Information-Revelation and Coordination Using Cheap Talk in a Battle of the Sexes with Two-Sided Private Information
Information-Revelation and Coordination Using Cheap Talk in a Battle of the Sexes with Two-Sided Private Information*

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Abstract

We consider a Battle of the Sexes game with two types, High and Low, for each player and allow cheap talk regarding players' types before the game. We prove that the unique fully revealing symmetric cheap talk equilibrium exists for a low range of prior probability of the High-type. This equilibrium has a desirable type-coordination property: it fully coordinates on the ex-post efficient pure Nash equilibrium when the players' types are different. Type-coordination is also obtained in a partially revealing equilibrium in which only the High-type is not truthful, for a medium range of prior probability of the High-type.

Keywords: Battle of the Sexes, Private Information, Cheap Talk, Coordination, Full Revelation.

JEL Classification Numbers: C72.

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1 INTRODUCTION

In games with multiple (Nash) equilibria, players need to coordinate their actions in order to achieve one of the equilibrium outcomes. Such a coordination problem is more severe in situations, as in the Battle of the Sexes (BoS, hereafter), where none of the equilibria can be naturally selected. In a seminal paper, Farrell (1987) showed that rounds of cheap talk regarding the intended choice of play reduces the probability of miscoordination; the probability of coordination on one of the two pure Nash equilibria increases with the number of rounds of communication (although, at the limit, may be bounded away from $1$).\footnote{Park (2002) identified conditions for achieving efficiency and coordination in a similar game with three players.} Parallel to the theory, the experimental literature also shows that cheap-talk and any pre-play non-binding communication can significantly improve coordination in games like BoS (Cooper \textit{et al} 1989; Crawford 1998; Costa-Gomes 2002; Camerer 2003; Burton \textit{et al} 2005).

The coordination problem in the BoS is even more complicated with incomplete information, where each player has private information about the “intensity of preference” for the other player’s favorite outcome (e.g., how strongly the “Husband” (dis)likes “Concert”). Apart from its applications,\footnote{The complete information BoS has many economic applications (see the Introduction in Cabrales \textit{et al} (2000)); the corresponding game of incomplete information is not just a natural extension but is also relevant in many of these economic situations where the intensity of preference and its prior probability are important factors.} the BoS with private information is of interest to theorists and experimentalists for two main reasons. One, it is not clear whether coordination using cheap talk in the complete information BoS would extend to this incomplete information version; moreover, it is not obvious at all whether truthful revelation and thereby separation of players’ types can be achieved in a cheap talk equilibrium.

To analyze the above two issues, we use the simplest possible version of the BoS (as in Banks and Calvert (1992)) with two types (“High” and “Low”) for each player regarding the payoff from the other player’s favorite outcome (e.g., the payoff from (Concert, Concert) of the “High-type Husband” is more than that of the “Low-type Husband”, although, the payoff from (Football, Football) is higher than that from (Concert, Concert) for either type of Husband).

The question we ask is whether in the above game, players will truthfully reveal their types in a direct cheap talk equilibrium and also coordinate on Nash equilibrium outcomes in different states of the world. With incomplete information, one might find it desirable to coordinate on the (ex-post) efficient outcome when the two players are of different types, in which the compromise is made by the player who suffers a smaller loss in utility (e.g., the “High-type Husband” and the “Low-type Wife” coordinate on (Concert, Concert)). Such efficiency is automatically obtained with coordination in the complete information cheap talk game but, with incomplete information, efficiency and coordination do not necessarily go together.

The aim of this paper thus is two-fold. We first look for a truthful cheap talk equilibrium in which
the players reveal their types truthfully before playing. Second, following Farrell (1987), we would like to check if the desirable coordination in this context is achieved in an equilibrium outcome. The answer to our question is “yes”, provided the prior probability of the High-type is moderately low.

We analyze a class of cheap talk equilibria for the BoS with incomplete information augmented by just one stage of direct unmediated cheap talk in which both players make simultaneous announcements on their possible types only. We prove that there does exist a fully revealing symmetric cheap talk equilibrium of this game in which the players announce their types truthfully; moreover, it is unique if we maintain symmetry (Theorem 1). The allowable range of the prior probability of the High-type for the fully revealing equilibrium to exist has to be moderately low - the upper bound being strictly less than $\frac{1}{2}$ - suggesting that full revelation is not a cheap talk equilibrium when the probability of a player being High-type is too high or too low. Also, this fully revealing cheap talk equilibrium has the desirable type-coordination property: when the players' types are different, it fully coordinates on the ex-post efficient pure Nash equilibrium, as they play (Concert, Concert) ((Football, Football)) when their true type profile is (High, Low) ((Low, High)).

We then ask whether partially revealing cheap talk equilibria (where the players are not truthful in their cheap talk) exist and possess the type-coordination property, particularly when the fully revealing equilibrium does not exist. Keeping the spirit of the fully revealing equilibrium, we consider a class of partially revealing cheap talk equilibria in which only the High-type is not truthful, while the Low-type is truthful. We analyze this particular type of partial revelation because under the type-coordination property, the High-type is expected to compromise and coordinate on his less preferred outcome when the other player claims to be of Low-type. We characterize the complete set of partially revealing cheap talk equilibria in which only the High-type is not truthful while the Low-type is truthful. We analyze this particular type of partial revelation because under the type-coordination property, the High-type is expected to compromise and coordinate on his less preferred outcome when the other player claims to be of Low-type. We characterize the complete set of partially revealing cheap talk equilibria in which only the High-type is not truthful while the Low-type is truthful (Theorem 2).

As a corollary (Corollary 1), we identify the unique equilibrium with the type-coordination property among this set and prove its existence based on the prior probability of the High-type being within a range that turns out to be non-overlapping and higher than that for the fully revealing equilibrium. We illustrate all these results using a running numerical example in different sections of the paper.

Following the seminal paper by Crawford and Sobel (1982), much of the cheap talk literature has focused on the sender-receiver framework whereby one player has private information but takes no action and the other player is uninformed but is responsible for taking a payoff-relevant decision. There indeed is a small but growing literature on games where both players have private information

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3Considering symmetric communication processes seems reasonable here (otherwise, one can always generate trivial asymmetric equilibria where only one player communicates). Since the players are identical, facing an identical symmetric situation ex-ante, we study (type and player) symmetric cheap talk equilibria, following the tradition in the literature (as in Farrell (1987) and Banks and Calvert (1992)).

4It is easy to check that the fully efficient symmetric outcome in all four states, involving a lottery over the Nash outcomes when players' types are identical, cannot be obtained as a fully revealing cheap talk equilibrium.
and can send cheap talk messages to each other.\footnote{Examples of information transmission using two-sided cheap talk under two-sided incomplete information can be found in Farrell and Gibbons (1989), Matthews and Postlewaite (1989), Baliga and Morris (2002), Doraszelski \textit{et al} (2003), Baliga and Sjostrom (2004), Chen (2009), Horner \textit{et al} (2010) and Goltsman and Pavlov (2013). Two-sided cheap talk using multiple stages of communication where only one of the players has incomplete information has also been studied by Aumann and Hart (2003) and Krishna and Morgan (2004).} We have contributed to this literature by analyzing symmetric cheap talk equilibria in a game with two-sided information and two-sided cheap talk.

Banks and Calvert (1992) characterized the (ex-ante) efficient symmetric incentive compatible direct mechanism for a similar game so that the players are truthful and obedient to the mechanism (mediator). Following Banks and Calvert (1992), one may check whether our cheap talk equilibria can be achieved as outcomes of such mediated equilibria using incentive compatible mechanisms. Considering such mediated mechanisms is useful because they inform us about the limits to communication possibilities via cheap talk. As expected, in the spirit of the “revelation principle”, we note that any equilibrium outcome of our cheap talk game can be obtained as an outcome of an incentive compatible mechanism. Moreover, rather intuitively, one can show that our cheap talk equilibrium outcomes may be achieved as outcomes of the corresponding incentive compatible mechanisms for a strictly larger range of the prior probability of the High-type (Theorem 3). This result indicates that there are priors for which an outcome can be obtained as an equilibrium via a direct mechanism but not using the unmediated one-round cheap talk that only allows direct communication between players of different types.

\section{MODEL}

\subsection{The Game}

We consider a version of the BoS with incomplete information as given below, in which each of the two players has two strategies, namely, $F$ (Football) and $C$ (Concert). The payoffs are as in the following table, in which the value of $t_i$ is the private information of player $i$, $i = 1, 2$, with $0 < t_1, t_2 < 1$. Also, the payoffs to both players from the miscoordinated outcome is normalized to 0, while the payoff to the Husband (Wife) from $(F, F)$ ($(C, C)$) is normalized to 1.

\begin{center}
\begin{tabular}{|c|c|c|}
\hline
 & Football ($F$) & Concert ($C$) \\
\hline
Husband (Player 1) & $1, t_2$ & 0, 0 \\
& 0, 0 & $t_1, 1$ \\
\hline
\end{tabular}
\end{center}

We assume that $t_i$ is a discrete random variable that takes only two values $L$ and $H$ (where, $0 < L < H < 1$), whose realization is only observed by player $i$. For $i = 1, 2$, we henceforth refer to the
values of $t_i$ as player $i$’s type (Low, High). We further assume that each player’s type is independently drawn from the set $\{L, H\}$ according to a probability distribution with $\text{Prob}(t_i = H) = p \in [0, 1]$.

The unique symmetric Bayesian-Nash equilibrium\(^6\) of this game can be characterized by $\sigma_i(s_i | t_i)$, the probability that player $i$ of type $t_i$ plays the pure strategy $s_i$. The unique symmetric Bayesian Nash equilibrium is given by the following strategy for player 1 (player 2’s strategy is symmetric and is given by $\sigma_2(F | t) = \sigma_2(C | t)$ when $t = H, L$):

- $\sigma_1(F | H) = 0$ and $\sigma_1(F | L) = \frac{1}{(1-p)(1+L)}$ when $p < \frac{L}{1+L}$,
- $\sigma_1(F | H) = 0$ and $\sigma_1(F | L) = 1$ when $\frac{L}{1+L} \leq p \leq \frac{H}{1+H}$,
- $\sigma_1(F | H) = 1 - \frac{H}{p(1+H)}$ and $\sigma_1(F | L) = 1$ when $p > \frac{H}{1+H}$.

To illustrate our game, consider the following numerical example, in which the payoff of the $H$-type Husband (Wife) from $(C, C)$ ($(F, F)$) is $\frac{2}{3}$, twice that of the Low-type as described in the following table.

<table>
<thead>
<tr>
<th></th>
<th>$F$</th>
<th>$C$</th>
<th>$F$</th>
<th>$C$</th>
<th>$F$</th>
<th>$C$</th>
<th>$F$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>$0$</td>
<td>$0$</td>
<td>$\frac{1}{16}$</td>
<td>$0$</td>
<td>$\frac{1}{16}$</td>
<td>$0$</td>
<td>$\frac{1}{16}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$C$</td>
<td>$1$</td>
<td>$0$</td>
<td>$\frac{15}{256}$</td>
<td>$0$</td>
<td>$\frac{15}{256}$</td>
<td>$0$</td>
<td>$\frac{15}{256}$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Types: $HH$ Types: $HL$ Types: $LH$ Types: $LL$

Suppose, in the above example the (independent) prior probability of the $H$-type, $\text{Prob}(t_i = \frac{2}{3})$ is $\frac{1}{5}$. Then, the unique symmetric Bayesian Nash equilibrium of this game is given by the following symmetric strategy profile: player 1 (Husband) plays $F$ with probability $\frac{15}{16}$ when the type is Low and plays the pure strategy $C$ when the type is High (player 2’s strategy is symmetric and is $C$ with probability $\frac{15}{16}$ when the type is Low and $F$ when the type is High) which generates the following distribution over the outcomes for different type profiles (states of the world).

<table>
<thead>
<tr>
<th></th>
<th>$F$</th>
<th>$C$</th>
<th>$F$</th>
<th>$C$</th>
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<tr>
<td>$F$</td>
<td>$0$</td>
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<td>$\frac{15}{16}$</td>
<td>$0$</td>
<td>$\frac{15}{16}$</td>
<td>$0$</td>
<td>$\frac{15}{256}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$C$</td>
<td>$1$</td>
<td>$0$</td>
<td>$\frac{15}{16}$</td>
<td>$0$</td>
<td>$\frac{15}{16}$</td>
<td>$0$</td>
<td>$\frac{15}{256}$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Types: $HH$ Types: $HL$ Types: $LH$ Types: $LL$

### 2.2 Cheap Talk

We study an extended game in which the players are first allowed to have a round of simultaneous canonical cheap talk intending to reveal their private information before they play the above BoS. In

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\(^6\)The corresponding game with complete information with commonly known values $t_1$ and $t_2$, has two pure Nash equilibria, $(F, F)$ and $(C, C)$, and a mixed Nash equilibrium in which player 1 plays $F$ with probability $\frac{1}{1+t_2}$ and player 2 plays $C$ with probability $\frac{1}{1+t_1}$. 

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the first (cheap talk) stage of this extended game, each player \(i\) simultaneously chooses a costless and nonbinding announcement \(\tau_i\) from the set \(\{L, H\}\). Then, given a pair of announcements \((\tau_1, \tau_2)\), in the second (action) stage of this extended game, each player \(i\) simultaneously chooses an action \(s_i\) from the set \(\{F, C\}\).

An announcement strategy in the first stage for player \(i\) is a function \(a_i : \{L, H\} \rightarrow \Delta(\{L, H\})\), where \(\Delta(\{L, H\})\) is the set of probability distributions over \(\{L, H\}\). We write \(a_i(H \mid t_i)\) for the probability that strategy \(a_i(t_i)\) of player \(i\) with type \(t_i\) assigns to the announcement \(H\). Thus, the announcement \(\tau_i\) of player \(i\) with type \(t_i\) is a random variable drawn from \(\{L, H\}\) according to the probability distribution with \(\text{Prob}(\tau_i = H) = a_i(H \mid t_i)\).

In the second (action) stage, a strategy for player \(i\) is a function \(\sigma_i : \{L, H\} \times \{L, H\} \times \{L, H\} \rightarrow \Delta(\{F, C\})\), where \(\Delta(\{F, C\})\) is the set of probability distributions over \(\{F, C\}\). We write \(\sigma_i(F \mid t_i; \tau_1, \tau_2)\) for the probability that strategy \(\sigma_i(t_i; \tau_1, \tau_2)\) of player \(i\) with type \(t_i\) assigns to the action \(F\) when the first stage announcements are \((\tau_1, \tau_2)\). Thus, player \(i\) with type \(t_i\)’s action choice \(s_i\) is a random variable drawn from \(\{F, C\}\) according to a probability distribution with \(\text{Prob}(s_i = F) = \sigma_i(F \mid t_i; \tau_1, \tau_2)\).

Given a pair of realized action choices \((s_1, s_2)\) \(\in \{F, C\} \times \{F, C\}\), the corresponding outcome is generated. Thus, given a strategy profile \(((a_1, \sigma_1), (a_2, \sigma_2))\), one can find the players’ actual payoffs from the induced outcomes in the type-specific payoff matrix of the BoS and hence, the (ex-ante) expected payoffs.

As the game is symmetric, in our analysis, we maintain the following notion of symmetry in the strategies, for the rest of the paper.

**Definition 1** A strategy profile \(((a_1, \sigma_1), (a_2, \sigma_2))\) is called announcement-symmetric (in the announcement stage) if \(a_i(H \mid t_i) = a_{-i}(H \mid t_i)\); a strategy profile is called action-symmetric (in the action stage) if \(\sigma_i(F \mid t; \tau_1, \tau_2) = \sigma_{-i}(C \mid t; \tau_2, \tau_1)\), for all \(t, \tau_1, \tau_2\). A strategy profile is called symmetric if it is both announcement-symmetric and action-symmetric.

Note that Definition 1 preserves symmetry for both players and the types for each player. We consider the following standard notion of equilibrium in this two-stage cheap talk game.

**Definition 2** A symmetric strategy profile \(((a_1, \sigma_1), (a_2, \sigma_2))\) is called a symmetric cheap talk equilibrium if (i) given the announcement strategies \((a_1, a_2)\), the action strategies \((\sigma_1, \sigma_2)\) constitute a Bayesian-Nash equilibrium in the simultaneously played second stage BoS with the posterior beliefs generated by \((a_1, a_2)\) and (ii) the announcement strategies \((a_1, a_2)\) are Bayesian-Nash equilibrium of the simultaneous announcement game given the action strategies \((\sigma_1, \sigma_2)\) to be followed.

Definition 2 suggests that a symmetric cheap talk equilibrium can be characterized by a set of (symmetric) equilibrium constraints (2 for the announcement stage and another possible 8 for the action stage).
3 RESULTS

We present some symmetric cheap talk equilibria in this section. We first consider the possibility of full revelation of the types as a result of our canonical cheap talk.

3.1 Full Revelation

We consider a specific class of strategies in this subsection where we impose the property that the cheap talk announcement should be fully revealing.

Definition 3 A symmetric strategy profile \(((a_1, \sigma_1), (a_2, \sigma_2))\) is called fully revealing if the announcement strategy \(a_i\) reveals the true types with certainty, i.e., \(a_i(H | H) = 1\) and \(a_i(H | L) = 0\).

Definition 3 simply asserts that, in the cheap talk stage, each player makes an announcement coinciding with their type: \(\tau_i = t_i\). Note that, under fully revealing announcements, for player \(i\) with type \(t_i\), the action strategy can be written as \(\sigma_i(F | t_i, t_j)\) that denotes the probability that player \(i\) assigns to the action \(F\) when the (true and announced) types are \(t_i\) and \(t_j\).

We now characterize fully revealing symmetric cheap talk equilibria. Using Definitions 2 and 3, we first observe the following facts.

Claim 1 In a fully revealing symmetric cheap talk equilibrium \(((a_1, \sigma_1), (a_2, \sigma_2))\), the players’ strategies in the action phase must constitute a (pure or mixed) Nash equilibrium of the corresponding complete information BoS, that is, \((\sigma_1(t_1, t_2), \sigma_2(t_1, t_2))\) is a (pure or mixed) Nash equilibrium of the BoS with values \(t_1\) and \(t_2\), \(\forall t_1, t_2 \in \{H, L\}\).

Claim 2 In a fully revealing symmetric cheap talk equilibrium \(((a_1, \sigma_1), (a_2, \sigma_2))\), conditional on the announcement profile \((H, H)\) or \((L, L)\), the strategy profile in the action phase must be the mixed strategy Nash equilibrium of the corresponding complete information BoS, that is, whenever \(t_1 = t_2\), \((\sigma_1(t_1, t_2), \sigma_2(t_1, t_2))\) is the mixed Nash equilibrium of the BoS with values \(t_1 = t_2\).

Based on the above claims, one can easily identify all the candidate equilibrium strategy profiles of the extended game that are fully revealing and symmetric. Claim 2 implies that these profiles are differentiated only by the actions played when \(t_1 \neq t_2\), that is, when the players’ types are \((H, L)\) and \((L, H)\).

We now consider a specific fully revealing (separating) strategy profile that we call \(S_{\text{separating}}\), influenced by the equilibrium action profile in Farrell (1987) for the complete information version of this game. In this strategy profile, the players announce their types truthfully and then in the action stage, they play the mixed Nash equilibrium strategies of the complete information BoS when both
players’ types are identical and they play \((C, C)\) \(((F, F))\), when only player 1’s type is \(H\) \((L)\). Formally, \(S_{\text{separating}}\) is characterized by \(\sigma_1 [F|H, L] = 0\), \(\sigma_1 [F|L, H] = 1\) and \(\sigma_i [F|t, t] = \sigma^i(tt)\) with \(t = H\), \(L\), where \(\sigma^i(tt)\) is the probability of playing \(F\) in the mixed Nash equilibrium strategy of player \(i\) of the complete information BoS with values \(t_1 = t_2 = t\), \(t = H\), \(L\) (see Footnote 6). We state our first result (the proof of which is in the Appendix).

**Theorem 1** \(S_{\text{separating}}\) is the unique fully revealing symmetric cheap talk equilibrium and it exists only for \(\frac{L^2 + L^2 H}{1 + L + L + LH + LH^2} \leq p \leq \frac{LH + LH^2}{1 + L + LH + LH^2} \).

One can easily check that the upper bound for \(p\) in Theorem 1, \(\frac{LH + LH^2}{1 + L + LH + LH^2}\) is always \(\frac{1}{2}\), since \(\frac{1}{2} - \frac{HL + H^2 L}{1 + L + LH + LH^2} = \frac{(1 + L - LH - LH^2)}{2(1 + L + LH + LH^2)} > 0\), as long as \(L < H < 1\). To understand why \(p\) must lie in such a low range for this equilibrium to exist, consider the incentives for deviations by player 1 at the announcement stage. By deviating and claiming to be an \(L\)-type, player \(1(H\text{-type})\) gains \(1 - \frac{H}{1+H} = \frac{1}{1+H}\) when player 2 is a \(H\)-type (with probability \(p\)) and loses \(H - \frac{H}{1+H} = \frac{H}{1+H}\) when player 2 is a \(L\)-type (with probability \(1-p\)). Since \(\frac{1}{1+H} - \frac{HL}{1+L} = \frac{(1 + L - LH - LH^2)}{(1 + H)(1 + L)} > 0\), the gain from the deviation when playing against player \(2(H\text{-type})\) is bigger than the loss when playing against player \(2(L\text{-type})\).

If a \(H\)-type is equally or more likely than a \(L\)-type, then player \(1(H\text{-type})\) will obviously deviate and truthful revelation will not be an equilibrium. So, \(p\) must be \(< \frac{1}{2}\). Indeed, \(p\) needs to be small enough to make the above deviation unattractive and the precise value of \(p\) for which this holds is \(\frac{HL + H^2 L}{1 + L + LH + LH^2}\) or less.

However, \(p\) cannot be too close to \(0\) either. This is because of incentives for deviations by player \(1(L\text{-type})\). By deviating and claiming to be a \(H\)-type, player \(1(L\text{-type})\) gains \(L - \frac{L}{1+L} = \frac{L^2}{1+L}\) when player 2 is a \(L\)-type (with probability \(1-p\)) and loses \(1 - \frac{H}{1+H} = \frac{1}{1+H}\) when player 2 is a \(H\)-type (with probability \(p\)). Since \(\frac{1}{1+H} - \frac{L^2}{1+L} = \frac{(1 + L - LH - L^2)}{(1 + H)(1 + L)} > 0\), the loss from the deviation when playing against player \(2(H\text{-type})\) is bigger than the gain when playing against player \(2(L\text{-type})\). The expected gain will outweigh the expected loss only if a \(L\)-type is much more likely than a \(H\)-type (and \(L\) is bigger than \(0\)). Hence, player \(1(L\text{-type})\) would deviate at the cheap talk stage only if \(p\) is too close to \(0\).

The ex-ante expected payoff for any player from \(S_{\text{separating}}\) is given by \(EU_{\text{separating}} = p^2 \frac{H}{1+H} + p(1-p)(1+H) + (1-p)^2 \frac{1}{1+H}\), which is increasing over the range of \(p\) where it exists. Hence, the best achievable payoff from \(S_{\text{separating}}\) (when \(p = \frac{LH + LH^2}{1 + L + LH + LH^2}\)) is \(\frac{L(1 + L + H + LH + 2H^2 + 2LH^2 + 2LH^3 + LH^4)}{(1 + L + LH + LH^2)^2}\).

To illustrate this equilibrium in the example in Section 2, let us take \(L = \frac{1}{3}\), \(H = \frac{2}{3}\). For these values, the range of the prior \(p\) for which \(S_{\text{separating}}\) exists is \(\frac{5}{11} \approx 0.12 \leq p \leq \frac{5}{22} \approx 0.22\). The (best) payoff from \(S_{\text{separating}}\) (at \(p = \frac{5}{22}\)) is \(\frac{241}{22^2} \approx 0.46\). This equilibrium generates the following
distribution over the outcomes for our example.

\[
\begin{array}{ccc|ccc|ccc}
F & F & C & F & F & C & F & F & C \\
\hline
F & \frac{6}{25} & \frac{9}{25} & F & 0 & 0 & F & 1 & 0 \\
C & \frac{4}{25} & \frac{6}{25} & C & 0 & 1 & C & 0 & 0 \\
\end{array}
\]

Types: HH \quad Types: HL \quad Types: LH \quad Types: LL

As the above example demonstrates, \( S \) separating features a specific form of coordination in which the players play \((C,C)\) when only player 1’s type is \(H\) (\(L\)), that is, when the players’ types are different, players fully coordinate on a pure Nash equilibrium outcome that generate the ex-post efficient payoffs of 1 and \(H\). We call this property “type-coordination”.

**Definition 4** A strategy profile is said to have the type-coordination property if the induced outcome is \((C,C)\) and \((F,F)\), when the players’ true type profile is \((H,L)\) and \((L,H)\), respectively.

Note that the property of type-coordination can be achieved in other kinds of equilibria. Indeed, as mentioned in the previous section, the Bayesian Nash equilibrium (of the BoS without the cheap talk) also satisfies type-coordination property when the prior \(p\) is between \(L\) and \(H\) (for example, between \(\frac{1}{4}\) and \(\frac{2}{5}\) for the parameters \(L = \frac{1}{3}, H = \frac{2}{3}\)). In the next subsection, we will see how this feature can be obtained in a partially revealing equilibrium.

### 3.2 Partial Revelation

In this subsection, we consider a class of partially revealing symmetric strategy profiles of the above cheap talk game. Keeping the spirit of the unique fully revealing equilibrium and the type-coordination property, we restrict our attention to announcement strategy profiles in which only the \(L\)-type truthfully reveals while the \(H\)-type does not (as under the type-coordination property, the \(H\)-type is expected to compromise, when the opponent is of \(L\)-type).

Formally, consider a symmetric announcement strategy profile in which the \(H\)-type of player \(i\) announces \(H\) with probability \(r\) and \(L\) with probability \((1 - r)\) and the \(L\)-type of player \(i\) announces \(L\) with probability 1, i.e., \(a_i(H|H) = r\) and \(a_i(H|L) = 0\).

Clearly, after the cheap talk phase, the possible message profiles \((\tau_1, \tau_2)\) that the \(H\)-type of player 1 may receive are \((H,H)\), \((H,L)\), \((L,H)\) or \((L,L)\) while the \(L\)-type of player 1 may receive either \((L,H)\) or \((L,L)\). Let us denote an action-strategy of Player 1 by \(\sigma_1(F|H;H,H) = q_0, \sigma_1(F|H;H,L) = q_1, \sigma_1(F|H;L,H) = q_2, \sigma_1(F|H;L,L) = q_3, \sigma_1(F|L;L,H) = q_4\) and \(\sigma_1(F|L;L,L) = q_5\). By symmetry, a partially revealing symmetric strategy profile \(((a_1, \sigma_1), (a_2, \sigma_2))\) in our set-up can thus be identified by \((r, q_0, q_1, q_2, q_3, q_4, q_5)\).
We characterize the set of partially revealing symmetric cheap talk equilibria in this set-up. First note that, on receiving the message profile \((H, H)\), the players know the true types and hence in any such equilibrium, \(q_0\) has to correspond to the mixed Nash equilibrium of the complete information BoS with values \(H\) and \(H\). Thus, in any such partially revealing symmetric cheap talk equilibrium, \(q_0 = \frac{1}{1 + r}\). Hence, \((r, q_1, q_2, q_3, q_4, q_5)\) fully characterizes such an equilibrium. The following theorem (proof of which is in the Appendix) describes this equilibrium set.

**Theorem 2** The following profiles are the only partially revealing symmetric cheap talk equilibria in which only the \(L\)-type is truthful:

(i) \(q_1 = \frac{1}{1 + r}, q_2 = q_3 = \frac{p + H + p - H}{p + H + 2 - H p}; q_4 = q_5 = 1\) with any \(0 < r \leq 1 - \frac{H (1 - p)}{p}\); exists when \(p > \frac{H}{1 + H}\).

(ii) \(q_1 = 0, q_2 = 1, q_3 = 0, q_4 = 1, q_5 = \frac{1}{1 + L + L H + L H^2 - p - p - L H p - L H^2 + p}; r = \frac{L H + L H^2}{p + L H + L H^2 + p}\); exists when \(\frac{1}{1 + H + H^2 + H^3} < p < \frac{1}{1 + L + L H + L H^2} + \frac{L H + L H^2}{p + L H + L H^2 + p}\).

(iii) \(q_1 = 0, q_2 = 1, q_3 = 0, q_4 = 1, q_5 = 1\) and \(r = \frac{H^2}{p + H^2 + H^2};\) exists when \(\frac{1}{1 + H + H^2 + H^3} < p < \frac{1}{1 + H + H^2 + H^3}\).

(iv) \(q_1 = 0, q_2 = 1, q_3 = 0, q_4 = 1, q_5 = 1\) and \(r = \frac{p + H - H}{p}\); exists when \(\frac{H}{1 + H} < p < 1\).

We are interested to know whether the type-coordination property can be obtained by any such partially revealing equilibrium. Note that for the type-coordination property to hold, we need profiles satisfying \(q_1 = 0, q_3 = 0\) and \(q_5 = 1\). Using symmetry, for player 2(\(H\)-type), we then must have \(\sigma_2(F; H; L, H) = 1 - q_1 = 1\). This implies that in any such equilibrium, \(q_2 = 1\) and \(q_4 = 1\). Thus, we find the unique equilibrium with the type-coordination property from the list in Theorem 2, namely (iv) \(q_1 = 0, q_2 = 1, q_3 = 0, q_4 = 1, q_5 = 1\) and \(r = \frac{H^2}{1 + H + H^2}\). Let us call this equilibrium profile \(S_{pooling}\). We summarize our result below.

**Corollary 1** \(S_{pooling}\) is the only partially revealing symmetric cheap talk equilibrium, where only the \(L\)-type is truthful, satisfying the type-coordination property; it exists only when \(\frac{L + L H + L H^2}{1 + L + L H + L H^2} < p < \frac{H + H^2 + H^3}{1 + H + H^2 + H^3}\).

Clearly, the upper bound of the range for \(p\) in Corollary 1 does not involve \(L\) and is increasing in \(H\) and is bounded by \(\frac{3}{4}\).

Note that for a fixed value of \(H\) and \(L\), the lower bound \(\frac{L + L H + L H^2}{1 + L + L H + L H^2}\) is bigger than the upper bound for \(p\) for \(S_{separating}\). This means that these two different equilibria with the type-coordination property exist for distinct values of \(p\). For \(\frac{L + L H + L H^2}{1 + L + L H + L H^2} < p < \frac{H + H^2 + H^3}{1 + H + H^2 + H^3}\), when \(S_{pooling}\) exists as an equilibrium, \(S_{separating}\) is not an equilibrium because the \(H\)-type does not want to truthfully reveal
his information. Allowing the $H$-type to reveal his information partially in the cheap talk stage helps sustain the partially revealing equilibrium.

To understand why $p$ must lie within such a range for $S_{\text{pooling}}$ to be an equilibrium, consider the incentives for deviations by player 1 at the action stage. According to the above strategy profile, on receiving the message profile $(L, L)$, player 1 needs to play $C$. Given that player 2 plays $F$ and player 2 plays $C$, player 1 will indeed play $C$ only if he believes that player 2 is more likely to be an $L$-type than an $H$-type. This means that player 1’s posterior belief about player 2 being an $H$-type should not be too high. If we denote this posterior belief by $p'$, then $p' = P(t_2 = H | \tau_1 = L) = \frac{1-rp}{1-r'p}$. Since this posterior $p'$ is an increasing function of the prior $p$, the constraint that $p'$ should not be too high implies that the prior $p$ cannot be very high either. Hence, there is an upper bound for $p$ that is strictly less than 1. Similarly, according to the above strategy profile, after receiving the message profile $(L, L)$, player 1 needs to play $F$. Again, given that player 2 plays $F$ and player 2 plays $C$, player 1 will play $F$ only if the posterior $p'$ is not too small which explains the lower bound on $p$.

The expected utility from $S_{\text{pooling}}$ is $p\frac{1+H}{1+H+H^2-p-H^2p}$. We illustrate $S_{\text{pooling}}$ in our running example using the parameter values $L = \frac{1}{3}$, $H = \frac{2}{3}$ with $p$ between $\frac{19}{46}$ ($\approx 0.41$) and $\frac{38}{65}$ ($\approx 0.58$). For such a game, $S_{\text{pooling}}$ exists in which the $L$-type is truthful but the $H$-type partially reveals his true type with probability $\frac{10}{19}$ ($\approx 0.53$). The corresponding equilibrium distribution over the outcomes, indicating type-coordination, is as follows.

<table>
<thead>
<tr>
<th></th>
<th>$F$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>141/361</td>
<td>36/361</td>
</tr>
<tr>
<td>$C$</td>
<td>27/361</td>
<td>141/361</td>
</tr>
</tbody>
</table>

Types: $HH$ 
Types: $HL$ 
Types: $LH$ 
Types: $LL$

In this example, the (best) payoff from such an equilibrium (at $p = \frac{38}{65}$) is $\frac{38}{65} (\approx 0.58)$.

4 REMARKS

4.1 Asymmetric Equilibria

We have characterized the unique fully revealing symmetric cheap talk equilibrium in the BoS with private information. There are of course many fully revealing but asymmetric cheap talk equilibria of this game. Clearly, babbling equilibria exist in which the players ignore the communication and

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7If $p$ were to be equal to 1, i.e., player 2 were certainly an $H$-type, player 1($H$-type) would then definitely have preferred playing $F$, not $C$. 

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just play one of the Nash equilibria of the complete information BoS for all type-profiles. There are other asymmetric equilibria as well. Consider for example, the following strategy profile in the action stage after full revelation:

\[
\sigma_1(F|H,H) = \sigma_1(F|H,L) = 0, \quad \sigma_2(F|H,H) = \sigma_2(F|H,L) = 1, \\
\sigma_1(F|L,H) = \sigma_1(F|L,L) = 1, \quad \sigma_2(F|L,H) = \sigma_2(F|L,L) = 0, \\
\sigma_i(F|L,L) = \sigma_i(LL),
\]

where \(\sigma_i(LL)\) is the probability of playing \(F\) in the mixed Nash equilibrium strategy of player \(i\) of the complete information BoS with values \(t_1 = t_2 = L\). It can be easily checked that the above profile is a fully revealing cheap talk equilibrium when \(L^2 \leq p \leq \frac{LH}{1+L-H}\).  

### 4.2 Bayesian-Nash Equilibrium

Our cheap talk equilibria, \(S_{\text{separating}}\) and \(S_{\text{pooling}}\) satisfy the type-coordination property. As noted earlier, it is possible to achieve type-coordination in the unique symmetric Bayesian Nash equilibrium of the BoS (without the cheap talk stage) itself when \(\frac{L}{1+L} \leq p \leq \frac{H}{1+H}\). It is also conceivable that for some parameter values of \(L\) and \(H\) (such as, \(L = 0.2, H = 0.9\)), the ranges of \(p\) where \(S_{\text{separating}}\) and \(S_{\text{pooling}}\) respectively exist, do separately overlap with the interval \(\left[\frac{L}{1+L}, \frac{H}{1+H}\right]\). Hence, there exist possible values of \(p\) for which both \(S_{\text{separating}}\) and the Bayesian Nash equilibrium achieve type-coordination (such as, \(p = 0.2\) for the parameter values \(L = 0.2, H = 0.9\)) and similarly, values of \(p\) for which both \(S_{\text{pooling}}\) and the Bayesian Nash equilibrium achieve type-coordination (such as, \(p = 0.4\) for the parameter values \(L = 0.2, H = 0.9\)). However, when \(\frac{L}{1+L} \leq p \leq \frac{H}{1+H}\), the structure of the Bayesian Nash equilibrium is such that the players fully miscoordinate when their true type profile is \((H,H)\) or \((L,L)\). This contrasts with \(S_{\text{separating}}\) where the players achieve some degree of coordination in both the states \((H,H)\) and \((L,L)\), and with \(S_{\text{pooling}}\) where the players manage to coordinate with positive probability when the state is \((H,H)\) although there is complete miscoordination when the true type profile is \((L,L)\). Hence, both \(EU_{\text{separating}}\) and \(EU_{\text{pooling}}\) are strictly greater than that of the Bayesian Nash equilibrium, confirming that cheap talk is strictly beneficial to both players.

### 4.3 One-sided Talk

One-sided cheap talk with two-sided private information has also been studied in the literature (see, for example, Seidmann (1990) and more recently, Moreno de Barreda (2012)). One thus may be interested to know whether the feature of the two-sided cheap talk equilibria analyzed in our paper, namely, the type-coordination property, can be achieved with one-sided cheap talk. One can show that this is not possible within our set-up when only one player (say, player 1) is allowed to talk.

Formally, we first note that the type-coordination property cannot be achieved as a fully revealing
equilibrium with one-sided cheap talk maintaining our set-up. This is because, if player 1 reveals his type truthfully, in order to achieve type-coordination, player 1 has to play $C (F)$ after announcing $H (L)$. Player 2’s best response would then be to play $C (F)$ after receiving $H (L)$ irrespective of her own type. This is not an equilibrium because player 1($H$-type) would then deviate and announce $L$.

Similarly, the type-coordination property cannot be achieved as a partially revealing equilibrium where player 1($H$-type) of announces $H$ with probability $r$ and $L$ with probability $(1 - r)$ while player 1($L$-type) announces $L$ with probability 1 followed by an action strategy (of player 1) of playing $C$ by the $H$-type and $F$ by the $L$-type. In order to achieve type-coordination, we must have that player 1($H$-type) plays $C$ after announcing either $H$ or $L$ and player 1($L$-type) plays $F$. This implies that, in an equilibrium, after receiving $H$, player 2($H$-type) and player 2($L$-type) should play $C$. To preserve type-coordination, after receiving $L$, player 2($H$-type) needs to play $F$ because the announcement could have been made by player 1($L$-type) and player 2($L$-type) needs to play $C$ because the announcement could have been made by player 1($H$-type). Given these strategies in the action stage, player 1($H$-type) cannot be indifferent between announcing $H$ and $L$ in the cheap talk phase because the expected payoff from announcing $H$ is equal to $H$ whereas the expected payoff from announcing $L$ is equal to $(1 - p)H$. This contradicts the fact that player 1($H$-type) uses a mixed strategy in the cheap talk stage. Hence, there is no one-sided cheap talk equilibrium with the type-coordination property.

This suggests that two-sided cheap talk achieves more type-coordination than one-sided cheap talk in our set-up; this is in contrast with the well-known experimental results for cheap talk in the complete information BoS (Cooper et al 1989).

4.4 Incentive Compatible Mechanisms

Banks and Calvert (1992) characterized the (ex-ante) efficient symmetric incentive compatible direct mechanism for a similar BoS game with private information. Following Banks and Calvert (1992), we now analyze a (direct) mechanism (as in Ganguly and Ray (2009)) which is a symmetric probability distribution over the product set of actions $\{(F,F), (F,C), (C,F), (C,C)\}$ for every profile of types. As in the earlier section, we impose symmetry and thus consider symmetric probability distributions only. In such a (direct) mediated communication process, the players first report their types ($H$ or $L$) to the mechanism (mediator) and then the mediator picks an action profile according to the given probability distribution and informs the respective action to each player privately; the players then play the game.

A symmetric direct mechanism is called a symmetric mediated equilibrium if it provides the players with incentives (i) to truthfully reveal their types to the mediator and (ii) to follow the mediator’s recommendations following their type-announcements. A symmetric mediated equilibrium thus can
be characterized by some incentive compatibility constraints. Among the class of symmetric mediated equilibria, Banks and Calvert (1992) have characterized the ex-ante efficient ones. We, however, are interested in achieving the cheap talk equilibrium outcomes in this paper as outcomes of incentive compatible mechanisms.

**Theorem 3**  
(i) The distribution over the outcomes generated by $S_{\text{separating}}$ can be achieved as a symmetric mediated equilibrium when 
\[ \frac{L^2 + 2L^2 H + L^2 H^2}{1 + L + H + LH + L^2 + L^2 H + H^2} \leq p \leq \frac{H - L + LH^2 + L^2 H + L^2 H^2 + H^2}{1 + L + H + LH + L^2 + L^2 H + H^2}. \]

(ii) The distribution over the outcomes generated by $S_{\text{pooling}}$ can be achieved as a symmetric mediated equilibrium when 
\[ \frac{L + LH + LH^2}{1 + L + LH + LH^2} \leq p \leq \frac{H + H^2 + H^3}{1 + H + H^2 + H^3}. \]

The proof of Theorem 3 is in the Appendix.

One might be interested in comparing the above ranges of $p$ with the corresponding ranges of $p$ for our cheap talk equilibria. Not surprisingly, the above ranges strictly contain the ranges for the cheap talk equilibria.

First, for $S_{\text{separating}}$, note that 
\[ \frac{L^2 + 2L^2 H + L^2 H^2}{1 + L + H + LH + L^2 + L^2 H + H^2} < \frac{L^2 + L^2 H}{1 + L + L^2 + L^2 H} \quad \text{and} \quad \frac{LH + LH^2}{1 + L + LH + LH^2} < \frac{H - L + LH^2 + L^2 H + L^2 H^2 + H^2}{1 + L + H + LH + L^2 + L^2 H + H^2}. \]

Also, comparing with the ranges for $S_{\text{pooling}}$, note that the upper bounds (for cheap talk equilibrium and mediated equilibrium) are the same and the lower bounds are nested (as, $\frac{L + LH + LH^2}{1 + L + LH + LH^2} < \frac{L + LH + LH^2}{1 + L + LH + LH^2}$), implying a larger range of $p$ for which the corresponding mechanism is in equilibrium.
5 APPENDIX

We collect the proofs of our results in this section.

Proof of Theorem 1. As the strategies are symmetric, it is sufficient to characterize these candidate profiles only by $\sigma_1[F|H,L]$. From Claim 1, there are only three possible candidates for $\sigma_1[F|H,L]$ as the complete information BoS with values $H$ and $L$ has three (two pure and one mixed) Nash equilibria. These profiles are (i) $\sigma_1[F|H,L] = \sigma^1(HL)$ where $\sigma^1(HL)$ is the probability of playing $F$ in the mixed Nash equilibrium strategy of player 1 of the complete information BoS with values $t_1 = H$ and $t_2 = L$, that we call $S_m$; (ii) $\sigma_1[F|H,L] = 1$, that we call $S_{ineff}$ and (iii) $\sigma_1[F|H,L] = 0$, which indeed is $S_{separating}$.

We first show that $S_m$ is not an equilibrium. Under $S_m$, $H$-type will announce his type truthfully only if $p(H) + (1 - p)(\frac{H}{H+L}) \geq p(H) + (1 - p)(\frac{H}{H+L})$, where the LHS is the expected payoff from truthfully announcing $H$ and the RHS is the expected payoff from announcing $L$ and choosing the corresponding optimal action strategy. This inequality implies $\frac{1}{H} \geq \frac{1}{L}$ which can never be satisfied as $H > L$.

The second candidate strategy profile, $S_{ineff}$ is an equilibrium only when $\frac{1}{1+H+\frac{H^2}{H+L+H'}} \leq p$ and $p \leq \frac{1+L+HL-H^2}{1+L+HL+H'}$. To see this, note that under $S_{ineff}$, $H$-type will announce his type truthfully only if $p(H) + (1 - p)(\frac{H}{H+L}) \geq p(H) + (1 - p)(\frac{H}{H+L})$ which implies $p \leq \frac{1+L+HL-H^2}{1+L+HL+H'}$. Similarly, $L$-type will announce his type truthfully only if $pL + (1 - p)(\frac{L}{H+L}) \geq p(H) + (1 - p)(\frac{H}{H+L})$ which implies $\frac{1+L+HL-H^2}{1+L+HL+H'} \leq p$. However, it can be shown that $\frac{1+L+HL-H^2}{1+L+HL+H'} > \frac{1}{1+L+HL+H'}$. Hence, $S_{ineff}$ cannot be an equilibrium.

Finally, we prove that $S_{separating}$ is an equilibrium only when $\frac{HL^2+L^2}{1+L+HL+H'} \leq p \leq \frac{HL^2+L^2}{1+L+HL+H'}$. Under $S_{separating}$, $H$-type will announce his type truthfully only if $p(H) + (1 - p)(\frac{H}{H+L}) \geq p(H) + (1 - p)(\frac{H}{H+L})$ which implies $p \leq \frac{HL+H^2L}{1+L+HL+H'}$. Similarly, $L$-type will announce his type truthfully only if $p + (1 - p)(\frac{L}{H+L}) \geq p(H) + (1 - p)(\frac{L}{H+L})$ which implies $\frac{HL^2+L^2}{1+L+HL+H'} \leq p$. □
Proof of Theorem 2. If \( q_1, q_2, q_3, q_4 \) and \( q_5 \) correspond to completely mixed strategies in the action stage, then we must have the following five conditions for player 1 to be indifferent between playing \( F \) and \( C \) (where the LHS in each equation is the expected payoff from \( F \) and the RHS in each equation is the expected payoff from \( C \)):

Player 1\((H\text{-type})\) receiving the message profile \((L, H)\):

\[
q_1 = q_1 H, \tag{1}
\]

Player 1\((H\text{-type})\) receiving the message profile \((H, L)\):

\[
\left(\frac{p - rp}{1 - rp}\right)(1 - q_2) + \left(1 - \frac{p - rp}{1 - rp}\right)(1 - q_4) = \left(\frac{p - rp}{1 - rp}\right)q_2 H + \left(1 - \frac{p - rp}{1 - rp}\right)q_4 H, \tag{2}
\]

Player 1\((H\text{-type})\) receiving the message profile \((L, L)\):

\[
\left(\frac{p - rp}{1 - rp}\right)(1 - q_3) + \left(1 - \frac{p - rp}{1 - rp}\right)(1 - q_5) = \left(\frac{p - rp}{1 - rp}\right)q_3 H + \left(1 - \frac{p - rp}{1 - rp}\right)q_5 H, \tag{3}
\]

Player 1\((L\text{-type})\) receiving the message profile \((L, L)\):

\[
\left(\frac{p - rp}{1 - rp}\right)(1 - q_3) + \left(1 - \frac{p - rp}{1 - rp}\right)(1 - q_5) = \left(\frac{p - rp}{1 - rp}\right)q_3 L + \left(1 - \frac{p - rp}{1 - rp}\right)q_5 L, \tag{4}
\]

Player 1\((L\text{-type})\) receiving the message profile \((L, H)\):

\[
q_1 = q_1 L. \tag{5}
\]

Also, in the cheap talk phase, player 1\((H\text{-type})\) should be indifferent between announcing \( H \) and \( L \), which implies

\[
(1 - p) (q_1 (1 - q_4) + (1 - q_1) q_4 H) + p (r \frac{H}{1 + H} + (1 - r) (q_1 (1 - q_2) + (1 - q_1) q_2 H))
\]

\[
= p \left( r (q_2 (1 - q_1) + (1 - q_2) q_1 H) + (1 - r) (q_3 (1 - q_3) + (1 - q_3) q_3 H) \right)
\]

\[
+ (1 - p) (q_3 (1 - q_5) + (1 - q_3) q_5 H). \tag{6}
\]

Finally, in the cheap talk phase, it should be incentive compatible for player 1\((L\text{-type})\) to announce \( L \), which implies

\[
(1 - p) (q_5 (1 - q_5) (1 + L)) + p (r (q_4 (1 - q_1) + (1 - q_4) q_4 L) + (1 - r) (q_5 (1 - q_3) + (1 - q_5) q_5 L))
\]

\[
\geq Max_{x} (1 - p) (x (1 - q_4) + (1 - x) q_4 L) + p (r \frac{H}{1 + H} + (1 - r) (x (1 - q_2) + (1 - x) q_2 L)), \tag{7}
\]

where \( x \) is the optimal probability of playing \( F \) in the action phase if player 1\((L\text{-type})\) deviates and announces \( H \) and receives the message profile \((H, L)\).

By virtue of symmetry, the conditions for player 2 are identical to the above.
It is obvious that only one of the equations between (1) and (5) can be satisfied. Similarly, both (3) and (4) cannot hold simultaneously. So, at least one of $q_2$ or $q_4$ and at least one of $q_3$ or $q_5$ must correspond to a pure strategy.

Considering equations (1) and (5), one can see that if $0 < q_4 < 1$ and hence (5) holds, then $q_2 = 0$. Similarly, from (1) and (5), if $0 < q_2 < 1$ and hence (1) holds, then $q_4 = 1$. Also, from (3) and (4), if $0 < q_5 < 1$ and hence (4) holds, then $q_3 = 0$ and if $0 < q_3 < 1$ and hence (3) holds, then $q_5 = 1$.

This gives us the following 15 candidate strategy profiles for possible equilibria, split into four cases.

Case 1. One of $q_2$ or $q_4$ and one of $q_3$ or $q_5$ correspond to a pure strategy.

Profile (i): $0 < q_1 < 1$, $q_2 = 0$, $q_3 = 0$, $0 < q_4 < 1$ and $0 < q_5 < 1$.

With these values, equations (2), (4) and (5) together imply that $q_1 = \frac{1}{1+L}$, $q_4 = \frac{1-rp}{1+H-rp}$ and $q_5 = \frac{1-rp}{1+L-rp}$. For these values, (6) is not satisfied. Hence, this profile is not an equilibrium.

Profile (ii): $0 < q_1 < 1$, $q_2 = 0$, $0 < q_3 < 1$, $0 < q_4 < 1$ and $q_5 = 1$.

With these values, the only solution to the equations (2), (3), (5) and (6) is given by $q_1 = \frac{1}{1+L}$, $q_3 = \frac{p+H}{p+H-rp-H} \cdot q_4 = \frac{1}{1+H-rp}$ and $r = 0$. Since $r = 0$, this cannot be an equilibrium.

Profile (iii): $0 < q_1 < 1$, $0 < q_2 < 1$, $q_3 = 0$, $q_4 = 1$ and $0 < q_5 < 1$.

With these values, equations (2), (1), (4) imply that $q_1 = \frac{1}{1+H}$, $q_2 = \frac{p+H}{p+H-rp-H} \cdot q_5 = \frac{1-rp}{1+L-rp}$. For these values, (6) becomes $\frac{H}{1+H} = \frac{H(1+L+Hrp)}{(1+L)(1+H)}$ which equals to $L (1 - rp) = H (1 - rp)$, which is not possible. Hence, this profile is not an equilibrium.

Profile (iv): $0 < q_1 < 1$, $0 < q_2 < 1$, $0 < q_3 < 1$, $q_4 = 1$ and $q_5 = 1$.

With these values, equations (2), (1), (3) imply that $q_1 = \frac{1}{1+L}$ and $q_2 = q_3 = \frac{p+H}{p+H-rp-H} \cdot q_5 = \frac{1-rp}{1+L-rp}$.

Then, using these, (6) becomes $\frac{H}{1+H} = \frac{H}{1+L}$ and hence is satisfied for any value of $r$. We also need to check that the incentive compatibility condition (7) is satisfied. The derivative of the RHS of (7) with respect to $x = (1-p)(1-q_4-q_4 L) + p (1-r) (1-q_2-q_2 L)$ which equals to $(H-L) \frac{1-rp}{1+H}$, as $q_4 = 1$ and $q_2 = \frac{p+H}{p+H-rp-H}$. For these values, (7) holds, then player (1-type), after receiving the message profile $(L,L)$, obtains a higher expected payoff from playing $C$ which is a contradiction.

If $q_3 = 1$ and $q_5 = 0$, then player 1(H-type), after receiving the message profile $(L,L)$, obtains a higher expected payoff from playing $F$ which is a contradiction.

If $q_3 = 1$ and $q_5 = 0$, then we must have $(1 - \frac{r+rp}{1+H}) > (\frac{r+rp}{1+H})$ and $(1 - \frac{r+rp}{1+H}) < (\frac{r+rp}{1+H}) \cdot L$, which is not possible.

Case 2. One of $q_2$ or $q_4$ corresponds to a pure strategy and both $q_3$ and $q_5$ correspond to pure strategies.

If $q_3 = 1$ and $q_5 = 1$, then player 1(H-type), after receiving the message profile $(L,L)$, obtains a higher expected payoff from playing $C$ which is a contradiction.

If $q_3 = 0$ and $q_5 = 0$, then player 1(H-type), after receiving the message profile $(L,L)$, obtains a higher expected payoff from playing $F$ which is a contradiction.

If $q_3 = 1$ and $q_5 = 0$, then we must have $(1 - \frac{r+rp}{1+H}) > (\frac{r+rp}{1+H}) \cdot L$ and $(1 - \frac{r+rp}{1+H}) < (\frac{r+rp}{1+H}) \cdot L$, which is not possible.
So, the only possible candidate is $q_3 = 0$ and $q_5 = 1$ which implies that $(\frac{q-3p}{1-q}) < (1 - \frac{q-3p}{1-q})H$ and
\(\frac{(p-rp)}{1-tp} > (1 - \frac{p-rp}{1-tp})L\).

Profile (v): $0 < q_1 < 1$, $0 < q_2 < 1$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$.

With these values, the only solution to the equations (2), (1) and (6) is given by $q_1 = \frac{1}{1+H}$, $q_2 = 0$ and $r = \frac{1}{p}(p + Hp - H)$. Since $q_2 = 0$, this profile cannot be an equilibrium.

Profile (vi): $0 < q_1 < 1$, $q_2 = 0$, $q_3 = 0$, $0 < q_4 < 1$ and $q_5 = 1$.

With these values, the only solution to the equations (2), (5) and (6) is given by $q_1 = \frac{1}{1+L}$, $q_4 = \frac{p+Hp+H+LH+LH+LH+H-2H-1}{p+Hp+H+LH+LH+LH+H-1}$ and $r = \frac{p+Hp+H+LH+LH+LH+H-2H-1}{p+Hp}$.

However, note that although $q_3 > 0$, $q_4 < 1$ requires $p < \frac{H}{1+H}$ while $r > 0$ requires $p > \frac{H+LH}{1+L+LH+LH} = \frac{H}{1+H}$. Hence, this profile cannot constitute an equilibrium.

Case 3. Both $q_2$ and $q_4$ correspond to pure strategies and one of $q_3$ or $q_5$ corresponds to a pure strategy.

Subcase 3a. Suppose $q_2 = q_4 = 1$.

Under this subcase, $q_1$ must be 0. This is because, player $1(H$-type), after receiving the message profile $(H, L)$, obtains a strictly higher payoff from playing $C$ than playing $F$, implying $0 < q_1 < 1$ is not possible in an equilibrium. Also, if $q_1 = 1$, then player $1(H$-type) and player $1(L$-type), after receiving the message profile $(L, H)$, would both strictly prefer playing $C$ than $F$, which is a contradiction. Hence, the possible candidate profiles under this subcase are as follows.

Profile (vii): $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$ and $0 < q_5 < 1$.

With these values, the only solution to the equations (4) and (6) is given by $r = \frac{LH+LH^2}{p+Lp+LHp+LHp}$. and $q_5 = \frac{1}{1+L+LH+LH^2+Lp+Lp+LHp+LHp} > 0$. However, $q_5 < 1$ requires $p < \frac{L+LH+LH^2}{1+L+LH+LH^2}$ while $r < 1$ implies $p > \frac{LH+LH^2}{1+L+LH+LH^2}$.

Finally, we need to check that the incentive compatibility condition (7) is also satisfied. To do so, note that the derivative of the RHS of (7) with respect to $x$ is $L+LH+LH^2+Lp+Lp+LHp+LHp$.

Thus, the condition (7) is satisfied as LHS of (7) = $\frac{L+LH+LH^2}{1+L+LH+LH^2}$ > RHS of (7) = $\frac{L+LH+LH^2}{1+L+LH+LH^2}$.

Hence, this constitutes an equilibrium if $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p < \frac{L+LH+LH^2}{1+L+LH+LH^2}$.

The expected utility from this equilibrium (to each player) = $EU_{(vii)} = \frac{L+LH+LH^2+Hp-Lp}{1+L+LH+LH^2}$.

Profile (viii): $q_1 = 0$, $q_2 = 1$, $0 < q_3 < 1$, $q_4 = 1$ and $q_5 = 1$.

With these values, the only solution to the equations (3) and (6) is given by $r = \frac{H^2}{p+Hp}$ and $q_3 = \frac{p+Hp+Hp+Hp+Hp+Hp+Hp+Hp}{p+Hp+Hp+Hp+Hp+Hp+Hp+Hp}$.

Here, $r < 1$ implies that $\frac{H^2}{1+H} < p$ which also guarantees that $q_3 < 1$; however, for $q_3 > 0$, we need $p > \frac{H^2+H^2}{1+H+H^2+H^2}$. Also, we need to check that the incentive compatibility condition (7) is satisfied. To do so, note that the derivative of the RHS of (7) with respect to $x$ is $(1-p)(1-q_4-q_4L)+p(1-r)(1-q_2-q_2L) = -\frac{L+(1+L)}{1+L+LH+LH^2} < 0$, which implies $x = 0$.

Thus, the condition (7) is satisfied (as LHS of (7) = $\frac{L+H^2}{1+L+LH+LH^2}$ > RHS of (7) = $\frac{L+LH+LH^2}{1+L+LH+LH^2}$). Since $\frac{L+H^2+H^2}{1+H+H^2+H^2} > H^2$, this profile constitutes an equilibrium if $p > \frac{H^2+H^2}{1+H+H^2+H^2}$.
The expected utility from this equilibrium (to each player) = \( EU_{(viii)} = \frac{H^2 + H^3 + H^4}{1 + H + H^2 + H^3}. \)

**Subcase 3b. Suppose** \( q_2 = q_4 = 0. \)

Under this subcase, \( q_1 \) must be 1. This is because, player 1(H-type), after receiving the message profile \((H, L)\) obtains a strictly higher payoff from playing \( F \) than playing \( C \) implying \( 0 < q_1 < 1 \) is not possible. Also, if \( q_1 = 0 \), then player 1(H-type) and player 1(L-type), after receiving the message profile \((L, H)\), would both strictly prefer playing \( F \) than \( C \), which is a contradiction. Hence, the possible candidate profiles under this subcase are as follows.

**Profile (ix):** \( q_1 = 1, q_2 = 0, q_3 = 0, q_4 = 0 \) and \( 0 < q_5 < 1 \).

With these values, the only solution to the equations (4) and (6) is given by \( r = \frac{1 + L + LH - H^2}{p + LP + LHP + LH^2} \) and \( q_5 = \frac{LH + LH^2}{1 + L + LH + LH^2} \). We need to check that the incentive compatibility condition (7) is satisfied. Note that the derivative of the RHS of (7) with respect to \( x = (1 - p) (1 - q_4 - q_1 L) + p (1 - r) (1 - q_2 - q_2 L) = \frac{H^2 (1 + L)}{1 + L + LH + LH^2} \) > 0, implying \( x = 1 \), in which case the condition (7) is not satisfied (as LHS of (7) = \( \frac{L + LH + H^3}{1 + L + LH + LH^2} \) < RHS of (7) = \( \frac{H^2 + H^3 + H^4}{1 + H + H^2 + H^3} \)). Hence, this profile cannot constitute an equilibrium.

**Profile (x):** \( q_1 = 1, q_2 = 0, q_3 < 1, q_4 = 0 \) and \( q_5 = 1 \).

With these values, the only solution to the equations (3) and (6) is given by \( r = \frac{1}{p + LH} \) and \( q_3 = \frac{H^2 + H^3 + H^4}{p + LP + LHP + LH^2} \). We also need to check that the incentive compatibility condition (7) is satisfied. Note that the derivative of the RHS of (7) with respect to \( x = (1 - p) (1 - q_4 - q_1 L) + p (1 - r) (1 - q_2 - q_2 L) = \frac{H^2}{1 + L + LH + LH^2} \) > 0, implying \( x = 1 \), in which case the condition (7) is not satisfied (as LHS of (7) = \( \frac{L + LH + H^3}{1 + L + LH + LH^2} \) < RHS of (7) = \( \frac{H^2 + H^3 + H^4}{1 + H + H^2 + H^3} \)). Hence, this profile cannot constitute an equilibrium.

**Subcase 3c. Suppose** \( q_2 = 1, q_4 = 0. \)

This requires \( 1 - q_1 < q_1 L \) and \( 1 - q_1 > q_1 H \), which is not possible.

**Subcase 3d. Suppose** \( q_2 = 0, q_4 = 1. \)

This requires \( 1 - q_1 < q_1 H \) and \( 1 - q_1 > q_1 L \), implying that \( \frac{1}{1 + H} < q_1 < \frac{1}{1 + L} \), which in turn means that player 1(H-type), after receiving the message profile \((H, L)\), should obtain the same expected payoff from playing \( C \) and playing \( F \). So, \( (\frac{H^2}{1 + H}) = (1 - \frac{H^2}{1 + L}) H \). Hence, the possible candidate profiles under this subcase are as follows.

**Profile (xi):** \( 0 < q_1 < 1, q_2 = 0, q_3 = 0, q_4 = 1 \) and \( 0 < q_5 < 1 \).

With these values, the only solution to the equations (2), (4) and (6) is given by \( r = \frac{\nu + HP - H}{p} \), \( q_1 = \frac{L + 2H + 2HP + 2H^2 + 2H^3 + 2H^4}{p + LP + 2HP + 2LHP + 2H^2P + 2H^3P + 2H^4P - LH - LH^2 - LH^3} \) and \( q_5 = \frac{1 + H}{1 + L} \). Since \( q_5 > 1 \), this cannot be an equilibrium.

**Profile (xii):** \( 0 < q_1 < 1, q_2 = 0, 0 < q_3 < 1, q_4 = 1 \) and \( q_5 = 1 \).

With these values, the only solution to the equations (2), (3) and (6) is given by \( r = \frac{\nu + HP - H}{p} \), \( q_1 = \frac{1}{1 + H} \), \( q_3 = 0 \). Since \( q_3 = 0 \), this cannot be an equilibrium.
Case 4. Both $q_2$ and $q_4$ correspond to pure strategies and both $q_3$ and $q_5$ correspond to pure strategies.

Using previous arguments, we must have $q_3 = 0$ and $q_5 = 1$ and so, the following are the only potential candidates for equilibrium.

Profile (xiii): $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$.

First we note that with these values, equation (6) implies that $r = \frac{H + H^2 + p}{1 + H + H^2}$. Next, we check that the incentive compatibility condition (7) is satisfied. The derivative of the RHS of (7) with respect to $x$ is $(1 - p) (1 - q_4 - q_4 L) + p (1 - r) (1 - q_2 - q_2 L) = L (p - 1) + L p (\frac{H + H^2}{1 + H + H^2} - 1) < 0$, which implies $x = 0$ and in turn shows that the condition (7) is satisfied (LHS of (7) $= p \geq$ RHS of (7) = $\frac{L + L H + L H^2 + H^2 p - L H p - L H^2 p}{1 + H + H^2}$) only when $p \geq \frac{L + L H + L H^2}{1 + H + H^2}$. We then note that to have $q_3 = 0$ and $q_5 = 1$, we need $(\frac{p - r}{1 - r}) < (1 - \frac{p - r}{1 - r}) H$ and $(\frac{p - r}{1 - r}) > (1 - \frac{p - r}{1 - r}) L$ which imply $p < \frac{H + H^2 + H^3}{1 + H + H^2}$ and $p > \frac{L + L H + L H^2}{1 + H + H^2}$. Since $\frac{H + H^2 + H^3}{1 + H + H^2} - \frac{L + L H + L H^2}{1 + H + H^2} > 0$, the above gives us a meaningful range for $p$. So, the above constitutes an equilibrium if $\frac{L + L H + L H^2}{1 + H + H^2} < p < \frac{H + H^2 + H^3}{1 + H + H^2}$.

The expected utility from this equilibrium (to each player) $= EU_{(xiii)} = \frac{b(1 + H)(1 + H^2 - p - H^2 p)}{1 + H + H^2}$.

Profile (xiv): $q_1 = 1$, $q_2 = 0$, $q_3 = 0$, $q_4 = 0$ and $q_5 = 1$.

First we note that with these values, equation (6) implies that $r = \frac{(1 + H)(1 + H^2 - p)}{p(1 + H + H^2)}$. We then check that the incentive compatibility condition (7) is satisfied. The derivative of the RHS of (7) with respect to $x$ is $(1 - p) (1 - q_4 - q_4 L) + p (1 - r) (1 - q_2 - q_2 L) = -\frac{H (p + H^2 - 2 H - 1)}{1 + H + H^2} > 0$, implying $x = 1$ and in turn shows that the condition (7) is satisfied (LHS of (7) = $\frac{L + p + H^2 + L H p + L H^2 p - L H^2 - 1}{1 + H + H^2}$) $> 0$, which implies $x = 1$ and in turn shows that the condition (7) is satisfied (LHS of (7) = $\frac{H (H - p + 2)}{1 + H + H^2}$) when $p \geq \frac{1 + L H^2 + 2 H - L}{1 + H + H^2}$. We also note that to have $q_3 = 0$ and $q_5 = 1$, we need $(\frac{p - r}{1 - r}) < (1 - \frac{p - r}{1 - r}) H$ $\implies p < \frac{1 + H^2 + H^3}{1 + H + H^2}$. However, as $\frac{1 + H + H^2}{1 + H + H^2} - \frac{1 + 2 H + L H^2 - L}{1 + H + H^2} = -\frac{(H - L)(1 + H + H^2)}{1 + H + H^2} < 0$, the above cannot constitute an equilibrium.

Profile (xv): $0 < q_1 < 1$, $q_2 = 0$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$.

With these values, the only solution to equations (2) and (6) is given by $r = \frac{p + H p - H}{p}$ and $q_1 = \frac{1}{1 + H}$. As $0 < r < 1$, we have $\frac{H}{1 + H} < p < 1$. We then check that the incentive compatibility condition (7) is satisfied. The derivative of the RHS of (7) with respect to $x$ is $(1 - p) (1 - q_4 - q_4 L) + p (1 - r) (1 - q_2 - q_2 L) = (H - L) (1 - p) > 0$, implying $x = 1$. One can now check that the condition (7) is satisfied (as, LHS of (7) = $\frac{H}{1 + H} =$ RHS of (7)). The above constitutes an equilibrium if $\frac{H}{1 + H} < p < 1$.

The expected utility from this equilibrium (to each player) $= EU_{(xv)} = \frac{H}{1 + H}$. ■
Proof of Theorem 3. A symmetric mediated equilibrium is a probability distribution over the set of outcomes of the BoS for every profile of reported types, as below,

\[
\begin{array}{|c|c|}
\hline
F & C \\
\hline
\frac{1-v_6-v_7}{2} & v_7 \\
v_6 & \frac{1-v_6-v_7}{2} \\
\hline
\end{array}
\begin{array}{|c|c|}
\hline
F & C \\
\hline
1-v_3-v_4-v_5 & v_5 \\
v_4 & v_3 \\
\hline
\end{array}
\begin{array}{|c|c|}
\hline
F & C \\
\hline
v_3 & v_5 \\
v_4 & 1-v_3-v_4-v_5 \\
\hline
\end{array}
\begin{array}{|c|c|}
\hline
F & C \\
\hline
1-v_1-v_2 & v_2 \\
v_1 & \frac{1-v_1-v_2}{2} \\
\hline
\end{array}
\begin{array}{|c|c|}
\hline
F & C \\
\hline
v_3 & (1-v_3-v_4-v_5)L + (1-p)(1-v_3-v_4-v_5) \geq pv_5L + (1-p)v_2L. \\
\end{array}
\]

where all \(v_i\)’s lie in the closed interval \([0, 1]\), such that the following incentive compatibility constraints are satisfied to ensure truthful reporting and obedience to recommendations.9

Incentive compatibility for \(H\)-type to report \(H \implies \)
\[
p \left( \frac{1-v_6-v_7}{2} \right) (1+H) + (1-p) \left[ (1-v_3-v_4-v_5) + v_3H \right] 
\geq p [v_3 + (1-v_3-v_4-v_5)H] + (1-p) \left[ \frac{1-v_1-v_2}{2} \right] (1+H); \quad \text{(IC1)}
\]

Incentive compatibility for \(L\)-type to report \(L \implies \)
\[
p \left[ \frac{1-v_6-v_7}{2} \right] (1+L) + (1-p) \left[ (1-v_3-v_4-v_5) + v_3L \right] 
\geq p [v_3 + (1-v_3-v_4-v_5)H] + (1-p) \left[ \frac{1-v_1-v_2}{2} \right] (1+L); \quad \text{(IC2)}
\]

Incentive compatibility for \(H\)-type to choose \(F\) when \(F\) has been recommended \(\implies \)
\[
p \left( \frac{1-v_6-v_7}{2} \right) + (1-p)(1-v_3-v_4-v_5) \geq pv_7H + (1-p)v_5H; \quad \text{(IC3)}
\]

Incentive compatibility for \(H\)-type to choose \(C\) when \(C\) has been recommended \(\implies \)
\[
p \left( \frac{1-v_6-v_7}{2} \right) H + (1-p)v_3H \geq pv_6 + (1-p)v_4; \quad \text{(IC4)}
\]

Incentive compatibility for \(L\)-type to choose \(F\) when \(F\) has been recommended \(\implies \)
\[
p v_3 + (1-p)\frac{1-v_1-v_2}{2} \geq pv_5L + (1-p)v_2L \quad \text{(IC5)}
\]

Incentive compatibility for \(L\)-type to choose \(C\) when \(C\) has been recommended \(\implies \)
\[
p(1-v_3-v_4-v_5)L + (1-p)\frac{1-v_1-v_2}{2}L \geq pv_4 + (1-p)v_1. \quad \text{(IC6)}
\]

9These constraints are for the player 1 and by symmetry, the set of constraints for player 2 is mathematically identical.
(i) The symmetric mediated mechanism which is equivalent to $S_{\text{separating}}$ is given by $v_1 = \frac{L^2}{(1+L)^2}$, $v_2 = \frac{1}{(1+L)^2}$, $v_3 = 1$, $v_4 = 0$, $v_5 = 0$, $v_6 = \frac{H^2}{(1+H)^2}$ and $v_7 = \frac{1}{(1+H)^2}$. Substituting these values into the above six incentive-compatibility constraints, one can easily check that IC3 (LHS = RHS = $\frac{H^2}{(1+H)^2}$) and IC6 (LHS = RHS = $\frac{L^2(1-p)}{(1+L)^2}$) are satisfied for all $p$. Also, IC4 (LHS - RHS = $H(1-p)$) holds for any $p \leq 1$ while IC5 (LHS - RHS = $p$) holds for any $p \geq 0$. Finally, we note that

LHS of IC2 - RHS of IC2 = $\frac{p+H_P+L_P+H^2+L^2+H^2L^2+H^2L^2+L^2}{(1+L)(1+H)^2}$, which requires

$p \geq \frac{1+H+L+L^2+H^2+L^2+H^2L^2}{1+H+L+L^2+H^2+L^2+H^2L^2}$. Similarly,

LHS of IC1 - RHS of IC1 = $\frac{H+H^2+L^2+H^2L^2+L^2-LH+L^2L^2+H^2L^2+H^2L^2}{(1+H)(1+L)^2}$,

which is satisfied if $p \leq \frac{H+H^2+L^2+H^2L^2+L^2-H^2-L}{1+H+L+L^2+H^2+L^2+H^2L^2}$.

The above two ICs are compatible with each other because

$1+H+L+L^2+H^2+L^2+H^2L^2-L^2 = H+H^2+L^2+H^2L^2+L^2-H^2-L$.

So, this mechanism will be in equilibrium if

$\frac{2H^2+H^2L^2+L^2}{1+H+L+L^2+H^2+L^2+H^2L^2} \leq p \leq \frac{H+H^2+L^2+H^2L^2+L^2-H^2-L}{1+H+L+L^2+H^2+L^2+H^2L^2}$.

(ii) The symmetric mediated mechanism which is equivalent to $S_{\text{pooling}}$ is given by $v_1 = 0$, $v_2 = 1$, $v_3 = 1$, $v_4 = 0$, $v_5 = 0$, $v_6 = \frac{H^2}{(1+H)^2} + (1-r)^2$ and $v_7 = \frac{r^2}{(1+H)^2}$, where $r = \frac{H^2}{1+H+H^2}$. Substituting these values into the six incentive-compatibility constraints, one can easily check that IC6 (LHS = RHS = $0$) and IC3 (LHS - RHS = $rp(1-r)$) are satisfied for all $p$. Also, note that IC5 will hold if

$p \geq \frac{L}{1+H}$.

Next, we note that LHS of IC4 - RHS of IC4 = $\frac{H+2H^2+3H^3+2H^4+H^5+H^6-p+H_P-H^2-P+H^2L^2+H^2L^2+H^2L^2}{(1+H)^2}$. So, IC4 will hold if $p \leq \frac{H+2H^2+3H^3+2H^4+H^5}{1+H+H^2+H^3+H^4+H^5}$.

Similarly, LHS of IC2 - RHS of IC2 = $\frac{(1+H)^2(p+Lp+H^2p+H^2Lp-H^2L-H^2L^2)}{1+H+H^2}$. So, IC2 will hold if

$p \geq \frac{L+H^2L^2}{1+H+H^2}$.

Finally, LHS of IC1 - RHS of IC1 = $\frac{(1+H)(H+H^2+H^3-p+H^2P-H^2P+H^2P)}{1+H+H^2}$. So, IC1 will hold if $p \leq \frac{H+H^2+H^3}{1+H+H^2}$.

We also note the following:

$\frac{H+H^2+H^3}{1+H+H^2+H^3} - \frac{L+H+L^2+H^2}{1+H+L^2+H^2} = \frac{(H-L)(1+H^2)}{1+H+L}(1+H^2) > 0$,

$\frac{L+H+L^2+H^2}{1+H+L^2+H^2} - \frac{L}{1+L} = \frac{(L-1)(1+L)}{1+L} > 0$,

$\frac{H+2H^2+3H^3+2H^4+H^5}{1+H+H^2+H^3+H^4+H^5} - \frac{H^2+H^3}{1+H+H^2+H^3+H^4+H^5} = \frac{H^2(1+H^2)}{(1+H^2)(1+H+H^2)^2} > 0$,

which imply that the mechanism will be in equilibrium if

$\frac{L+H+L^2+H^2}{1+H+L^2+H^2} \leq p \leq \frac{H+H^2+H^3}{1+H+H^2}$.
6 REFERENCES


