2. Thus, $\bar{p} \geq \tilde{p}$ divides her priors into three zones: those in which she will not tenure at $t = T$, regardless of performance; probationary hiring; and permanent hiring. The relatively high strike price, $L$, thus seems to attach value to retaining all three possible outcomes.

When, on the other hand, $V \geq L$, the probationary hiring zone disappears: the committee member opposes hiring unless she is convinced from the outset that the candidate should be granted tenure. This case presents an apparent paradox: Figure 2’s diagonal now passes through region II. In this region, $p_{0k} \in (\bar{p}, \tilde{p})$, the tenure weather-vane opposes hiring but will vote in favour of tenure even after a bad probationary performance. Her behaviour can be explained by the time costs that are sunk ex post, once the tenure meeting is reached.

When inequality 8 holds, a weaker version of this reasoning applies in region IV: $p_{0k} \in (\bar{p}, \tilde{p})$ implies that the tenure weather-vane votes against hiring, although she would vote in favour of tenure if the candidate was hired and generated $\theta = 1$.

The lemmata above clearly give tenure weather-vanes an incentive to dissemble, abstain or delegate their later tenure votes to others. While clearly plausible phenomena, they are not allowed by the present model.

As every agent $i$ compares her $p_{0i}$ to the same cut points, voting ‘yes’ when above it and ‘no’ otherwise, a hiring weather-vane at $t = 0$ may be defined analogously to the tenure weather-vane at $t = T$:

**Definition 4.** Member $j$ is a weather-vane voter at time 0 for beliefs $p_0$ and voting rule $\delta_0$ when

$$j = \begin{cases} \lceil \delta_0 N \rceil & \text{if } \delta_0 N \text{ is not an integer;} \\ \lceil \delta_0 N \rceil + 1 & \text{otherwise;} \end{cases}$$

Thus, the outcome of the hiring process is a function of the beliefs of weather-vanes $j$ and $k$, as shown in Figure 3. The panel on the right refers
to the case when the inequalities in equation 8 do not hold. Refer to this two dimensional space as weather-vane space.

In general, only half of the weather-vane space is accessible. Which half is depends on the voting rule adopted. When, for example, $j < k$, the tenure weather-vane, by definition, has at least as high a level of conviction about a candidate as will the hiring weather-vane. In this case, the lower triangles are accessible. When $j = k$, only the diagonal line is accessible.

Candidates’ success therefore depends on the beliefs of no more than two committee members, who are determined on the basis of their ordinal beliefs and the voting rule. This ordinality means that a candidate’s success cannot easily be expressed in terms of the moments of $p_{0i}$. As cases in which higher mean priors help a candidate are obvious, we present examples of three less obvious effects.

First, higher mean priors may harm a candidate through the inverse enthusiasm effect mentioned above:

**Example 2.** Let $N = 5$, $\delta_0 = 1$, $\delta_T = \frac{1}{2}$ and $V < L$. Then $j = 5$ and $k = 3$. Let $1 > p_{01} - 5\varepsilon = p_{02} - 4\varepsilon = p_{03} - 2\varepsilon = p_{04} - \varepsilon = p_{05} > \bar{p}$, where $\varepsilon > 0$, $p_{03} \in (\bar{p} - \varepsilon, \bar{p})$ and $p_{04} < \bar{p}$. The candidate is hired unanimously on a probationary basis. Now increase the mean prior while holding variance constant by adding $\varepsilon$ to $p_{03}$, so that $p_{03} > \bar{p}$. The tenure weather-vane’s increased enthusiasm means that hiring would be permanent. The two more skeptical committee members block the hire.
Next consider an example in which increased variance hurts hiring possibilities:

**Example 3.** Let \( N = 5, \delta_0 = \frac{1}{2}, \delta_T = \frac{2}{3} \) and \( V < L \). Then \( j = 3 \) and \( k = 4 \). Let \( p_{01} = p_{02} = p_{03} = \hat{p} + \varepsilon < 1 \) and \( \tilde{p} < p_{04} = p_{05} = \hat{p} - \varepsilon < \tilde{p} \) where \( \varepsilon > 0 \). The candidate is hired with a sample variance of \( \frac{30}{25}\varepsilon^2 \). Now increase the most optimistic beliefs to \( p_{01} = p_{02} = \hat{p} + 2\varepsilon \), but reduce the hiring weather-vane’s to \( p_{03} = p_{04} = p_{05} = \hat{p} - \varepsilon \): the mean prior remains unchanged, the variance more than doubles, and the candidate is not hired.

Finally consider the opposite:

**Example 4.** Consider the committee and voting rule of Example 3 and let \( V < L \). Now let \( p_{01} = p_{02} = \hat{p} + \varepsilon < 1 \) and \( \tilde{p} < p_{03} = p_{04} = p_{05} = \hat{p} - \varepsilon < \tilde{p} \), where again \( \varepsilon > 0 \). The priors’ variance is as in the initial case considered in Example 3, but the candidate is not hired. Now let the hiring weather-vane join the optimists, so that \( p_{03} = \hat{p} + \varepsilon \), but make the pessimists gloomier, so that \( p_{04} = p_{05} = \hat{p} - 2\varepsilon \): the sample mean prior remains the same, the variance again more than doubles, but the candidate is hired.

Thus, the role of variance can be reduced to how it influences the weather-vane: losing the hiring weather-vane loses the candidate the job; winning the hiring weather-vane wins the job. Recasting analysis in terms of the two weather-vanes allows us to restate our discussion of Lemmata 4 and 5 in terms of them:

**Lemma 6.** When \( V \geq L, j \geq k \) implies that there do not exists priors, \( p_0 \), that allow probationary hiring. When \( V \geq L \) and \( j \leq k \), and when \( V \leq L \), such priors exist.

This follows from inspection of Figure 3. Thus, if a voting rule that sets the candidate a rising threshold over time \( (j > k) \) generates a high value, \( V \), it does so by discarding the possibility of probationary hire. In other words, it does so by discarding the possibility of buying options on candidates. This suggests that \( V \) in this region should be independent of \( \sigma \). For \( j < k \) and \( V > L(\beta, \sigma, T) \), on the other hand, the value function should depend on \( \sigma \). These observations are confirmed in our explicit computations with of \( N = 1 \) and \( N = 2 \), below. The phenomenon whereby a committee chooses not to use informative signals is further explored in our discussion of optimal voting rules (Section 5.5).
We conclude by revisiting our initial discussion of degeneracy, based on the decision tree in Figure 1, in the light of Figures 2 and 3. All of the conclusions presented initially can now be stated in terms of threshold beliefs, $V$ and $L$.

Setting $\sigma = \frac{1}{2}$ implies $p = \bar{p} = V > L = -\infty$. Thus, the possibility of probationary hiring disappears. Setting $T = \infty$ sets $\hat{p}$ and $\bar{p}$ to infinity as well and $L < 0$. Thus, the committee always rejects candidates.

Finally, when $\beta = 1$ so does $L$, so that $L > V$. Thus, probationary hiring is retained. As $\sigma \to 1, p \to 0, \bar{p} \to 1$ and $\hat{p} \to 0$. In the limit, all candidates are probationarily hired.

Up to this point, $V$ has been treated as a parameter. In fact, $V$ depends on the committee’s voting behaviour. This presents a fixed point problem: the cut points $\hat{p}, \bar{p}, \underline{p}$ and $\bar{p}$ depend on $V$ which, in turn, depends on the cut points. The next two sections address this problem. To build intuition, we first address it in the context of a single committee member.

4 A single committee member: $N = 1$

When $N = 1$, an expression for $V$ may be constructed from the diagonal in Figure 3. Depending on the inequalities in equation 8, this contains either two or three relevant intervals. We consider the simpler case first.

When the inequalities in equation 8 do not hold, $V$ is bounded below. Thus, as displayed in the right panel of Figure 3, there are only two relevant intervals: probationary hire is excluded.

We assume, for tractability’s sake, that the committee member imagines her priors over the as yet unknown next candidate, $p_0 \in [0, 1]$, to be drawn from the uniform distribution. Therefore,

$$V = \int_{\bar{p}}^{\hat{p}} \beta V dp_0 + \int_{\hat{p}}^{1} \beta T p_0 dp_0 \Rightarrow V = \beta T - 2 \left[ 1 \pm \sqrt{1 - \beta^2} \right] \geq L (\beta, \sigma, T).$$

The larger of these solutions may be eliminated by the following lemma, which applies more generally to $N > 1$: 

**Lemma 7.** When $\beta < 1, \hat{p} > 1 \Rightarrow V = 0$.

**Proof.** By Lemma 2, $\hat{p} > 1 \Rightarrow \hat{p} > \bar{p}$. This, by inequalities 8, in turn implies that $\hat{p} \geq \hat{p} > 1$. Thus, the committee never hires a candidate. As it is impatient, it therefore expects no return. \[\square\]
Corollary 1. \( V > \beta^{T-1} \) is inadmissible.

Proof. \( V > \beta^{T-1} \Rightarrow \tilde{p} > 1 \). By Lemma 7, this implies \( V = 0 < \beta^{T-1} \), a contradiction.

As predicted by Lemma 6, the admissible solution to equation 10 is a function of \( \beta \) and \( T \) alone: when probationary hire is discarded, so is the value function’s dependence on \( \sigma \).

Now consider the case in which the inequalities in expression 8 hold, bounding \( V \) above. This involves all three intervals. Thus,

\[
V = \hat{p} \beta V + \int_{\hat{p}}^{\bar{p}} \left\{ [(1 - \sigma) p_0 + \sigma (1 - p_0)] V + \sigma p_0 \right\} \beta^T dp_0 + \int_{\hat{p}}^{1} \beta^T p_0 dp_0 \leq L.
\]

(11)

Integration reveals a quintic in \( V \), whose coefficients are themselves polynomials in \( \beta, \sigma \) and \( T \), linearly independent and generically non-zero. In general, quintics and higher order polynomials with rational coefficients alone are not solvable in radicals and hence a fortiori not solvable for quintics with real coefficients such as the above.\(^{10}\) Although formulae for the roots of polynomials with rational coefficients involving multivariate hypergeometric functions exist, we do not know of generalizations of such methods to the case of polynomials with real coefficients. In any case, numerical methods accurately approximate the behaviour of polynomials like \( V \); thus, we see no advantage to using hypergeometric methods even in the case of rational coefficients (q.v. Stewart, 1989, p.135). We therefore numerically project \( V \) onto the \((\beta, \sigma)\) plane for \( T = 2 \).\(^{11}\) We focus on the \( T = 2 \) case from now on.

The results are combined with those from equation 10 and displayed in Figure 4. Above the bisector (the \( V = L \) curve at the intersection of the \( V \) and \( L \) surfaces) \( V < L \), producing equation 11; below it, \( V > L \), producing equation 10 - the region without probationary hire. As \( \beta \) and \( \sigma \) increase, the costs of probationary hire decrease: mistakes are less costly, probationary hire.

\(^{10}\)Some quintics with rational coefficients define solvable Galois groups and are therefore solvable in radicals. However, such methods do not apply to polynomials with real coefficients, much less the large class of those generated here.

\(^{11}\)The Maple code used is available at www.economics.bham.ac.uk/rowat/research/ARZ-maple.zip.

\(^{12}\)The results for \( T > 2 \) are as expected: the contours become compressed toward \( \beta = 1 \).
periods more informative. Thus, the region in which the committee discards probationary hire shrinks.

The other statics are also appealing. Value strictly increases in $\beta$. It increases strictly in $\sigma$ when equation 11 holds, but is otherwise insensitive to $\sigma$, as noted above. Thus, higher values are attainable when $V < L$ than when $V > L$: informative signals are valuable. In larger committees, we shall see that this is not always so.

5 Committees of two: $N = 2$

Now consider $N = 2$, the simplest case in which voting rules may be compared. The value of the game now generally depends on both committee members’ priors, $(p_{01}, p_{02})$. This implies two dimensional integration. We
therefore generalise the assumption in the previous section so that $p_0 \sim UID[0,1]^2$. As committee members 1 and 2 can only be identified as such once their priors have been realised, refer to them as $a$ and $b$ ex ante.

This generates four separate cases to consider, corresponding to the possible combinations of weather-vanes, $(j,k)$. Without loss of generality, as committee members are identical before their priors are realised, we concentrate on the payoffs expected by $b$. To indicate that value varies by voting rule, let $V_{jk}$ be the value function under weather-vanes $j$ and $k$.

5.1 Double (weak) majority: $(j,k) = (1,1)$

The case of $N=2$ and $(j,k) = (1,1)$ is that of (weak) majority rule at both stages. This corresponds to any voting rule, $(\delta_0, \delta_T)$, in which $\delta_0, \delta_T \in (0, \frac{1}{2})$. The more optimistic committee member is the weather-vane at both stages.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.png}
\caption{Payoffs expected by member $b$ when $N=2, (j,k) = (1,1)$}
\end{figure}

Again, first consider the case of $V_{11} \geq L(\beta, \sigma, T)$. The payoffs expected by committee member $b$ are displayed in the right panel of Figure 5. Below its diagonal, $a$ is the more optimistic member, so that $p_{01} = p_{0a}$; above the diagonal, the roles are reversed and $p_{01} = p_{0b}$.

In the lower inner triangle, weather-vane $a$ rejects the candidate, so that $b$ expects a payoff of $\beta V_{11}$. In the lower quadrilateral, weather-vane $a$ hires and tenures the candidate; the payoff expected by $b$ now depends on her prior. The situation is symmetric in the upper triangle. In this case, while $b$ still expects a payoff of $\beta^T p_{0b}$ if the candidate is tenured, this is higher than it would be were $a$ the weather-vane.
Formally,
\[
V_{11} = \hat{p}^2 \beta V_{11} + \int_{\hat{p}}^{1} \int_{\hat{p}}^{0} \beta^T p_{0b} d p_{0a} d p_{0a} + \int_{0}^{1} \int_{\text{max} \{ \hat{p}, p_{0b} \}}^{1} \beta^T p_{0b} d p_{0b} d p_{0a}
\]
\[
= \hat{p}^2 \beta V_{11} + \frac{1}{2} \beta^T (1 - \hat{p}^3) \geq L(\beta, \sigma, T).
\]

We may also write this as the cubic
\[
V_{11} = \hat{p}^2 \beta V_{11} + \frac{1}{2} \beta^T (1 - \hat{p}^3) \geq L(\beta, \sigma, T).
\]

5.2 Double unanimity: \((j, k) = (2, 2)\)

The expected payoffs when \(j = k = 2\) and \(V_{22} > L(\beta, \sigma, T)\) are depicted in Figure 6. Its space is divided into the same regions as those in Figure 5, and by the same cut point, \(\hat{p}\). The difference between the two is that the boundaries in Figure 5 are demarcated by the beliefs of the more optimistic agent; the present ones are demarcated by the less optimistic agent. Thus,
\[
V_{22} = [1 - (1 - \hat{p})^2] \beta V_{22} + \frac{1}{2} (1 + \hat{p}) (1 - \hat{p}) \beta^T \geq L(\beta, \sigma, T).
\]
Figure 6: Payoffs expected by member $b$ when $N = 2, (j, k) = (2, 2), V_{22} > L(\beta, \sigma, T)$

Again, this is $\sigma$ independent.

When $V_{22} < L(\beta, \sigma, T)$ the map of expected payoffs resembles that in the left panel of Figure 5, but with the boundaries demarcated by the beliefs of the less optimistic agent. Thus:

$$V_{22} = [1 - (1 - \hat{p})^2] \beta V_{22} + \frac{1}{2} (1 + \hat{p}) (1 - \hat{p})^2 \beta T + \beta^T \sigma V_{22} [2 - (\bar{p} + \hat{p})] (\bar{p} - \hat{p})$$

$$+ \frac{1}{2} [(1 - 2\sigma) V_{22} + \sigma] \beta T [(\bar{p} + \hat{p}^2 - \bar{p}^3) - (\hat{p} + \bar{p}^2 - \hat{p}^3)] \leq L(\beta, \sigma, T).$$

Again, the resulting contours are qualitatively similar to those found in the $j = k = 1$ case.
5.3 Falling threshold: \((j, k) = (2, 1)\)

When \((j, k) = (2, 1)\) and \(V_{21} > L(\beta, \sigma, T)\), the map of expected payoffs is the same as that in Figure 6. The value function over this range is therefore defined as in equation 12.

When \(V_{21} < L(\beta, \sigma, T)\), expected payoffs are as displayed in Figure 7; they are symmetric about \(p_{0a} = p_{0b}\). Now, the central square corresponds to probationary hiring; the object to its upper right corresponds to permanently hiring; and the remaining one to rejection at the initial stage.
In this case,

\[ V_{21} = \left[ \hat{p}^2 + 2(\bar{p} - \hat{p})\hat{p} + 2(1 - \bar{p})\hat{p} \right] \beta V_{21} \]

\[ + \beta^T \left\{ \sigma V_{12} (\bar{p} - \hat{p})^2 + \frac{1}{2} [(1 - 2\sigma) V_{12} + \sigma] (\bar{p} - \hat{p})^2 (\bar{p} + \hat{p}) \right\} \]

\[ + \frac{1}{2} \beta^T (1 - \bar{p}) [(1 + \bar{p})(1 - \bar{p}) + (\hat{p}^2 - \bar{p}^2)] \leq L(\beta, \sigma, T). \]

As the contours of \(V\) are qualitatively similar to those already derived, they are not displayed.

5.4 Rising threshold: \((j, k) = (1, 2)\)

![Figure 8: Payoffs expected by member b when \(N = 2, (j, k) = (1, 2)\)](image)

When \((j, k) = (1, 2)\), expected payoffs are as depicted in Figure 8; the left panel corresponds to \(V_{12} \leq L(\beta, \sigma, T)\) and the right to its complement.

Thus,

\[ V_{12} = \left[ \hat{p}^2 + 2(\bar{p} - \hat{p})\hat{p} + 2(1 - \bar{p})\hat{p} \right] \beta V_{12} \]

\[ + \beta^T \left\{ \frac{1}{2} [(1 - 2\sigma) V_{12} + \sigma] (1 - \hat{p}) (\bar{p} - p) (1 + \hat{p} + \bar{p} + p) + 2\sigma V_{12} (\bar{p} - p) (1 - \hat{p}) \right\} \]

\[ + \frac{1}{2} \beta^T (1 - \bar{p}) [(1 + \bar{p})(1 - \bar{p}) + \hat{p}^2 - \bar{p}^2] \geq L(\beta, \sigma, T); \]

25
and

\[ V_{12} = \left[ \hat{\rho}^2 + 2 (1 - \bar{p}) p \right] \beta V_{12} + \frac{1}{2} \beta^T (1 + \bar{p}) (1 - \bar{p})^2 \]
\[ + \frac{1}{2} \beta^T [(1 - 2\sigma) V_{12} + \sigma] \left[ \bar{p} (1 + \bar{p} - \hat{p}^2) - (p + \bar{p}^2) + \hat{p} (p^2 + pp - \hat{p}^2) \right] \]
\[ + \beta^T \sigma V_{12} \left[ (1 - \hat{p}) (\bar{p} + \hat{p} - 2p) + (1 - \bar{p}) (\bar{p} - \hat{p}) \right] \leq L(\beta, \sigma, T). \]

Thus, this is the first in which the value function contains a term in \( \sigma \) when \( V > L \).

Figure 9: Value function contours \( V_{12} = \{ \frac{1}{10}, \ldots, \frac{9}{10} \} \) when \( N = 2, j = 1, k = 2 \)

The contours of \( V_{12} \) are displayed in Figure 9. The most significant feature of this diagram is the behaviour of the contours when \( V_{12} \geq L(\beta, \sigma, T) \). In the \( N = 1 \) case, and under the other voting rules, value was insensitive
to the signal quality: as predicted by Lemma 6, a failure of inequality 8 corresponded to a rejection of probationary hiring in these cases.

In contrast, not only is the signal taken into account when \( V_{12} \geq L(\beta, \sigma, T) \), but decreased signal quality improves the value of the problem to the committee. The effect is very weak: that the relevant contours in Figure 9 curve backward near \( \sigma = \frac{1}{2} \) only becomes obvious under magnification. Technically, as \( \sigma \to \frac{1}{2}, \bar{p} \to \tilde{\bar{p}} \), causing probationary hiring to disappear. Intuitively, as signal quality decreases, an optimistic hiring weather-vane ceases to hope that a good performance by a marginal candidate will convince the skeptics with his probationary performance.

### 5.5 The optimal voting rule

The optimal voting rule for \( N = 2 \) is determined by pairwise comparison of the voting rules analysed above. Each pair of value functions is implicitly plotted in \((\beta, \sigma, V)\) space. This is implemented by implicitly plotting the value surfaces on a \((100, 100, 100)\) grid. \(^{13}\) Projection onto the \((\beta, \sigma)\) plane then allows identification of the function with the greater value for any \((\beta, \sigma)\) combination.

Comparison reveals \((1, 2)\) to at least weakly dominate \((2, 2)\) for all \((\beta, \sigma)\). Similarly, \((2, 1)\) is at least weakly dominated by \((1, 2)\) for all \((\beta, \sigma)\). Before discarding the dominated voting rules, we suggest intuitions for these results.

Consider first the dominance of \((2, 2)\) by \((1, 2)\). The two rules agree at the tenure meeting: unanimity is required. The first rule is, however, more restrictive at the hiring meeting. When a committee is perceptive, this more restrictive rule gives it fewer opportunities to use its perception: fewer candidates are hired. When a committee is imperceptive, its tenure decision largely reflects its priors; a good rule should therefore get candidates to that decision point quickly - which \((2, 2)\) does less well than does \((1, 2)\).

As to the dominance of \((2, 1)\) by \((1, 2)\) for all \((\beta, \sigma)\), neither rule is more stringent than the other in the sense of requiring the candidate to garner more votes over the two periods. However, the dominated rule imposes its stricter vote at the outset, when priors have not yet been informed by probationary performance. The preferred rule allows its stricter vote to condition on this performance.

\(^{13}\)Maple 9.5 code available at www.economics.bham.ac.uk/rowat/research/ARZ-maple.zip.
The only remaining pairwise comparison is between (1,1) and (1,2). Figure 10 displays the result, which may be partitioned into four regions:\textsuperscript{14}

I in this region, $L > V_{12} > V_{11}$. This corresponds to the left panel in Figure 8. Thus, it allows probationary hiring.

II in this region, $V_{12} > V_{11}$ and $V_{12} > L$. This corresponds to the right panel in Figure 8. Probationary hiring is again possible, and is subject to the perverse effect of decreased signal quality noted above.

\textsuperscript{14}As some of the plotted values reflect poor numerical approximation, we ignore two features. First, the effects along $\beta = 0$ and in the $\beta = \sigma = 1$ corner. Second, the ragged $V = L$ boundary. We expect that these reflect difficulties associated with poor conditioning.
III in this region, $L > V_{11} > V_{12}$. This corresponds to the left panel in Figure 5. It also allows probationary hiring.

IV finally, $V_{11} > V_{12}$ and $V_{11} > L$. This corresponds to the right panel in Figure 5. It does not allow probationary hiring.

In regions I and II, the optimal rule is to allow the optimists to ‘buy the option’ on a candidate, but the skeptics to choose whether or not to exercise it. For a patient committee, the mistake to avoid is not a delayed hire, but a bad hire. With an informative signal, even a skeptic will be convinced by a good performance. Signal quality deterioration into region II reduces the use of probationary hiring.

In regions III and IV, the double (weak) majority rule is preferred. Intuitively, skepticism about candidates when a committee is impatient and imperceptive is very costly, offering the possibility of repeated returns to the pool. Thus, optimists are made pivotal in both meetings, quickly hiring and tenuring. When the committee is particularly impatient or imperceptive - region IV - it even rejects probationary hiring, reducing its problem to a one stage problem.

Thus, as a committee becomes more impatient or imperceptive, its optimal rule progressively speeds the expected time to tenure a candidate: as patience or perceptiveness decrease, the majority requirement at tenure is eased, and then the committee gives up the ability to engage in probationary hiring.

Similar pairwise comparisons also show that the two optimal voting rules for $N = 2$ are dominated by the $N = 1$ committee. This comparison is the wrong one, however: the problem here is not one of aggregating information about a candidate, but one of social choice. A single committee member does not have less relevant information, but does not need to compromise.

Thus, the more appropriate comparison is between the optimal voting rules presented above, and another one in which $j = k \forall (\beta, \sigma)$: one committee member making the decisions, but on behalf of both. We have shown that $(j, k) = (1, 1)$ is optimal for some values of $(\beta, \sigma)$, but not others.

6 Discussion

The options interpretation suggested above is outlined more explicitly by reference to Cox, Ross, and Rubinstein (1979) (CRR). That presented a
discrete-time model for valuing European options managed by risk neutral unitary investors.

In CRR, the current value of an asset on which a call is written either moves up or down to \( p^\ast \) by an exogenously determined amount in the next period. In our case, priors - expected values - are exogenous, and the extent of their updating to posteriors determined by the variance parameter, \( \sigma \). Their time to maturity is our \( T \). Their valuation formula does not depend on “the probability that the stock price will rise or fall”; our \( V_{jk} \) is independent of \( p_0 \) and the realisation of \( \theta \).

There are, however, two principle differences between our model and the standard model, as exemplified by CRR. First is the way in which it is closed. In CRR, the interest rate - the return to the market portfolio - is exogenous. With \( K \), the option’s strike price, the underlying asset’s prices, and a no-arbitrage condition, this allows calculation of the cost of the call option.

In our case, \( V \), the expected return to the ‘market’, is endogenous. Endogenising this requires giving something up to avoid over-determining the system. This explains the absence of the no-arbitrage assumption in our model: the market for candidates is not perfectly competitive. We could free the return to a ‘good’ candidate, currently set at 1, if we wished to both use a no-arbitrage condition and not over-determine the system.

The second difference is that our option is managed by a committee. Thus, ‘rational exercise policy’ gives way to a ‘strategic exercise policy’ in our environment. In CRR, the option’s exercise depends on whether the option is in or out of the money:

\[
\max \{0, p^\ast - K\}.
\]  
(13)

Versions of this appear in our more complicated environment. First, the committee’s decision to exercise the option on a probationary candidate, granting him tenure, is governed by

\[
\max \{0, p_{Tk} - V_{jk}\}.
\]

The condition on when to buy the option - hire a candidate - is more complicated:

\[
\max \{0, p_{0j} - \min \{\hat{p}, \tilde{p}\}\}.
\]  
(14)

Nevertheless, its interpretation is the same: when expression 14 is ‘in the money’, a candidate is hired. Which of the terms in the min operator is
smaller depends, in turn, on
\[ \text{max} \{0, V_{jk} - L\} . \]  

(15)

As a function of exogenous parameters, but not of the voting rule, \( L \) seems to parallel \( K \) in CRR. This offers an interpretation of the upper bound of unity on \( L \): a strike price above the underlying asset’s maximum price does not make sense.

Remembering, from expression 8, that \( V_{jk} > L \iff \bar{p} > \tilde{p} > \hat{p} \), we may obtain an options interpretation from expression 15 as well. The committee’s decision when the expression is ‘in the money’ depends on the voting rule. In general, it may be interpreted as an option on probationary hiring: hiring with the possibility of not granting tenure.

Now consider the assumption of Bayesian committee members. A weaker form of updating suffices for the results: committee members’ only need to be able to predict their future relative assessments of a candidate and form expectations over the weather-vane’s posteriors. This departure from rationality, however, might call into question the otherwise rational play. A second weakening is also possible: members could also receive private signals, perhaps informing the non-commonly held elements of their values. In this case, analysis would be similar, but committee members would be forced to vote on the basis of ‘expected weather-vanes’. This complicates analysis without clearly adding insight.

Tighter empirical tests may also be posed. Variance in \( \delta \) and \( T \) across ranked institutions would allow estimation of the share of their ranking that can be explained by the discrepancy between institutions’ voting rules and the optimal ones.\(^\text{15}\)

A second empirical observation may be made. The probability with which a department grants tenure may be empirically observed, as may its voting rule. Knowing these may allow inference about the committee’s \( \beta \) and \( \sigma \) parameters on the basis of Figure 3. The promotion rate to tenure in law schools, for example, is substantially higher than that of other departments in major research universities (compare Siow (1998) with Chused (1988)). Thus, law schools engage in less probationary hiring. Assuming that discount factors do not vary significantly by discipline, this difference could be accounted for if \( \sigma \) was lower in law schools: \( \sigma \rightarrow \frac{1}{2} \) causes the zone of probationary hiring disappears.

\(^{15}\)Empirical testing may not be straightforward: a higher ranked institution will attract more highly qualified candidates, and possibly have more perceptive committee members.
Finally, we turn to possible extensions to the model. Without altering its analytical structure at all, that of most obvious interest is to extend the analysis of optimal rules to committees with more than two members. The techniques applied above can be extended to $N > 2$ on a case-by-case basis, but it would be preferable to have a technique that allowed assessment for a general $N$.

A second, more involved extension would be to make $T$ a meaningful variable. At present, $T = 2$ is optimal for all committees: the signal that they receive on a probationary candidate is no worse over a short probationary period than it is over a long one; further, larger values of $T$ delay the committee’s ability to return to the pool if a probationary hire emits a bad signal. A more satisfactory treatment might address two issues.

First, longer probationary periods could allow the committee to receive more realisations of $\theta$; alternatively, the accuracy of $\sigma$ could increase in $T$. Second, payoffs could be allowed to accrue over the probationary period, rather than just upon granting tenure. This approach alters the interpretation of the $p_i$: once payoffs are actually observed, they are no longer expected payoffs.

A third extension would be to consider an American option version of the model, wherein the committee has the right to ‘exercise’ at every point up to $T$. Setting $T$ to a high value would allow analysis of jobs subject to regular review.

A fourth, deeper extension, would specify a richer environment within which the candidate presented himself to the committee. A candidate is a rival good, and thus may face bids from other institutions. Additionally, our candidates are inert, not even, for example, choosing effort levels. This abstraction may not have important analytical consequences as the signal may represent a reduced form of the effort choice. Perhaps more importantly, a candidate might seek to alter the distribution of committee members’ priors.

Finally, two stage decision processes may be seen as forms of incomplete contracts. While these processes are observed in practice, no motives for them from, for example, Tirole (1999), have been presented here. Indeed, as the model is structured, they seem precluded: if agents know that they all are updating their priors on the basis of a common signal, they could write a contract at the probationary decision offering permanent employment, contingent upon satisfying performance targets. Rent seeking by committee members might be one reason for retaining discretion, but that is not modelled.
References


Sayantan Ghosal and Ben Lockwood. Information aggregation, costly voting and common values. mimeo, January 2003.


