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Dynamic Prudential Regulation

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Abstract

This paper investigates regulations for banks that covered by deposit insurance in a dynamic setting where bankruptcy entails social costs. Regulatory policy operates through rules governing the bank's capital structure and asset allocation that may be adjusted each period. Throughout, the regulator must take into account that the bank is better informed about the inherent risks of its assets (adverse selection) and may forgo unobservable and costly actions to improve asset quality (moral hazard). Under the optimal regulatory policy under banks face risk-adjusted capital requirements but also hard-caps on size and leverage. In addition, the optimal policy counteracts procyclical bank behaviour through the use of capital buffers. Overall, the optimal policy broadly supports major elements of the proposed Basel III regulatory framework.

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1 Introduction

This paper determines the optimal dynamic regulatory policy for insured deposit-taking banks that persistently have private information regarding credit-market conditions and their risk-mitigation activities, and where bankruptcy generates large externalities. Imperfect information results in mis-priced deposit insurance giving banks incentives to maximize profits by increasing the risk of their loan portfolios to maximize the value of the deposit insurance. Specifically the paper examines how, under the optimal regulatory policy, banks choose to adjust their capital and asset structures in response to changing credit-market conditions. Furthermore, recent proposals for changes to the current international banking regulatory framework (Basel II), that are applicable in the context of the model, are evaluated qualitatively against the optimal regulatory policy.

The main contribution of the paper is the characterization of the optimal regulatory policy. A key finding is that under the optimal stationary regulatory policy, banks face restrictions on loan volume, portfolio quality as well as minimum capital requirements. However, banks with loan portfolios above a threshold quality level face a strict regulatory regime while banks with lower quality portfolios are afforded flexibility. Specifically, the loan limit imposed on higher quality banks does not change with further improvements in the quality of their portfolios, nor is a reduction in capital permitted. By contrast, for lower quality banks, an increase in quality raises the allowable loan limit while raising the capital requirement somewhat. Overall, under the optimal policy, leverage is (weakly) increasing in loan quality[1], banks face risk-based capital adequacy requirements and must adhere to limits on size (in terms of loan volume), and leverage.

Qualitatively, the optimal regulatory policy supports elements in the Basel III accord. While, the need for quality-adjusted capital requirements are a standard policy tool, controls on leverage are broadly consistent with Basel III proposals that seek to introduce a leverage ratio requirement. Moreover, the size restrictions in the optimal policy support the broader macro-prudential goal in Basel III of "leaning against excessive credit growth."[2] In addition, the adjustment behaviour of banks under the optimal regulatory policy calls for decline in loan volume but also for declines in capital requirements following an adverse aggregate shock. This is consistent with the Basel III proposals calling for banks to maintain capital buffers than can be drawn down following adverse shocks.

One of the first theoretical models studying the inter-temporal effects of capital constraints is given by Blum (1999). In a discrete time model he studies the incentives for asset substitution coming from the reduction in expected profits imposed by the requirement. In order to raise the amount of equity in the following period, a bank may find it optimal to increase risk today, in which case strengthening the requirement would have the opposite effects for which it was designed, to curb bank risk taking. This paper similarly concludes that simple capital constraints may be ineffective in a dynamic setting but does not characterize the optimal regulatory policy.

[1]Leverage is (weakly) pro-cyclical in the terminology of Adrian and Shin (2010).

[2]http://www.bis.org/speeches/sp101011.htm.

A number of papers such as Caleb and Robb (1999), Dangl and Lehar (2004), Shim (2006) and Zhu (2007) study dynamic models of bank portfolio choice comparing risk-based capital requirements to risk-insensitive ones in addressing the moral hazard problem resulting from deposit-insurance. These papers broadly favour imposing risk-sensitive requirements however do not address the informational constraints faced by regulator. For instance, in a continuous time setting Dangl and Lehar (2004) study a model in which the regulator sets capital requirements as well as an audit policy. Well-adjusted risk-based capital requirements are optimal and benefit both depositors and equity holders, as they allow commitment to portfolio choices that increase the bank's charter value. In comparison, this paper takes a broader view of the supervisory role encompassing both capital requirements and portfolio restrictions nevertheless emphasizing informational constraints as a barrier to implementing policy. As a result, the regulatory regime differs markedly for higher quality banks for whom the optimal regulatory policy is inflexible and not responsive to asset quality.

With respect to the pro-cyclicality of regulatory policy, an early contribution is Blum and Hellwig (1995).The authors show that rigid capital adequacy regulation for banks may reinforce macroeconomic fluctuations. Repullo and Suarez (2010) defend the pro-cyclical nature of policy while offering important modifications that may help mitigate the severity of credit crunches following an aggregate shock. In line with their arguments this paper finds that bank balance sheet adjustments under the optimal policy are pro-cyclical but in addition, using the characterization of the optimal policy, the optimal adjustments are also determined.

2 Model

Banks emerge to improve upon outcomes in environments plagued by frictions. For instance, banks reduce the informational asymmetries between borrowers and lenders by screening investment projects and monitoring borrowers. By providing demandable deposits, they also insure consumers against idiosyncratic liquidity risk. Consequently, bank failures generate negative externalities by exposing bank customers to the underlying frictions. As a result, governments regulate and monitor the activities of banks to reduce the possibility of bank default but nevertheless also explicitly guarantee deposits.

To capture the salient feature of this scenario, consider a risk-neutral bank that raises funds from depositors and outside shareholders to allocate across risky loans and reserves.[3] Through deposits, the bank provides liquidity services and through loans, the bank funds entrepreneurial projects. The bank has private information regarding the credit-risk of its loan portfolio and can also raise portfolio quality through costly effort. When the bank is unable to meet its obligations, as is the case when returns are sufficiently low, the bank defaults. A defaulting bank imposes social costs on society that are proportional to bank losses but are not internalized by the bank.

There is a regulator that provides deposit insurance and maximizes social welfare.[4]

[3]The stage game is an extension of Giammarino, Lewis, and Sappington (1993).

[4]More generally, guarantees for debt or debt-like instruments issued a financial intermediary.

The presence of the regulator stems from the existence of externalities imposed on society due to bank failure. The regulator is unable to observe the effort by the bank manager to improve loan quality (moral hazard) nor the private information regarding credit-risk on the loans held by bank (adverse selection). As a result, the relationship between the bank and the regulator is plagued with a moral hazard problem that interacts with the adverse selection problem as in Laffont and Tirole (1986). These informational problems result in mis-priced deposit insurance giving banks incentives to maximize profits by increasing the risk of their loan portfolios to maximize the value of the deposit insurance. The regulator imposes restrictions on both the capital structure and asset structure of the bank to influence bank behaviour.

More specifically, there are four types of actors: depositors, firms, banks and a regulator. These actors are described in detail below.

2.1 Investors / Depositors

There is a large number of risk-averse investors or depositors. They are willing to invest in a risky security for return $r^e > r^f$ where r^f is the return on a risk-less asset, normalized to 1.

2.2 Firms/Investments

The bank has sole access to a number of investment opportunities (or projects) and is the sole source of outside financing in this economy. The projects vary in risk and size. The average return on a portfolio of loans in the amount L is rL with $r \in [0, \infty]$. Due to its monopoly position in the lending market, the bank captures all the surplus from loans. Therefore, bank profits capture total welfare from loan-making activities. This assumption is not critical but will simplify the regulator's welfare maximization problem because including the bank's profits in the welfare function will be sufficient to capture total welfare. Therefore, the bank's gross profit from issuing an amount of loans L is just rL.

2.3 Bank

The bank holds an outstanding share of equity at the start of the period. The bank raises deposits D and (outside) equity E by selling a fraction z of its equity share. It uses these funds to invest L in risky loans while holding R in reserves. Thus the bank equates assets and liabilities ex-ante as follows:

$$D + E = L + R \tag{1}$$

The average gross rate of return, r, on the bank's loan portfolio is a random variable with smooth distribution function $G(r|q)$ where q is the quality of the bank's loan portfolio. Higher levels of q imply more profits in a first-order stochastic dominance sense. Formally, G has positive support on $[0, \infty]$ and that an increase in q increases returns in a FOSD sense:

Assumption 1. *$G_q(r|q) \leq 0 \; \forall r \in [\underline{r}, \overline{r}]$ with strict inequality for some r.*

The quality of the loan portfolio, q, is comprised of the "innate" quality θ and effort e so that $q = \theta + e$. The innate quality θ can be viewed as factors beyond the control of the bank such as the credit-worthiness of the borrowers or the state of the financial markets. The bank privately observes this innate quality while the regulator believes it to be drawn from a distribution $F(\theta)$ with support on $\Theta \equiv [\underline{\theta}, \overline{\theta}]$. On the other hand, effort e, can be viewed as all the factors *within* the control of the bank but that are costly to implement such as risk-management and loan monitoring. The cost of effort is given by $C(e)$ where $C', C'' > 0$. In addition, the processing of loans entails additional administrative costs $\gamma(L)$ as a function of loans L where $\gamma'(\cdot), \gamma''(\cdot) > 0$.

The bank's revenues after paying depositors for a realization of a return r on loans are:

$$rL - \gamma(L) + R - D$$

The cutoff for returns, r^b, below which the banks defaults is given by:

$$r^b L - \gamma(L) + R - D = 0$$

so that

$$r^b = \frac{D + \gamma(L) - R}{L} \tag{2}$$

At the end of the period, the bank pay claims to depositors and shareholders. If profits exceed D, then depositors are paid in full and the rest of the profits are distributed among shareholders. If profits are insufficient to pay depositors, the regulator pays the depositors. As a result, the return to (outside) equity E is equated with the fraction of expected profits entitled to shareholders:

$$r^e(E)E = z \int_{r^b}^{\overline{r}} [rL - \gamma(L) + R - D] dG(r|q) \tag{3}$$

The return to equity here is a decreasing function of the amount of equity E issued (i.e. $dr^e/dE < 0$) because a larger equity cushion lowers the risk of default.

Expected profits for the bank are:

$$(1 - z) \int_{r^b}^{\overline{r}} [rL - \gamma(L) + R - D] dG(r|q) - C(e)$$

subject to (1) and (2). Following Giammarino, Lewis, and Sappington (1993), assume $D = L$, so that cheap deposits fund loans, and $E = R$ so that equity provides a "cushion" for losses. Using expression (3) , the bank's problem can be written as:

$$\max_{e,L,R} \pi[e, L, R] \equiv \int_{r^b}^{\overline{r}} [(r - 1)L - \gamma(L) + R] dG(r|q) - C(e). \tag{4}$$

2.4 The regulator's problem

The regulator maximizes social welfare while also providing deposit insurance. However, the regulator does not observe the effort nor the innate quality of the loan portfolio and so cannot condition the policies on these variables. However, following the bank's report of its innate loan quality, the regulator determines the loan volume L, reserves R and quality q that the bank must adhere to. Then, the bank raises outside funds and chooses effort e. Note that as $q = \theta + e$, without loss of generality, we can view the bank as choosing q rather than e. After assets are in place, the regulator carries out an inspection. Following the inspection, only if $\{q, L, R\}$ are consistent with regulation, is the bank allowed to continue operations. The underlying assumption is that inspection conveys perfect information about the realized quality of the loan portfolio.

Formally, the regulator offers a policy or contract $\mu(\cdot) = \{q(\cdot), L(\cdot), R(\cdot)\}$ to the bank, where the functions $q(\cdot), L(\cdot)$ and $R(\cdot)$ are all piecewise-C^1 functions. The bank observes the innate loan quality and using a reporting strategy $m(\theta)$ issues a report $\hat{\theta} = m(\theta) \in \Theta$. The policy for the bank when it reports its innate loan quality as $\hat{\theta}$ will be denoted by $\mu(\hat{\theta}) = \{q(\hat{\theta}), L(\hat{\theta}), R(\hat{\theta})\}$. The timing is summarized below:

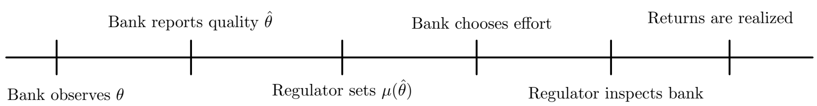

Figure 1: Timing

In equilibrium, the expected cost of bankruptcy to the regulator (i.e. the cost of providing deposit insurance) for a bank with innate quality θ is:

$$S[\mu(\theta)] \equiv (1+b) \int_{\underline{r}}^{r^b} [(r-1)L(\theta) - \gamma(L(\theta)) + R(\theta)] dG(r|q(\theta)).$$

The integral is the expected loss borne by the regulator when bank profits fail to cover the depositor's claims where we have used $D = L$ to simplify the expression. Here the parameter b captures negative externalities from bankruptcy that are proportional to losses.[5]

The expected profits of the bank (and thereby the total surplus from lending) when it has innate quality θ but reports $\hat{\theta}$ is

$$\pi[\mu(\hat{\theta}), \theta] \equiv \int_{r^b}^{\overline{r}} [(r-1)L(\hat{\theta}) - \gamma(L(\hat{\theta})) + R(\hat{\theta})] dG(r|q(\hat{\theta})) - C(q(\hat{\theta}) - \theta) - r^e(E)R(\hat{\theta}). \quad (5)$$

As is clear from this expression, the bank makes profits from loan-making activities and holds reserves to offset losses.

Applying an extended revelation principle[6], we can write the regulator's problem as follows:

$$\max_{\mu(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} \{S[\mu(\theta)] + \pi[\mu(\theta), \theta]\} dF(\theta) \quad (6)$$

[5] As noted in Giammarino, Lewis, and Sappington (1993), alternative specifications of the social costs of default are possible. For example, they could be made proportional to the level of loans issued. These yield qualitatively similar results.

[6] Myerson (1982) provides a Revelation Principle for environments that feature both adverse selection and hidden actions such as the one above.

subject to

$$\pi[\mu(\theta), \theta] \geq 0 \tag{7}$$

$$\pi[\mu(\theta), \theta] \geq \pi[\mu(\hat{\theta}), \theta] \tag{8}$$

for all $\theta, \hat{\theta} \in \Theta$, where $S[\mu(\theta)] + \pi[\mu(\theta), \theta]$ is the combined welfare from loan-making activities and the provision of deposit insurance for a bank with innate quality θ that reports truthfully. The first set of constraints are the limited-liability constraints for the bank. The second set are the incentive compatibility constraints, ensuring that a bank of type θ does indeed prefer to report θ rather than some $\hat{\theta} \neq \theta$.

3 The Optimal Regulation

We first analyze the regulatory problem with complete information in order to establish a pair of welfare benchmarks. Under complete information, the regulator is able to perfectly assess the credit risk associated with the bank's loan portfolio and so is able to play the role of an external ratings agency. A bank that employs an external agency to assess its credit risk is said to be following the standardized approach in assessing such risk. Formally, the standardized policy $\mu^S(\theta)$ maximizes welfare (6) subject to the limited liability constraints (7) without the incentive constraints (8). It must satisfy the first-order conditions for the regulator's problem:

$$\frac{\partial S[\mu^S(\theta)]}{\partial \mu_k} + \frac{\partial \pi[\mu^S(\theta), \theta]}{\partial \mu_k} = 0 \qquad \forall \mu_k = q, L, R. \tag{9}$$

Clearly, $\mu^S(\theta)$ varies directly with the innate quality of the bank's loan portfolio, imposing a risk-based capital structure $(D(\theta), E(\theta))$ and leverage ratio $(L(\theta)/E(\theta))$. Moreover, $\mu^S(\theta)$ achieves the highest welfare possible while leaving no surplus to the bank.

At the other end of the spectrum, the regulator may choose to ignore the innate loan quality of the bank altogether when setting its policy. Such a policy imposes a fixed capital structure, fixed asset portfolio and fixed deposit insurance premium on the bank *regardless* of the bank's asset quality. We label such a policy a full hard-cap policy, denoted μ^{FHC}, as it does not permit flexibility in any policy variable. Formally, it solves the following problem:

$$\max_{\mu} \int_{\underline{\theta}}^{\overline{\theta}} \{S[\mu] + \pi[\mu, \theta]\} dF(\theta) \tag{10}$$

subject to the limited liability constraints $\pi[\mu, \theta] \geq 0$. It is clear that this policy achieves a lower level of welfare than $\mu^S(\theta)$.

3.1 Policies and Welfare Under Private Information

When the bank has private information, the regulator must rely on the bank to provide an overall assessment of its credit-risk. Then, in contrast to the perfect-information case, the ability of the regulator to adjust the regulatory regime in accordance with the private information of the bank is curtailed by the presence of the incentive-constraints. Indeed, this adjustment can be performed only be surrendering a certain amount of surplus in eliciting the information.

To solve the regulator's problem we first replace the global incentive-constraints (8) with appropriate local versions of them. To this end, we view the bank as choosing a report $\hat{\theta}$ to solve $\max_{\hat{\theta}} \pi[\mu(\hat{\theta}), \theta)]$. The necessary condition for truthful reporting to be optimal is then:

$$\sum_k \frac{\partial \pi[\mu(\hat{\theta}), \theta]}{\partial \mu_k} \frac{d\mu_k(\hat{\theta})}{d\hat{\theta}} \bigg|_{\hat{\theta}=\theta} = \sum_k \frac{\partial \pi[\mu(\theta), \theta]}{\partial \mu_k} \frac{d\mu_k(\theta)}{d\theta} = 0 \tag{11}$$

The sufficient condition is equivalent to $\frac{\partial^2 \pi[\mu(\hat{\theta}),\theta)]}{\partial \hat{\theta} \partial \theta} \Big|_{\hat{\theta}=\theta} \geq 0$ or

$$\sum_k \frac{\partial^2 \pi[\mu(\hat{\theta}), \theta]}{\partial \mu_k \partial \theta} \frac{d\mu_k(\hat{\theta})}{d\hat{\theta}} \bigg|_{\hat{\theta}=\theta} = \sum_k \frac{\partial^2 \pi[\mu(\theta), \theta]}{\partial \mu_k \partial \theta} \frac{d\mu_k(\theta)}{d\theta} \geq 0 \tag{12}$$

Under the following weak single-crossing assumption:

Assumption 2.

$$\frac{\partial^2 \pi[\mu(\theta), \theta]}{\partial \mu_k \partial \theta} \geq 0 \tag{13}$$

for all k with strict inequality for at least one k.

the local-incentive constraints (11) are equivalent to the incentive-constraints (8) if and only if the regulatory policy is weakly monotonic: $d\mu_k(\theta)/d\theta \geq 0$ for all k. We also make the following assumption on the hazard rate:

Assumption 3.

$$\frac{d}{d\theta} \left\{ \frac{1 - F(\theta)}{f(\theta)} \right\} \leq 0 \textit{ for all } \theta \in [\underline{\theta}, \overline{\theta}] \tag{14}$$

Standard techniques[7] then ensure that given the assumptions above, the regulator's relaxed problem can be written as

$$\max_{\mu(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \left[S[\mu(\theta)] + \left(\frac{1 - F(\theta)}{f(\theta)} \right) \frac{\partial \pi[\mu(\theta), \theta]}{\partial \theta} \right] dF(r|q(\theta)) \tag{15}$$

subject to incentive-constraints (11) and the limited liability constraint $\pi[\mu(\underline{\theta}), \underline{\theta}] = 0$. The necessary conditions for optimality are:

$$\frac{\partial S[\mu(\theta)]}{\partial \mu_k} + \left(\frac{1 - F(\theta)}{f(\theta)} \right) \frac{\partial^2 \pi[\mu(\theta), \theta]}{\partial \mu_k \partial \theta} = 0 \qquad \forall k \tag{16}$$

The above formulation of the problem shows that the regulator choses a policy that maximizes the *virtual* social surplus which takes into account the costs of providing the

[7] See for example the methodology outlined in Fundenberg and Tirole (1991). The use of this methodology also requires making some assumptions on the third derivatives of the profit function corresponding to A8 in section 7.3 of Fundenberg and Tirole (1991) that we have omitted here for simplicity.

incentives for the bank to truthfully report its credit risk. As the credit risk is assessed by the bank, we refer to the optimal policy in this case as the *Internal Rating-Based or IRB* policy, denoted $\mu^{IRB}(\theta)$. An important comparative static result is that $\mu_k^{IRB}(\theta)$ is strictly increasing in θ for all k so that monotonicity conditions are not binding.

Proposition 1. *Given assumptions 1 - 3, the optimal policy variables are increasing in θ and so is leverage:*

$$\frac{d\mu_k^{IRB}(\theta)}{d\theta} > 0 \text{ for all } k \text{ and for all } \theta \in \Theta$$

and

$$\frac{d[L(\theta)/R(\theta)]}{d\theta} > 0 \text{ for all } \theta \in \Theta$$

Proof. See Appendix B. □

From a welfare standpoint, $\mu^{IRB}(\theta)$ yields an equilibrium with the highest social surplus with private information which is nonetheless smaller than the surplus achieved by $\mu^S(\theta)$.

4 The Dynamic Setting

We now analyze the infinitely repeated version of the stage-game described in the previous section. At each $t \geq 0$, the regulator offers a policy $\mu_t = \{q_t, L_t, R_t\}$. The bank observes private information concerning loan characteristics, $\theta_t \in \Theta$. Then, the bank sends a report $\hat{\theta}_t \in \Theta$ regarding its private information to the regulator resulting in the outcome, $\mu_t(\hat{\theta}_t) = \{q_t(\hat{\theta}_t), L_t(\hat{\theta}_t), R_t(\hat{\theta}_t)\}$. In order to specify strategies for the bank and the regulator we need to describe histories on which these strategies may be conditioned. A history at time t is defined as

$$h_t = (\theta_t, \hat{\theta}_t, q_t, L_t, R_t)$$

The history h_t is just the bank's report together with the prescribed policy of the regulator within period t. The space of all possible period t histories h_t is denoted by H_t. The initial date is $t = 0$.

Definition 1. *The history up to the period t will be denoted by $h^t \equiv \{h_0, h_1, \ldots, h_t\}$.*

The set of all possible histories up through time t will be denoted by $H^t \equiv H_0 \times H_1 \cdots \times H_t$. The regulator does not observe the history h^t because he does not observe the private information θ_t each period. Instead, the regulator only observes

$$s_t \equiv (\hat{\theta}_t, q_t, L_t, R_t) \subset h_t$$

in each period t.

Definition 2. *We will denote the public history up to date t as $s^t \in S_t$ which will be drawn from the set $S^t \equiv S_0 \times S_1 \times \cdots \times S_t$.*

As the regulator observes a subset of the bank's history, that is $s_t \subset h_t$ for every t, we can write the regulator's history as a function of the bank's history: $s^t(h^t)$. To complete the definition of histories we take $h_{-1} = s_{-1} \equiv \emptyset$ as there is no information prior to period 0.

We can now define strategies for the bank and the regulator that make use of the histories defined above:

Definition 3. *A reporting strategy for the bank is a sequence of functions*

$$m \equiv \{m_t(\theta_t, h^{t-1})\}_{t=0}^{\infty} \text{ for all } h^{t-1} \in H^{t-1}, \theta_t \in \Theta$$

that map the bank's history h^{t-1} and period t private information θ_t into a report to be sent to the regulator at the beginning of period t.

Definition 4. *The regulator offers the bank a sequence of policies*

$$\mu \equiv \{q_t(\hat{\theta}_t, s^{t-1}), L_t(\hat{\theta}_t, s^{t-1}), R_t(\hat{\theta}_t, s^{t-1})\}_{t=0}^{\infty}$$

for all histories $s^{t-1} \in S^{t-1}$ and reports $\hat{\theta}_t \in \Theta$, that map the regulator's history together with the bank's message into an outcome.

The regulator observes s_{t-1} prior to setting a regulatory policy in period t and so can condition the policy on the *public history* s^{t-1}. Obviously he can not condition the policy on the *private history* h^{t-1} as he does not observe h_{t-1}. Consequently, the regulator's policy specifies the quantity of loans, reserves, insurance premium and loan quality (implicitly giving an effort recommendation) each period as a function of the public history s^{t-1} and the bank's period t message. On the other hand, the bank is free to use the full history h^{t-1} when considering its choice of report each period.

Any reporting strategy (m) and regulatory policy (μ) generate values, one each for the bank and the regulator (society). Denote by σ the strategy profile (m, μ). Then, the value to the bank of following the reporting strategy m when the regulatory offers a policy μ is equal to the discounted stream of profits arising from σ at time zero:

$$V_b(\sigma) = (1-\beta)\sum_{t=0}^{\infty} \beta^t \pi[\mu_t(\hat{\theta}_t, s^{t-1}), \theta_t] \tag{17}$$

where the bank uses its reporting strategy m to report $\hat{\theta}_t = m_t(\theta_t, h^{t-1}) \in \Theta$ for all $\theta_t \in \Theta$ in each period t, and

$$\begin{aligned}
&\pi[\mu_t(\hat{\theta}_t, s^{t-1}), \theta_t] \equiv \\
&\quad \int_{r^b}^{\bar{r}} [rL_t(m_t(\theta_t, h^{t-1}), s^{t-1}) - \gamma(L_t(m_t(\theta_t, h^{t-1}), s^{t-1})) + R_t(m_t(\theta_t, h^{t-1}), s^{t-1}) \\
&\quad - L_t(m_t(\theta_t, h^{t-1}), s^{t-1})]dG(r(q(m_t(\theta_t, h^{t-1}), s^{t-1}))) - r^e R_t(m_t(\theta_t, h^{t-1}), s^{t-1}) \\
&\qquad\qquad\qquad\qquad - C(q(m_t(\theta_t, h^{t-1}), s^{t-1}) - \theta_t),
\end{aligned}$$

the time-variant cutoff is given by

$$r^b = \frac{L_t(m_t(\theta_t, h^{t-1}), s^{t-1}) + \gamma(L_t(m_t(\theta_t, h^{t-1}), s^{t-1})) - R_t(m_t(\theta_t, h^{t-1}), s^{t-1})}{L_t(m_t(\theta_t, h^{t-1}), s^{t-1})}.$$

The corresponding value for the regulator is:

$$V_r(\sigma) = (1-\beta)\sum_{t=0}^{\infty}\int_{\underline{\theta}}^{\overline{\theta}} \beta^t \left[S[\mu_t(\hat{\theta}_t, s^{t-1})] + \pi[\mu_t(\hat{\theta}_t, s^{t-1}), \theta_t]\right] dF(\theta_t). \tag{18}$$

where

$$S[\mu_t(\hat{\theta}_t, s^{t-1})] \equiv r^e P(\hat{\theta}_t, s^{t-1}) - (1+b)\int_{\underline{r}}^{r^b} [rL(\hat{\theta}_t, s^{t-1}) - \gamma(L(\hat{\theta}_t, s^{t-1})) + R(\hat{\theta}_t, s^{t-1}) - L(\hat{\theta}_t, s^{t-1})]dG(r|q(\hat{\theta}_t, s^{t-1}))$$

The factor $(1-\beta)$ normalizes the payoffs to current period units. Future histories h^t and s^t are generated recursively as follows: in period t, following history s^{t-1} and the report $\hat{\theta}_t = m_t(\theta_t, h^{t-1})$, the regulator responds with the policy $\mu_t(\hat{\theta}_t, s^{t-1})$ so that the bank's history $h^t = (h^{t-1}, \theta_t, \hat{\theta}_t, \mu_t(\hat{\theta}_t, s^{t-1}))$. The regulator's history is then $s^t = (s^{t-1}, \hat{\theta}_t, \mu_t(\hat{\theta}_t, s^{t-1}))$.

When the bank reports its private information truthfully each period, that is $m_t(\theta_t, h^{t-1}) = \theta_t$ for all $t \geq 0, \theta_t \in \Theta$ and $h^{t-1} \in H^{t-1}$, the social value from providing deposit insurance is:

$$V_r(\sigma) = (1-\beta)\sum_{t=0}^{\infty}\int_{\underline{\theta}}^{\overline{\theta}} \beta^t [S[\mu_t(\theta_t, s^{t-1})] + \pi[\mu_t(\theta_t, s^{t-1}), \theta_t]dF(\theta_t). \tag{19}$$

The bank's payoff from following a truthful strategy starting at $t = 0$:

$$V_b(\sigma) = (1-\beta)\sum_{t=0}^{\infty} \beta^t \pi[\mu_t(\theta_t, s^{t-1}), \theta_t] \tag{20}$$

and the payoff when history h^{k-1} has been realized is:

$$V_b(\sigma|h^{k-1}) = (1-\beta)\sum_{t=k}^{\infty} \beta^{t-k}\pi[\mu_t(m_t(\theta_t, h^{t-1}|h^{k-1}), s^{t-1}), \theta_t].$$

where the bank's (truthful) reports are functions of the full history.

We require that the bank's truthful strategy be optimal from any history of the regulator s^{k-1} onwards:

$$\begin{aligned} V_b(m, \mu|s^{k-1}) \equiv& (1-\beta)\sum_{t=k}^{\infty} \beta^{t-k}\pi[\mu_t(m_t(\theta_t, h^{t-1}|s^{k-1}), s^{t-1}|s^{k-1}), \theta_t] \\ \geq& (1-\beta)\sum_{t=k}^{\infty} \beta^{t-k}\pi[\mu_t(\hat{m}_t(\theta_t, h^{t-1}|s^{k-1}), s^{t-1}|s^{k-1}), \theta_t] \equiv V_b(\hat{m}, \mu|s^{k-1}) \end{aligned}$$

for any reporting strategy $\hat{m}$ such that $\hat{m}_t(\theta_t, h^{t-1}|s^{k-1}) = \hat{\theta}_t \neq \theta_t$ for at least one t.

The first point to note is that we can restrict attention to public strategies. That is, given that the regulator is using public histories to formulate the policy, the bank cannot do better when using its private history to formulate its reporting strategy.

Lemma 1. *The bank cannot improve upon its payoffs by conditioning its reporting strategy on private histories rather than public histories:*

$$(1-\beta)\sum_{t=k}^{\infty}\beta^{t-k}\pi[\mu_t(m_t(\theta_t,s^{t-1}|s^{k-1}),s^{t-1}|s^{k-1}),\theta_t]$$
$$\geq (1-\beta)\sum_{t=k}^{\infty}\beta^{t-k}\pi[\mu_t(m_t(\theta_t,h^{t-1}|s^{k-1}),s^{t-1}|s^{k-1}),\theta_t] \quad (21)$$

for all $t \geq k$, $\hat{\theta}_t \in \Theta$ and histories $s^{k-1} \in S^{k-1}$.

Proof. Conditioning history s^{k-1} enters the incentive constraints only by affecting the payoffs through the history-dependent outcome function $\mu_t(\cdot,\cdot)$. These payoffs are identical for all h^{k-1} that coincide in the s^{k-1} part once θ_k is realized. The bank's private information on past innate quality realizations θ_t affects the present only through the different policies proposed by the regulator. Imposing a separate constraint for each h^{k-1} is therefore not more restrictive. □

The intuition is that in equilibrium, only the bank's current private information will be relevant for payoffs in the current period. Given the regulator's policy, the bank will then find it optimal to report its information truthfully in the current period regardless of its reports in past periods.

We can now define an equilibrium for the infinitely repeated game in public strategies (i.e. where both the regulator and the bank condition their strategies on public histories):

Definition 5. *A perfect Bayesian equilibrium (PBE) of the infinitely repeated game is a reporting strategy, $m = \{m_t(\theta_t, s^{t-1})\}_{t=0}^{\infty}$, a regulatory policy, $\mu = \{\mu_t(\theta_t, s^{t-1})\}_{t=0}^{\infty}$ such that*

- *the bank prefers to report $m_t(\theta_t, s^{t-1}) = \theta_t$ rather than $m_t \neq \theta_t$, that is*

$$V_b(m,\mu|s^{k-1}) \geq V_b(\hat{m},\mu|s^{k-1})$$

- *limits on the bank's liability are met:*

$$\pi[\mu_t(\theta_t, s^{t-1}|s^{k-1}),\theta_t] \geq 0$$

for every history $s^{k-1} \in S^{k-1}$ and realization of the bank's private information $\theta_t \in \Theta$ and alternative reporting strategy $\hat{m}$.

The definition effectively ensures that in all perfect Bayesian equilibria, the regulator is able to elicit truth-telling and participation in equilibrium. As there will typically be many such equilibria, the mechanism design problem is to choose a reporting strategy and regulatory policy such that they will be incentive-compatible, ensure participation and that the resulting outcomes will maximize social welfare. Restricting ourselves to direct mechanisms, the dynamic mechanism design problem for the regulator [DRP] can be formulated as follows:

$$\max_{\{\mu_t(\cdot,\cdot)\}_{t=0}^{\infty}} (1-\beta)\sum_{t=0}^{\infty}\int_{\underline{\theta}}^{\overline{\theta}}\beta^t[S[\mu_t(\theta_t,s^{t-1})]+\pi[\mu_t(\theta_t,s^{t-1}),\theta_t]dF(\theta_t) \quad (22)$$

subject to

$$V_b(m,\mu|s^{k-1}) \geq V_b(\hat{m},\mu|s^{k-1}) \tag{23}$$

$$\pi[\mu_t(\theta_t,s^{t-1}|s^{k-1}),\theta_t] \geq 0 \tag{24}$$

for all histories $s^{k-1} \in S^{k-1}$ and $\theta_t, \hat{\theta}_t \in \Theta$. Moreover, although we have restricted ourselves to direct mechanisms in the formulation of the regulator's problem, this restriction does not limit the set of feasible allocations as the Revelation Principle holds in this environment. This is shown formally in the following proposition:

Proposition 2. *Given a general reporting space $\mathcal{M}$, consider a reporting strategy $m^* = \{m_t^*(\theta_t, s^{t-1})\}_{t=0}^{\infty}$ where $m_t^*(\theta_t, s^{t-1}) \in \mathcal{M}$ $\forall t$, an effort strategy $\eta^* = \{\eta_t^*(\theta_t, s^{t-1})\}_{t=0}^{\infty}$ where $\eta_t^*(\theta_t, s^{t-1}) \in \Theta$ $\forall t$, a regulatory policy $\mu^* = \{\mu_t^*(m_t^*, \hat{\eta}_t, s^{t-1})\}_{t=0}^{\infty}$ where $\hat{\eta}_t$ is an effort recommendation, that are optimal, there is a corresponding incentive-compatible direct mechanism in which the bank obeys the effort recommendation that is also optimal.*

Proof. See Appendix B. □

4.1 Recursive Formulation of the Regulator's Problem

We will use the apparatus of Abreu, Pearce, and Staccehetti (1990) to solve the regulator's problem recursively. First, we show how all PBE strategies can be decomposed into a current period outcome and a continuation strategy. Then, we show how we can characterize the *value* associated with each PBE in terms of a current period payoff and continuation values. By characterizing all PBE strategies in terms of values, we can ignore the former and focus on the latter. This focus on the continuation values permits a recursive formulation of the dynamic mechanism design problem using these continuation values as state variables. That is, by using continuation values as state variables we can show that a version of the principle of optimality holds so that the solution to the dynamic mechanism design problem can be found by solving an appropriate *static* mechanism design problem.

Given any PBE strategy profile $\sigma = (m, \mu)$, we can view this strategy profile as inducing a first-period outcome $(\theta_0, \mu_0(\theta_0, \emptyset))$ (as s^{-1} is the null history) and a continuation strategy $\sigma|s^0$ to be played after the first-period history $s_0 = s^0 = (\theta_0, q_0, L_0, R_0, P_0)$. Here,

$$\sigma|s^0 = \{m_t(\theta_t, s^{t-1}|s^0), \mu_t(m_t, s^{t-1}|s^0)\}_{t=1}^{\infty} \tag{25}$$

where $m_t(\theta_t, s^{t-1}|s^0) = \theta_t$ and $\mu_t(m_t, s^{t-1}|s^0) = \mu_t(\theta_t, s^{t-1}|s^0)$ are the equilibrium strategies prescribed by σ following the history s^0.

Then, we can break-down the *value* from this strategy profile into the sum of values from the first period outcome and continuation values drawn from the set of incentive-compatible and limited-liability payoffs. For example, we can write the value to the regulator from the profile σ as follows:

$$V_r(\sigma) = (1-\beta)\left[S[\mu_0(\theta_0,\emptyset)] + \pi[\mu_0(\theta_0,\emptyset),\theta_0]\right] + \beta V_r(\sigma|s^0)$$

where the first term is the value from the equilibrium (i.e. truthful) first-period strategy and the second term is the value from following the continuation strategy $\sigma|s^0$. Hence, we can decompose the value associated with any PBE into a current period value and a continuation value. In fact, we can completely characterize all the PBE in this manner. We prove this formally in the proposition below:

Proposition 3. *If σ is a PBE than so is the continuation strategy $\sigma|s^0$. Moreover, σ is a PBE if and only if*

- *for every first period history s^0 that is incentive-compatible and satisfies limited-liability, $\sigma|s^0$ is a PBE*
- $(1-\beta)\pi[\mu_0(\theta_0,\emptyset),\theta_0]+\beta V_b(\sigma|s^0) \geq (1-\beta)\pi[\mu(\hat{\theta}_0,\emptyset),\theta_0]+\beta V_b(\sigma|\hat{s}^0)$ *for all* $\hat{s}^0 \in S^0$ *where* $\hat{s}^0 = (\hat{\theta}_0, \mu_0(\hat{\theta}_0,\emptyset))$ *for all* $\theta_0, \hat{\theta}_0 \in \Theta$.

Proof. See Appendix B. □

This proposition characterizes all PBE strategies in terms of a first-period history s^0 and continuation values $V_b(\sigma|s^0)$ and $V_b(\sigma|\hat{s}^0)$ for the bank to induce it to adhere to the strategy or punish it when it deviates. Now that we have the characterization of all PBE in terms of values in hand, we would like to characterize the boundary of the set of values associated with all PBE.

Denote by V the set of values associated with all PBE of the infinitely repeated game:

$$V = \{V_b(\sigma) \mid \sigma \text{ is an PBE }\}$$

Obviously, $V \subset \mathbb{R}$. Now, as per the proposition, for every PBE σ with first-period outcome $(\theta_0, \mu_0(\theta_0,\emptyset))$ there exist $v(\hat{\theta}_0) \in V$ such that:

$$(1-\beta)\pi[\mu_0(\theta_0),\emptyset|\theta_0,\emptyset)] + \beta v(\theta_0) \geq (1-\beta)\pi[\mu_0(\hat{\theta}_0,\emptyset|\theta_0,\emptyset)] + \beta v(\hat{\theta}_0) \text{ for all } \theta_0, \hat{\theta}_0 \in \Theta$$

Let $\sigma^1(\theta_0)$ and $\sigma^2(\hat{\theta}_0)$ be the PBEs for which $v(\theta_0) = V_b(\sigma^1(\theta_0)), v(\hat{\theta}_0) = V_b(\sigma^2(\hat{\theta}_0))$. The PBE that supports the first period outcome $(\theta_0, \mu_0(\theta_0,\emptyset))$ is completed by specifying $\sigma|s^0 = \sigma^1(\theta_0)$ and $\sigma|\hat{s}^0 = \sigma^2(\hat{\theta}_0)$ for all $\theta_0, \hat{\theta}_0 \in \Theta$. The values $v(\hat{\theta}_0) \in V$, for all $\hat{\theta}_0$ produce then a PBE σ with value $v \in V$ given by

$$v = (1-\beta)\pi[\mu_0(\theta_0,\emptyset),\theta_0] + \beta v(\theta_0) \tag{26}$$

Therefore, the construction here maps values $v(\hat{\theta}_0)$ into a strategy profile σ with first period outcome $(\theta_0, \mu_0(\theta_0,\emptyset))$ and a value $v = V_b(\sigma)$. To obtain the set V we apply this construction to all values in a sufficiently large candidate set W, obtaining a set $B(W)$. The largest fixed point of the operator $B(W)$ is then the set V.

Formally, we can encapsulate the above construction in the following notion of *enforceability*:

Definition 6. *The first-period outcome $(\theta_0, \mu_0(\theta_0,\emptyset))$ and the continuation values $w(\cdot)$ are enforceable by W if*

- $w(\hat{\theta}_0) \in W$ *for all* $\hat{\theta}_0 \in \Theta$
- $(1-\beta)\pi[\mu_0(\theta_0,\emptyset),\theta_0] + \beta v(\theta_0) \geq (1-\beta)\pi[\mu_0(\hat{\theta}_0,\emptyset),\theta_0] + \beta v(\hat{\theta}_0)\ \forall \theta_0, \hat{\theta}_0 \in \Theta$

Of course, whenever $W \subset V$, the enforceable first-period outcome $(\theta_0, \mu_0(\theta_0, \emptyset))$ and continuation values $w(\cdot)$ determine a PBE strategy profile σ and value v given by (26). We need not characterize the entire set V but can rather focus on the finding the largest value in V. This largest value will be the value achieved by the regulator after solving the dynamic mechanism design problem. The largest value is the solution to the following static mechanism design problem:

$$\bar{v} = \max_{\mu_0(\theta_0), w(\theta_0)} \int (1-\beta)\left[S[\mu_0(\theta_0, \emptyset)] + \pi[\mu_0(\theta_0, \emptyset), \theta_0] + \beta w(\theta_0)\right] dF(\theta_0)$$

subject to the restriction that $(\theta_0, \mu_0(\theta_0, \emptyset))$ and $w(\cdot)$ be enforceable by W.

5 The Optimal Dynamic Regulatory Policy

5.1 The optimal mechanism

Rather than characterizing the set W, we will solve the best-value problem and characterize the optimal mechanism. Following Athey, Atkeson, and Kehoe (2005) we define the mechanism to be static if $w(\theta) = \bar{w}$ for θ and dynamic if $w(\theta) < \bar{w}$ on a subset of Θ of positive measure. The key result is that under an appropriate set of single-crossing and monotone hazard conditions, the optimal mechanism is static. This result is a direct extension of Proposition 1 in Athey, Atkeson, and Kehoe (2005) to an environment in which the decision space[8] is multidimensional.

The major steps in the proof as follows. First, we replace the global incentive constraints with appropriate local incentive constraints. This requires assuming that appropriate single-crossing as well as monotonicity conditions on the regulatory policy variables hold. Typically, one then proceeds to solve the relaxed problem in which the monotonicity and feasibility constraints are ignored and then checks to see if these are violated. Despite making an assumption on an appropriately defined hazard-rate, the relaxed problem yields non-monotonic policy functions meaning that at least one of the ignored constraints binds. The constraint that binds turns out to be the feasibility constraints and this is shown using the variational approach of Athey, Atkeson, and Kehoe (2005).

5.2 Preliminaries

Dropping the time subscripts, we can write the regulator's problem as follows:

$$\bar{v} = \max_{\mu(\theta), w(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} \left[\tilde{S}[\mu(\theta)] + \tilde{\pi}[\mu(\theta), \theta] + \beta w(\theta)\right] dF(\theta) \quad \text{subject to}$$

[8] Following the terminology in section 7.3 of Fundenberg and Tirole (1991), the decision space here is $\{q, L, R, P\}$.

$$\tilde{\pi}[\mu(\theta), \theta] + \beta w(\theta) \geq \tilde{\pi}[\mu(\hat{\theta}), \theta] + \beta w(\hat{\theta}) \tag{27}$$

$$\tilde{\pi}[\mu(\theta), \theta] + \beta w(\theta) \geq 0 \tag{28}$$

$$w(\hat{\theta}) \in W \tag{29}$$

for all $\theta, \hat{\theta} \in \Theta$ where we have used $\tilde{S}, \tilde{\pi}$ to denote $(1-\beta)S, (1-\beta)\pi$, $w(\cdot)$ is a piecewise-C^1 function and $\mu(\cdot)$ is a vector of piecewise-C^1 functions.

Following standard methodology[9], we will first replace the global incentive-constraints in (27) with equivalent local versions. To derive these, we can formulate the bank's problem as choosing a report $\hat{\theta}$ that solves $\max_{\hat{\theta}} \Phi(\hat{\theta}, \theta) \equiv \tilde{\pi}[\mu(\hat{\theta}), \theta] + \beta w(\hat{\theta})$. The first-order necessary condition for the truthful report $\hat{\theta} = \theta$ to be global incentive-compatible is

$$\begin{aligned} \left.\frac{\partial \Phi(\hat{\theta}, \theta)}{\partial \hat{\theta}}\right|_{\hat{\theta}=\theta} &= \sum_k \frac{\partial \tilde{\pi}[\mu(\hat{\theta}), \theta]}{\partial \mu_k} \frac{d\mu_k(\hat{\theta})}{d\hat{\theta}} + \beta \left.\frac{dw(\hat{\theta})}{d\hat{\theta}}\right|_{\hat{\theta}=\theta} \\ &= \sum_k \frac{\partial \tilde{\pi}[\mu(\theta), \theta]}{\partial \mu_k} \frac{d\mu_k(\theta)}{d\theta} + \beta \frac{dw(\theta)}{d\theta} = 0 \end{aligned} \tag{30}$$

while the second-order sufficient condition is $\frac{\partial^2 \Phi(\hat{\theta}, \theta)}{\partial \hat{\theta}^2} \leq 0$. Differentiating the first-order condition at the optimum with respect to θ, we have $\frac{\partial^2 \Phi(\hat{\theta}, \theta)}{\partial \hat{\theta}^2} + \frac{\partial^2 \Phi(\theta, \theta)}{\partial \hat{\theta} \partial \theta}|_{\hat{\theta}=\theta} = 0$. Hence, the second-order condition is equivalent to $\frac{\partial^2 \Phi(\hat{\theta}, \theta)}{\partial \hat{\theta} \partial \theta} \geq 0$ or

$$\sum_k \frac{\partial}{\partial \theta} \left(\frac{\partial \tilde{\pi}[\mu(\theta), \theta]}{\partial \mu_k} \right) \frac{d\mu_k(\theta)}{d\theta} \geq 0$$

Following from the weak single-crossing assumption, Assumption 2, we have:

$$\frac{\partial}{\partial \theta} \left(\frac{\partial \tilde{\pi}}{\partial \mu_k} \right) \geq 0 \quad \forall k \tag{31}$$

with strict inequality for at least one k. Then, the second-order condition is satisfied as long as the monotonicity constraints, $\frac{d\mu_k(\theta)}{d\theta} \geq 0$, hold for all k.

We say an allocation $(\mu(\cdot), w(\cdot))$ is *locally incentive-compatible* if both (30) and the monotonicity constraints hold. Furthermore, we have just shown that the allocation $(\mu(\cdot), w(\cdot))$ is incentive-compatible in the sense of (27) if and only if it is locally incentive-compatible. Note that when the bank is risk-neutral, Assumption 2 is automatically satisfied as $\frac{\partial}{\partial \theta} \left(\frac{\partial \tilde{\pi}}{\partial q} \right) = C''(\cdot) > 0$ and $\frac{\partial}{\partial \theta} \left(\frac{\partial \tilde{\pi}}{\partial L} \right) = \frac{\partial}{\partial \theta} \left(\frac{\partial \tilde{\pi}}{\partial R} \right) = \frac{\partial}{\partial \theta} \left(\frac{\partial \tilde{\pi}}{\partial P} \right) = 0$ so that only (30) and the monotonicity constraints are needed for incentive-compatibility.

We will now incorporate the local incentive constraints (30) into the regulator's objective. First, denote the value at the optimum to a bank of type θ by $U(\theta)$ where:

$$U(\theta) = \tilde{\pi}[\mu(\theta), \theta] + \beta w(\theta). \tag{32}$$

[9] See for example section 7.3 in Fundenberg and Tirole (1991).

Notice that $U(\theta)$ is piecewise-C^1 as both $\tilde{\pi}[\mu(\theta),\theta]$ and $w(\theta)$ are piecewise-C^1 and that:

$$\begin{aligned}
U'(\theta) &= \frac{\partial}{\partial\theta}\tilde{\pi}[\mu(\theta),\theta] + \beta\frac{dw(\theta)}{d\theta} \\
&= \frac{\partial\tilde{\pi}}{\partial\theta} + \sum_k \frac{\partial\tilde{\pi}[\mu(\theta),\theta]}{\partial\mu_k}\frac{d\mu_k}{d\theta} + \beta\frac{dw(\theta)}{d\theta} \\
&= \frac{\partial\tilde{\pi}}{\partial\theta}
\end{aligned}$$

where the last line follows from the local incentive-compatibility requirement (30). Integrating from $\underline{\theta}$ to θ we have:

$$U(\theta) - U(\underline{\theta}) = \int_{\underline{\theta}}^{\theta} \frac{\partial\tilde{\pi}[\mu(z),z]}{\partial\theta}dz \tag{33}$$

Using this result, we can first rewrite the continuation values as:

$$\beta w(\theta) \equiv U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \frac{\partial\tilde{\pi}[\mu(z),z]}{\partial\theta}dz - \tilde{\pi}[\mu(\theta),\theta] \tag{34}$$

Then, we can rewrite the regulator's objective as follows:

$$\begin{aligned}
&\int_{\underline{\theta}}^{\overline{\theta}} \left[\tilde{S}[\mu(\theta)] + \tilde{\pi}[\mu(\theta),\theta] + \beta w(\theta)\right] f(\theta)d\theta \\
&\quad= \int_{\underline{\theta}}^{\overline{\theta}} \left[\tilde{S}[\mu(\theta)] + \int_{\underline{\theta}}^{\theta} \frac{\partial\tilde{\pi}[\mu(z),z]}{\partial\theta}dz + U(\underline{\theta})\right] f(\theta)d\theta \\
&\quad= \int_{\underline{\theta}}^{\overline{\theta}} \left[\tilde{S}[\mu(\theta)] + U(\underline{\theta})\right] f(\theta)d\theta + \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} \left(\frac{\partial\tilde{\pi}[\mu(z),z]}{\partial\theta}dz\right) f(\theta)d\theta \\
&\quad= U(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} \left[\tilde{S}[\mu(\theta)] + \left(\frac{1-F(\theta)}{f(\theta)}\right)\frac{\partial\tilde{\pi}[\mu(\theta),\theta]}{\partial\theta}\right] f(\theta)d\theta
\end{aligned}$$

after an integration by parts. We now make the following joint assumption on the distribution of types and the welfare function:

Assumption 4.

$$\frac{\partial\tilde{S}[\mu(\theta)]}{\partial\mu_k} + \left(\frac{1-F(\theta)}{f(\theta)}\right)\frac{\partial^2\tilde{\pi}[\mu(\theta),\theta]}{\partial\mu_k\partial\theta} \tag{35}$$

is strictly decreasing in θ for at least one k.

This assumption states that the marginal benefit of increasing any one of the policy variables is decreasing with the bank's innate quality. In other words, tightening the policy for banks with low innate quality portfolios, for instance by raising their reserve requirements or realized loan quality, improves welfare more than similar increases aimed at higher quality banks. This assumption is motivated by the observation that prior to the recent financial crisis, society would probably have benefited more from tighter regulatory policy vis-a-vis banks with high-risk portfolios such as Bears Sterns or Lehman Brothers than corresponding policy towards banks with safer portfolios.

We can now view the regulator's formal problem as choosing a regulatory policy $\mu(\cdot)$ that maximizes welfare given by

$$U(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} \left[\tilde{S}[\mu(\theta)] + \left(\frac{1 - F(\theta)}{f(\theta)} \right) \frac{\partial \tilde{\pi}[\mu(\theta), \theta]}{\partial \theta} \right] f(\theta) d\theta \tag{36}$$

subject to the following monotonicity, feasibility and limited-liability constraints:

$$\begin{aligned}
&d\mu_k/d\theta \geq 0 \text{ for all } k \\
&w(\theta) \equiv U(\underline{\theta})/\beta + \int_{\underline{\theta}}^{\theta} \frac{\partial \tilde{\pi}[\mu(z), z]}{\partial \theta} dz - \tilde{\pi}[\mu(\theta), \theta]/\beta \leq \bar{w} \text{ for all } \theta \in \Theta \\
&\tilde{\pi}[\mu(\underline{\theta}), \underline{\theta}] + \beta w(\underline{\theta}) = 0.
\end{aligned}$$

The main result is as follows:

Proposition 4. *Under Assumptions 2 and 4, the optimal mechanism is static, that is $w(\theta) = \bar{w}$ for all $\theta \in \Theta$.*

Proof. See Appendix B. □

5.3 Characterizing the Optimal Policy

We can now characterize the optimal regulatory policy. To suitably narrow the class of optimal policies, we can impose the following necessary conditions derived earlier:

1. $\mu_k(\theta)$ is continuous for all k
2. $\mu_k(\theta)$ is weakly increasing for
3. given that the optimal mechanism is static, $w(\theta) = \bar{w}$, from (30) optimality requires that $\mu(\theta)$ satisfy

$$\sum_k \frac{\partial \tilde{\pi}[\mu(\theta), \theta]}{\partial \mu_k} \frac{d\mu_k(\theta)}{d\theta} = 0 \tag{37}$$

From the last condition, it is clear that the optimal policy potentially consists of flat regions in one or more policy variables (i.e. where $\frac{d\mu_k}{d\theta}$ is zero for one or more k) together with non-flat regions for the other policy variables. We systematically analyze these cases via the following sequence of lemmas.

Lemma 2. *When the optimal policy is flat everywhere, $d\mu_k(\theta)/d\theta = 0$ for all k and $\theta \in \Theta$, it must be the μ^{FHC} policy.*

Proof. When $d\mu_k/d\theta = 0$ for all k on the entire interval $[\underline{\theta}, \overline{\theta}]$, the optimal policy is of the form $\mu^* = \{q^*, L^*, R^*, P^*\}$ where q^*, L^*, R^* and P^* are constants. Thus, the incentive-constraints (37) hold trivially and

$$\mu^* = \arg\max_{\mu} \int_{\underline{\theta}}^{\overline{\theta}} [\tilde{S}[\mu] + \tilde{\pi}[\mu, \theta] + \beta \bar{w}] dF(\theta) \text{ s.t. } \tilde{\pi}[\mu, \theta] \geq 0 \text{ for all } \theta \in \Theta.$$

so that from (10) it is clear that μ^* coincides with μ^{FHC} as $\beta\bar{w}$ is a constant. □

When the optimal policy is not μ^{FHC}, there are potentially a large number of cases to consider as the policy can fail to be flat in many ways. The next result simplifies much of the analysis. It states that a policy that is flat in a limited set of policy variables cannot be optimal over any interval as the IRB policy is welfare improving over such a policy.

Lemma 3. *Under assumptions 2, 3 and 4, if $d\mu_k/d\theta > 0$ for some but not all k on $I \subset \Theta$, then we can find an alternative policy that improves welfare on I and for which $d\mu_k/d\theta > 0$ for all k, namely the IRB policy.*

Proof. First, it is useful to define the set of *partially-flat* optimal policies. Consider a policy $\mu(\theta)$. Let $K_0^I(\mu)$ be the set of policy variables of μ that are flat over the set I. That is, $K_0^I(\mu) = \{k : d\mu_k/d\theta = 0 \text{ for } \theta \in I\}$. When $K_0^I(\mu)$ is not the empty set, we say μ is a partially-flat policy. As an example, consider $K_0^I = \{R\}$ so that the policy is of the form $\mu(\theta) = \{q(\theta), L(\theta), R\}$. Now, the set of partially-flat optimal policies consists of all policies that are flat in some variable and solve the regulator's problem. For example, the policy $\mu^L(\theta)$ given by

$$\mu^L(\theta) = \underset{q(\theta),L(\theta),R}{\arg\max} \int_{\underline{\theta}}^{\overline{\theta}} [\tilde{S}[q(\theta), L(\theta), R] + \tilde{\pi}[q(\theta), L(\theta), R, \theta] + \beta\bar{w}]dF(\theta)$$

$$\text{s.t. } \tilde{\pi}[q(\theta), L(\theta), R, \theta] \geq 0 \text{ and (37) holds for all } \theta \in \Theta.$$

is a partially-flat optimal policy.

Now, if a policy is flat in some variables and is optimal over some interval I then it must coincide with one of the partially-flat optimal policies for not-flat variables over I. Note that this is a direct result of the incentive-constraint (37). However, as we are limiting the admissible policies to those that are flat in some variables, selecting policies from from a larger admissible set (i.e. those that may not be flat in any variable) will yield higher welfare. That is, we can engineer a welfare improving relative to the partially-flat optimal policy by using the IRB policy over I. For instance, in the example above, allowing $R(\theta)$ to vary with θ is welfare-improving (over all intervals) as the optimal policy is then the IRB policy. Moreover, from Proposition 1 we know that the policy variables for the IRB policy are strictly increasing. Therefore, no partially-flat policy can be optimal over any interval. □

The main result regarding the characterization of the optimal policy can be stated as follows:

Proposition 5. *Unless $\mu(\theta)$ is flat everywhere (and thus equal to μ^{FHC}), the optimal policy is of the following form:*

$$\mu(\theta) = \begin{cases} \mu^{IRB}(\theta) & \text{if } \theta \in [\underline{\theta}, \theta') \\ \mu^{IRB}(\theta') & \text{if } \theta \in (\theta', \overline{\theta}] \end{cases} \tag{38}$$

Proof. See Appendix B. □

6 Policy Implications

The first implication of the characterization of the optimal regulatory policy is that all banks face a limit on loan volume and must adhere to quality and capital requirements. The second implication is that the regulatory policy for banks below the threshold quality level θ' will feature the same degree of flexibility as in the static environment. Namely, loan volume, quality and capital requirements will increase with the quality of the bank's portfolio, as per Proposition 1. Moreover, loan volume will increase more rapidly than capital requirements so that leverage increases with quality as well. The third implication is that for banks that have portfolios of higher quality than θ', the regulatory policy is very rigid. Although they are allowed to issue the highest quantity of loans, they are not permitted to exceed that amount irrespective of improvements to loan quality. Hence, regulation can be seen as imposing a maximum size on banks by assets, namely $L(\overline{\theta})$. Moreover, further quality improvements also do not permit a decrease in the capital requirement so that effectively regulation can also be seen as imposing a cap on leverage given by $L(\overline{\theta})/R(\overline{\theta})$. Overall, leverage is weakly increasing in quality as it increase for all $\theta < \theta'$ and is capped for all $\theta > \theta'$. Thus, in the terminology of Adrian and Shin (2010), leverage is weakly pro-cyclical.

The optimal regulatory policy is broadly in line with Basel III. First, it supports risk-based capital requirements for all banks. Second, it finds support for a broad supervisory role that sets regulation concerning both the asset and liability sides of the bank's balance sheet while allowing banks the use of internal risk-based models for assessing credit-risk. Third, it supports the use of leverage as an additional measure of inherent bank risk as leverage in the model evolves directly with quality. Finally, through the use of loan volume caps the optimal policy also support the Basel III goal of protecting against excessive credit buildup.

6.1 Cyclical Adjustments

Both the academic literature and policy makers have noted[10] that de-leveraging by all banks through fire-sales of assets at the same time may exacerbate the credit problems following an aggregate adverse shock.[11] As a result, a key provision of Basel III is the requirement for banks to maintain a capital buffer that can be drawn down in bad times so as to avoid excessive declines in credit. The optimal regulatory policy also incorporates the notion of a capital buffer in mitigating de-leveraging by banks. Furthermore, by design, it also spells out the optimal capital and asset structure to be held following an adverse shock by through carefully balancing the social costs of avoiding default against the social costs of bankruptcy.

In the model, a bank's innate quality can be interpreted as factors that affect credit-worthiness but that are beyond the control of the bank. A natural way to interpret θ is as the aggregate state of the economy. The state affects all entrepreneurs simultaneously and so banks are able to ascertain it as a result of their intermediation

[10] For a discussion of these issues see Hanson, Kashyap, and Stein (forthcoming).

[11] A number of such implications have been termed "pro-cyclical" effects of regulatory policy. See Wellink (2010).

role while the regulator learns the state with a delay. Then, given an innate quality θ_t in period t, a realization of $\theta_{t+1} > \theta_t$ in period $t+1$ would be interpreted as a positive aggregate shock while a realization of $\theta_{t+1} < \theta_t$ would be viewed as an adverse aggregate shock. Then, comparing the behaviour of banks with innate qualities θ_t and θ_{t+1} under the optimal policy, provides a means for understanding the adjustments that would be required following a positive or adverse aggregate shock. For instance, the balance sheet adjustments required by the optimal regulatory policy after an adverse shock are a reduction in the size and quality of the loan portfolio and a *reduction* in capital in the amount $R(\theta_t) - R(\theta_{t+1})$. Effectively, banks hold a capital reserve buffer, by design, of $R(\theta)$. In addition, loans would decline by $L(\theta_t) - L(\theta_{t+1})$ while portfolio quality would also suffer. As leverage also decreases following an adverse shock, loans are reduced more than capital. The important point to note here is that the process of de-leveraging that is typically observed in economic downturns following severe adverse shocks is mitigated by declines in the capital requirement or in other words through the maintenance of a capital buffer.

7 Conclusion

This paper has investigated the optimal regulatory policy for deposit-taking institutions in a dynamic setting. It has found that the optimal policy is stationary and requires controls on both the asset and liability side of the bank's balance sheet. Specifically, it requires banks to adhere to a risk-adjusted capital requirement, a quality requirement along with limits on loan volume. Furthermore, policy accords lower quality banks a limited degree of flexibility in adjusting their balance sheets in response to quality improvements. Such flexibility is markedly absent for higher quality banks.

The optimal regulatory policy also dictates a hard cap on loan volume and leverage across all banks. This is in broad agreement with the Basel III goals of limiting excessive credit buildup and the use of leverage as an additional risk measure. Regulatory policy also characterizes the optimal balance sheet adjustment strategy for banks following (independent and identically distributed) aggregate shocks over time. Specifically, banks will optimally adjust their balance sheets through reductions in both loan volume and capital nevertheless leading to a decline in leverage. This adjustment process fully takes into account the social costs stemming from declines in the availability of credit as a result of bank de-leveraging following an adverse aggregate shock. The reductions in capital as part of the adjustment process accord well with the proposals for capital buffers to counteract pro-cyclicality in the Basel III framework.

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A Feasibility

When $\Delta(a) \geq 0$ we cannot use the variation described earlier. In this case, we must resort to an alternative variation. Notice that to maintain incentives after applying the variation, we can either change the promised values of banks with type above θ_1 *or* change the continuation values for all types below θ_2. To see the later, note that given $U'(\theta) = \partial\tilde{\pi}[\mu(\theta), \theta]\partial\theta$, by integrating down from $\overline{\theta}$ to θ, we have:

$$U(\theta) = U(\overline{\theta}) - \int_{\theta}^{\overline{\theta}} \frac{\partial\tilde{\pi}[\mu(z), z]}{\partial\theta} dz \tag{39}$$

Let $\mu(\theta)$ be the optimal policy. Again, consider the following alternate policy:

$$\tilde{\mu}(\theta) = \begin{cases} \tilde{\mu} & \text{if } \theta \in (\theta_1, \theta_2) \\ \mu(\theta) & \text{otherwise} \end{cases}$$

To ensure that the variation we have constructed is feasible, we need to adjust the continuation values in such way that $w(\theta; a) \leq \bar{w}$ for some $a \in [0, 1]$. The continuation values under the variation are given by:

$$w(\theta; a) = U(\overline{\theta}) - \int_{\theta}^{\overline{\theta}} \frac{\partial\tilde{\pi}[\mu(z; a), z]}{\partial\theta} dz - \tilde{\pi}[\mu(\theta; a), \theta]. \tag{40}$$

Notice that we can rewrite them as

$$\beta w(\theta; a) = \begin{cases} \beta w(\theta) & \text{for } \theta \leq \theta_1 \\ \beta w(\theta) - \Delta(a) & \text{for } \theta > \theta_1 \end{cases} \tag{41}$$

where $\Delta(a)$ is given by

$$\Delta(a) = \int_{\theta_1}^{\theta} \left[\frac{\partial\tilde{\pi}[\mu(z; a), z]}{\partial\theta} - \frac{\partial\tilde{\pi}[\mu(z), z]}{\partial\theta} \right] dz. \tag{42}$$

From there we can see that we can now use this variation whenever $\Delta(a) \leq 0$ as we can apply this variation to reduce the promised values below θ_1 and leave all other promised values unchanged.

B Proofs

B.1 Proof of Proposition 1

The proof follows directly from the proof of Proposition 2 in Giammarino, Lewis, and Sappington (1993).

B.2 Proof of Proposition 2

The proof follows from Proposition 2 of Myerson (1982).

B.3 Proof of Proposition 3

We first show that $\sigma|s^0$ is a PBE given that σ is a PBE. We need to verify Definition 5 for $\sigma|s^0$ in the continuation game. Essentially, writing out the definition of the continuation game and strategy makes this obvious. Now, if σ is a PBE, then incentive-compatibility in definition 5 implies truthful revelation in the first-period. Then, as we have shown above $\sigma|s^0$ is a PBE. Finally, if $\sigma|s^0$ is a PBE and the first period history is incentive-compatible then $\sigma = (m_0, \mu_0(m_0, \emptyset), \sigma|s^0)$ is a PBE.

B.4 Proof of Proposition 4

The standard approach to the above mechanism design problem is to first solve the relaxed problem and then verify that the monotonicity and feasibility constraints hold for the solution to that problem. As we show below, this approach fails here because the solution to the relaxed problem violates one of the constraints. To demonstrate this we analyze the first-order conditions to the regulator's problem. In order to derive these, consider the perturbed policy $\mu(\theta)+\delta h(\theta)$ where $h(\theta)$ is a vector of piecewise-C^1 functions, δ is a small number and $\mu(\theta)$ is assumed to be optimal. Welfare under this perturbed policy is denoted by $g(\delta)$ where

$$g(\delta) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\tilde{S}[\mu(\theta) + \delta h(\theta)] + \left(\frac{1 - F(\theta)}{f(\theta)} \right) \frac{\partial \tilde{\pi}[\mu(\theta) + \delta h(\theta), \theta]}{\partial \theta} \right] f(\theta) d\theta$$

For the policy $\mu(\theta)$ to be optimal, welfare should remain unchanged for small perturbations around it. Formally: $g'(0) = 0$ or

$$\int_{\underline{\theta}}^{\overline{\theta}} \sum_k \left[\frac{\tilde{S}[\mu(\theta)]}{\partial \mu_k} + \left(\frac{1 - F(\theta)}{f(\theta)} \right) \frac{\partial^2 \tilde{\pi}[\mu(\theta), \theta]}{\partial \mu_k \partial \theta} \right] h_k f(\theta) d\theta = 0 \tag{43}$$

for *all* vectors of piecewise-C^1 functions $h(\theta)$. The integral in (43) can be zero for all such $h(\theta)$ only if the integrand is zero at all $\theta \in \Theta$. Formally,

$$\left[\frac{\tilde{S}[\mu(\theta)]}{\partial \mu_k} + \left(\frac{1 - F(\theta)}{f(\theta)} \right) \frac{\partial^2 \tilde{\pi}[\mu(\theta), \theta]}{\partial \mu_k \partial \theta} \right] = 0 \tag{44}$$

for all k and all $\theta \in \Theta$. These are the necessary first-order conditions for the solution to the regulator's problem. To interpret them notice that for example when $\mu_k = q$, the LHS of the first-order condition

$$\left[\frac{\tilde{S}[\mu(\theta)]}{\partial q} + \left(\frac{1 - F(\theta)}{f(\theta)} \right) \frac{\partial^2 \tilde{\pi}[\mu(\theta), \theta]}{\partial q \partial \theta} \right]$$

is the marginal increase in welfare from regulating an increase in the quality of the bank's loan portfolio. Under Assumption 4, this marginal increase in welfare is greater for banks of low innate quality than for those with high innate quality. Therefore, a policy that is decreasing with innate quality policy is optimal. Such a policy however violates the monotonicity constraint $dq/d\theta \geq 0$. Hence, the solution to the relaxed problem is invalid.

In fact, the constraint that is binding is the feasibility constraint $w(\theta) \leq \bar{w}$. To show this we adapt the variational approach in Athey, Atkeson, and Kehoe (2005). The key idea is that as long as $\mu(\theta)$ is strictly increasing in any one policy variable, we can find a variation that improves welfare by flatten it. We then use this idea to show that $w(\theta)$ must a step function because it weren't then incentive-compatibility would require $\mu(\theta)$ to be strictly increasing in some policy variable so that a welfare improving variation would exist. Finally, we show that $w(\theta)$ is also continuous so that it must be constant.

We would like to show that the feasibility constraint binds. The analysis of the relaxed problem suggests that whenever $\mu(\theta)$ is increasing in some direction, a variation that reduces the policy for higher types in that direction and increases it for lower types should be welfare improving. This idea is formalized in the following lemma:

Lemma 4. *Let $(\mu(\theta), w(\theta))$ be an allocation where both $\frac{d\mu_k}{d\theta} > 0$ and Assumption 4 are satisfied for at least one k on the interval (θ_1, θ_2). Then, there is a variation that improves welfare, assuming that it is feasible.*

Proof. Let $\mu(\theta)$ be the optimal policy. Consider the following alternate policy:

$$\tilde{\mu}(\theta) = \begin{cases} \tilde{\mu} & \text{if } \theta \in (\theta_1, \theta_2) \\ \mu(\theta) & \text{otherwise} \end{cases}$$

where $\tilde{\mu}$ is a vector of constants with typical element

$$\tilde{\mu}_k = \frac{\int_{\theta_1}^{\theta_2} \mu_k(\theta) f(\theta) d\theta}{F(\theta_2) - F(\theta_1)}.$$

Of course, $\tilde{\mu}_k$ is just the conditional mean of $\mu_k(\theta)$ for on the interval (θ_1, θ_2). Whenever, $d\mu_k(\theta)/d\theta > 0$, the alternate policy differs from the old one by reducing $\mu(\theta)$ in the policy variable μ_k when the latter is above its conditional mean and raising it when it is below. We say $\tilde{\mu}(\theta)$ is flatter than $\mu(\theta)$, possibly along more than one policy variable. Now, consider the following variation of $\mu(\theta)$ that mixes the two policies:

$$\mu(\theta; a) = a\tilde{\mu}(\theta) + (1-a)\mu(\theta) \qquad \text{for } a \in [0, 1]$$

With this variation we can analyze the marginal effect of flattening $\mu(\theta)$. The value to the bank under this variation is:

$$U(\theta; a) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \frac{\partial \tilde{\pi}[\mu(z; a), z]}{\partial \theta} dz \tag{45}$$

and the continuation values are:

$$w(\theta; a) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \frac{\partial \tilde{\pi}[\mu(z; a), z]}{\partial \theta} dz - \tilde{\pi}[\mu(\theta; a), \theta]. \tag{46}$$

Welfare under this variation is given by:

$$\begin{aligned} V(a) &= \int_{\underline{\theta}}^{\overline{\theta}} \left[\tilde{S}[\mu(\theta;a)] + \left(\frac{1-F(\theta)}{f(\theta)} \right) \frac{\partial \tilde{\pi}[\mu(\theta;a),\theta]}{\partial \theta} \right] f(\theta) d\theta + U(\underline{\theta}) \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \left[\tilde{S}[a\tilde{\mu}(\theta) + (1-a)\mu(\theta)] \right. \\ &\qquad \left. + \left(\frac{1-F(\theta)}{f(\theta)} \right) \frac{\partial \tilde{\pi}[a\tilde{\mu}(\theta) + (1-a)\mu(\theta),\theta]}{\partial \theta} \right] f(\theta) d\theta + U(\underline{\theta}) \end{aligned}$$

Then the change in the welfare from a small variation is:

$$\begin{aligned} \frac{dV(0)}{da} &= \int_{\underline{\theta}}^{\overline{\theta}} \sum_k \left[\frac{\partial \tilde{S}}{\partial \mu_k} [\tilde{\mu}_k(\theta) - \mu_k(\theta)] + \left(\frac{1-F(\theta)}{f(\theta)} \right) \frac{\partial^2 \tilde{\pi}}{\partial \mu_k \partial \theta} [\tilde{\mu}_k(\theta) - \mu_k(\theta)] \right] f(\theta) d\theta \\ &= \int_{\theta_1}^{\theta_2} \sum_k \left[\frac{\partial \tilde{S}}{\partial \mu_k} [\tilde{\mu}_k - \mu_k(\theta)] + \left(\frac{1-F(\theta)}{f(\theta)} \right) \frac{\partial^2 \tilde{\pi}}{\partial \mu_k \partial \theta} [\tilde{\mu}_k - \mu_k(\theta)] \right] f(\theta) d\theta \\ &= \int_{\theta_1}^{\theta_2} \sum_k [\tilde{\mu}_k - \mu_k(\theta)] \left[\frac{\partial \tilde{S}}{\partial \mu_k} + \left(\frac{1-F(\theta)}{f(\theta)} \right) \frac{\partial^2 \tilde{\pi}}{\partial \mu_k \partial \theta} \right] f(\theta) d\theta \\ &= [F(\theta_2) - F(\theta_1)] \sum_k \frac{\int_{\theta_1}^{\theta_2} [\tilde{\mu}_k - \mu_k(\theta)] \left[\frac{\partial \tilde{S}}{\partial \mu_k} + \left(\frac{1-F(\theta)}{f(\theta)} \right) \frac{\partial^2 \tilde{\pi}}{\partial \mu_k \partial \theta} \right] f(\theta) d\theta}{[F(\theta_2) - F(\theta_1)]} \\ &= [F(\theta_2) - F(\theta_1)] \sum_k E \left\{ [\tilde{\mu}_k - \mu_k(\theta)] \left[\frac{\partial \tilde{S}}{\partial \mu_k} + \left(\frac{1-F(\theta)}{f(\theta)} \right) \frac{\partial^2 \tilde{\pi}}{\partial \mu_k \partial \theta} \right] \right\} \\ &= [F(\theta_2) - F(\theta_1)] \sum_k Cov \left\{ \tilde{\mu}_k - \mu_k(\theta), \left[\frac{\partial \tilde{S}}{\partial \mu_k} + \left(\frac{1-F(\theta)}{f(\theta)} \right) \frac{\partial^2 \tilde{\pi}}{\partial \mu_k \partial \theta} \right] \right\} \end{aligned}$$

where the expectation and covariance are being taken with respect to the conditional density $f(\theta)/[F(\theta_2)-F(\theta_1)]$ and the last line follows from noting that $E[\tilde{\mu}_k - \mu_k(\theta)] = 0$ for all k by construction.

Now, to see that this marginal change in welfare is positive, recall that from Assumption 4 that $\left[\frac{\partial \tilde{S}}{\partial \mu_k} + \left(\frac{1-F(\theta)}{f(\theta)} \right) \frac{\partial^2 \tilde{\pi}}{\partial \mu_k \partial \theta} \right]$ is strictly decreasing in θ and $d\mu_k/d\theta > 0$ for at least one k, say $k^{'}$. Then, also notice that $\tilde{\mu}_k - \mu_k(\theta)$ is also decreasing, by construction, on the interval (θ_1, θ_2) for all k including $k^{'}$. Therefore, the $k^{'}$ term in the sum of the covariance terms is positive implying that the *sum* of the covariances is positive as the other terms are either also positive or zero. Thus the marginal increase in welfare from the variation is also positive. □

Therefore, as long as $\mu(\theta)$ is strictly increasing in some policy variable, we can construct a welfare-improving variation by flattening $\mu(\theta)$ along that variable. We will now use this idea to show that $w(\theta)$ must be a step-function. The proof is by contradiction and the argument is as follows: if the optimal $w(\theta)$ is not a step-function then we can show that $\mu(\theta)$ must increasing in at least one policy variable but that then we can find a variation (i.e. by flattening $\mu(\theta)$) that improves welfare, so a $w(\theta)$

that is not a step-function could not have been optimal. In the following lemma we present this argument formally while also establishing that such a variation is always feasible.

Lemma 5. *Given Assumption 4, $w(\theta)$ is a step-function.*

Proof. Suppose $w(\theta)$ is not a step-function. Then, the incentive-compatibility condition expresses the derivate of $w(\theta)$ in terms of the derivatives of the policy function:

$$\sum_k \frac{\partial \tilde{\pi}[\mu(\theta), \theta]}{\partial \mu_k} \frac{d\mu_k}{d\theta} + \frac{dw(\theta)}{d\theta} = 0.$$

As $\mu_k(\theta)$ is a piecewise-C^1 function for all k, this implies that $w(\theta)$ is also a piecewise-C^1 function. Then, $w(\theta)$ is differentiable everywhere except on a set of measure zero so we can always find an interval over which $dw/d\theta$ is strictly positive or negative. This entails that over this interval the feasibility constraint must be slack or $w(\theta) \leq \bar{w} - \epsilon$ for some $\epsilon > 0$. Incentive-compatibility also implies that whenever $dw(\theta)/d\theta \neq 0$ and $\partial\tilde{\pi}/\partial\mu_k \neq 0$ for some k, then the monotonicity constraint for μ_k is strict: $d\mu_k(\theta)/d\theta > 0$. Appealing then to the previous lemma, there exists a variation that improves welfare (assuming that it is feasible) so that any $w(\theta)$ that is not a step-function could not be optimal.

To ensure that the variation we have constructed is feasible, we need to adjust the continuation values in such way that $w(\theta; a) \leq \bar{w}$ for some $a \in [0, 1]$. The continuation values under the variation are given by (46). Notice that we can rewrite them as

$$\beta w(\theta; a) = \begin{cases} \beta w(\theta) & \text{for } \theta \leq \theta_1 \\ \beta w(\theta) + \Delta(a) & \text{for } \theta > \theta_1 \end{cases} \tag{47}$$

where $\Delta(a)$ is given by

$$\Delta(a) = \int_{\theta_1}^{\theta} \left[\frac{\partial \tilde{\pi}[\mu(z; a), z]}{\partial \theta} - \frac{\partial \tilde{\pi}[\mu(z), z]}{\partial \theta} \right] dz. \tag{48}$$

Whenever, $\Delta(a) \leq 0$ for $\theta \geq \theta_2$, the continuation values from the variation are equal to or lower than the continuation values for $\mu(\theta)$ outside the interval (θ_1, θ_2). An increase in continuation values then only occurs on (θ_1, θ_2). However, recall that $w(\theta) \leq \bar{w} - \epsilon$ on this interval and as $\tilde{\pi}$ is continuous we can find a small enough $a \in [0, 1]$ so that $w(\theta; a) \leq \bar{w}$. Therefore, the welfare-improving variation we constructed above is feasible. The intuition behind this procedure is that potentially raising the continuation values for types on the interval (θ_1, θ_2), by at most $\Delta(a)$, weakens their incentives to report their types as above θ_2. To fix this, we can lower the continuation values for all types above θ_2 by $\Delta(a)$. Now, whenever $\Delta(a) \geq 0$ for $\theta \geq \theta_2$, we can apply an analogous variation that gives us the same result. □

Having shown that $w(\theta)$ is a step-function, if we can show that it is continuous we know that it must be constant. Optimally then necessitates that this constant be $\bar{w}$. The continuity of both $\mu(\theta)$ and $w(\theta)$ is shown in the lemma below, the proof of which we relegate to Appendix C as it is largely technical in nature.

Lemma 6. *Given Assumptions 2 and 4, $\mu_k(\theta)$ is continuous for all k. Also, $w(\theta)$ is continuous and therefore $w(\theta) = \bar{w}$ for all $\theta \in \Theta$.*

Proof. See Appendix A in Athey, Atkeson, and Kehoe (2005). $\square$

B.5 Proof of Proposition 5

If the optimal policy is not μ^{FHC}, then given the lemmas above the optimal policy must coincide with the IRB policy over some interval. That is, $d\mu_k/d\theta > 0$ for all k on some $I \subset [\underline{\theta}, \overline{\theta}]$. This is clear from observing that when $w(\theta) = \bar{w}$ is imposed, the optimal policy solves a problem that is an affine transformation of the problem solved by the IRB policy so that both must satisfy (37).

On the rest of the interval, as $d\mu_k/d\theta = 0$ for all k in order for (37) to hold, the optimal policy is flat. In fact, the subset on which $\mu(\theta) = \mu^{IRB}$ is a connected set. If the set was disconnected, the optimal policy would be discontinuous contrary to what was shown in Lemma 6. To see this last point, note that between the two disconnected segments where the optimal policy coincides with the IRB policy, it would be flat. Thus, the optimal policy would need to jump up at the start of the second segment in order to coincide again with $\mu^{IRB}(\theta)$ as the latter is strictly increasing in θ. Thus, $\mu(\theta)$ has the following form:

$$\mu(\theta) = \begin{cases} \mu_1 = \mu^{IRB}(\theta_1) & \text{if } \theta \in [\underline{\theta}, \theta_1) \\ \mu^{IRB}(\theta) & \text{if } \theta \in [\theta_1, \theta') \\ \mu_2 = \mu^{IRB}(\theta') & \text{if } \theta \in (\theta', \overline{\theta}] \end{cases} \tag{49}$$

We need to now show that $\theta_1 = \underline{\theta}$. To see this consider an alternate policy $\tilde{\mu}(\theta)$ that is has the same form as $\mu(\theta)$ above except that it lowers θ_1. Then, $\tilde{\mu}(\theta)$ improves welfare for all types $\theta < \theta_1$ as $\mu^{IRB}(\theta)$ is strictly increasing in θ. Formally, the change in welfare from lowering θ_1 is given by:

$$\begin{aligned}
\frac{dV}{d\theta_1} &= \frac{d}{d\theta_1}\left[\int_{\underline{\theta}}^{\overline{\theta}} [\tilde{S}[\mu(\theta)] + \left(\frac{1-F(\theta)}{f(\theta)}\right)\frac{\partial\tilde{\pi}[\mu(\theta),\theta]}{\partial\theta} + \beta\bar{w}]dF(\theta)\right] \\
&= \frac{d}{d\theta_1}\left[\int_{\underline{\theta}}^{\theta_1} [\tilde{S}[\mu^{IRB}(\theta_1)] + \left(\frac{1-F(\theta)}{f(\theta)}\right)\frac{\partial\tilde{\pi}[\mu^{IRB}(\theta_1),\theta]}{\partial\theta} + \beta\bar{w}]dF(\theta)\right. \\
&\qquad\qquad \left. + \int_{\theta_1}^{\overline{\theta}} [\tilde{S}[\mu^{IRB}(\theta)] + \left(\frac{1-F(\theta)}{f(\theta)}\right)\frac{\partial\tilde{\pi}[\mu^{IRB}(\theta),\theta]}{\partial\theta} + \beta\bar{w}]dF(\theta)\right] \\
&= \frac{d}{d\theta_1}\left[[\tilde{S}[\mu^{IRB}(\theta_1)] + \left(\frac{1-F(\theta)}{f(\theta)}\right)\frac{\partial\tilde{\pi}[\mu^{IRB}(\theta_1),\theta_1]}{\partial\theta} + \beta\bar{w}]f(\theta_1)\right] \\
&\qquad\qquad + \int_{\underline{\theta}}^{\theta_1} \frac{d}{d\theta_1}\left[\tilde{S}[\mu^{IRB}(\theta_1)] + \left(\frac{1-F(\theta)}{f(\theta)}\right)\frac{\partial\tilde{\pi}[\mu^{IRB}(\theta_1),\theta]}{\partial\theta} + \beta\bar{w}\right] dF(\theta) \\
&\qquad\qquad\qquad - \frac{d}{d\theta_1}\left[[\tilde{S}[\mu^{IRB}(\theta_1)] + \left(\frac{1-F(\theta)}{f(\theta)}\right)\frac{\partial\tilde{\pi}[\mu^{IRB}(\theta_1),\theta_1]}{\partial\theta} + \beta\bar{w}]f(\theta_1)\right] \\
&= \int_{\underline{\theta}}^{\theta_1} \sum_k \left[\left(\frac{\partial\tilde{S}[\mu^{IRB}(\theta_1)]}{\partial\mu_k} + \left(\frac{1-F(\theta)}{f(\theta)}\right)\frac{\partial^2\tilde{\pi}[\mu^{IRB}(\theta_1),\theta]}{\partial\mu_k\partial\theta}\right)\frac{\partial\mu_k^{IRB}(\theta_1)}{\partial\theta}\right] dF(\theta)
\end{aligned}$$

To see that this change is positive note that $\frac{\partial \mu_k^{IRB}(\theta_1)}{\partial \theta} > 0$ for all k as the IRB policy is strictly increasing. Also $\mu^{IRB}(\theta_1)$ is suboptimal at $\theta < \theta_1$, so from (16) $\frac{\partial \tilde{S}[\mu^{IRB}(\theta_1)]}{\partial \mu_k} + \left(\frac{1-F(\theta)}{f(\theta)}\right) \frac{\partial^2 \tilde{\pi}[\mu^{IRB}(\theta_1),\theta]}{\partial \mu_k \partial \theta} < 0$ for all k. Finally, as $dF(\theta) > 0$ and $d\theta_1 < 0$ (we are decreasing θ_1),

$$dV = \left(\int_{\underline{\theta}}^{\theta_1} \sum_k \left[\left(\frac{\partial \tilde{S}[\mu^{IRB}(\theta_1)]}{\partial \mu_k} + \left(\frac{1-F(\theta)}{f(\theta)}\right) \frac{\partial^2 \tilde{\pi}[\mu^{IRB}(\theta_1),\theta]}{\partial \mu_k \partial \theta}\right) \frac{\partial \mu_k^{IRB}(\theta_1)}{\partial \theta}\right] dF(\theta)\right) d\theta_1 > 0$$