Stochastic Consistent Expectations Equilibria in a Regime Switching Model of Inflation Determination

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This Draft: April 15, 2018

PRELIMINARY AND INCOMPLETE

Abstract

We propose a general framework to study the existence of Stochastic Consistent Expectations Equilibria (SCEE) in linear Markov regime switching (MRS) models. Boundedly rational agents form expectations using a simple univariate model. A SCEE obtains when the model-implied mean and correlation coincide with those predicted by the agents. We apply our method to a Fisherian model of inflation determination where the inflation coefficient in the Taylor rule switches across regimes. We find that the range of parameters for which at least one SCEE exists in the MRS model is very large, and includes the parametric space for which Davig-Leeper (2007) find a unique MRS rational expectations equilibrium. Most of SCEE exhibit higher inflation persistence and volatility than under rational expectations.

Interstingly, while under fixed coefficients, highly persistent SCEE can be ruled out by a sufficiently active rule, this task appear more daunting in a MRS set-up. Excess inflation persistence and volatility may still occur if the second regime is passive and sufficiently long-lasting.

Keywords: Taylor Rule, Multiple Equilibria, Regime Switching, Bounded Rationality, Consistent Expectations

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1 Introduction

There is a long tradition in macroeconomics in assuming that economic agents populating our models are endowed with Rational Expectations (henceforth, RE), that is, they have full knowledge of the equilibrium probability distribution of economic variables. While this has brought much desirable discipline in the way macroeconomists were previously modeling expectations, it comes with the unrealistic assumption that agents have unlimited computational power in the form of complete knowledge about each others' preferences, incentives, beliefs, and market clearing conditions. In addition to this, we have come to understand that unless we impose structural frictions and other forms of sluggish adjustments of sufficient magnitude (often exceeding what observed in reality) RE models have a hard time replicating some salient features in the data, such has the excess persistence and volatility of key macro indicators (e.g. stock prices, inflation, etc...).

Starting with the seminal work of George Evans and Seppo Honkapohja in the '80s (see Evans and Honkapohja, 2001, for an extensive review), a viable alternative to RE has been to model economic agents as econometricians: they try to infer the laws of motion of economic variables by estimating simple (linear) forecasting models where parameters are constantly updated as new information comes available. Such form of adaptive learning can then be used to assess whether the underlying RE equilibrium is attainable (namely, learnable or stable under leaning) and/or whether learning can act as an additional propagation mechanism, thus complementing/substituting more structural mechanisms.

Due to the self-referentiality of equilibrium systems - namely, the fact that current endogenous variables depend on subjective expectations and viceversa - other types of near-rational equilibria are possible within most macroeconomic models. In particular, agents may form forecasts based on a mis-specified perceived law of motion (PLM), which they will not be able to reject based on simple statistical testing: a *self-confirming* restricted perception equilibrium would then arise instead of a REE. Within this class of equilibria, a very interesting concept is the Stochastic Consistent Expectation Equilibrium (henceforth, SCEE) originally developed by Hommes and Sorger (1998) and Hommes et al. (2002). Boundedly rational agents make forecasts using a parsimonious linear model, say a stationary AR(1) process. A SCEE arises when the unconditional mean and autocorrelation of the forecasting model coincide with those generated by the unknown true data generating process - namely, the unknown self-referential structural model. This equilibrium concept is rather appealing as it imposes minimum knowledge on behalf of agents, and, most importantly, yields serially uncorrelated expectational errors.[1] In a recent paper, Hommes and Zhu (2014, henceforth HZ) provide a general analysis of *first-order* SCEE in linear stochastic self-referential models, showing its potential to generate excess persistence and volatility with respect to RE.

The objective of this paper is to study the existence and quantitative properties of SCEE in the context of a simple Fisherian model of inflation determination where the monetary policy's responsiveness to inflation in a Taylor-type interest rate rule can be subject to Markov regime switching (henceforth, MRS). In this respect, we merge the above cited literature on SCEE with that on MRS in monetary policy, as pioneered by Davig and Leeper (2007, 2009) and Farmer et al. (2009, 2010). Our contribution is twofold. First, we provide a general framework to study SCEE in MRS models. In particular, we derive analytical expression for unconditional first and second moments of mean-square-stable MRS models, which we can then use to show existence of SCEE. Second, we show how the existence, uniqueness and persistence properties of a SCEE heavily depend on the degree of aggressiveness of policy towards inflation, as well as on the probability that the latter switches across regimes (say from a more active to a less active, possibly passive, response).

To better understand the implications of MRS, we first provide an extensive analysis for the case of no MRS in policy. We fully characterize the policy space for which either a unique, multiple or no SCEE exist. In particular, we show that an *active* response to inflation (almost) always yields a unique SCEE. The latter tends to be more persistent than under RE - that is, it displays a higher first order autocorrelation - when policy is not very active and the key driving exogenous fundamental displays sufficient persistence. Under a *passive* policy, when it exists, a SCEE is not unique. For this to occur, the fundamental have to be not too persistent. Along a SCEE, there can be excess inflation volatility with respect to RE. Similarly to HZ, excess volatility is non-monotonic in the fundamental's persistence, and requires the latter to be sufficiently large. We show that by granting a more aggressive policy response to inflation the central bank can guarantee that the SCEE is at most as volatile as the REE. For instance, we find that the standard Taylor rule parameterization - having the response coefficient to inflation equal to 1.5 - is just enough.

[1] The occurrence of autocorrelated forecasting errors was the main critique to the adaptive learning literature which then spurred the RE revolution of Muth (1961) and Lucas (1971).

When extending the analysis to a MRS framework similar to Davig and Leeper (2007, henceforth DL), several interesting results appear. First, we find that within the parametric space for which DL find a unique MRS-REE there always exists a unique SCEE. The latter exhibits higher inflation persistence than under RE for parameterizations that remain close to the long-run Taylor principle (the determinacy threshold under RE) identified by DL. When both regimes are highly persistence, no SCEE exists for policy parameterizations leading to RE indeterminacy.

Second, if the incumbent central banker (regime 1) faces the possibility of being replaced by a not so hawkish one (regime 2), even a currently very active policy might not be enough to rule out a high persistence/high volatility SCEE (as it is instead the case under fixed coefficients).

Third, as long as its expected duration is not too long, any policy parameterization of regime 2, even a very passive one, can lead to a SCEE. This stands in stark contrast with what found under fixed coefficients, where, under a baseline calibration, a passive response to inflation never leads to a SCEE.

As for the case of a fixed regime, excess SCEE volatility is non-monotonic in fundamentals' persistence. In contrast to that, being moderately active in the current regime might not rule out excess volatility. This appears to be the case when the alternative regime is sufficiently long-lived, and features an inflation response very close to unity (though still active) or passive.

The paper is organized as follows. Section 2 lays out the Fisherian model of inflation determination. Section 3 provide an extensive analysis on the existence and uniqueness of SCEE for the case of no regime change in monetary policy. Section 4 does the same for the case of Markob regime switching. The stability under learning of SCEE is assessed in Section 5, while the conditions under which the model can generate excess volatility (with respect to its RE counterpart) has studied in Section 6. Section 7 concludes highlighting our main findings and current work in progres.

2 Model

The model is composed of a Fisher equation, a AR(1) process governing the real interest rate, and a Taylor-type interest rate rule used by the central bank to set the nominal rate. Letting lower case letters denote log-deviations from steady state values, in its log-linearized form, the Fisher equation reads:

$$i_t = \tilde{E}_t \pi_{t+1} + r_t \tag{1}$$

where i_t is the nominal rates, π_{t+1} is the inflation rate between t and $t+1$, and r_t denotes the real interest rate.[2] The expectation operator $\tilde{E}$ in front of π_{t+1} is not necessarily rational (see below), and is conditional on information available to agents at time t. In the rest of the analysis we will denote by E the rational expectations (RE) operator. For simplicity, the real interest rate r_t is assumed to follows a stationary AR(1) process:

$$r_t = \rho r_{t-1} + \varepsilon_t \tag{2}$$

for $|\rho| < 1$ and $\varepsilon_t \sim$iid$N(0, \sigma_\varepsilon^2)$. The nominal interest rate is set according to the following linear Taylor rule:

$$i_t = \phi(s_t)(\pi_t - \pi_t^*) \tag{3}$$

The response coefficient to inflation $\phi(s_t)$ switches stochastically according to a 2-state Markov chain with transition probabilities $p_{ij} \equiv \mathcal{P}(s_t = j \mid s_{t-1} = i)$ for $i, j \in \{1, 2\}$:

$$\phi(s_t) = \begin{cases} \phi_1 \text{ for } s_t = 1 \\ \phi_2 \text{ for } s_t = 2 \end{cases} \tag{4}$$

with $\phi_i \geq 0$ for $i = 1, 2$. For reasons that we will clarify below, we allow for temporary disturbances to the inflation target by assuming that $\pi_t^* \sim$iid$N(0, \sigma_{\pi^*}^2)$.[3] After combining (1) and (3), we obtain

$$\pi_t = \frac{1}{\phi(s_t)} \tilde{E}_t \pi_{t+1} + \frac{1}{\phi(s_t)} r_t + \pi_t^* \tag{5}$$

Notice that, under RE, the stochastic difference equation (5) would be identical to the one studied by Davig and Leeper (2007) (henceforth, DL), except for the presence of an additional *iid* shock to the inflation target. We instead assume that agents are boundedly rational: due to

[2] Such linearized Fisher equation can be obtained by log-linearizing an endowment economy or a flexible price economy around a non-stochastic steady state. Hence, the real interest rate r_t captures demand-side disturbances, such as shocks to TFP, government spending, preferences, etc...As outlined by Benhabib, Schmitt-Grohe and Uribe in a series of papers, a Taylor rule that responds actively to inflation around the target steady state induces a second lower (possibly deflationary) steady state. For the time being, we ignore this issue.

[3] Since our model a log-linear approximation of an underlying non-linear model, it is natural to assume that transitory shocks have zero means.

cognitive limitations, they do not recognize that inflation is determined by (5) - in particular the fact that inflation is driven by exogenous fundamentals, r_t and π_t^*, and that monetary policy is subject to exogenous regime switching - but rather use a simple parsimonious univariate model to forecast future inflation. Their perceived law of motion (PLM) is:

$$\pi_t = \alpha + \beta(\pi_{t-1} - \alpha) + \delta_t \tag{6}$$

where δ_t is a white noise error, while α and β (both real numbers with $\beta \in (-1, 1)$) are, respectively the unconditional mean and the first order autocorrelation of π_t. As agents are not aware of the regime switching, neither α nor β are functions of the state s_t.[4] Assuming π_t is unknown at time of forecast, from (6), the boundedly rational inflation expectation is

$$\tilde{E}_t\pi_{t+1} = \alpha + \beta^2(\pi_{t-1} - \alpha) \tag{7}$$

Using (5) and (7), one can derive the actual law of motion (ALM)

$$\pi_t = c(s_t) + a(s_t)\pi_{t-1} + b(s_t)r_t + \pi_t^* \tag{8}$$

where $c(s_t) \equiv \frac{\alpha(1-\beta^2)}{\phi(s_t)}$, $a(s_t) \equiv \frac{\beta^2}{\phi(s_t)}$, and $b(s_t) \equiv \frac{1}{\phi(s_t)}$. The ALM (8) is a univariate Markov Regime Switching (MRS) model.

In order to pin down α and β, we appeal to the concept of *Stochastic Consistent Expectation Equilibrium* (SCEE), as defined in Hommes and Zhu (2014) (henceforth, HZ), suitably adapted to our MRS framework. In general, given the forecasting rule (6), a SCEE exists if and only if the ALM has unconditional mean equal to α and unconditional first order autocorrelation coefficient equal to β. In other words, agents think that their PLM is correct if and only if their perceived mean and first-order autocorrelation are equal to the ones they observe in (estimate from) the data. We start the analysis by discussing the SCEE in the context of a simpler fixed-coefficients monetary policy model

3 SCEE without MRS

Assume that $\phi(s_t) = \phi$ in both states. The ALM (8) becomes

$$\pi_t = \alpha(1 - \beta^2)\phi^{-1} + \beta^2\phi^{-1}\pi_{t-1} + \phi^{-1}r_t + \pi_t^* \tag{9}$$

[4] This assumption could be relaxed by allowing both α and β to be state dependent. We will consider this possibility later.

In order to possess finite mean and autocorrelation, the univariate model (9) needs to be asymptotically covariance stationary: given stationarity of r_t and π_t^*, this requires that $\left|\beta^2\phi^{-1}\right| < 1$. With β restricted to lie inside the unit circle, this always holds when the Taylor rule is active, $\phi > 1$,in which case the unconditional mean of π_t is simply

$$\bar{\pi} = \frac{\alpha(1-\beta^2)}{\phi - \beta^2}, \tag{10}$$

Hence, the first consistency requirement of the SCEE, $\bar{\pi} = \alpha$, implies that $\alpha = 0$. Straightforward computation gives the following expression for the first-order autocorrelation coefficient:

$$Corr\left(\pi_t, \pi_{t-1}\right) = F(\beta) \equiv \frac{\beta^2 + \rho\phi + (1-\rho^2)\phi\beta^2(\phi - \rho\beta^2)\frac{\sigma^2_{\pi^*}}{\sigma^2_{\varepsilon}}}{\rho\beta^2 + \phi + (1-\rho^2)\phi^2(\phi - \rho\beta^2)\frac{\sigma^2_{\pi^*}}{\sigma^2_{\varepsilon}}} \tag{11}$$

The second consistency requirement is then $G\left(\beta\right) \equiv F\left(\beta\right) - \beta = 0$ for some $\beta^* \in \left(0, 1\right)$, with $\beta^* < \sqrt{\phi}$. Due to the complexity of (11), analytical results on the existence and uniqueness of SCEE are attainable only for specific cases. Proposition 1 establishes some with respect to the the degree of policy aggressiveness and the persistence of the real interest rate.

Proposition 1 *Let $G\left(\beta\right) \equiv F\left(\beta\right) - \beta$, with $F\left(\beta\right)$ defined in (11). The following results hold.*

1. *Degree of Policy Aggressiveness*

- *Under an extremely active response to inflation, $\phi \to \infty$, there exists a unique SCEE with $\beta \to 0$.*
- *Under an extremely passive response to inflation, $\phi \to 0$, no SCEE exists.*

2. *Real Interest Rate Persistence*

- *For $\rho \to 1$, there exists a unique SCEE with $\beta \to 1$ if $\phi > 1$, while no SCEE exists if $\phi \in \left(0, 1\right)$.*
- *For $\rho = 0$, there exists a unique SCEE with $\beta = 0$ if $\phi > 1$, while two SCEE exists (one with $\beta = 0$ and a second one with $\beta = \phi$) if $\phi \in \left(0, 1\right)$.*

Proof. See Appendix. ■

More general cases are characterized numerically. For the moment, we restrict the analysis to the case of an active policy rule. Figure 1 plots $G(\beta)$ for three alternative values of ϕ (1.1, 1.5 and 2), and for three alternative values of ρ (0.1, 0.5 and 0.8), while setting $\sigma_\varepsilon = \sigma_{\pi^*} = 0.01$. The first order autocorrelation coefficient under rational expectations, $Corr^{RE}$, is obtained from the minimum state variable solution for inflation, $\pi_t = \frac{1}{\phi-\rho} r_t + \pi_t^*$. Simple algebra gives

$$Corr^{RE} = \frac{\rho}{1 + (1-\rho^2)(\phi-\rho)^2 \frac{\sigma^2_{\pi^*}}{\sigma^2_\varepsilon}} < \rho \tag{12}$$

Few results emerge. First, for each parameterization, a SCEE always exists and is unique. This equilibrium coexists with a unique REE. Second, for given $\phi > 1$, increasing the persistence parameter ρ yields a higher autocorrelation coefficient β^* in the SCEE. Third, for given ρ, a more aggressive central bank lowers the persistence of inflation in the SCEE.

We also notice that for the SCEE to exhibit higher first order autocorrelation than the underlying REE - that is for $\beta^* > Corr^{RE}$ - we need r_t to be sufficiently persistent and monetary policy to be not very *active*. This is evident from panel a), where β^* appears very close to unity, while the autocorrelation under RE is only 0.77. As policy becomes more aggressive, the persistence of inflation in the REE and the SCEE are almost indistinguishable. This result hints to the possibility of having the central bank choosing ϕ appropriately to induce a certain (desirable) degree of inflation persistence (hence volatility) irrespective of the prevailing type of equilibrium, or, equivalently, of how agents form inflation expectations. We discuss this more in details in a later section.

To get a better sense of the interplay between ϕ and ρ, Figure 2 provides a visual description of the set of SCEE for $\phi \in (0, 2)$ - hence, we also consider the case of a passive rule - and $\rho \in (0, 0.95)$.[5] For a policy rule that responds actively to inflation (which in turn implies that the REE is always unique), the SCEE always exists and is unique. However, depending on the combinations of ϕ and ρ, the unique SCEE can display either higher (light grey) or lower (darker grey) first order autocorrelation with respect to the REE solution. In particular, we find that for ρ sufficiently large (higher than about 0.4), monetary policy can ensure that the SCEE is less persistent than under REE by responding sufficiently aggressively to inflation. This result is clearly consistent with what displayed in Figure 1. Under a passive rule, the set of equilibria is rather different: either

[5] While stationarity of (9) is always guaranteed for any $\beta \in (0, 1)$ when $\phi > 1$, under a passsive rule ($\phi \in (0, 1)$), for a SCEE to exist we also check that indeed $\left|\beta^2 \phi^{-1}\right| < 1$ holds.

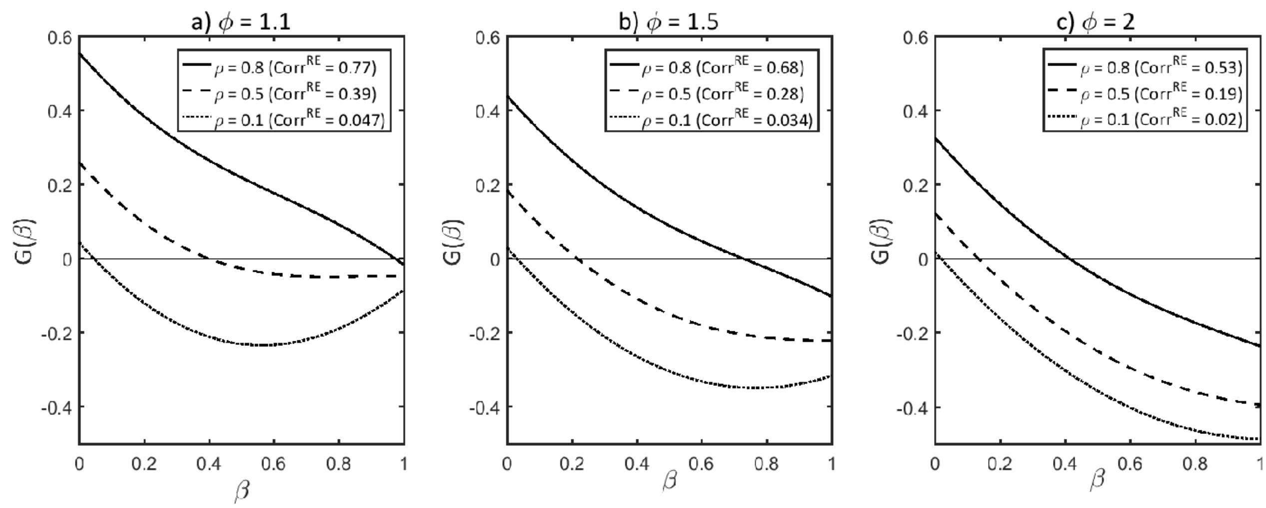


Figure 1: Plot of $G(\beta)$ for degrees of policy aggressiveness and shock persistence.

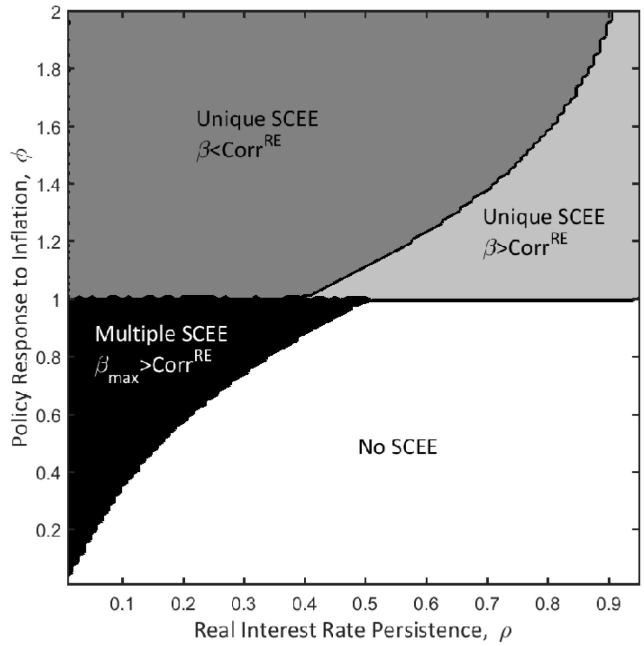


Figure 2: Existence, Uniqueness and Persistence of SCEE

there is none (white area) or there are multiple SCEE (black area), in which at least one displays autocorrelation larger than what observed in the *fundamental* MSV-RE equilibrium.[6] Notice that for the extreme cases of $\rho = 0$ and $\rho \to 1$ the type of equilibria displayed in the figure are consistent with the analytical results of Proposition 1.

The existence, uniqueness and persistence properties of the SCEE are significantly affected by the relative variance of the inflation target π_t^* to the real interest rate noise ε_t, i.e. $\kappa \equiv \sigma_{\pi^*}^2 / \sigma_{\varepsilon}^2$. As Figure 3 shows, making the inflation target relatively less volatile ($\kappa = 0.25$ in panel a), while the ratio is unity in Figure 2), enlarges the space of active policy rules for which the unique SCEE deliver high persistence in inflation. The exact opposite occurs if instead the inflation target becomes relatively more volatile ($\kappa = 4$ in panel b)). The following Proposition states analytical results for two extreme cases.

Proposition 2 *Let $\kappa \equiv \sigma_{\pi^*}^2 / \sigma_{\varepsilon}^2$, and assume $\phi > 1$. For $\kappa \to 0$ (which nests the case of a constant*

[6] Under a passive rule, the REE is locally indeterminate. The unique (fundamental) MSV solution coexists with a continuum of sunspot-driven equilibria.

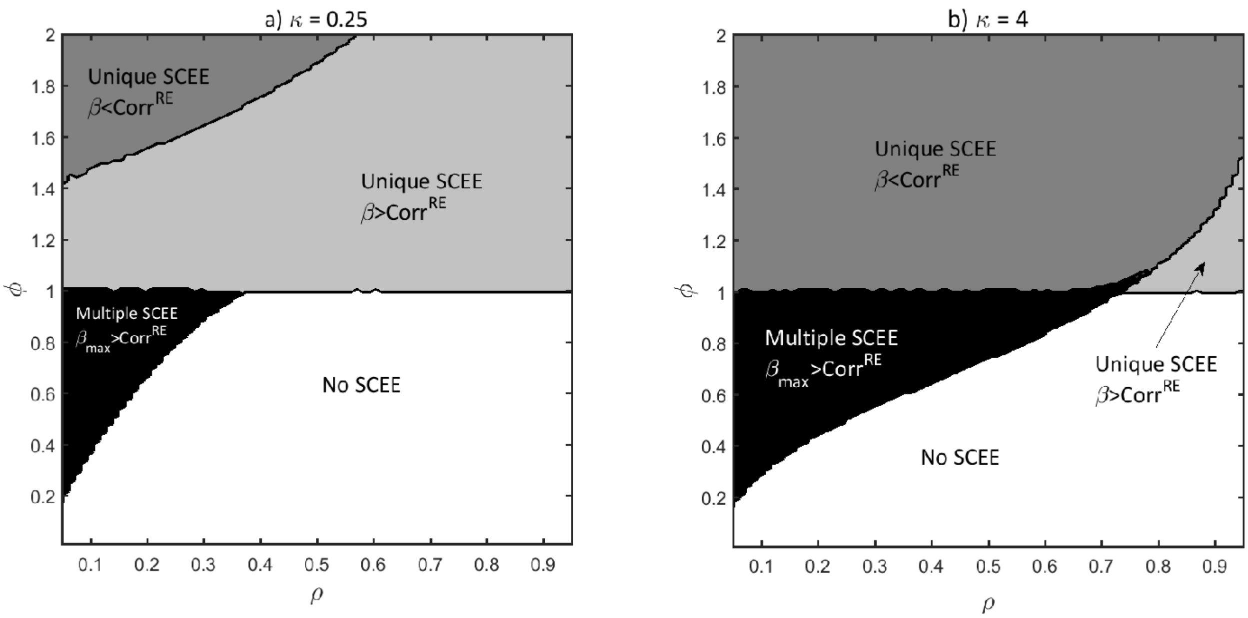


Figure 3: SCEE and Relative Variance of Fundamentals: $\kappa \equiv \sigma^2_{\pi^*}/\sigma^2_{\varepsilon}$

inflation target) - such that, under RE, the first order autocorrelation corresponds to the persistence of r_t, i.e. $Corr^{RE} = \rho$ - there exists a unique SCEE with $\beta > \rho$. For $\kappa \to \infty$, there exists a unique SCEE with $\beta = 0$.

Proof. See Appendix. ■

4 SCEE with MRS

We now move back to the MRS model. First, it is worth revisiting the conditions for existence and uniqueness of a REE (henceforth, MRS-REE), as studied in DL. Using their notation, let $\pi_{it} \equiv \pi_t(s_t = i, r_t, \pi_t^*)$ denote the solution to (5) when $s_t = i$. We can then write the contingent inflation process as follows:

$$\begin{bmatrix} \pi_{1t} \\ \pi_{2t} \end{bmatrix} = \underbrace{\begin{bmatrix} \phi_1^{-1} & 0 \\ 0 & \phi_2^{-1} \end{bmatrix} \begin{bmatrix} p_{11} & (1-p_{11}) \\ (1-p_{22}) & p_{22} \end{bmatrix}}_{M} E_t \begin{bmatrix} \pi_{1t+1} \\ \pi_{2t+1} \end{bmatrix} + \begin{bmatrix} \phi_1^{-1} & 0 \\ 0 & \phi_2^{-1} \end{bmatrix} \begin{bmatrix} r_t \\ r_t \end{bmatrix} + \begin{bmatrix} \pi_t^* \\ \pi_t^* \end{bmatrix} \tag{13}$$

When $\phi_i > 0$ for $i = 1, 2$, a necessary and sufficient condition for the existence of a unique bounded solution for inflation in (13) is that all the eigenvalues of matrix M lie within the unit circle. For sake of completeness, we state without proving the following proposition, which closely corresponds to Proposition 2 in DL.

Proposition 3 *Assume $\phi_i > p_{ii}$ for $i = 1, 2$. There exists a unique bounded solution to (13) if and only if $\phi_i > 1$ for some $i = 1, 2$, and*

$$\phi_1\phi_2 + (1 - \phi_2)p_{11} + (1 - \phi_1)p_{22} > 1 \text{ (Long-Run Taylor Principle)} \tag{14}$$

For what concerns the existence of a SCEE instead, we first need to make sure that the ALM under MRS (8) possesses finite mean and autocorrelation. For this purpose, we follow Farmer et al. (2009) (henceforth, FWZ) and appeal to the concept of Mean Square Stability (MSS). The MRS process (8) is mean-square-stable if it possess finite ergodic first and second moments. Let

$$N \equiv \beta^4 \begin{bmatrix} p_{11}/\phi_1^2 & (1 - p_{22})/\phi_1^2 \\ (1 - p_{11})/\phi_2^2 & p_{22}/\phi_2^2 \end{bmatrix}$$

Applying results from FWZ and Cho (2016), the ALM (8) satisfies MSS if all eigenvalues of N lie within the unit circle in the complex plane. Proposition 4 establishes a sufficient condition for MSS.

Proposition 4 *Given $\beta \in [0, 1)$ and $\phi_i > \sqrt{p_{ii}}$ for $i = 1, 2$, a sufficient condition for the ALM (A.1) to be MSS is that the following inequality is satisfied*

$$\phi_1^2\phi_2^2 + (1 - \phi_2^2)p_{11} + (1 - \phi_1^2)p_{22} > 1 \tag{15}$$

Proof. See Appendix. ■

The inequality in (15) is indeed quite similar to the long-run-Taylor-principle (LRTP) defined in (14). As the following proposition shows, if the sufficient condition for MSS stated in Proposition 4 holds, so does the LRTP.

Proposition 5 *Assume that $\phi_i > \sqrt{p_{ii}}$ for $i = 1, 2$. If condition (15) holds, so does (14).*

Proof. See Appendix. ■

Summarizing, in our MRS context, a SCEE is exists when the ALM (8) satisfied the following three conditions**:**

1. it is mean-square stable, such that first and second ergodic moments exists and are finite;

2. has unconditional mean equal to α;

3. has unconditional first-order autocorrelation equal to β.

Appendix A derives analytical expressions for both the unconditional mean and the first order autocorrelation coefficient of (8). Since both of them are functions of α and β, we denote them, respectively, as $D(\alpha,\beta)$ and $H(\alpha,\beta)$. Hence, according to our definition, a SCEE exist when $D(\alpha,\beta)=\alpha$ and $H(\alpha,\beta)=\beta$.

For sake of comparrison, consider the solution under RE. For parameters configurations for which determinacy holds, we can write the MSV solution of (13) as

$$\pi_t = \psi(s_t)r_t + \pi_t^* \tag{16}$$

where, following DL, the state-dependent coefficients $\psi_1 \equiv \psi(s_t=1)$ and $\psi_2 \equiv \psi(s_t=2)$ are

$$\psi_1 = \psi_1^F\left(\frac{1+\rho p_{12}\psi_2^F}{1-\rho^2 p_{12}\psi_2^F p_{21}\psi_1^F}\right) \tag{17}$$

$$\psi_2 = \psi_2^F\left(\frac{1+\rho p_{21}\psi_1^F}{1-\rho^2 p_{21}\psi_1^F p_{12}\psi_2^F}\right) \tag{18}$$

where $\psi_i^F \equiv \frac{1}{\phi_i-\rho p_{ii}}$ for $i=1,2$ correspond to the fixed regime MSV solutions. By stationarity of r_t and π_t^*, and ergodicity of the Markov chain, the MSV (16) is MMS. Notice that (16) is nested into the more general form (A.1) studied in the Appendix (simply set $c(s_t)$ and $a(s_t)$ equal to zero in all states). We can therefore apply the same procedure to find an expression for the first-order autocorrelation under RE. Straightforward but tedious algebra gives:

$$Corr_{MSV}^{RE} = \frac{p_{11}(1-p_{22})\psi_1^2 + (1-p_{11})p_{22}\psi_2^2 + 2(1-p_{11})(1-p_{22})\psi_1\psi_2}{\left[(1-p_{22})\psi_1^2 + (1-p_{11})\psi_2^2\right] + (2-p_{11}-p_{22})(1-\rho^2)\frac{\sigma_\pi^2}{\sigma_\varepsilon^2}}\rho < \rho \tag{19}$$

Corollary 6 *The mean in the determinate REE is the same as in a SCEE. Namely,* $\alpha = 0$.

The corollary implies that for both types of agents, the (log) inflation series will be fluctuating around zero. As the analytical expression for the first-order autocorrelation of (8) is rather complicated, we study the existence of SCEE by numerical methods. As for the fixed coefficients case, we set $\sigma_\varepsilon = \sigma_{\pi^*} = 0.01$, and consider alternative parameterizations for the persistence parameter ρ.

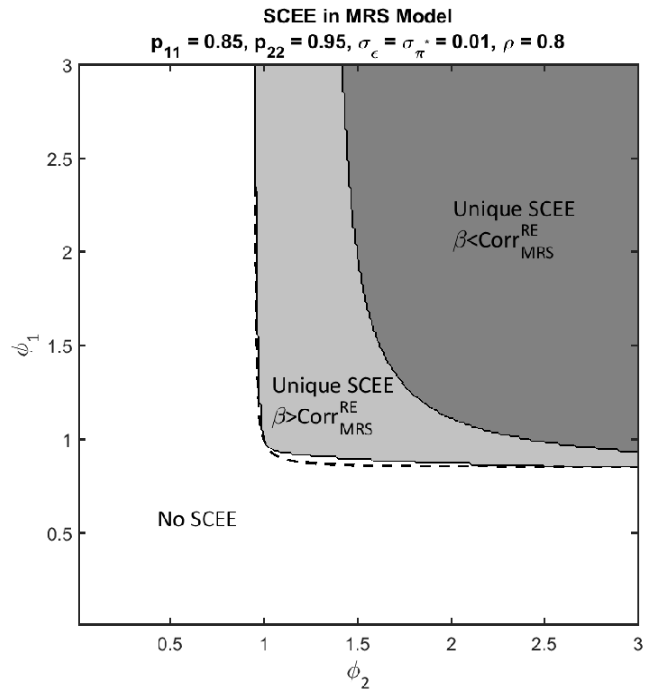


Figure 4: **SCEE in MRS Model: Baseline Calibration**

To start with, we set $\rho = 0.8$. Key parameters are the transition probabilities. As baseline values we use $p_{11} = 0.85$ and $p_{22} = 0.95$, which are very close to what used in the previously cited works on MRS models of monetary policy.

Figure 4 displays the results, for ϕ_1 and ϕ_2 ranging between 0 and 3. Three types of regions emerge: one where a SCEE does not exists (white area); one where the SCEE is unique and displays an autocorrelation coefficient larger than under RE (light grey), and one where the SCEE is again unique but displays a lower autocorrelation than under RE (darker grey). The dashed line corresponds to the LRTP of DL: under RE, the MRS model is locally determinate for combinations of ϕ_1 and ϕ_2 above the dashed line.

First of all, under this baseline calibration, within the RE determinacy area, if it exists, the SCEE is always unique, while no SCEE exists when the MRS-REE is locally indeterminate. This implies that the LRTP can rule out RE multiplicity but cannot eliminate the possibility of fluctuations driven by bounded rationality.

Second, there is a wide range of combinations for which the unique SCEE delivers higher persistence (and hence volatility) compared to the RE counterpart. With respect to the fixed coefficient case, the possibility of a switch makes it harder for a central bank to rule out these higher persistence equilibria. Consider the case of $\rho = 0.8$ in Figure 2: a central bank which is permanently in power can eliminate the persistent SCEE by responding to inflation with a coefficient roughly above 1.5. Now let a regime switching occur with transition probabilities $p_{11} = 0.85$ and $p_{22} = 0.95$, that is regime 1 (where the current central bank is operating) is very likely to persist, but once the switch occurs regime 2 could last quite long. If the central bank operating in regime 2 adopts a mildly active response to inflation, $\phi_2 \in (1, 1.5))$, the regime 1 central bank will not be able to eliminate the persistent SCEE, not even by granting a very aggressive response to inflation. Setting $\phi_1 = 1.5$ might do it, but only if, once in power, the regime 2 central bank will be sufficiently aggressive. In this sense, the introduction of MRS can enhance the possibility of persistent SCEE.

Figure 5 shows the consequences of lowering p_{22}, hence shortening the expected duration of the second regime. The three different colors have the same interpretation of the Figure 4. As p_{22} gets smaller, the region where a SCEE exists expands beyond the LRTP threshold. A unique SCEE can then coexist with multiple REE. As the second regime becomes sufficiently short-lasting, a SCEE becomes possible for virtually any policy parameterization in the second regime, even a very passive one. This stands in stark contrast with what found under fixed coefficients, where, under the same calibration, a passive response to inflation never leads to a SCEE.

Looking at the results displayed in Figures 4 and 5, one might conclude that, under the assumption of (almost) symmetric persistence across states - namely, $p_{11} \approx p_{22}$ - the policy spaces for which a unique REE and a unique SCEE exist are equivalent. This is not correct. In Figure 6 we consider three cases with $p_{11} = p_{22}$. When both regimes are very persistence (panel a), it is indeed the case that policy combinations for which a REE is locally determinate also yield a unique SCEE. As persistence declines (but symmetry remains), the existence/uniqueness region for a SCEE spills beyond the LRTP (dashed line). We can therefore have policy combinations for which a unique SCEE coexists with a continuum of REE. This becomes even more remarkable as the persistence of both regimes approaches zero.

Figure 7 consider the case of a more persistent regime 1 ($p_{11} = 0.95$), while restricting regime 2 to adopt only a passive monetary policy, $\phi_2 \in (0, 1)$. In panel a), we have $p_{22} = 0$, such that after

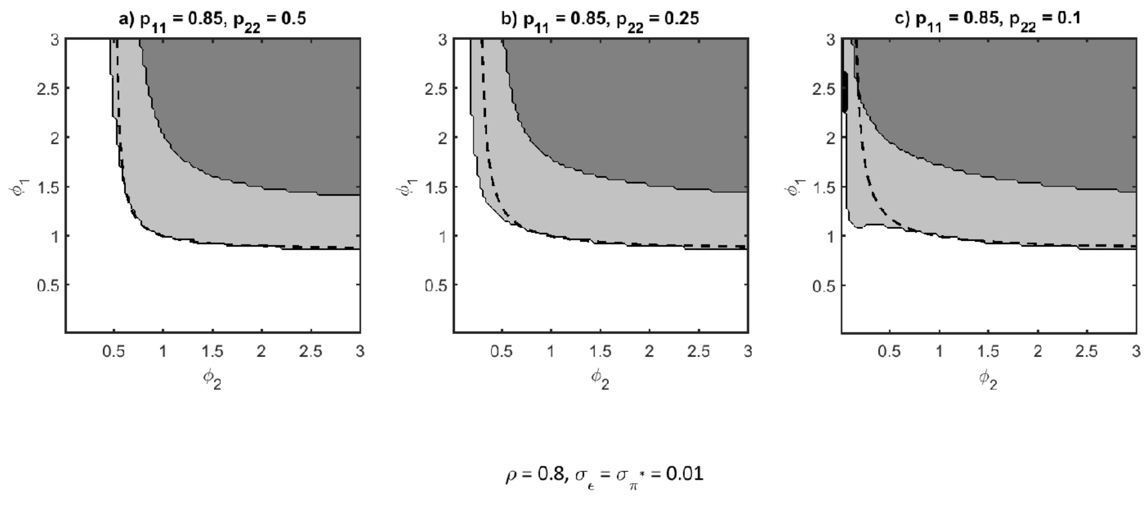


Figure 5: **SCEE in MRS Model: Lowering Regime 2 Duration**

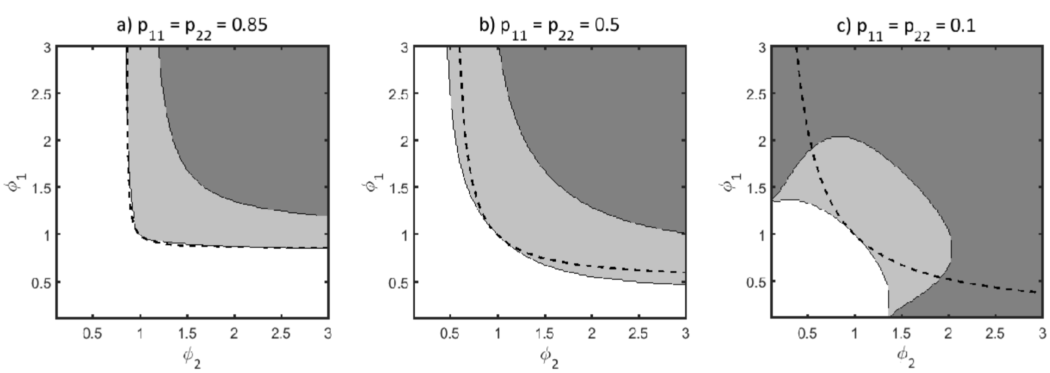


Figure 6: **SCEE in MRS Model: Symmetric Regime Switching**

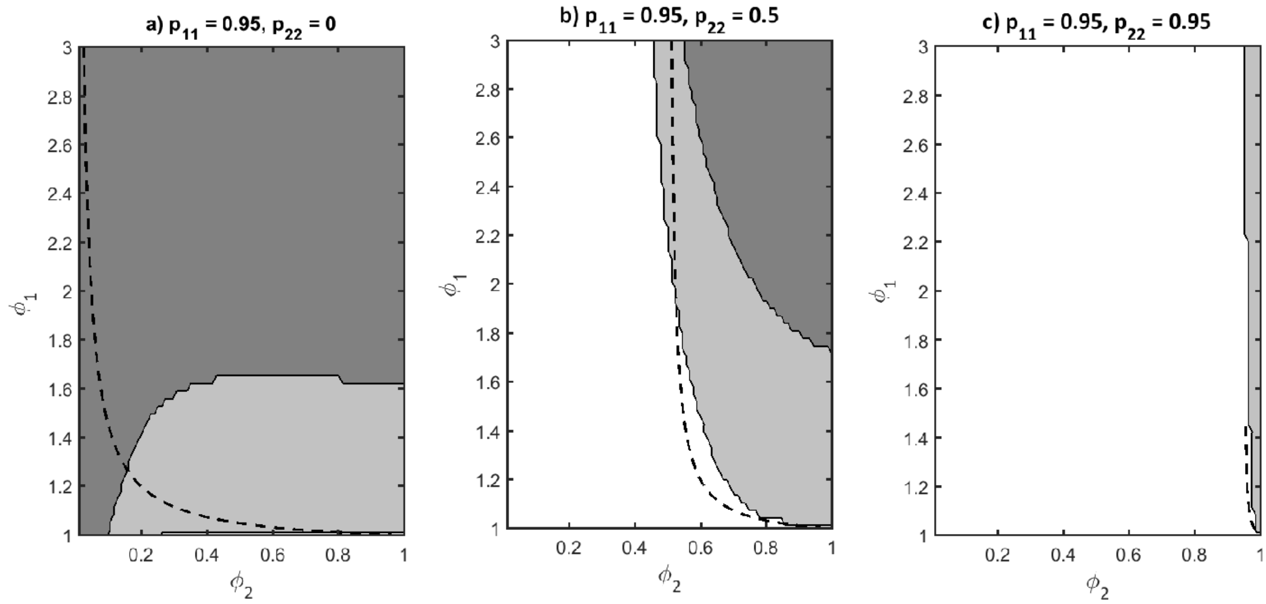


Figure 7: **SCEE in MRS Model: Passive Regime 2**

hitting regime 2 the economy immediately reverts back to regime 1. In this case a unique SCEE always exists, for any $\phi_2 \in (0, 1)$ and any $\phi_1 > 1$. Whenever the active regime adopts a policy response lower than about 1.5, the SCEE displays higher persistence than under RE for almost any passive policy rule. In panel b), the expected duration is 2 periods. In this case, a very passive response (for the specific calibration, $\phi_2 < 0.5$) never leads to a SCEE (while multiple REE are possible). For values of ϕ_2 yielding a SCEE, the regime 1 response has to be quite aggressive to eliminate the more persistent SCEE. When the second regime becomes extremely persistent (panel c), a unique persistent SCEE can still occur, but requires ϕ_2 to be very close to unity. In this case it is also very likely to observe multiple REE.

5 Stability under Learning

Following HZ, we assume agents use sample autocorrelation (SAC) learning to estimate values for α and β. More specifically, given a sample of observed inflation rates up to period t, $\{\pi_0, \pi_1, ..., \pi_t\}$, their estimates correspond, respectively, to the sample average and first-order autocorrelation, as given by

$$\alpha_t = \frac{1}{t+1}\sum_{j=0}^{t}\pi_j, \tag{20}$$

$$\beta_t = \frac{\sum_{j=0}^{t-1}(\pi_j - \alpha_t)(\pi_{j-1} - \alpha_t)}{\sum_{i=0}^{t}(\pi_i - \alpha_t)^2} \tag{21}$$

As more data on inflation comes available, estimates are updated accordingly. Letting $R_t \equiv \frac{1}{t+1}\sum_{i=0}^{t}(x_i - \alpha_t)^2$, it is relatively straightforward to derive the following recursive expressions:

$$\alpha_t = \alpha_{t-1} + \frac{1}{t+1}(\pi_t - \alpha_{t-1}) \tag{22}$$

$$\beta_t = \beta_{t-1} + \frac{1}{t+1}R_t^{-1}\left[(\pi_t - \alpha_{t-1})(\pi_{t-1} + \frac{\pi_0}{t+1} - \frac{t^2+3t+1}{(t+1)^2}\alpha_{t-1} - \frac{1}{(t+1)^2}\pi_t) - \frac{t}{t+1}\beta_{t-1}(\pi_t - \alpha_{t-1})^2\right] \tag{23}$$

$$R_t = R_{t-1} + \frac{1}{t+1}\left[\frac{t}{t+1}(\pi_t - \alpha_{t-1})^2 - R_{t-1}\right] \tag{24}$$

With time-varying mean and autocorrelation, under SAC learning, the ALM becomes

$$\pi_t = \frac{\alpha_{t-1}(1-\beta_{t-1}^2)}{\phi(s_t)} + \frac{\beta_{t-1}^2}{\phi(s_t)}\pi_{t-1} + \frac{1}{\phi(s_t)}r_t + \pi_t^* \tag{25}$$

The ALM defines a map $T : R^2 \to R^2$ from perceived to actual sample moments, $T(\alpha, \beta) = (D(\alpha, \beta), H(\alpha, \beta))$, with associated ordinary differential equations (ODE)

$$\frac{d\alpha}{d\tau} = D(\alpha, \beta) - \alpha \tag{26}$$

$$\frac{d\beta}{d\tau} = H(\alpha, \beta) - \beta \tag{27}$$

A SCEE is a fixed point $(0, \beta^*)$ of the system (26)-(27).

Proposition 7 *When the SCEE is unique it is always SAC-stable.*

Proof. See Appendix. ■

Figure 8 plots the simulated dynamics for the sample mean α_t and autocorrelation β_t under SAC-learning. Convergence (hence stability) occurs under both the case of fixed and MRS policy coefficients.

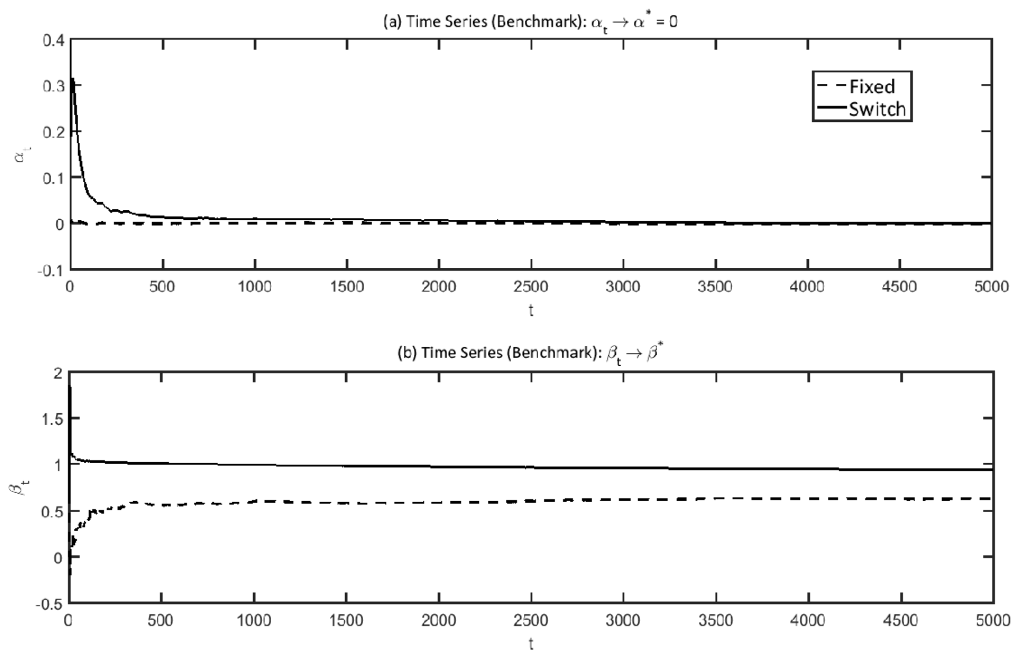


Figure 8: Stability under SAC-learning

6 Excess Volatility and Persistence

As found in the previous sections, a SCEE can generate excess inflation persistence with respect to a REE. Does it lead to excess inflation volatility as well? Can monetary policy be set in a way to replicate the persistence and volatility properties of a REE?

We begin with the case of fixed policy coefficients by comparing the volatility of inflation in a SCEE with what obtained under RE, $\frac{Var(\pi^{SCEE})}{Var(\pi^{REE})}$. Figure 9 displays the results with respect to the persistence parameter ρ, which each panel corresponding to a different inflation coefficient ϕ. Within each panel we consider three alternative values for the relative volatility of the inflation target π_t^* to the real interest rate noise ε_t, namely we let $\kappa \equiv \sigma_{\pi^*}^2/\sigma_\varepsilon^2$ be either 0.25, 1 or 4. Consider panel a), and, in particular, our benchmark case of $\kappa = 1$. The relationship between the relative volatility of inflation across the two equilibrium concepts and ρ is strongly non-monotonic. First, inflation appears equally volatile in a SCEE and in a REE for $\rho = 0$ and $\rho \to 1$. Second, a SCEE can generate higher volatility than under RE for intermediate-to-high degrees of persistence in fundamentals. Interestingly, the relationship between $\frac{Var(\pi^{SCEE})}{Var(\pi^{REE})}$ and ρ is hump-shaped, with $\rho \approx 0.8$ yielding the highest relative volatility. Lowering (respectively, increasing) κ enlarges (respectively, restricts) the range of ρ values yielding excess SCEE volatility. Moving across panels, we notice how a more aggressive policy response to inflation tends to eliminate the excess volatility caused by bounded rationality. For $\phi = 1.5$ - the most common Taylor rule parameterization - a SCEE is at most as volatile as the REE.

To further stress the dampening effect of policy aggressiveness on the excess volatility due to bounded rationality, in Figure 10, we display the relationship between the relative volatility and ϕ, fixing $\rho = 0.8$. Along a SCEE inflation is remarkably more volatile than under RE when the central bank's active response to inflation is close to unity. This result holds irrespective of the value we assign to κ. The relationship is highly non-monotonic. In the limit case of an infinite policy response to inflation, relative volatility converges to unity.

We pursue the same analysis for the MRS model. Panel a) in Figure 6 shows the case of a switch between two active policies, $\phi_1 = 1.5$ and $\phi_2 = 1.05$. We set $p_{11} = 0.85$ and consider three alternative values of p_{22}. Recall from Figure 9 (panel c) that in a fixed regime set-up - i.e. for $p_{11} = 1$ - and for the baseline calibration $\phi_1 = 1.5$, inflation is never more volatile than under RE. Allowing for regime switching, we find instead that, as long as the second regime is sufficiently long-

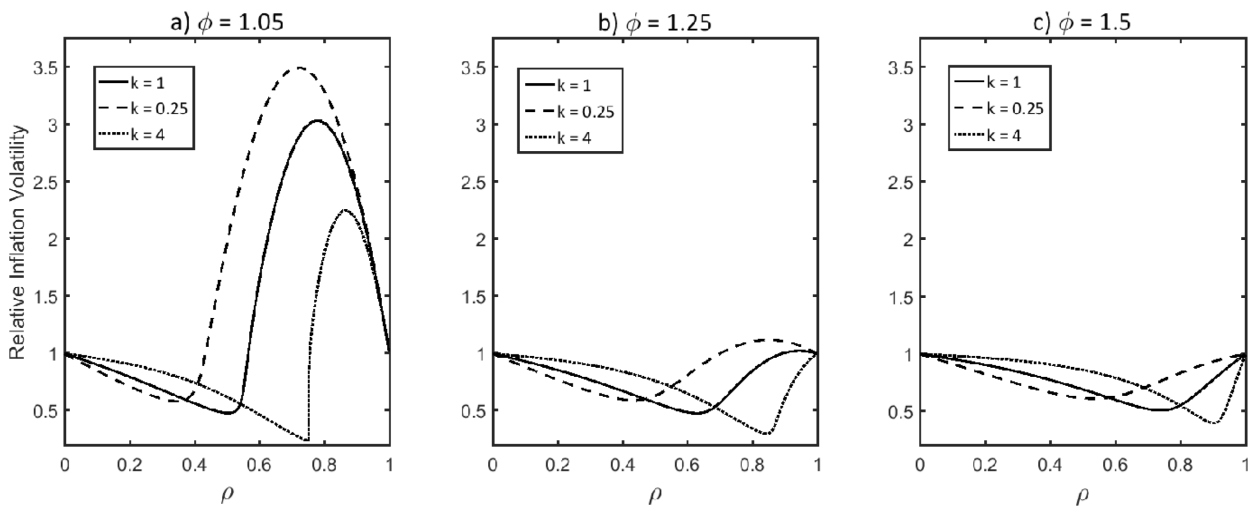


Figure 9: Relative Inflation Volatility of SCEE versus RE, $Var\left(\pi_t^{SCEE}\right)/Var\left(\pi_t^{RE}\right)$: Impact of Fundamental Persistence ρ

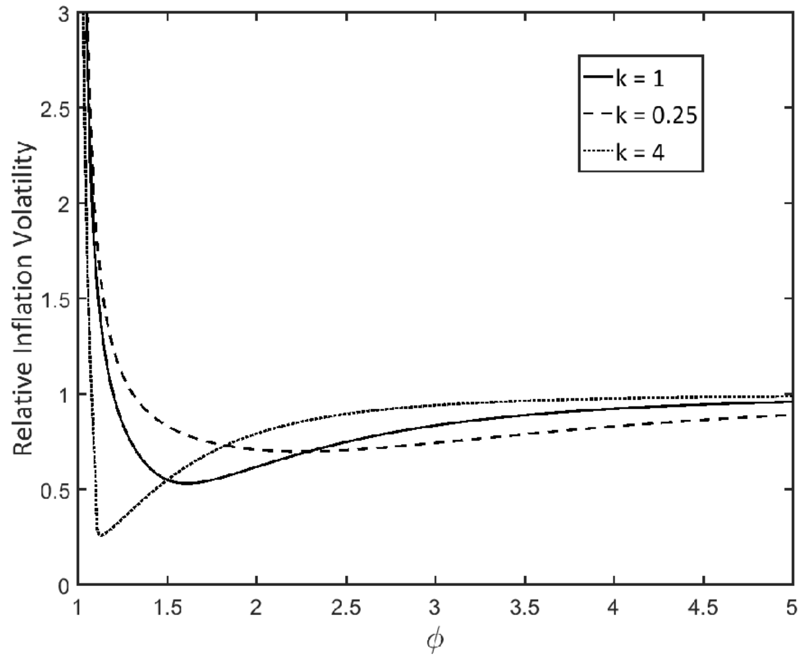
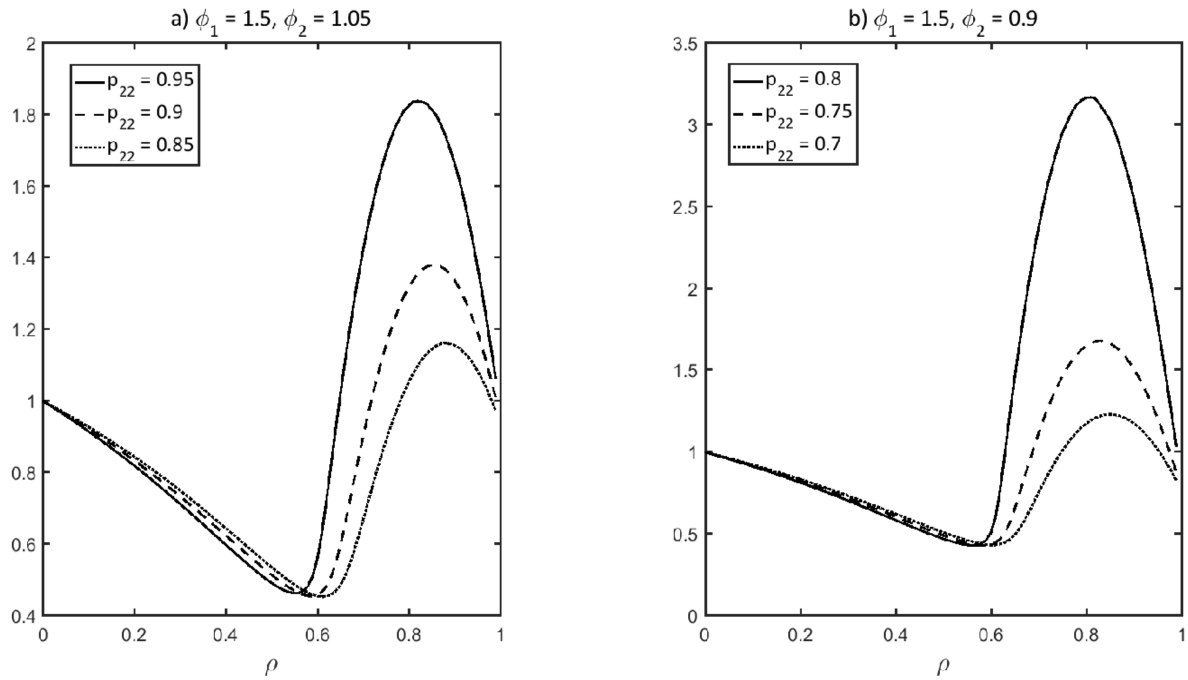


Figure 10: Relative Inflation Volatility of SCEE versus RE, $Var\left(\pi_t^{SCEE}\right)/Var\left(\pi_t^{RE}\right)$: Impact of Policy Aggressiveness ϕ



lived, such policy cannot rule out excess volatility. For instance, let $\rho = 0.8$. A SCEE delivers an almost twice as volatile inflation for $p_{22} = 0.95$ (a standard parameterization used in DL). Similar results hold for the case of a switch to a passive policy where $\phi = 0.9$ (panel b). In this case, excess volatility obtains for even lower values of p_{22}.[7]

Based on these results on unconditional moments, we simulate a time series for inflation along the REE and the SCEE, setting $\phi_1 = 1.5$. Due to the lack of regime change, in the top panel of Figure 11, inflation behaves very similarly along both equilibria. This is consistent with panel c) in Figure 10. As the middle and bottom panels show, excess volatility and persistence is possible with regime switching as long as the passive regime (with $\phi_2 = 0.9$) is sufficiently long-lived.

Following these exercises, a natural question that may arise is whether a central bank could pick ϕ_1 appropriately to make the SCEE coincide with the REE along some statistical dimensions. For instance, could a central bank operating under fixed regime replicate the autocorrelation observed under RE? Figure 12 displays the first order autocorrelation coefficients along both the unique REE and the SCEE as functions of ϕ_1. In the top panel, the relative volatility $\kappa \equiv \sigma^2_{\pi^*}/\sigma^2_{\varepsilon}$ is unity. We observe a coincidence for ϕ_1 around 1.6. In the bottom panel we have instead $\kappa = 0$, which

[7] We find that by further increasing p_{22} above 0.8 no SCEE exists as the MRS model losses mean-square-stability.

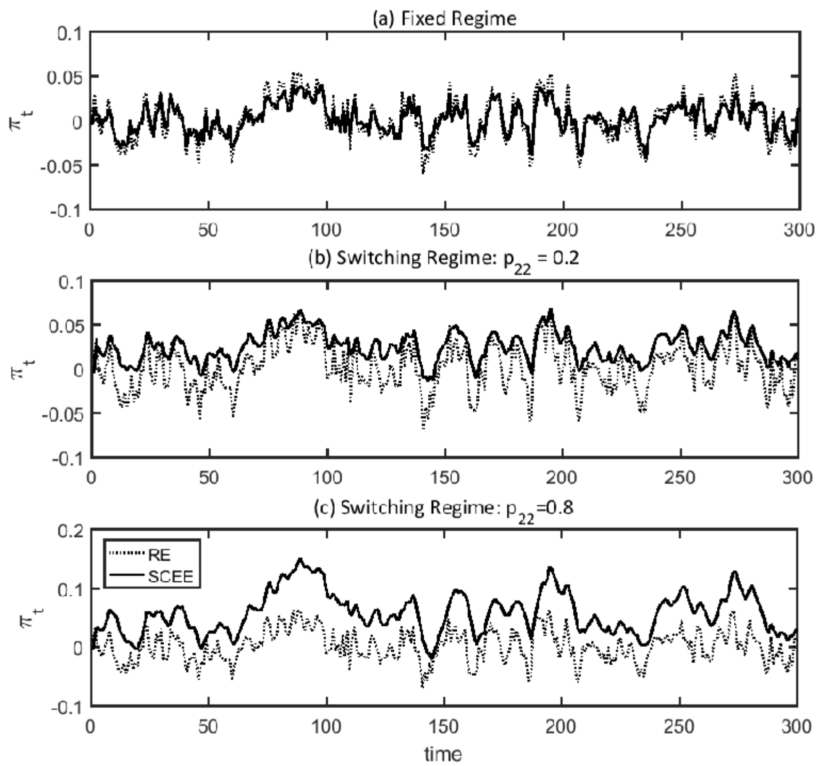


Figure 11: Model Simulated Inflation

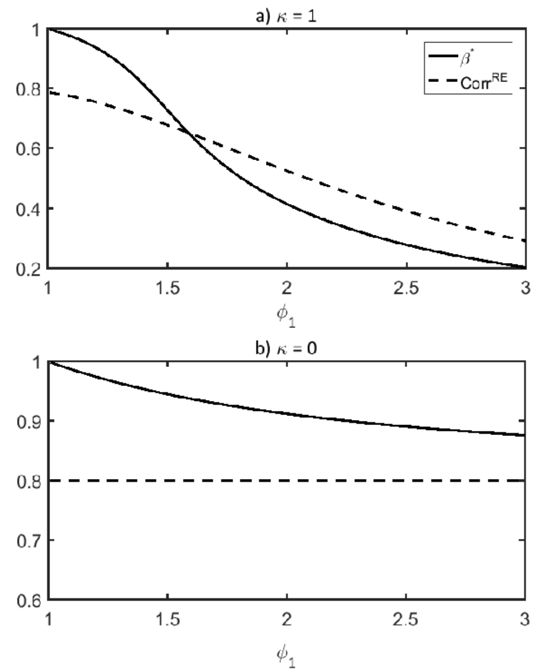


Figure 12: First Order Autocorrelation as Function of ϕ_1: REE versus SCEE under Fixed Regime

corresponds to the case of a constant inflation target. In this case the SCEE displays always higher persistence than the REE, for any policy response to inflation. Figure 13 replicates the analysis for the MRS case. Panel a) considers the baseline calibration. A current central bank (regime 1) facing the possibility of being replaced by a passive central banker ($\phi_2 = 0.9$ and $p_{22} = 0.2$ in regime 2) with a 15% probability ($p_{11} = 0.85$) can still mimic the REE by setting ϕ_1 equal to about 1.8. This challenge appears daunting when the inflation target is constant and/or when the alternative central bank might remain sufficiently long in power.

7 Conclusions

We have studied the existence of Stochastic Consistence Expectations Equilibria (SCEE) in a simple Fisherian model of inflation determination, allowing for the possibility of a Markov regime switch (MRS) in the Taylor-rule responsiveness to inflation. We have studied the stability under learning of SCEE, as well has analyzed its dynamic properties, in particular for what concerns the possibility

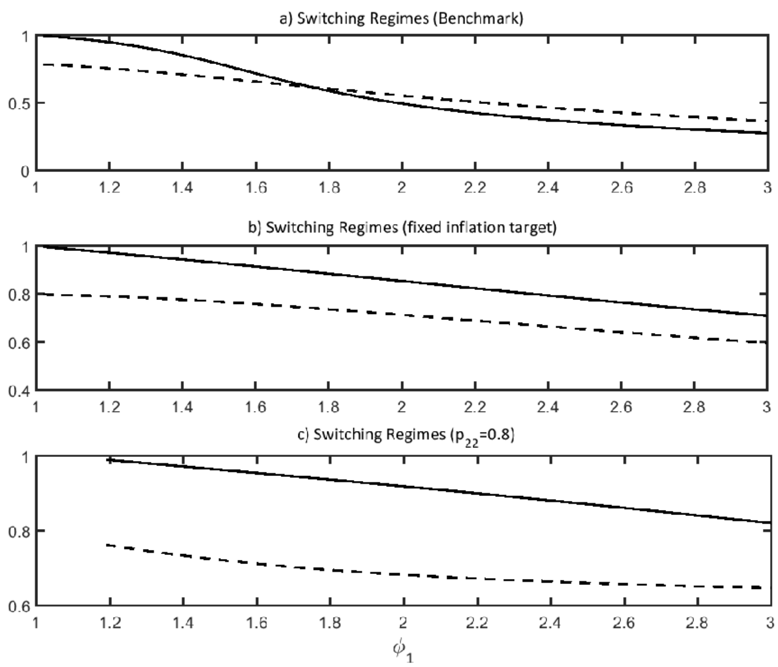


Figure 13: First Order Autocorrelation as Function of ϕ_1: REE versus SCEE under MRS Regime

of excess volatility and persistence with respect to rational expectations.

Within a fixed regime environment, we have found that as long as the Taylor rule's response to inflation is active, a SCEE always exists and is unique. Such equilibrium might feature higher or lower persistence compared to rational expectations depending on how active the response is in relation to the degree of persistence in the underlying fundamentals (the real interest rate). Under a passive response instead, either there is none or there are multiple SCEE (some more and some less persistent than under RE). We also find a highly non-monotonic relationship between the excess inflation volatility of a SCEE (with respect to the REE) and the degree of persistence in fundamentals. The latter needs to lie above 0.5 (but sufficiently below unity), and requires a not too active response to inflation for the model to generate excess volatility. Setting the response to inflation to 1.5 - a common parameterization in the literature - guarantees that the SCEE is at most as volatile as the REE.

Under MRS, we find that the Long-Run-Taylor-Principle identified by Davig and Leeper (2007) (as a criterion to eliminate rational sunspot-driven equilibria) cannot eliminate the possibility of fluctuations driven by bounded rationality. Namely, within their RE determinacy region, a unique SCEE always exists. Moreover, there is a wide range of policy combinations across regimes for which such equilibrium delivers excess persistence. Interestingly we find that, with respect to the fixed coefficient case, the possibility of a switch makes it harder for a central bank to rule out these higher persistence equilibria. Compared to the case of RE, a regime switch makes the model more prone to induce excess inflation volatility. An active policy response in regime 1 might not suffice if regime 2 is passive and sufficiently persistent.

We are currently working on extending the model in several directions. First, to better capture the liquidity traps episodes post the 2007-9 financial crisis, we are currently studying the case where the policy switch is endogenous: policy might move from active to passive as inflation falls below a certain threshold and/or as a fundamental shock make the Taylor rule violate the zero-lower-bound. Second, we plan to extend the analysis to the case of nominal rigidities. This will allow us to think more deeply about optimal policy by doing appropriate welfare comparisons.

Appendix

A Univariate Markov Regime Switching Model

Consider the following generic univariate regime switching (RS) model

$$x_t = c(s_t) + a(s_t)\, x_{t-1} + b(s_t)\, y_t + u_t \tag{A.1}$$

where $c(s_t)$, $a(s_t)$ and $b(s_t)$ are regime-dependent coefficients (s_t denoting the state or regime), while

$$y_t = \rho y_{t-1} + \varepsilon_t, \qquad |\rho| < 1, \ \varepsilon_t \sim iid\left(\bar{\varepsilon}, \sigma_\varepsilon^2\right) \tag{A.2}$$

$$u_t \sim iid\left(\bar{u}, \sigma_u^2\right) \tag{A.3}$$

are regime-independent stationary shocks.[8]. The model we develop in Section 2 and study in Section 4 has $c(s_t) \equiv \frac{\alpha(1-\beta^2)}{\phi(s_t)}$, $a(s_t) \equiv \frac{\beta^2}{\phi(s_t)}$, and $b(s_t) \equiv \frac{1}{\phi(s_t)}$, with $y_t = r_t$ and $u_t = \pi_t^*$.

For simplicity, we assume the state s_t can take only two values, i.e. $s_t \in \{1, 2\}$, with transition probabilities $p_{ij} \equiv \mathcal{P}(s_t = j \mid s_{t-1} = i)$, for $i, j = 1, 2$. The associated ergodic Markov-chain transition matrix is

$$P \equiv \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix} \tag{A.4}$$

We assume the RS model (A.1) satisfies *Mean Square Stability* (MSS), that is, asymptotically, it possess finite ergodic first and second moments:

$$\lim_{t\to\infty} E(x_t) = \mu$$
$$\lim_{t\to\infty} E\left(x_t^2\right) = \sigma_x^2$$

[8] We keep things general and assume that both the noise term ε_t and the *iid* shock u_t have non-zero unconditional means. In our application to a Fisherian model of inflation determination - where the linearity comes from approximating the model around a non-stochastic steady state - we have that $\bar{\varepsilon} = \bar{u} = 0$.

Given that the Markov chain is ergodic, and that the exogenous shocks y_t and u_t are covariance-stationary, the RS model (A.1) is also asymptotically covariance stationarity.[9] Under MSS, for any $k \geq 0$ and $i = 1, 2$, the followings ergodic properties hold:

$$
\begin{aligned}
E\left(x_t \mid s_t = i\right) &= E\left(x_{t-k} \mid s_{t-k} = i\right) && \text{(A.5)}\\
E\left(x_t^2 \mid s_t = i\right) &= E\left(x_{t-k}^2 \mid s_{t-k} = i\right) && \text{(A.6)}\\
E\left(x_t y_t \mid s_t = i\right) &= E\left(y_{t-k} x_{t-k} \mid s_{t-k} = i\right) && \text{(A.7)}
\end{aligned}
$$

Following Costa et al. (2004), given the stationarity of the stochastic terms y_t and u_t, and the ergodicity of the MC, the process (A.1) is MSS if and only if the homogeneous part $x_t = a\left(s_t\right) x_{t-1}$ is MSS. Letting a_1 and a_2 denote, respectively, the values of $a\left(s_t\right)$ for $s_t = 1$ and $s_t = 2$, we can apply the results in Costa et al. (2004), as seen also in Farmer et al. (2009) and Cho (2016): (A.1) is mean-square stable if and only if all eigenvalues of the following matrix H lie inside the unit circle:

$$
H = \begin{bmatrix} p_{11} a_1^2 & p_{21} a_1^2 \\ p_{12} a_2^2 & p_{22} a_2^2 \end{bmatrix} \tag{A.8}
$$

For the rest of the analysis, we will assume that indeed this is the case.

A.1 First Moment

The first moment of x_t is defined as

$$
E\left(x_t\right) = E(x_t \mid s_t = 1)\mathcal{P}\left(s_t = 1\right) + E(x_t \mid s_t = 2)\mathcal{P}\left(s_t = 1\right) \tag{A.9}
$$

where the stationary probabilities $\mathcal{P}\left(s_t = 1\right)$ and $\mathcal{P}\left(s_t = 2\right)$ solve the following system

$$
\begin{aligned}
\mathcal{P}\left(s_t = 1\right) &= p_{11}\mathcal{P}\left(s_{t-1} = 1\right) + p_{21}\mathcal{P}\left(s_{t-1} = 2\right)\\
\mathcal{P}\left(s_t = 2\right) &= p_{12}\mathcal{P}\left(s_{t-1} = 1\right) + p_{22}\mathcal{P}\left(s_{t-1} = 2\right)\\
\mathcal{P}\left(s_t = 1\right) + \mathcal{P}\left(s_t = 2\right) &= 1
\end{aligned}
$$

[9] See Section 5 in Farmer et al. (2009). Castro et al. (2004) provide a general analysis of linear markov regime-switching (also known as Markov-jump linear) processes and MSS.

for $\mathcal{P}(s_t = 1) = \mathcal{P}(s_{t-1} = 1) = P_1$ and $\mathcal{P}(s_t = 2) = \mathcal{P}(s_{t-1} = 2) = P_2$. Simple algebra gives:

$$P_1 = \frac{1 - p_{22}}{2 - p_{11} - p_{22}} \tag{A.10}$$

$$P_2 = \frac{1 - p_{11}}{2 - p_{11} - p_{22}} = 1 - P_1 \tag{A.11}$$

Let $\mu_i \equiv E(x_t \mid s_t = i)$ denote the first moment of x_t, when state s_t is in the $i-$th regime. Using the definition of x_t in (A.1), we have that:

$$\begin{aligned}
\mu_i &\equiv E(x_t \mid s_t = i) \\
&= E[c(s_t) + a(s_t)x_{t-1} + b(s_t)y_t + u_t \mid s_t = i] \\
&= \sum_{j=1}^{2} E[c(s_t) + a(s_t)x_{t-1} + b(s_t)y_t + u_t \mid s_t = i, s_{t-1} = j]\, q_{ij}
\end{aligned} \tag{A.12}$$

where $q_{ij} \equiv \mathcal{P}(s_{t-1} = j \mid s_t = i)$ is the transition probability of the associated reversed MC. In particular, using standard results from MC, we have that

$$q_{ij} = \frac{\mathcal{P}(s_t = i \mid s_{t-1} = j)\, \mathcal{P}(s_{t-1} = j)}{\mathcal{P}(s_t = i)} = p_{ji}\frac{P_j}{P_i}. \tag{A.13}$$

Using the latter, together with the expressions for P_1 and P_2 in (A.10)-(A.11), and the fact that $p_{ji} = 1 - p_{jj}$, simple algebra yields

$$q_{ij} = p_{ij}, \tag{A.14}$$

that is, the MC is time-reversible. Letting $\bar{y} = \frac{\bar{\varepsilon}}{1-\rho}$ denote the unconditional mean of y_t, from (A.12), we can find μ_1:

$$\begin{aligned}
\mu_1 &= \sum_{j=1}^{2} E[c_1 + a_1 x_{t-1} + b_1 y_t + u_t \mid s_{t-1} = j]\, q_{1j} \\
&= c_1 + b_1\bar{y} + \bar{u} + a_1 E(x_{t-1} \mid s_{t-1} = 1)\, q_{11} + a_1 E(x_{t-1} \mid s_{t-1} = 2)\, q_{12} \\
&= c_1 + b_1\bar{y} + \bar{u} + a_1[\mu_1 p_{11} + \mu_2(1 - p_{11})]
\end{aligned} \tag{A.15}$$

where the last equality makes use of (A.14) and (A.5). Similarly, we can find μ_2:

$$\mu_2 = c_2 + b_2\bar{y} + \bar{u} + a_2\left[\mu_1\left(1-p_{22}\right) + \mu_2 p_{22}\right] \tag{A.16}$$

Letting $A \equiv diag\left(a_1, a_2\right)$ denote a diagonal matrix with non-zero elements a_1 and a_2, $c \equiv \left[c_1, c_2\right]'$, and $b \equiv \left[b_1, b_2\right]'$, the unconditional mean vector $\mu \equiv \left[\mu_1, \mu_2\right]'$ is $\bar{u}$

$$\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \left(I_2 - AP\right)^{-1}\left(\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\bar{y} + \begin{bmatrix} \bar{u} \\ \bar{u} \end{bmatrix}\right)$$

The first moment of x_t defined in (A.9) is therefore

$$E\left(x_t\right) = P_1\mu_1 + \left(1-P_1\right)\mu_2 \tag{A.17}$$

By the definition of c_1 and c_2 at the beginning of this Appendix, the unconditional mean $E\left(x_t\right)$ is a function of α and β. We therefore let $D\left(\alpha,\beta\right) \equiv E\left(x_t\right)$. Notice that α only enters c_1 and c_2 in a linear form, while β enters c_i,and a_i, for $i = 1, 2$. Hence $D\left(\alpha,\beta\right)$ takes the following form $\alpha\tilde{D}\left(\beta\right)$, where $\tilde{D}\left(\beta\right)$ is a non-linear function of β.

In the RS model of interest, we have $c_1 = c_2 = 0$, which combined with the fact that the exogenous shocks have zero unconditional means, $\bar{\varepsilon} = \bar{u} = 0$, implies that $E\left(x_t\right) = 0$.

A.2 Second Moment and Variance

The unconditional second moment $E\left(x_t^2\right)$ is defined as follows:

$$E\left(x_t^2\right) = E(x_t^2 \mid s_t = 1)P_1 + E(x_t^2 \mid s_t = 2)P_2 \tag{A.18}$$

Let $f_i\left(0\right) \equiv E(x_t^2 \mid s_t = i)$ for $i = 1, 2$. Using (A.1) again:

$$\begin{aligned} f_1\left(0\right) &= E\left[\left(c\left(s_t\right) + a\left(s_t\right)x_{t-1} + b\left(s_t\right)y_t + u_t\right)^2 \mid s_t = 1)\right] \\ &= E\left[\left(c_1 + a_1 x_{t-1} + b_1 y_t + u_t\right)^2 \mid s_{t-1} = 1)\right]\mathcal{P}\left(s_{t-1} = 1 \mid s_t = 1\right) \\ &\quad + E\left[\left(c_1 + a_1 x_{t-1} + b_1 y_t + u_t\right)^2 \mid s_{t-1} = 2)\right]\mathcal{P}\left(s_{t-1} = 2 \mid s_t = 1\right) \\ &= E\left[\left(c_1 + a_1 x_{t-1} + b_1 y_t + u_t\right)^2 \mid s_{t-1} = 1)\right]p_{11} \\ &\quad + E\left[\left(c_1 + a_1 x_{t-1} + b_1 y_t + u_t\right)^2 \mid s_{t-1} = 2)\right]\left(1-p_{11}\right) \end{aligned} \tag{A.19}$$

where the third equality makes use of time-reversibility of MC. Consider the first term:

$$
\begin{aligned}
& E\left[\left(c_1+a_1x_{t-1}+b_1y_t+u_t\right)^2 \mid s_{t-1}=1\right)\right] \\
= \;& c_1^2+b_1^2\sigma_y^2+\sigma_u^2+a_1^2E\left(x_{t-1}^2 \mid s_{t-1}=1\right)+2c_1a_1E\left(x_{t-1} \mid s_{t-1}=1\right) \\
& +2c_1b_1E\left(y_t \mid s_{t-1}=1\right)+2c_1E\left(u_t \mid s_{t-1}=1\right)+2a_1E\left(x_{t-1}u_t \mid s_{t-1}=1\right) \\
& +2b_1E\left(y_tu_t \mid s_{t-1}=1\right)+2a_1b_1E\left(x_{t-1}y_t \mid s_{t-1}=1\right) \\
= \;& c_1^2+b_1^2\sigma_y^2+\sigma_u^2+a_1^2f_1(0)+2c_1a_1\mu_1 \\
& +2c_1b_1\bar{y}+2c_1\bar{u}+2a_1\bar{u}\mu_1 \\
& +2b_1\bar{y}\bar{u}+2a_1b_1\rho E\left(x_{t-1}y_{t-1} \mid s_{t-1}=1\right)+2a_1b_1\bar{\varepsilon}E\left(x_{t-1} \mid s_{t-1}=1\right) \\
= \;& c_1^2+b_1^2\sigma_y^2+\sigma_u^2+2c_1a_1\mu_1+2c_1b_1\bar{y}+2c_1\bar{u}+2a_1\bar{u}\mu_1+2a_1b_1\bar{\varepsilon}\mu_1+2b_1\bar{y}\bar{u} \qquad \text{(A.20)} \\
& +2a_1b_1\rho E\left(x_{t-1}y_{t-1} \mid s_{t-1}=1\right)+a_1^2f_1(0)
\end{aligned}
$$

where the second equality follows from the ergodic properties in (A.5)-(A.6), the independence of y_t and u_t from the state, and the process (A.2). Similarly, we have that

$$
\begin{aligned}
& E\left[\left(c_1+a_1x_{t-1}+b_1y_t+u_t\right)^2 \mid s_{t-1}=2\right)\right] \\
= \;& c_1^2+b_1^2\sigma_y^2+\sigma_u^2+2c_1a_1\mu_2+2c_1b_1\bar{y}+2c_1\bar{u}+2a_1\bar{u}\mu_2+2a_1b_1\bar{\varepsilon}\mu_2+2b_1\bar{y}\bar{u} \qquad \text{(A.21)} \\
& +2a_1b_1\rho E\left(x_{t-1}y_{t-1} \mid s_{t-1}=2\right)+a_1^2f_2(0)
\end{aligned}
$$

Let $n_i \equiv E\left(x_{t-1}y_{t-1} \mid s_{t-1}=i\right)$, for $i=1,2$. Using (A.1) for the definition of x_{t-1}, together with the stochastic properties of the exogenous shocks y_t and u_t, we can write n_1 as follows:

$$
\begin{aligned}
n_1 &= E\left[\left(c_1 + a_1 x_{t-2} + b_1 y_{t-1} + u_{t-1}\right) y_{t-1} \mid s_{t-1} = 1\right] \\
&= c_1 E\left(y_{t-1} \mid s_{t-1} = 1\right) + a_1 E\left[\left(\rho y_{t-2} + \varepsilon_{t-1}\right) x_{t-2} \mid s_{t-1} = 1\right] \qquad \text{(A.22)} \\
&\quad + b_1 E\left(y_{t-1}^2 \mid s_{t-1} = 1\right) + E\left(y_{t-1} u_{t-1} \mid s_{t-1} = 1\right) \\
&= c_1 \bar{y} + b_1 \sigma_y^2 + \bar{y}\bar{u} \\
&\quad + a_1 \rho E\left(y_{t-2} x_{t-2} \mid s_{t-1} = 1\right) + a_1 \bar{\varepsilon} E\left(x_{t-2} \mid s_{t-1} = 1\right) \\
&= c_1 \bar{y} + b_1 \sigma_y^2 + \bar{y}\bar{u} \\
&\quad + a_1 \rho E\left(y_{t-2} x_{t-2} \mid s_{t-1} = 1, s_{t-2} = 1\right) \mathcal{P}\left(s_{t-2} = 1 \mid s_{t-1} = 1\right) \\
&\quad + a_1 \rho E\left(y_{t-2} x_{t-2} \mid s_{t-1} = 1, s_{t-2} = 2\right) \mathcal{P}\left(s_{t-2} = 2 \mid s_{t-1} = 1\right) \\
&\quad + a_1 \bar{\varepsilon} E\left(x_{t-2} \mid s_{t-1} = 1, s_{t-2} = 1\right) \mathcal{P}\left(s_{t-2} = 1 \mid s_{t-1} = 1\right) \\
&\quad + a_1 \bar{\varepsilon} E\left(x_{t-2} \mid s_{t-1} = 1, s_{t-2} = 2\right) \mathcal{P}\left(s_{t-2} = 2 \mid s_{t-1} = 1\right) \\
&= c_1 \bar{y} + b_1 \sigma_y^2 + \bar{y}\bar{u} \\
&\quad + a_1 \rho \left[n_1 p_{11} + n_2 \left(1 - p_{11}\right)\right] \qquad \text{(A.23)} \\
&\quad + a_1 \bar{\varepsilon} \left[\mu_1 p_{11} + \mu_2 \left(1 - p_{11}\right)\right]
\end{aligned}
$$

where the last equality uses of the definition $\mu_i \equiv E\left(x_t \mid s_t = i\right)$, for $i = 1, 2$, together with the ergodic properties stated in (A.5) and (A.7), and time-reversibility. Similarly, we can find n_2:

$$
\begin{aligned}
n_2 &= c_2 \bar{y} + b_2 \sigma_y^2 + \bar{y}\bar{u} \\
&\quad + a_2 \rho \left[n_1 \left(1 - p_{22}\right) + n_2 p_{22}\right] \qquad \text{(A.24)} \\
&\quad + a_2 \bar{\varepsilon} \left[\mu_1 \left(1 - p_{22}\right) + \mu_2 p_{22}\right]
\end{aligned}
$$

We can then combine (A.23) and (A.24) in matrix form to get analytical expressions for n_1 and n_2:

$$
\begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \begin{bmatrix} 1 - \rho a_1 p_{11} & -a_1 \rho \left(1 - p_{11}\right) \\ -a_2 \rho \left(1 - p_{22}\right) & 1 - \rho a_2 p_{22} \end{bmatrix}^{-1} \begin{bmatrix} c_1 \bar{y} + b_1 \sigma_y^2 + \bar{y}\bar{u} \\ c_2 \bar{y} + b_2 \sigma_y^2 + \bar{y}\bar{u} \end{bmatrix} \tag{A.25}
$$

Once we have found the latter, we can combine (A.20) and (A.21) back in (A.19), we get an expression for $f_1(0)$:

$$
f_1(0) = d_1 + a_1^2 \left[p_{11} f_1(0) + \left(1 - p_{11}\right) f_2(0)\right] \tag{A.26}
$$

where we have defined

$$\begin{aligned} d_1 &\equiv c_1^2 + b_1^2\sigma_y^2 + \sigma_u^2 + 2\left[c_1\left(b_1\bar{y} + \bar{u}\right) + b_1\bar{y}\bar{u}\right] \\ &+2\left[p_{11}\mu_1 + \left(1 - p_{11}\right)\mu_2\right] a_1\left(c_1 + \bar{u} + b_1\bar{\varepsilon}\right) \\ &+2a_1 b_1 \rho\left[p_{11} n_1 + \left(1 - p_{11}\right) n_2\right] \end{aligned}$$

Following a similar procedure, we find $f_2(0)$:

$$f_2(0) = d_2 + a_2^2\left[\left(1 - p_{22}\right) f_1(0) + p_{22} f_2(0)\right] \tag{A.27}$$

where

$$\begin{aligned} d_2 &\equiv c_2^2 + b_2^2\sigma_y^2 + \sigma_u^2 + 2\left[c_2\left(b_2\bar{y} + \bar{u}\right) + b_2\bar{y}\bar{u}\right] \\ &+2\left[\left(1 - p_{22}\right)\mu_1 + p_{22}\mu_2\right] a_2\left(c_2 + \bar{u} + b_2\bar{\varepsilon}\right) \\ &+2a_2 b_2 \rho\left[\left(1 - p_{22}\right) n_1 + p_{22} n_2\right] \end{aligned}$$

We can then solve explicitly for $f_1(0)$ and $f_2(0)$ after putting (A.26)-(A.27) in matrix form:

$$\begin{bmatrix} f_1(0) \\ f_2(0) \end{bmatrix} = \left[I_2 - A^2 P\right]^{-1} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \tag{A.28}$$

Given these, from (A.18), we have the unconditional second moment $E\left(x_t^2\right)$,

$$E\left(x_t^2\right) = f_1(0) P_1 + f_2(0) P_2, \tag{A.29}$$

which, in turn, can be combined with $E(x_t)$ to give the unconditional variance

$$\begin{aligned} Var(x_t) &= E\left(x_t^2\right) - \left[E(x_t)\right]^2 \\ &= f_1(0) P_1 + f_2(0) P_2 - \left(\mu_1 P_1 + \mu_2 P_2\right)^2 \end{aligned} \tag{A.30}$$

A.3 First-Order Autocovariance and Autocorrelation

The unconditional first-order autocovariance is defined as follows:

$$Cov(x_t, x_{t-1}) = E(x_t x_{t-1}) - \left[E(x_t)\right]^2 \tag{A.31}$$

where

$$E\left(x_t x_{t-1}\right) = E\left(x_t x_{t-1} \mid s_t = 1\right) P_1 + E\left(x_t x_{t-1} \mid s_t = 2\right) P_2 \tag{A.32}$$

Let $f_i(1) \equiv E\left(x_t x_{t-1} \mid s_t = i\right)$. Using (A.1), we have:

$$\begin{aligned} f_1(1) &= E\left[\left(c\left(s_t\right) + a\left(s_t\right) x_{t-1} + b\left(s_t\right) y_t + u_t\right) x_{t-1} \mid s_t = 1\right] \\ &= c_1 E\left[x_{t-1} \mid s_t = 1\right] + a_1 E\left[x_{t-1}^2 \mid s_t = 1\right] \\ &\quad + b_1 E\left[y_t x_{t-1} \mid s_t = 1\right] + E\left[u_t x_{t-1} \mid s_t = 1\right] \end{aligned} \tag{A.33}$$

where

$$E\left[x_{t-1} \mid s_t = 1\right] = \mu_1 p_{11} + \mu_2\left(1 - p_{11}\right) \tag{A.34}$$

$$E\left[x_{t-1}^2 \mid s_t = 1\right] = f_1(0) p_{11} + f_2(0)\left(1 - p_{11}\right) \tag{A.35}$$

$$E\left[u_t x_{t-1} \mid s_t = 1\right] = \bar{u}\left[\mu_1 p_{11} + \mu_2\left(1 - p_{11}\right)\right] \tag{A.36}$$

and

$$\begin{aligned} E\left[y_t x_{t-1} \mid s_t = 1\right] &= E\left[\left(\rho y_{t-1} + \varepsilon_t\right) x_{t-1} \mid s_t = 1\right] \\ &= \rho E\left[y_{t-1} x_{t-1} \mid s_t = 1\right] + \bar{\varepsilon}\left[\mu_1 p_{11} + \mu_2\left(1 - p_{11}\right)\right] \end{aligned} \tag{A.37}$$

Consider the first term $E\left[y_{t-1} x_{t-1} \mid s_t = 1\right]$. We have that:

$$\begin{aligned} E\left[y_{t-1} x_{t-1} \mid s_t = 1\right] &= E\left[y_{t-1} x_{t-1} \mid s_t = 1, s_{t-1} = 1\right] \mathcal{P}\left(s_{t-1} = 1 \mid s_t = 1\right) \\ &\quad + E\left[y_{t-1} x_{t-1} \mid s_t = 1, s_{t-1} = 2\right] \mathcal{P}\left(s_{t-1} = 2 \mid s_t = 1\right) \\ &= E\left[y_{t-1} x_{t-1} \mid s_{t-1} = 1\right] p_{11} + E\left[y_{t-1} x_{t-1} \mid s_{t-1} = 2\right]\left(1 - p_{11}\right) \\ &= n_1 p_{11} + n_2\left(1 - p_{11}\right) \end{aligned} \tag{A.38}$$

where the last equality follows from the definition $n_i \equiv E\left(x_{t-1} y_{t-1} \mid s_{t-1} = i\right)$, for $i = 1, 2$, with expressions given in (A.23) and (A.24). We can then inset (A.34)-(A.38) into (A.33) to find

$$\begin{aligned} f_1(1) &= c_1\left[\mu_1 p_{11} + \mu_2\left(1 - p_{11}\right)\right] + a_1\left[f_1(0) p_{11} + f_2(0)\left(1 - p_{11}\right)\right] \\ &\quad + b_1 \rho\left[n_1 p_{11} + n_2\left(1 - p_{11}\right)\right] + b_1 \bar{\varepsilon}\left[\mu_1 p_{11} + \mu_2\left(1 - p_{11}\right)\right] \\ &\quad + \bar{u}\left[\mu_1 p_{11} + \mu_2\left(1 - p_{11}\right)\right] \end{aligned} \tag{A.39}$$

Following similar steps we find $f_2(1) \equiv E(x_t x_{t-1} \mid s_t = 2)$:

$$\begin{aligned} f_2(1) \;\; = \;\; & c_2 [\mu_1 (1-p_{22}) + \mu_2 p_{22}] + a_2 [f_1(0)(1-p_{22}) + f_2(0) p_{22}] \\ & + b_2 \rho [n_1 (1-p_{22}) + n_2 p_{22}] + b_1 \bar{\varepsilon} [\mu_1 (1-p_{22}) + \mu_2 p_{22}] \\ & + \bar{u} [\mu_1 (1-p_{22}) + \mu_2 p_{22}] \end{aligned} \quad \text{(A.40)}$$

The unconditional first order covariance is

$$Cov(x_t, x_{t-1}) = P_1 f_1(1) + (1-P_1) f_2(1) - [\mu_1 P_1 + \mu_2 (1-P_1)]^2$$

The first order autocorrelation coefficient is

$$Corr(x_t, x_{t-1}) = \frac{Cov(x_t, x_{t-1})}{Var(x_t)} \quad \text{(A.41)}$$

In the context of our model, given the definitions of c_1, c_2, a_1 and a_2, the unconditional first order autocorrelation (A.41) is again a function of α and β. We therefore let $H(\alpha, \beta) \equiv Corr(x_t, x_{t-1})$.

B Proof of Propositions and Corollaries

B.1 Proof of Proposition 1

Degree of Policy Aggressiveness. Consider the case of an extremely aggressive response to inflation, $\phi \to \infty$. It is easy to show that $\lim_{\phi \to \infty} G(\beta) = -\beta$. Hence, the unique SCEE features $\beta = 0$. Now consider the opposite case of an interest rate peg, $\phi \to 0$. In this case $\lim_{\phi \to 0} G(\beta) = \frac{1}{\rho} - \beta$. Since $\rho \in (0,1)$, no SCEE exists.

Real Interest Rate Persistence. Suppose the real interest rate is extremely persistent, $\rho \to 1$. In this case $\lim_{\rho \to 1} G(\beta) = 1 - \beta$. Hence the unique solution to $G(\beta) = 0$ is $\beta = 1$. However, since stationarity of (9) requires $|\beta^2 \phi^{-1}| < 1$, this is a SCEE if and only if $\phi > 1$. Suppose instead the real interest rate follows an *iid* process, $\rho = 0$. In this case $G(\beta) = \frac{\beta(\beta - \phi)}{\phi}$. Hence, under an active rule, there exists a unique SCEE with $\beta = 0$. Under a passive rule, two SCEE are possible: one with $\beta = 0$, and a second one with $\beta = \phi$.

B.2 Proof of Proposition 2

Let $\kappa \equiv \sigma_{\pi^*}/\sigma_\varepsilon$, and suppose $\kappa \to 0$. Then, $F(\beta) = \frac{\beta^2 + \rho\phi}{\rho\beta^2 + \phi} = \frac{\frac{\beta^2}{\phi} + \rho}{\rho\frac{\beta^2}{\phi} + 1}$. Notice that, if we let $b_1 \equiv \phi^{-1}$, this is equivalent to expression (3.11) in HZ. Assuming that $\phi > 1$, such that $b_1 \in (0,1)$, we can directly apply their Proposition 2 to show that there exists a unique SCEE with $\beta > \rho$.

Suppose now that $\kappa \to \infty$. In this case $G(\beta) = \frac{\beta(\beta-\phi)}{\phi}$, as we had for the case of $\rho = 0$. Hence, by the results of our Proposition 1, we can state that, for $\phi > 1$, there exists a unique SCEE with $\beta = 0$.

B.3 Proof of Proposition 4

Let

$$N_0 \equiv \begin{bmatrix} p_{11}/\phi_1^2 & (1-p_{22})/\phi_1^2 \\ (1-p_{11})/\phi_2^2 & p_{22}/\phi_2^2 \end{bmatrix}$$

Then, if $\beta < 1$ and the eigenvalues of N_0 are within the unit circle, so will the eigenvalues of N. Let $q_i = \frac{1}{\phi_i}$ for $i \in \{1, 2\}$. The eigenvalues of N_0 are

$$\lambda_1 = \frac{p_{11}q_1^2 + p_{22}q_2^2 + \sqrt{(p_{11}q_1^2 + p_{22}q_2^2)^2 + 4q_1^2q_2^2(1-p_{11}-p_{22})}}{2}$$

$$\lambda_2 = \frac{p_{11}q_1^2 + p_{22}q_2^2 - \sqrt{(p_{11}q_1^2 + p_{22}q_2^2)^2 + 4q_1^2q_2^2(1-p_{11}-p_{22})}}{2}$$

With λ_1 always positive, we need to establish conditions under which $0 < \lambda_1 < 1$ and $-1 < \lambda_2 < 1$. Namely:

$$\lambda_1 < 1 \iff \sqrt{(p_{11}q_1^2 + p_{22}q_2^2)^2 + 4q_1^2q_2^2(1-p_{11}-p_{22})} < 2 - p_{11}q_1^2 - p_{22}q_2^2 \quad \text{(B.1)}$$

$$\lambda_2 > -1 \iff \sqrt{(p_{11}q_1^2 + p_{22}q_2^2)^2 + 4q_1^2q_2^2(1-p_{11}-p_{22})} < 2 + p_{11}q_1^2 + p_{22}q_2^2 \quad \text{(B.2)}$$

$$\lambda_2 < 1 \iff -\sqrt{(p_{11}q_1^2 + p_{22}q_2^2)^2 + 4q_1^2q_2^2(1-p_{11}-p_{22})} < 2 - p_{11}q_1^2 - p_{22}q_2^2 \quad \text{(B.3)}$$

Condition (B.1) implies (B.3). Squaring both (B.1) and (B.2) and simplifying yields

$$q_1^2q_2^2(1-p_{11}-p_{22}) < 1 - (p_{11}q_1^2 + p_{22}q_2^2)$$

$$q_1^2q_2^2(1-p_{11}-p_{22}) < 1 + (p_{11}q_1^2 + p_{22}q_2^2)$$

Since the first inequality implies the second, the only relevant condition for λ_1 and λ_2 to be within the unit circle is

$$q_1^2q_2^2(1-p_{11}-p_{22}) < 1 - (p_{11}q_1^2 + pq_2^2) \iff \phi_1^2\phi_2^2 + (1-\phi_2^2)p_{11} + (1-\phi_1^2)p_{22} > 1$$

B.4 Proof of Proposition 5

The area for which the LRTP condition (14) holds is anything above $h(\phi_1)$, where

$$h(\phi_1) \equiv \frac{1 - p_{11} - p_{22} + p_{22}\phi_1}{\phi_1 - p_{11}}$$

Simple algebra shows that $h(\phi_1) > 0$ for any $\phi_1 > p_{11}$. On the other hand, the area above which the sufficient condition MSS (15) holds is anything above $l(\phi_1)$, where

$$l(\phi_1) \equiv \left(\frac{1 - p_{11} - p_{22} + p_{22}\phi_1^2}{\phi_1^2 - p_{11}}\right)^{\frac{1}{2}}$$

Simple algebra shows that $l(\phi_1) > 0$ for any $\phi_1 > \sqrt{p_{11}}$. Moreover, by simple calculus, one can easily verify the following properties:

i) both h and l are decreasing and convex;

ii) $\lim_{\phi_1 \to \infty} h(\phi_1) = p_{22}, \qquad \lim_{\phi_1 \to p_{11}} h(\phi_1) = \infty;$

iii) $\lim_{\phi_1 \to \infty} l(\phi_1) = \sqrt{p_{22}} > p_{22}, \qquad \lim_{\phi_1 \to \sqrt{p_{11}}} l(\phi_1) = \infty$

iv) $h(1) = l(1) = 1, \qquad h'(1) = l'(1) = -\frac{1-p_{22}}{1-p_{11}}$.

From the stated properties, it immediately follows that $\lim\limits_{\phi_1 \to \sqrt{p_{11}}} h(\phi_1) < \lim\limits_{\phi_1 \to \sqrt{p_{11}}} l(\phi_1) = \infty$. Hence $l(\phi_1) > h(\phi_1)$ for any $\phi_1 \in \left(\sqrt{p_{11}}, 1\right)$. Now consider $\phi_1 > 1$. Simple algebra show that there exists a point $\breve{\phi}_1 \equiv \frac{1-p_{11}-p_{22}+\sqrt{p_{22}}p_{11}}{\sqrt{p_{22}}-p_{22}} > 1$ such that $h\left(\breve{\phi}_1\right) \leq \sqrt{p_{22}}$ - where $\sqrt{p_{22}} < l(\phi_1)$ - for $\phi_1 \geq \breve{\phi}_1$. By the convexity of both functions, and the fact that they are tangent for $\phi_1 = 1$, it follows that $h(\phi_1) < l(\phi_1)$ for any $\phi_1 > 1$ as well. Hence, $h(\phi_1) < l(\phi_1)$ for any $\phi_1 > \sqrt{p_{11}}$.

B.5 Proof of Proposition 7

To show that $(0, \beta^*)$ is stable in the SAC-learning sense, one has to show that the real parts of both eigenvalues of the Jacobian matrix associated with (26)-(27), evaluated at $(0, \beta^*)$, are negative. Such Jacobian is:

$$J(\alpha, \beta) = \begin{bmatrix} \frac{\partial D(\alpha,\beta)}{\partial \alpha} - 1 & \frac{\partial D(\alpha,\beta)}{\partial \beta} \\ \frac{\partial H(\alpha,\beta)}{\partial \alpha} & \frac{\partial H(\alpha,\beta)}{\partial \beta} - 1 \end{bmatrix}$$

where each element is a real number. From the first moment's derivation in Appendix A.1, we know that $D(\alpha, \beta) = \alpha \tilde{D}(\beta)$, where $\tilde{D}(\beta)$ is some function of β. Hence,

$$\frac{\partial D(\alpha, \beta)}{\partial \beta} \mid_{(\alpha,\beta)=(0,\beta^*)} = 0 * \tilde{D}'(\beta^*) = 0$$

The two eigenvalues of interest are then $(\frac{\partial D(\alpha,\beta)}{\partial \alpha} \mid_{(\alpha,\beta)=(0,\beta^*)} -1)$ and $(\frac{\partial H(\alpha,\beta)}{\partial \beta} \mid_{(\alpha,\beta)=(0,\beta^*)} -1)$. It follows that a SCEE $(0,\beta^*)$ is SAC-learnable if and only if $\frac{\partial D(\alpha,\beta)}{\partial \alpha} \mid_{(\alpha,\beta)=(0,\beta^*)} = \tilde{D}(\beta) \mid_{\beta=\beta^*} < 1$ and $\frac{\partial H(\alpha,\beta)}{\partial \beta} \mid_{(\alpha,\beta)=(0,\beta^*)} < 1$.

Consider the latter. If β^* is unique, then to establish that $\frac{\partial H(\alpha,\beta)}{\partial \beta} \mid_{(\alpha,\beta)=(0,\beta^*)} < 1$ all we need to show is that $H(\alpha,\beta) \mid_{(\alpha,\beta)=(0,0)} > 0$. This will guarantee that $H(\alpha,\beta)$ will cross the 45 degree line at β^* from above. For $\beta = 0$ (and $\alpha = 0$) the coefficients of the ALM in equation (A.1) are $a_i = 0$, $b_i = \frac{1}{\phi_i}$ and $c_i = 0$, for $i = 1, 2$. After substituting these expressions for a_i, b_i and c_i in Section A.2, simple algebra gives

$$Var(\pi_t) = (b_1^2\sigma_\varepsilon^2 + \sigma_{\pi^*}^2)P_1 + (b_2^2\sigma_\varepsilon^2 + \sigma_{\pi^*}^2)P_2 > 0$$

Similarly, substituting for a_i, b_i and c_i in Section A.3, one can derive

$$Cov(\pi_t, \pi_{t-1}) = \rho\sigma_\varepsilon^2 b_1(p_{11}b_1 + (1-p_{11})b_2)P_1 + \rho\sigma_\varepsilon^2 b_2(p_{22}b_2 + (1-p_{22})b_1)P_2 > 0$$

It then follows that

$$H(\alpha,\beta) = Corr(\pi_t, \pi_{t-1}) = \frac{Cov(\pi_t, \pi_{t-1})}{Var(\pi_t)} \mid_{\beta=0} > 0$$

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