Coordination and Cheap Talk in a Battle of the Sexes with Private Information

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Abstract

We consider a Battle of the Sexes game with incomplete information and allow cheap talk regarding players’ private information before the game is played. We prove that the unique fully revealing symmetric cheap talk equilibrium has a desirable coordination property. Such coordination can also be obtained as a partially revealing cheap talk equilibrium. These outcomes can also be achieved using corresponding incentive compatible mechanisms, however, for different ranges of the prior probability.

Keywords: Battle of the Sexes, Private Information, Cheap Talk, Coordination, Mechanism.

JEL Classification Numbers: C 7 2.

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1 INTRODUCTION

In games with multiple (Nash) equilibria, players need to coordinate their actions in order to achieve one of the equilibrium outcomes. Such a coordination problem is more severe in situations, as in the Battle of the Sexes (BoS, hereafter), where none of the equilibria can be naturally selected. In a seminal paper, Farrell (1987) showed that rounds of cheap talk regarding the intended choice of play reduces the probability of miscoordination; at the limit, the probability of coordination on one of the two pure Nash equilibria is 1. Park (2002) considered a similar game with three players and identified conditions for achieving efficiency and coordination.

We analyse a class of cheap talk equilibria in the BoS with incomplete information, where each player has private information about the “intensity of preference” for the other player’s favorite outcome (e.g., how strongly the “Husband” (dis)likes “Concert”). As is well-known, the complete information BoS has many economic applications; the corresponding game of incomplete information is not just a natural extension but is also very relevant in many of these economic situations where the intensity of preference and its prior probability are important factors of the model (for possible applications of the BoS to IO models, see the Introduction in Cabrales et al. 2000).

The version of the BoS we consider has two types (‘High’ and ‘Low’) for each player regarding the payoff from the other player’s favourite outcome (e.g., the payoff of the “High-type Husband” from (Concert, Concert) is more than that of the “Low-type Husband”, although, both types’ payoff from (Football, Football) is higher than that from (Concert, Concert)). This simplest form of the BoS with private information is clearly of interest for its applications and also because it is not clear whether the result from the complete information BoS would extend to this incomplete information version.

We study the BoS with incomplete information augmented by an initial stage of cheap talk in which both players make simultaneous announcements on their possible types only and we look for (type and player) symmetric cheap talk equilibria. We would like to check if coordination and efficiency are achieved in the equilibrium outcomes in this context (efficiency is automatically obtained with coordination in the complete information game but, with incomplete information, efficiency and coordination do not necessarily go together). With incomplete information, one might be interested in coordination on the (ex-post) efficient outcome when the two players are of different types, in which the compromise is made by the player who suffers a smaller loss in utility (e.g., the “High-type Husband” and the “Low-type Wife” coordinate on (Concert, Concert)). Our aim here is two-fold. We look for such coordination as an equilibrium of this extended game, using a round of pre-play cheap talk. Moreover, we would like the players to reveal their types truthfully before coordinating.

We first prove that there indeed exists, for a certain range of the prior probability of the High-type, a (unique) fully revealing symmetric cheap talk equilibrium and it has the desirable type-coordination
property: when the players’ types are different, it fully coordinates on the ex-post efficient pure Nash equilibrium (Theorem 1). We then ask whether such type-coordination can also be obtained when the fully revealing equilibrium does not exist and players are not truthful. Keeping the spirit of the fully revealing equilibrium, we consider a class of partially revealing equilibria in which only the High-type is not truthful, while the Low-type is truthful. Analysing this class of possible partially revealing equilibria seems reasonable because under the type-coordination property, the High-type is expected to compromise and coordinate on his less preferred outcome when the other player claims to be the Low-type. We identify the equilibria with the type-coordination property and prove their existence based on the prior probability of the High-type (Theorem 2).

Finally, we check whether our cheap talk equilibria can be achieved as mediated outcomes using incentive compatible mechanisms, as studied in Banks and Calvert (1992) who characterised the (ex-ante) efficient incentive compatible mechanism for this game. As expected, we note that the outcome of the fully revealing cheap talk equilibrium can be obtained as an outcome of an incentive compatible mechanism for a strictly larger range of the prior probability of the High-type. On the other hand, what is less intuitive and somewhat surprising is that a partially revealing equilibrium outcome may be achieved as an outcome of an incentive compatible mechanism for a strictly smaller range of the prior probability of the High-type. A probable reason might be that the incentive compatible mechanism is restricted to truthful revelations in the announcement stage whereas our cheap talk equilibrium allows partial revelation by High-type players.

2 MODEL

2.1 The Game

We consider a version of the BoS with incomplete information as given below, in which each of the two players has two strategies, namely, F (Football) and C (Concert). The payoffs corresponding to the outcomes are as in the following table, in which the value of $t_i$ is the private information for player $i$, with $0 < t_1, t_2 < 1$.

<table>
<thead>
<tr>
<th>Wife (Player 2)</th>
<th>Football</th>
<th>Concert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Husband (Player 1) Football</td>
<td>$1, t_2$</td>
<td>$0, 0$</td>
</tr>
<tr>
<td>Concert</td>
<td>$0, 0$</td>
<td>$t_1, 1$</td>
</tr>
</tbody>
</table>

We assume that $t_i$ is a discrete random variable that takes only two values $L$ and $H$ (where, $0 < L < H < 1$), whose realisation is only observed by player $i$. For $i = 1, 2$, we henceforth refer to the
values of \( t_i \) as player \( i \)'s type (Low, High). We further assume that each player’s type is independently drawn from the set \( \{L, H\} \) according to a probability distribution with \( \text{Prob}(t_i = H) = p \in [0, 1] \).

The unique symmetric Bayesian-Nash equilibrium\(^1\) of this game can be characterised by \( \sigma_i(s_i | t_i) \), the probability that player \( i \) of type \( t_i \) plays the pure strategy \( s_i \). The unique symmetric Bayesian Nash equilibrium is given by the following strategy for player 1 (player 2’s strategy is symmetric and is given by \( \sigma_1(F | t) = \sigma_2(C | t), t = H, L \)):

\[
\begin{align*}
\sigma_1(F | H) &= 0 \text{ and } \sigma_1(F | L) = \frac{1}{(1-p)(1+L)} \text{ when } p < \frac{L}{1+L} \\
\sigma_1(F | H) &= 0 \text{ and } \sigma_1(F | L) = 1 \text{ when } \frac{L}{1+L} \leq p \leq \frac{H}{1+H} \\
\sigma_1(F | H) &= 1 - \frac{H}{p(1+H)} \text{ and } \sigma_1(F | L) = 1 \text{ when } p > \frac{H}{1+H}.
\end{align*}
\]

### 2.2 Cheap Talk

We study an extended game (as presented in Ganguly and Ray (2009)) in which the players are first allowed to have a round of simultaneous canonical cheap talk intending to reveal their private information before they play the above BoS. In the first (cheap talk) stage of this extended game, each player \( i \) simultaneously chooses a costless and nonbinding announcement \( \tau_i \) from the set \( \{L, H\} \). Then, given a pair of announcements \( (\tau_1, \tau_2) \), in the second (action) stage of this extended game, each player \( i \) simultaneously chooses an action \( s_i \) from the set \( \{F, C\} \).

An announcement strategy in the first stage for player \( i \) is a function \( a_i : \{L, H\} \rightarrow \Delta(\{L, H\}) \), where \( \Delta(\{L, H\}) \) is the set of probability distributions over \( \{L, H\} \). We write \( a_i(H | t_i) \) for the probability that strategy \( a_i(t_i) \) of player \( i \) with type \( t_i \) assigns to the announcement \( H \). Thus, the announcement \( \tau_i \) of player \( i \) with type \( t_i \) is a random variable drawn from \( \{L, H\} \) according to the probability distribution with \( \text{Prob}(\tau_i = H) = a_i(H | t_i) \).

In the second (action) stage, a strategy for player \( i \) is a function \( \sigma_i : \{L, H\} \times \{L, H\} \times \{L, H\} \rightarrow \Delta(\{F, C\}) \), where \( \Delta(\{F, C\}) \) is the set of probability distributions over \( \{F, C\} \). We write \( \sigma_i(F | t_i; \tau_1, \tau_2) \) for the probability that strategy \( \sigma_i(t_i; \tau_1, \tau_2) \) of player \( i \) with type \( t_i \) assigns to the action \( F \) when the first stage announcements are \( (\tau_1, \tau_2) \). Thus, player \( i \) with type \( t_i \)'s action choice \( s_i \) is a random variable drawn from \( \{F, C\} \) according to a probability distribution with \( \text{Prob}(s_i = F) = \sigma_i(F | t_i; \tau_1, \tau_2) \).

Given a pair of realised action choices \( (s_1, s_2) \in \{F, C\} \times \{F, C\} \), the corresponding outcome is generated. Thus, given a strategy profile \( ((a_1, \sigma_1), (a_2, \sigma_2)) \), one can find the players’ actual payoffs from the induced outcomes in the type-specific payoff matrix of the BoS and hence, the (ex-ante) expected payoffs.

\(^1\)The corresponding game with complete information with commonly known values \( t_1 \) and \( t_2 \), has two pure Nash equilibria, \( (F, F) \) and \( (C, C) \), and a mixed Nash equilibrium in which player 1 plays \( F \) with probability \( \frac{1}{1+\sqrt{2}} \) and player 2 plays \( C \) with probability \( \frac{1}{1+\sqrt{2}} \).
As the game is symmetric, in our analysis, we maintain the following notion of symmetry in the strategies, for the rest of the paper.

**Definition 1** A strategy profile \(((a_1, \sigma_1), (a_2, \sigma_2))\) is called announcement-symmetric (in the announcement stage) if \(a_i(H | t_i) = a_{-i}(H | t_i)\); a strategy profile is called action-symmetric (in the action stage) if \(\sigma_i(F | t; \tau_1, \tau_2) = \sigma_{-i}(C | t; \tau_2, \tau_1)\), for all \(t, \tau_1, \tau_2\). A strategy profile is called symmetric if it is both announcement-symmetric and action-symmetric.

Note that Definition 1 preserves symmetry for both players and the types for each player. As explained in the Introduction, we are interested in a specific form of coordination in which the players play \((C, C) ((F, F))\) when only player 1’s type is \(H (L)\), that we call type-coordination.

**Definition 2** A strategy profile \(((a_1, \sigma_1), (a_2, \sigma_2))\) is said to have the type-coordination property if the induced outcome is \((C, C)\) when the players’ true type profile is \((H, L)\) and is \((F, F)\) when the players’ true type profile is \((L, H)\).

Note that in such a profile, when the players’ types are different, players fully coordinate on a pure Nash equilibrium outcome that generate the ex-post efficient payoffs of 1 and \(H\).

We consider the following standard notion of equilibrium in this two-stage cheap talk game.

**Definition 3** A symmetric strategy profile \(((a_1, \sigma_1), (a_2, \sigma_2))\) is called a symmetric cheap talk equilibrium if (i) given the announcement strategies \((a_1, a_2)\), the action strategies \((\sigma_1, \sigma_2)\) constitute a Bayesian-Nash equilibrium in the simultaneously played second stage BoS with the posterior beliefs generated by \((a_1, a_2)\) and (ii) the announcement strategies \((a_1, a_2)\) are Bayesian-Nash equilibrium of the simultaneous announcement game given the action strategies \((\sigma_1, \sigma_2)\) to be followed.

Definition 3 suggests that a symmetric cheap talk equilibrium can be characterised by a set of (symmetric) equilibrium constraints (2 for the announcement stage and another possible 8 for the action stage).

## 3 RESULTS

We first consider the possibility of full revelation of the types as a result of our canonical cheap talk.

### 3.1 Full Revelation

We consider a specific class of strategies in this section where we impose the property that the cheap talk announcement should be fully revealing.
**Definition 4** A symmetric strategy profile \( ((a_1, \sigma_1), (a_2, \sigma_2)) \) is called fully revealing if the announcement strategy \( a_i \) reveals the true types with certainty, i.e., \( a_i(H|H) = 1 \) and \( a_i(H|L) = 0 \).

Definition 4 simply asserts that, in the cheap talk stage, each player makes an announcement that coincides with that player’s type: \( \tau_i = t_i \). Note that, under fully revealing announcements, for player \( i \) with type \( t_i \), the action strategy can be written as \( \sigma_i(t_i, t_j) \).

Consider a specific fully revealing (separating) strategy profile that we call \( S_{\text{separating}} \), influenced by the equilibrium action profile in Farrell (1987) for the complete information version of this game. In this strategy profile, the players announce their types truthfully and then in the action stage, they play the mixed Nash equilibrium strategies of the complete information BoS when both players’ types are identical and they play \((C, C) ((F, F))\), when only player 1’s type is \( H \) \((L)\). Formally, \( S_{\text{separating}} \) is characterised by \( \sigma_1[F|H, L] = 0, \sigma_1[F|L, H] = 1 \) and \( \sigma_1[F|t, t] = \sigma^i(tt) \) with \( t = H, L \), where \( \sigma^i(tt) \) is the mixed Nash equilibrium of the complete information BoS with values \( t_1 = t_2 = t, t = H, L \) (see Footnote 1).

We now characterise fully revealing symmetric cheap talk equilibria. Using Definition 3, we first observe the following facts.

**Claim 1** In a fully revealing symmetric cheap talk equilibrium \( ((a_1, \sigma_1), (a_2, \sigma_2)) \), the players’ strategies in the action phase must constitute a (pure or mixed) Nash equilibrium of the corresponding complete information BoS, that is, \( (\sigma_1(t_1, t_2), \sigma_2(t_1, t_2)) \) is a (pure or mixed) Nash equilibrium of the BoS with values \( t_1 \) and \( t_2, \forall t_1, t_2 \in \{H, L\} \).

**Claim 2** In a fully revealing symmetric cheap talk equilibrium \( ((a_1, \sigma_1), (a_2, \sigma_2)) \), conditional on the announcement profile \((H, H)\) or \((L, L)\), the strategy profile in the action phase must be the mixed strategy Nash equilibrium of the corresponding complete information BoS, that is, whenever \( t_1 = t_2 \), \((\sigma_1(t_1, t_2), \sigma_2(t_1, t_2)) \) is the mixed Nash equilibrium of the BoS with values \( t_1 = t_2 \).

Based on the above claims, one can easily identify all the candidate equilibrium strategy profiles of the extended game that are fully revealing and symmetric. Claim 2 implies that these profiles are differentiated only by the actions played when \( t_1 \neq t_2 \), that is, when the players’ types are \((H, L)\) and \((L, H)\).

As the strategies are symmetric, it is sufficient to characterise these candidate profiles only by \( \sigma_1 [F|H, L] \). From Claim 1, there are only three possible candidates for \( \sigma_1 [F|H, L] \) as the complete information BoS with values \( H \) and \( L \) has three (two pure and one mixed) Nash equilibria. These profiles are (i) \( \sigma_1 [F|H, L] = \sigma^i(HL) \) where \( \sigma^i(HL) \) is the probability of playing \( F \) in the mixed Nash equilibrium strategy of player 1 of the complete information BoS with values \( t_1 = H \) and \( t_2 = L \), that we call \( S_m \); (ii) \( \sigma_1 [F|H, L] = 1 \), that we call \( S_{\text{ineff}} \) and (iii) \( \sigma_1 [F|H, L] = 0 \), which indeed is \( S_{\text{separating}} \). We now prove our first result.
Theorem 1 \( S_{\text{separating}} \) is the unique fully revealing symmetric cheap talk equilibrium and it exists only for \( \frac{L^2+2LH}{1+L+LH+LH^2} \leq p \leq \frac{LH+LH^2}{1+L+LH+LH^2} \).

Proof. We first show that \( S_m \) is not an equilibrium. Under \( S_m \), \( H \)-type will announce his type truthfully only if \( p(H|\text{H})+(1-p)(L|\text{H}) \geq p(H|\text{H})+(1-p)(L|\text{H}) \), where the LHS is the expected payoff from truthfully announcing \( H \) and the RHS is the expected payoff from announcing \( L \) and choosing the corresponding optimal action strategy. This inequality implies \( \frac{1}{1+H} \geq \frac{1}{1+L} \) which can never be satisfied as \( H > L \).

The second candidate strategy profile, \( S_{\text{ineff}} \) is an equilibrium only when \( \frac{1+H}{1+L+LH+LH^2} \leq p \) and \( p \leq \frac{1+L+H+L^2}{1+L+LH+LH^2} \). To see this, note that under \( S_{\text{ineff}} \), \( H \)-type will announce his type truthfully only if \( p(H|\text{H})+(1-p) \geq p(H|\text{H})+(1-p)(H|\text{H}) \) which implies \( p \leq \frac{1+L+H+L^2}{1+L+LH+LH^2} \). Similarly, \( L \)-type will announce his type truthfully only if \( p(L+1-p)(L|\text{L}) \geq p(H|\text{H})+(1-p)(L|\text{L}) \) which implies \( \frac{1+H}{1+L+LH+LH^2} \leq p \). However, it can be shown that \( \frac{1+L+H+L^2}{1+L+LH+LH^2} > \frac{1+L+LH+LH^2}{1+L+LH+LH^2} \). Hence, \( S_{\text{ineff}} \) cannot be an equilibrium.

Finally, we prove that \( S_{\text{separating}} \) is an equilibrium only when \( \frac{L^2+2LH}{1+L+LH+LH^2} \leq p \leq \frac{LH+LH^2}{1+L+LH+LH^2} \).

Under \( S_{\text{separating}} \), \( H \)-type will announce his type truthfully only if \( p(H|\text{H})+(1-p)H \geq p + (1-p)(H|\text{H}) \) which implies \( p \leq \frac{H+H^2}{1+L+LH+LH^2} \). Similarly, \( L \)-type will announce his type truthfully only if \( p + (1-p)(H|\text{L}) \geq p(H|\text{H})+(1-p)L \) which implies \( \frac{L^2+LH}{1+L+LH+LH^2} \leq p \).

Clearly, \( S_{\text{separating}} \) satisfies the type-coordination property as defined in Definition 2. One can also prove that the upper bound for \( p \) in Theorem 1, \( \frac{LH+LH^2}{1+L+LH+LH^2} \) is always \( \frac{1}{2} \). The ex-ante expected payoff for any player from \( S_{\text{separating}} \) is given by \( EU_{\text{separating}} = p^2 \frac{H}{1+H} + p(1-p)(1+H) + (1-p)^2 \frac{L}{1+H}, \) which is increasing over the range of \( p \) where it exists. Hence, the best achievable payoff from \( S_{\text{separating}} \) (when \( p = \frac{LH+LH^2}{1+L+LH+LH^2} \)) is \( \frac{L(1+L+LH+2LH^2+2LH^3+LH^4+H^2+2H^3+H^4)}{(1+L+LH+LH^2)^2} \).

3.2 Partial Revelation

We now consider a class of partially revealing symmetric strategy profiles of the cheap talk game. Keeping the spirit of the unique fully revealing equilibrium and the type-coordination property, we restrict our attention to announcement strategy profiles in which only the \( L \)-type truthfully reveals while the \( H \)-type does not (as under the type-coordination property, the \( H \)-type is expected to coordinate on \( (C,C) \), when the opponent is of \( L \)-type).

Formally, consider a symmetric announcement strategy profile in which the \( H \)-type of player \( i \) announces \( H \) with probability \( r \) and \( L \) with probability \( (1-r) \) and the \( L \)-type of player \( i \) announces \( L \) with probability \( 1 \), i.e., \( a_i(H|H) = r \) and \( a_i(H|L) = 0 \).

Clearly, after the cheap talk phase, the possible message profiles \((\tau_1, \tau_2)\) that the \( H \)-type of player 1 may receive are \((H, H), (H, L), (L, H) \) or \((L, L)\) while the \( L \)-type of player 1 may receive either \((L, H)\) or \((L, L)\). Let us denote an action-strategy of Player 1 by \( \sigma_1(F|H; H, H) = q_0, \sigma_1(F|H; H, L) = q_1, \)
\( \sigma_1(F|H; L, H) = q_2, \sigma_1(F|H; L, L) = q_3, \sigma_1(F|L; L, H) = q_4 \) and \( \sigma_1(F|L; L, L) = q_5 \). By symmetry, a partially revealing symmetric strategy profile \(((a_1, \sigma_1), (a_2, \sigma_2))\) in our set-up can thus be identified by \((r, q_0, q_1, q_2, q_3, q_4, q_5)\).

We now characterise the set of partially revealing (pooling) symmetric cheap talk equilibria in this set-up. First note that, on receiving the message profile \((H, H)\), the players know the true types and hence in any such equilibrium, \(q_0\) has to correspond to the mixed Nash equilibrium of the complete information BoS with values \(H\) and \(H\). Thus, in any partially revealing symmetric cheap talk equilibrium, \(q_0 = \frac{1}{1+r}\) (as in Footnote 1). Hence, \((r, q_1, q_2, q_3, q_4, q_5)\) characterises such a pooling equilibrium.

We are interested to know whether the type-coordination property can be obtained by any such pooling equilibrium. Note that for the type-coordination property to hold, we just need to look for equilibria satisfying \(q_1 = 0\) and \(q_3 = 0\). Using symmetry, for player 2 \((H\text{-type})\), we then must have \(\sigma_2(F|H; L, H) = 1 - q_1 = 1\). This implies that in any such equilibrium, \(q_2 = 1\) and \(q_4 = 1\). Thus, any partially revealing symmetric cheap talk equilibrium satisfying the type-coordination property can be characterised by only \(q_5\) and \(r\). We find two such equilibrium profiles, one involving a pure action-strategy \(q_5\) and the other a mixed strategy for \(q_5\). We state our main result below.

**Theorem 2** The only partially revealing symmetric cheap talk equilibria (where only the L-type is truthful) satisfying the type-coordination property are given by:

(i) pure-action: \(q_5 = 1\) and \(r = \frac{H+H^2}{1+H+H^2}\) and it exists only when \(p \geq \frac{L+LH+LH^2}{1+H+H^2}\).

(ii) mixed-action: \(q_5 = \frac{1}{1-p+L+LH-LH^2-LH^2p-LH^2} = \frac{1}{1-p+LH+LH^2} = \frac{LH+LH^2}{L+LH+LH^2}\) and \(r = \frac{LH+LH^2}{p+Lp+LH^2+LH^2} = \frac{LH+LH^2}{1+L+LH+LH^2}\) and it exists only when \(\frac{LH+LH^2}{1+L+LH+LH^2} < p < \frac{LH+LH^2}{1+H+H+H^2}\).

**Proof.** First note that, if \(q_3 = 0\) and \(q_5 = 0\), player 1 \((H\text{-type})\), after receiving message \((L, L)\), obtains a higher expected payoff (of 1) from playing \(F\) than the expected payoff (of 0) from playing \(C\), which is a contradiction. So, under \(q_3 = 0\), the only possible values of \(q_5\) are either \(q_5 = 1\) or \(0 < q_5 < 1\).

(i) We first consider \(q_5 = 1\). Thus, we have \(q_1 = q_3 = 0\), \(q_2 = q_4 = q_5 = 1\). Here, to be in equilibrium, in the announcement phase, player 1 \((H\text{-type})\) should be indifferent between announcing \(H\) and \(L\). This implies \((1-p) H + p \left( \frac{H}{1+p} + (1-r) H \right) = (1-p) H + pr\), which gives us \(r = \frac{H+H^2}{1+H+H^2}\).

Also, to be in equilibrium, in the announcement phase, player 1 \((L\text{-type})\) should prefer to announce \(L\). This requires \(p \geq \max (1-p)(1-x) L + p \left( \frac{H}{1+p} + (1-r)(1-x) L \right)\), where \(x\) is the optimal probability of playing \(F\) in the action phase if he announces \(H\) and receives the message profile \((H, L)\). Note that the derivative of the RHS with respect to \(x\) (with \(r = \frac{H+H^2}{1+H+H^2}\)) is \(-(1-p) L - pL \left(1 - \frac{H+H^2}{1+H+H^2}\right) < 0\) which implies that the argmax is attained at \(x = 0\) and thus the maximum value of the RHS is \(\frac{1}{1+H+H^2} (L + LH - LH^2 + p + LH^2 - LH^2p)\). Hence, the equilibrium condition for
player 1 (L-type) to announce $L$ becomes $p \geq \frac{1}{1+L+H} (L + LH - LHp + H^2p + LH^2 - LH^2p)$, that is, $p \geq \frac{L+LH+LH^2}{1+L+H+LH}$. We thus have the pure-action equilibrium which exists for $p \geq \frac{L+LH+LH^2}{1+L+H+LH}$.

(ii) Next we consider $0 < q_5 < 1$. Here, we have two indifference equations, one for the announcement of player 1 (H-type) and the other for the action of player 1 (L-type) after the message profile $(L, L)$. These equations respectively become

$$
(1 - p)H + p\left(\frac{H}{1+p} + (1 - r)H\right) = (1 - p)Hq_5 + pr
$$

and

$$
\frac{p - pr}{1 - pr} + (1 - q_5)\left(\frac{1 - p - pr}{1 - pr}\right) = Lq_5\left(\frac{1 - p - pr}{1 - pr}\right).
$$

Solving the above equations, we get $r = \frac{LH + LH^2}{1 + L + LH + LH^2 + LH^2p}$ and $q_5 = \frac{1}{1 - p + L + LH - pL + LH^2 - LH^2p}$. We need to make sure that both $0 < r < 1$ and $0 < q_5 < 1$. Obviously, $r > 0$ and $q_5 = \frac{LH + LH^2}{1 + L + LH + LH^2}$ is also $> 0$. Now, it can be shown that $r < 1$ when $\frac{LH^2 + LH}{1 + L + LH + LH^2} < p < \frac{L + LH^2 + LH}{1 + L + LH + LH^2}$.

As in the earlier case, we need to check the equilibrium condition in the announcement phase for player 1 (L-type), which in this case is given by

$$(1 - p)q_5 (1 - q_5) (1 + L)p (r + (1 - r)q_5) \geq \max_x (1 - p) (1 - x) L + p\left(\frac{H}{1+p} + (1 - r) (1 - x) L\right).$$

The derivative of the RHS with respect to $x$ is $-L (1 - p) - Lp (1 - r) < 0$, which implies that the argmax is attained at $x = 0$ and thus the maximum value of the RHS becomes $\frac{L(1 + LH + L^2)}{1 + L + LH + LH^2}$. It can be shown that the LHS at $r = \frac{LH + LH^2}{1 + L + LH + LH^2}$ and $q_5 = \frac{1}{1 - p + L + LH - pL + LH^2 - LH^2p}$ is $\frac{L(1 + L + H^2)}{1 + L + LH + LH^2}$, which implies that LHS $> RHS$. Hence, we have the mixed-action equilibrium that exists only when $\frac{LH^2 + LH}{1 + L + LH + LH^2} < p < \frac{L + LH^2 + LH}{1 + L + LH + LH^2}$.

Note that the lower bound of $p$ for the mixed-action equilibrium is actually the upper bound of $p$ for $S_{separating}$, while the upper bound for the mixed-action equilibrium is higher than the (lower) bound for the pure-action equilibrium.

The expected utility from the pure-action equilibrium is $EU_{pure} = \frac{p(1 + H)}{1 + H + H^2} \cdot (1 - p + H + H^2 - H^2p)$ and from the mixed-action equilibrium is $EU_{mixed} = \frac{L + LH - Lp + H + H^2}{1 + L + LH + LH^2}$. One can prove that $EU_{mixed} > EU_{pure}$ for the range of $p$ where both of these equilibria exist, which is not surprising as there is complete miscoordination in the pure-action equilibrium when both players are of Low-type.

There do exist other partially revealing equilibria. Indeed, we fully characterised this set of equilibria and found 4 other equilibria (details of which are available upon request).

4 REMARKS

We analysed cheap talk equilibria in a BoS with private information. Following the seminal paper by Crawford and Sobel (1982), much of the cheap talk literature has focused on the sender-receiver framework whereby one player has private information but takes no action and the other player is uninformed but is responsible for taking a payoff-relevant decision. We have contributed to this
literature by analysing a game with two-sided information and two-sided cheap talk.

4.1 Mechanism

We now analyse a (direct) mechanism which is a symmetric probability distribution over the set of outcomes of BoS, for every profile of types (as in Banks and Calvert (1992), Ganguly and Ray (2009)). In such a mediated communication process, the players first report their types (H or L) to the mechanism (mediator) and then the mediator picks an action profile according to a given probability distribution and informs the respective action to each player privately. The players then play the game.

A symmetric direct mechanism is a probability distribution over the set of outcomes of the BoS for every profile of reported types, as below:

\[
\begin{array}{c|c|c}
 F & C \\
\hline
 F & \frac{1-v_3-v_4}{2} & v_7 \\
 C & v_6 & \frac{1-v_3-v_7}{2}
\end{array}
\]

\[
\begin{array}{c|c|c}
 F & C \\
\hline
 F & 1-v_3-v_4-v_5 & v_5 \\
 C & v_4 & v_3
\end{array}
\]

\[
\begin{array}{c|c|c}
 F & C \\
\hline
 F & v_3 & v_5 \\
 C & v_4 & 1-v_3-v_4-v_5
\end{array}
\]

\[
\begin{array}{c|c|c}
 F & C \\
\hline
 F & \frac{1-v_1-v_2}{2} & v_2 \\
 C & v_1 & \frac{1-v_1-v_2}{2}
\end{array}
\]

\[
\begin{array}{c|c|c}
 F & C \\
\hline
 F & \frac{1-v_3-v_4}{2} & v_7 \\
 C & v_6 & \frac{1-v_3-v_7}{2}
\end{array}
\]

\[
\begin{array}{c|c|c}
 F & C \\
\hline
 F & 1-v_3-v_4-v_5 & v_5 \\
 C & v_4 & v_3
\end{array}
\]

\[
\begin{array}{c|c|c}
 F & C \\
\hline
 F & v_3 & v_5 \\
 C & v_4 & 1-v_3-v_4-v_5
\end{array}
\]

\[
\begin{array}{c|c|c}
 F & C \\
\hline
 F & \frac{1-v_1-v_2}{2} & v_2 \\
 C & v_1 & \frac{1-v_1-v_2}{2}
\end{array}
\]

where all \(v_i\)'s lie in the closed interval \([0, 1]\). A symmetric direct mechanism is called a symmetric mediated equilibrium if it provides the players with incentives (i) to truthfully reveal their types to the mediator and (ii) to follow the mediator's recommendations following their type announcements. A symmetric mediated equilibrium thus can be characterised by 6 incentive compatibility constraints (see Banks and Calvert (1992) for details).

Among the class of symmetric mediated equilibria, Banks and Calvert (1992) have characterised the ex-ante efficient ones. We, however, are interested in achieving the cheap talk equilibria in this paper as outcomes of incentive compatible mechanisms. We observe the following facts (detailed proofs of which are available upon request).

First, we note that the outcome of \(S_{\text{separating}}\) can be obtained as a symmetric mediated equilibrium when
\[
\frac{L^2+2L^2H+L^2H^2}{1+L+H+LH^2+L^2H+L^2H^2+H^2} \leq p \leq \frac{-L+H+LH^2+L^2H+L^2H^2+H^2}{1+L+H+LH^2+L^2H+L^2H^2+H^2}.
\]
One might be interested in comparing the above range of $p$ with the range of $p$ for $S_{separating}$ in Theorem 1. Not surprisingly, the above range strictly contains the range for $S_{separating}$, i.e.,

\[
\frac{L^2 + L^2 H + L^2 H^2}{1 + L + L H + L^2 H + L^2 H^2} < \frac{L^2 + L^2 H}{1 + L + L^2 H} \text{ and } \frac{L H + L H^2}{1 + L + L H + L^2 H + L^2 H^2} < \frac{L + L H + L H^2 + L^2 H^2 + H^2}{1 + L + L H + L^2 H + L^2 H^2 + H^2}.
\]

Our partially revealing equilibrium outcomes can also be obtained as symmetric mediated equilibria. However, rather surprisingly, the range for $p$ may not be larger here. For example, the outcome of the pure-action equilibrium in Theorem 2 can be obtained as a symmetric mediated equilibrium when

\[
\frac{L + L H + L H^2}{1 + L + L H + L^2 H^2} \leq p \leq \frac{H + H^2 + H^3}{1 + H + H^2 + H^3}.
\]

This range for $p$ is strictly contained in the range for $p$ for the pure-action equilibrium in Theorem 2 (as $\frac{L + L H + L H^2}{1 + L + L H + L^2 H} < \frac{L + L H + L H^2}{1 + L + L H^2 + H^2}$).

### 4.2 Examples

We may illustrate all our results using specific parameter values. Take for example, $L = \frac{1}{3}$, $H = \frac{2}{3}$.

For these values, the range of the prior $p$ for which $S_{separating}$ exists is $\frac{5}{21} (\approx 0.24) \leq p \leq \frac{5}{21} (\approx 0.22)$. The (best) payoff from $S_{separating}$ (at $p = \frac{5}{21}$) is $\frac{24}{32}$ (approx. 0.46). The range of $p$ for which this outcome can be obtained as a symmetric mediated equilibrium is given by $\frac{25}{229} (\approx 0.11) \leq p \leq \frac{85}{229} (\approx 0.37)$, with the corresponding (best) payoff (at $p = \frac{85}{229}$) of $\frac{28474}{521401} (\approx 0.54)$.

For these parameter values, the pure-action equilibrium in Theorem 2 exists as a partially revealing cheap talk equilibrium where the High-type reveals its true type with probability $\frac{10}{39} (r \approx 0.53)$ when $p > \frac{10}{39} (\approx 0.35)$. This outcome can be obtained as a symmetric mediated equilibrium when $\frac{19}{57} (\approx 0.37) \leq p \leq \frac{38}{65} (\approx 0.58)$. Finally, the mixed-action equilibrium is given by $q_5 = \frac{27}{40(1-p)}$, $r = \frac{5}{23p}$ and it exists for $\frac{5}{37} (\approx 0.22) < p < \frac{18}{30} (\approx 0.61)$, with $EU_{mixed} = \frac{5}{37} H + \frac{10}{37}$; thus, the payoff from the mixed-action equilibrium is $\frac{252}{10038} (\approx 0.24)$ at $p = \frac{5}{37}$ and is $\frac{10445}{221549} (\approx 0.49)$ at $p = \frac{10}{37}$.

### 4.3 Asymmetric Equilibrium

We have characterised the unique fully revealing symmetric equilibrium of the cheap talk game. There are of course many fully revealing but asymmetric equilibria of this extended game. Clearly, babbling equilibria exist in which the players ignore the communication and just play one of the Nash equilibria of the complete information BoS for all type-profiles. There are other asymmetric equilibria. Consider for example, the following strategy in the action stage after full revelation: $\sigma_1(H, H) = \sigma_1(H, L) = 0$, $\sigma_1(L, H) = 1$ and $\sigma_1(L, L) = \sigma_1(LL)$, where $\sigma'(LL)$ is the mixed Nash equilibrium of the complete information BoS when $t_1 = t_2 = L$. Call it $S_{asym}$.

One can prove (proof is available upon request) that $S_{asym}$ is a fully revealing cheap talk equilibrium when $L^2 \leq p \leq \frac{L H}{1 + L + H}$.
4.4 One-sided Talk

Finally, we note that the compromise feature of the cheap talk equilibria analysed here cannot be obtained with one-sided cheap talk where only player 1 is allowed to talk. One can prove (proof is available upon request) that the type-coordination property cannot be achieved as either a fully revealing (the $H$-type announces $H$ and the $L$-type announces $L$) or a partially revealing equilibrium (the $H$-type announces $H$ with probability $r$ and $L$ with probability $(1-r)$ while the $L$-type announces $L$ with probability 1), followed by an action strategy of player 1 where $H$-type compromises to play $C$ and the $L$-type plays $F$ in the action stage. This suggests that two-sided cheap talk achieves more type-coordination than one-sided cheap talk in our setup; this is in contrast with the well-known experimental results for cheap talk in the complete information BoS (Cooper, et al. 1989).

5 REFERENCES


