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JEL Classification: E63, E52, E58

Keywords: Adaptive Learning, Limited Credibility, Inflation targeting, Zero Interest Rate Lower Bound

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Price Level Targeting with Evolving Credibility[¤]

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19 February 2018

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1 Introduction

Inflation targeting (IT) as a good monetary policy framework was shaken by the global financial crisis in 2007. The crisis resulted in policy interest rates stuck near zero levels for a very long time in the US and Europe. An earlier crisis in Japan led to very low rates since the mid 1990s. This so-called zero lower bound (ZLB) constraint for policy interest rates led to new interest in ways for avoiding or getting out of the ZLB regime. Some prominent central bankers made calls to reform the monetary policy framework. One particular suggestion has been that price level targeting (PLT) can be a more appropriate framework for monetary policy rather than IT. Evans (2012) discusses the need for additional guidance for the price level and argues that price level targeting might be used to combat the liquidity trap. A related suggestion is Carney (2012) that with policy rates at ZLB "there could be a more favorable case for nominal GDP targeting" (nominal GDP targeting is related to PLT).

More recently, John C. Williams, President and CEO of the FRB of San Francisco and Ben Bernanke, former Chairman of the US Federal Reserve have also come out forcefully in support of flexible price level targeting. Williams (2016) reviews monetary policy in a low natural rate of interest world and suggests either a higher inflation target or a move to price level or nominal GDP targeting as possible new policy frameworks. Williams (2017) suggests that flexible price level targeting would be a good monetary policy framework in a world with the a natural rate of interest. Bernanke (2017) also suggests flexible PLT as the best policy in times when short term interest rates are near zero; in particular he advocates temporary PLT as an alternative framework for monetary policy (and argues against a higher inflation target under IT).

Interestingly, despite the strong advocacy mentioned above, there is in fact very little actual experience with PLT. Historically, the closest example to our knowledge is Sweden which way back in the 1920s and 30s briefly flirted with monetary policy somewhat akin to PLT. Jonung (1979) and Berg and Jonung (1999) discuss two episodes of price level stabilization in Sweden in 1921-22 and in the 1930's. Lack of actual experience with PLT probably explains why the discussion about this policy framework has been mostly confined to the academic literature.[1] Moreover, not surprisingly, most of

[1] Price-level targeting has received a fair amount of attention in monetary theory, see

the academic literature around PLT has been conducted within the rational expectations (RE) framework. A seminal paper, Eggertsson and Woodford (2003), considers optimal monetary policy and a modified form of PLT under RE in a liquidity trap. They argue that PLT gives guidance in terms of history-dependence of monetary policy and is a good policy under the ZLB constraint.

Rational expectations (RE) is, however, a very strong assumption about the agents' knowledge of the economy. This is so especially if the economy is in a recession and faces risks of deflation while policy makers contemplate a move from IT to PLT. The assumption of RE becomes informationally very demanding in this scenario. In this paper we relax the RE assumption and analyze PLT as a monetary policy framework under imperfect knowledge and learning and compare it to IT.[2]

The key novelty in this paper is that performance of PLT is assessed in the presence of *endogenously evolving credibility* of PLT monetary policy, taking into account the self-referential feature of the model. The evolution of credibility is formally modeled as reinforcement learning on the part of economic agents. The main question we ask is whether introduction of PLT in the presence of ZLB and sluggish economic activity can induce the economy to escape from the recessionary scenario towards the desired steady state (with inflation, output and interest rates converging to the targeted levels).

We conduct this analysis in a non-linear micro-founded New Keynesian (NK) model where the ZLB on interest rates is explicitly taken into account. The PLT regime, like IT, can be subject to global indeterminacy problems caused by the ZLB.[3] There are two steady states, the targeted steady state and a low-inflation steady state at which the policy interest rate is at the ZLB. Circumstances that are conducive to a successful escape from the ZLB regime are elucidated by focusing on di¤erent possibilities in the announced aspects of PLT and its evolving credibility. To obtain our results and intuition as starkly as possible we keep the NK model simple in other respects e.g. by ignoring financial market frictions. One interpretation of our analysis is that financial frictions leading to appearance of a credit spread have caused the economy to be stuck in a deflationary (low inflationary) scenario with interest

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for example Svensson (1999) and Vestin (2006). Ambler (2009), Cournède and Moccero (2009) and Hatcher and Minford (2014) survey the literature on PLT.

[2] See Section 8 for discussion and references of the learning approach.

[3] References to the literature on indeterminacy in these models are given in Section 4.1.

rates at the ZLB.[4]

Interestingly, our results are consistent with the experience of Sweden with episodes of price level stabilization mentioned above. According to Jonung (1979) and Berg and Jonung (1999), the Swedish experience during the 1920s was unfavourable whereas the experience with the 1930s episode was much more successful and this was partly due to di¤erences in the initial price level targeted by the authorities. We believe this is the first paper to make these theoretical arguments for the Swedish experience and to pay attention to appropriate setting of the initial value of the target price level.

A further issue in a possible move to PLT from IT is whether a future target path for the price level should be announced by the policy maker or not. Opacity about the price level target path yields no new guidance in comparison to IT. If instead the target path is made known, then the significance of additional guidance about the future depends on how much weight this information has in the agents' forecasts for inflation. Private agents can combine inflation forecasts partly based on knowledge about the target price path with forecasts based purely on inflation data. Credibility of PLT is defined in terms of the weight of the former forecasts relative to the latter. Credibility is assumed to evolve over time in an endogenous way that depends on a relative performance measure.

We then assess the robustness of each monetary policy regime by comparing the sizes of the domain of attraction of the targeted steady state under learning for each policy regime.[5] This criterion answers the question of how far from the targeted steady state the initial conditions can be and still deliver convergence to the target. Intuitively, an initial condition away from the targeted steady state represents a shock to the economy. A large domain of attraction for a policy regime means that the economy will eventually get back to the target even after a large shock. Domains of attraction have been computed for a given policy regime in the literature[6], but to our knowledge

[4] Most recently, even slightly negative policy rates have been seen. On the other hand, a positive credit spread due to financial frictions can imply that the lower bound on market rates can be positive, see Curdia and Woodford (2010) and Curdia and Woodford (2015). For brevity, we do not explicitly consider these possibilities.

[5] Formally, the domain of attraction is the set of all initial conditions from which learning dynamics converge to the steady state.

[6] Global aspects of monetary (and fiscal) policy in nonlinear models have recently been studied under both RE and adaptive learning. See e.g. Eusepi (2007), Benhabib and Eusepi (2005), Eusepi (2010), Benhabib, Evans, and Honkapohja (2014) and the references therein.

its size in different regimes has not been used as a desideratum.

The key general result of the paper is that the dynamic performance of learning in the PLT regime strongly depends on nature of communication about the target price path in PLT and degree of credibility of the regime if the target path is made known.[7]

As a starting point, Section 4 considers the case where the target price level path is not communicated. Price-level targeting is inferior to inflation targeting in terms of the robustness criterion. The targeted steady state is only locally stable under learning and the deflationary steady state locally unstable for the PLT regime. Numerical analysis of the domain-of-attraction criterion for the two policy regimes indicates that PLT without information about the target price path performs worse than IT.

The analysis of credibility of PLT begins in Section 5 by looking at situations where the target price level path is communicated to private agents. The latter can build this information into their inflation forecasting. Whether they actually do so depends on the credibility of the PLT regime. In Section 5 the extreme (or steady-state) case of full credibility of PLT is analyzed. PLT policy is then excellent as the economy will converge back to the targeted steady state from a very large set of possible initial conditions far away from the target. There is even convergence to the target from initial conditions arbitrarily close to the low steady state and when the ZLB is binding. Thus PLT policy regime is superior to IT in this case.

Our main focus is on imperfect initial credibility of the newly introduced PLT policy. This is taken up in Section 6. As mentioned above, the degree of credibility is assumed to depend on the relative accuracy of two ways of inflation forecasting, one of which employs the target price level path while the other just uses past data on inflation. Inflation forecasts of private agents are a weighted average of these two forecasts. The weights evolve endogenously over time in accordance with a standard model of reinforcement learning.[8] We examine how the domain of attraction depends on the initial

[7] Importance of communication about the policy instrument rule in inflation targeting policies is emphasized in Eusepi (2010) and Eusepi and Preston (2010). In PLT we show the key issue is actually the announcement of the future target path of the price level by the central bank (rather than transparency of the interest rule *per se*).

[8] Imperfect credibility of monetary policy has been introduced in different ways in the literature. Imperfection is thought to arise, for example, as deviation from RE optimal policy due to the ZLB constraint, see Bodenstein, Hebden, and Nunes (2012), or from policy maker's doubt about its model in an RE setting, see Dennis (2014), or as weighting

weight (credibility) of the PLT.

Numerical results show surprisingly that even a small positive degree of initial credibility for PLT can have big benefits in the sense that the domain of attraction is significantly larger than in the case of PLT with opacity. Less surprising is the result that a higher degree of initial credibility leads to a larger domain of attraction and is thus conducive to escape from the ZLB region and eventual convergence of the economy to the targeted steady state.

These results are sensitive on the ratio of initial target and actual price levels. The ratio should not be set too high. We also examine other aspects of the dynamics including comparison of IT and PLT with limited credibility when the economy is currently in boom situation to examine their performance in more normal circumstances.

In Section 7 we show how our theoretical results can match the di¤erential experience of Sweden during these two episodes as arising from di¤erential settings of the initial target price level (as discussed in Jonung (1979) and Berg and Jonung (1999)). Section 8 contains further material about learning. Section 9 concludes. The Appendix contains various technical details and discussion of some further issues.

2 Analytical Framework

2.1 A New Keynesian Model

We employ a standard New Keynesian model as the analytical framework. The same model has been used earlier, so we just summarize the key parts of the model.[9]

There is a continuum of household-firms, which produce a di¤erentiated consumption good under monopolistic competition and price-adjustment costs. There is also a government which uses monetary policy, buys a fixed amount of output, finances spending by taxes and issues of public debt, see below.

The objective for agent s is to maximize expected, discounted utility subject to a standard flow budget constraint (in real terms) over the infinite

of di¤erent models, see Gibbs and Kulish (2017) and Kryvtsov, Shukayev, and Ueberfeldt (2008), but with the weights remaining constant or evolving exogenously.

[9] See Benhabib, Evans, and Honkapohja (2014), Evans and Honkapohja (2010) or Evans, Guse, and Honkapohja (2008).

horizon:

$$Max\ E_{0,s} \sum_{t=0}^{\infty} \beta^t U_{t,s}\left(c_{t,s}, \frac{M_{t-1,s}}{P_t}, h_{t,s}, \frac{P_{t,s}}{P_{t-1,s}} - 1\right) \tag{1}$$

$$st.\ c_{t,s} + m_{t,s} + b_{t,s} + \tau_{t,s} = m_{t-1,s}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1,s} + \frac{P_{t,s}}{P_t}y_{t,s}, \tag{2}$$

where $c_{t,s}$ is the consumption aggregator, $M_{t,s}$ and $m_{t,s}$ denote nominal and real money balances, $h_{t,s}$ is the labor input into production, and $b_{t,s}$ denotes the real quantity of risk-free one-period nominal bonds held by the agent at the end of period t. $\tau_{t,s}$ is the lump-sum tax collected by the government, R_{t-1} is the nominal interest rate factor between periods $t-1$ and t, $P_{t,s}$ is the price of consumption good s, $y_{t,s}$ is output of good s, P_t is the aggregate price level, and the inflation rate is $\pi_t = P_t/P_{t-1}$. The subjective discount factor is denoted by β. The utility function has the parametric form

$$U_{t,s} = \frac{c_{t,s}^{1-\sigma_1}}{1-\sigma_1} + \frac{\chi}{1-\sigma_2}\left(\frac{M_{t-1,s}}{P_t}\right)^{1-\sigma_2} - \frac{h_{t,s}^{1+\varepsilon}}{1+\varepsilon} - \frac{\gamma}{2}\left(\frac{P_{t,s}}{P_{t-1,s}} - 1\right)^2,$$

where $\sigma_1, \sigma_2, \varepsilon, \gamma > 0$. For the most part we analyze the widely considered case when $\sigma_1 = \sigma_2 = 1$ and $\varepsilon = 1$. The final term parameterizes the cost of adjusting prices in the spirit of Rotemberg (1982). We use the Rotemberg formulation rather than the Calvo model of price stickiness because it enables us to study global dynamics in the nonlinear system. The household decision problem is also subject to the usual "no Ponzi game" (NPG) condition. In (1) the expectations $E_{0,s}(.)$ are in general subjective and they may not be rational. This approach is called anticipated utility maximization over the infinite horizon (IH). See Section 8 for comments and references of IH learning.

Production function for good s is given by

$$y_{t,s} = h_{t,s}^{\alpha},$$

where $0 < \alpha < 1$. Output is differentiated and firms operate under monopolistic competition. Each firm faces a downward-sloping demand curve

$$P_{t,s} = \left(\frac{y_{t,s}}{y_t}\right)^{-1/\nu} P_t. \tag{3}$$

Here $P_{t,s}$ is the profit maximizing price set by firm s consistent with its production $y_{t,s}$. The parameter ν is the elasticity of substitution between two goods and is assumed to be greater than one. y_t is aggregate output, which is exogenous to the firm.

The government's flow budget constraint in real terms is

$$b_t + m_t + \tau_t = g_t + m_{t-1}\pi_t^{-1} + R_{t-1}\pi_t^{-1}b_{t-1}, \tag{4}$$

where g_t denotes government consumption of the aggregate good, b_t is the real quantity of government debt, and τ_t is the real lump-sum tax collected. We assume that fiscal policy follows a linear tax rule for lump-sum taxes as in Leeper (1991)

$$\tau_t = \gamma_0 + \gamma b_{t-1}, \tag{5}$$

where we assume that $\beta^{-1} - 1 < \gamma < 1$. Thus fiscal policy is "passive" in the terminology of Leeper (1991) and implies that an increase in real government debt leads to an increase in taxes sufficient to cover the increased interest and at least some fraction of the increased principal.

We assume that g is stochastic

$$g_t = \bar{g} + \tilde{g}_t,$$

where random part is an observable exogenous AR(1) process $\tilde{g}_t = \rho\tilde{g}_{t-1} + v_t$ with zero mean. For simplicity, it is assumed that ρ is a known parameter (if not it could be estimated during learning).[10] From market clearing we have

$$c_t + g_t = y_t. \tag{6}$$

2.1.1 The Phillips curve and the consumption function

To determine the infinite-horizon (nonlinear) Phillips curve, the following assumptions are made for reasons of tractability and simplicity (see also Benhabib, Evans, and Honkapohja (2014) for further details). It is assumed that (i) agents have point expectations, (ii) anticipate that $P_{t+j,s} = P_{t+j}$ in the future since this relation has held in the past, (iii) know the per capita market clearing equation and (iv) utilities are logarithmic i.e. $\sigma_1 = \sigma_2 = 1$.

[10] Only one shock is introduced in the paper in order to have a simple exposition of the basics of least squares learning. One could introduce other random shocks, but they are not needed for our purposes.

In Appendix A it is shown that the Phillips curve takes the form

$$
\begin{aligned}
Q_t = {} & \frac{\nu}{\alpha\gamma} y_t^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} y_t (y_t - (\bar{g} + g_t))^{-1} + \\
& \frac{\nu}{\gamma} \sum_{j=1}^{\infty} \alpha^{-1} \beta^{j} \left(y_{t+j}^{e}\right)^{(1+\varepsilon)/\alpha} - \frac{\nu - 1}{\gamma} \sum_{j=1}^{\infty} \beta^{j} \left(\frac{y_{t+j}^{e}}{y_{t+j}^{e} - (\bar{g} + \rho^{j} g_t)} \right) \\
\equiv {} & \tilde{K}(y_t, y_{t+1}^{e}, y_{t+2}^{e}, \ldots),
\end{aligned}
\tag{7}
$$

where the notation $Q_t = (\pi_t - 1)\pi_t$ is used. The expectations in (7) are formed at time t and based on information about the endogenous variables at the end of period $t - 1$. Current value of the observable exogenous random shock is assumed to be known. Actual variables at time t are assumed to be in the information set of the agents when they make current decisions. We will treat (7) with the definition of Q_t as the temporary equilibrium equations that determine π_t, given expectations $\{y_{t+j}^{e}\}_{j=1}^{\infty}$.[11]

To derive the consumption function it is assumed that consumers are Ricardian in the sense that they amalgamate their own intertemporal budget constraint and that of the governments (where the latter is evaluated at the price expectations of the consumer). In Appendix A it is shown that the consumption function takes the form

$$
c_t = (1 - \beta) \left(y_t - \bar{g} + \sum_{j=1}^{\infty} (D_{t,t+j}^{e})^{-1} (y_{t+j}^{e} - (\bar{g} + \rho^{j} g_t)) \right), \tag{8}
$$

where the discount factor is

$$
D_{t,t+j}^{e} = \frac{R_t}{\pi_{t+1}^{e}} \prod_{i=2}^{j} \frac{R_{t+i-1}^{e}}{\pi_{t+i}^{e}}. \tag{9}
$$

It is seen from (9) that private agents form expectations about future interest rates as we focus on the non-transparent case (the case of transparency is also considered briefly). The monetary policy frameworks are discussed next.

[11] One might wonder why inflation does not also depend directly on the expected future aggregate inflation rate in the Phillip's curve relationship (7). (There is an indirect effect of expected inflation on current inflation via current output.) Using (3) in the first-order conditions to eliminate relative prices and the representative agent assumption, each firm's output equals average output in every period. Since firms can be assumed to have learned this to be the case, we obtain (7).

2.2 Monetary Policy Frameworks

It is assumed for the bulk of the paper that agents do not know the interest rate rule or even its functional form. This assumption is surely the realistic case as in practice central banks do not make their policy instrument rules known. This is especially the case when the central bank is contemplating a change in monetary policy (say from IT to PLT). Nevertheless, the implications of transparency are briefly considered for the case of evolving credibility in Appendix C.5.[12]

2.2.1 Inflation targeting (IT)

For concreteness and simplicity of comparisons we model IT in terms of the standard Taylor rule

$$R_t = 1 + \max[\bar{R} - 1 + \psi_\pi(\pi_t - \pi^*) + \psi_y[(y_t - y^*)/y^*], 0], \qquad (10)$$

where $\bar{R} = \beta^{-1}\pi^*$ is the gross interest rate at the target and we have introduced the ZLB, so that the gross interest rate cannot fall below one. For analytical ease, we adopt a piecewise linear formulation of the interest rate rule. The inflation target π^* for the medium to long run is assumed to be known to private agents but agents do not know the rule (10).[13]

2.2.2 Price-level targeting (PLT)

We consider a simple formulation, where (i) the policy maker sets an exogenous target path for the price level $\{\bar{P}_t\}$ as a medium to long run target and (ii) sets the policy instrument with the intention to move the actual price level gradually toward a targeted price level path.

The target path $\{\bar{P}_t\}$ is assumed to involve constant inflation, so that

$$\bar{P}_t/\bar{P}_{t-1} = \pi^* \geq 1. \qquad (11)$$

The Wicksellian interest rate rule takes the form

$$R_t = 1 + \max[\bar{R} - 1 + \psi_p[(P_t - \bar{P}_t)/\bar{P}_t] + \psi_y[(y_t - y^*)/y^*], 0], \qquad (12)$$

[12] Consequences of transparency about the policy rule are analyzed in Honkapohja and Mitra (2015) in the special case of full credibility.

[13] As noted above, an effective interest rate lower bound greater than one due to a credit spread could be introduced as in Woodford (2011). Neither the theoretical results nor the qualitative aspects of numerical results would be changed.

where the max operation takes account of the ZLB on the interest rate. To have comparability to the IT rule (10), we adopt a piecewise linear formulation of the interest rate rule and the same level for target inflation.

Rules like (12) are called Wicksellian, see pp.260-61 of Woodford (2003) and Giannoni (2012) for discussions of Wicksellian rules. In particular, Giannoni (2012) analyses a number of different versions of the Wicksellian rules. A number of other formulations of PLT exist in the literature.[14]

According to (12), the interest rate is set above (below, respectively) the targeted steady-state value of the instrument when the actual price level is above (below, respectively) the targeted price-level path $\bar{P}_t$, as measured in percentage deviations. The interest rate is also allowed to respond to the percentage gap between targeted and actual levels of output. The target level of output y^* is the steady state value associated with π^*. This formulation could be called flexible price-level targeting (recently suggested by John Williams and Ben Bernanke, as mentioned in the Introduction).

It will be seen below that the starting value $\bar{P}_0$ for the target price level path plays an important role in the performance of PLT. The choice of the initial value of the price target was widely discussed in the two historical episodes of price level stabilization in Sweden in 1921-22 and in 1930's. Our results accord with the Swedish experience as discussed in Section 7 below.

In the PLT regime the policy maker may or may not announce the target path $\{\bar{P}_t\}$ for the price level. (Recall that the interest rate rule (12) is assumed to be unknown to the private agents in all cases.) We consider a range of possibilities here.

(i) The target path $\{\bar{P}_t\}$ is not made known to the private agents. This is called the case of PLT with opacity. In this case private agents continue to forecast inflation using only past data on inflation (and other observable variables).[15]

(ii) The target path $\{\bar{P}_t\}$ is made known to the private agents. In this case private agents can make use of the information about $\{\bar{P}_t\}$ and apply a second method for forecasting inflation (details are discussed further below).

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[14] In the literature, PLT is sometimes advocated as a way to achieve optimal policy with timeless perspective under RE locally near the targeted steady state. The learnability properties of this form of PLT depend on the implementation of the corresponding interest rate rule. See Evans and Honkapohja (2013), section 2.5.2 for an overview and further references. Global properties of this case have not been analyzed.

[15] This assumption is plausible as lacking any prior experience of PLT, agents might forecast inflation the same way they did under IT.

(iii) The degree of credibility of the PLT regime influences the way agents forecast inflation even if the target path is announced. In general there is imperfect credibility. In this case private agents are assumed to form their inflation forecasts as a weighted average of the forecasts based on preceding cases of (i) and (ii) above.

If the announcement of the target path $\{P^*_t\}$ has full credibility then private agents make full use of the announced target price level path in inflation forecasting (as mentioned in case (ii) above), and zero weight on pure statistical forecasting from inflation data, i.e. case (i) above.

Use of the relative weights of the two forecasting methods as measures of the degree of credibility for the policy regimes and modeling the evolution of limited credibility as endogenous movements over time are the crucial elements in our analysis.

3 Learning and Temporary Equilibrium

In adaptive learning it is assumed that each agent has a model for perceived dynamics of state variables, also called the perceived law of motion (PLM), to make his forecasts of relevant variables. In any period the PLM parameters are estimated using available data and the estimated model is used for forecasting. The PLM parameters are then re-estimated when new data becomes available in the next period. A common formulation is to postulate that the PLM is a linear regression model where endogenous variables depend on intercepts, observed exogenous variables and possibly lags of endogenous variables. The estimation would then be based on least squares or related methods.

We now summarize the formal setting of learning used in this paper. See Section 8 for further details about the setting used and references to the literature.

Our model is purely forward-looking while the observable exogenous shock g_t is an AR(1) process. Then the appropriate PLM is a linear projection of the state variables $(y_{t+1}, \pi_{t+1}, R_{t+1})$ onto an intercept and the exogenous shock. In this setting convergence of learning to a fixed point is fully governed by the dynamics of intercepts. Thus, computation of the domains of attraction can be fully studied in the special case where the shock g_t is taken to be zero identically. The agents then estimate the mean values of the state variables. This is is called "steady state learning" in the literature.

It is therefore assumed that agents form expectations using so-called steady state learning with point expectations which is formalized as

$$s^e_{t+j} = s^e_t \text{ for all } j \geq 1, \text{ and } s^e_t = s^e_{t-1} + \omega_t(s_{t-1} - s^e_{t-1}) \tag{13}$$

for the relevant variables $s = y, \pi, R$. It should be noted that in this notation expectations s^e_t refer to future periods (and not the current one). When forming s^e_t the newest available data point is s_{t-1}, i.e. expectations are formed in the beginning of the current period. In this paper we assume 'constant gain' learning, so that the gain parameter $\omega_t = \omega$, for $0 < \omega \leq 1$. Here ω is assumed to be small.

We now return to the economic model. The temporary equilibrium equations with steady-state learning are:

1. The aggregate demand

$$\begin{aligned} y_t &= \bar{g} + (\beta^{-1} - 1)(y^e_t - \bar{g})\left(\frac{\pi^e_t}{R_t}\right)\left(\frac{R^e_t}{R^e_t - \pi^e_t}\right) \\ &\equiv Y(y^e_t, \pi^e_t, R_t, R^e_t) \end{aligned} \tag{14}$$

is obtained by combining (6) and (8). Here it is assumed that consumers make forecasts of future output, inflation and nominal interest rates (y^e_t, π^e_t, R^e_t) which are perceived as constants for all future periods, given that we are assuming steady-state learning. As agents do not know the interest rate rule of the monetary policy maker, they need to forecast future interest rates.

2. The nonlinear Phillips curve

$$\pi_t = Q^{-1}[\tilde{K}(y_t, y^e_t, y^e_t, \ldots)] \equiv Q^{-1}[K(y_t, y^e_t)] \equiv \Lambda(y_t, y^e_t), \tag{15}$$

where $\tilde{K}(.)$ is defined in (7) and

$$Q(\pi_t) \equiv (\pi_t - 1)\pi_t, \tag{16}$$

$$\begin{aligned} K(y_t, y^e_t) &\equiv \frac{\nu}{\gamma}\left(\alpha^{-1} y_t^{(1+\varepsilon)/\alpha} - \left(1 - \nu^{-1}\right)\frac{y_t}{(y_t - \bar{g})}\right) \\ &\quad + \frac{\nu}{\gamma}\left(\beta(1-\beta)^{-1}\left(\alpha^{-1}(y^e_t)^{(1+\varepsilon)/\alpha} - \left(1 - \nu^{-1}\right)\frac{y^e_t}{y^e_t - \bar{g}}\right)\right). \end{aligned} \tag{17}$$

is obtained from (7) under steady state learning and assuming $g_t = \bar{g}$.

There are also dynamics for b_t and m_t. With Ricardian consumers the dynamics for bonds and money do not influence the dynamics of inflation, output and the interest rate. As noted, the system in general has three expectational variables: output y_t^e, inflation π_t^e, and the interest rate R_t^e. The evolution of expectations is then given by

$$y_t^e = y_{t-1}^e + \omega(y_{t-1} - y_{t-1}^e); \tag{18}$$

$$\pi_t^e = \pi_{t-1}^e + \omega(\pi_{t-1} - \pi_{t-1}^e); \tag{19}$$

$$R_t^e = R_{t-1}^e + \omega(R_{t-1} - R_{t-1}^e) \tag{20}$$

in accordance with (13).

4 Expectation Dynamics

4.1 Steady States and Stability

A non-stochastic steady state (y, π, R) under PLT must satisfy the Fisher equation $R = \beta^{-1}\pi$, the interest rate rule (12), and steady-state form of the equations for output and inflation (14) and (15). One steady state clearly obtains when the actual inflation rate equals the inflation rate of the price-level target path, see equation (11). Then $R = \bar{R}$, $\pi = \pi^*$ and $y = y^*$, where y^* is the unique solution to the equation

$$\pi^* = \Pi(Y(y^*, \pi^*, \bar{R}, \bar{R}), y^*)].$$

Moreover, for this steady state $P_t = \bar{P}_t$ for all t.

The targeted steady state under the PLT rule is, however, not unique.[16] Intuitively, the Fisher equation $R = \beta^{-1}\pi$ is a key equation for a nonstochastic steady state and $\bar{R}, \pi^*$ satisfies the equation. If policy sets $R = 1$, then $\bar{\pi} = \beta < 1$ becomes a second steady state as the Fisher equation also

[16] The ZLB and multiple equilibria for an inflation targeting framework and a Taylor-type interest rate rule has been analyzed in Reifschneider and Williams (2000), Benhabib, Schmitt-Grohe, and Uribe (2001) and Benhabib, Schmitt-Grohe, and Uribe (2002). These issues have been considered under learning, e.g., in Evans and Honkapohja (2010),, Benhabib, Evans, and Honkapohja (2014) and Evans, Honkapohja, and Mitra (2016). Existence of the two steady states under PLT was pointed out in Evans and Honkapohja (2013), section 2.5.3.

holds at that point. Formally, there is a second steady state in which the ZLB condition is binding:[17]

Remark 1 *Assume that $\beta^{-1}\pi^* - 1 < \tilde{A}_p$. Under the Wicksellian PLT rule (12), there exists a ZLB-constrained steady state in which $\hat{R} = 1$, $\hat{\pi} = \beta$, and $\hat{y}$ solves the equation*

$$\hat{\pi} = \omega(Y(\hat{y}, \hat{\pi}, 1, 1), \hat{y}). \tag{21}$$

The proof of the remark stated as Proposition 4 with some discussion is in Appendix B.

We now start to consider dynamics of the economy in the IT and PLT regimes under the hypothesis that agents form expectations of the future using adaptive learning as described above. The first step in the analysis is to consider local stability or instability of the steady states.

We begin with the IT regime. In our model expectations of output, inflation and the interest rate influence their behavior as is evident from equations (14) and (15). Then agents' expectations are given by equations (18)-(20) in accordance with steady-state learning. The local stability conditions under learning for the IT regime (10) are given by the well-known Taylor principle for various versions of the model and formulations of learning.[18] We summarize them here, formal details and proofs are in Appendix B.

Remark 2 *(i) The targeted steady state is expectationally stable if $\tilde{A}_\pi > \beta^{-1}$ under IT, provided γ is not too large.*
(ii) The ZLB-constrained steady state is not expectationally stable under IT.

For PLT regime we start with the case of opacity and develop analytical results about stability and instability. The system under PLT with opacity consists of equations (14), (15), (12) and (23), together with the adjustment of output, inflation and interest rate expectations given by (18), (19) and (20). Theoretical learning stability conditions for the PLT regime are available in the limiting case $\gamma \to 0$ of small price adjustment costs. Appendix B contains the statement and proof for the following results:

[17] In what follows $\hat{R} = 1$ is taken as a steady state equilibrium. In principle, we then need to impose a finite satiation level in money demand or assume that the lower bound is slightly above one, say $\hat{R} = 1 + \varepsilon$. The latter assumption is used below in the numerical analysis.

[18] The seminal paper is Bullard and Mitra (2002) and recent summaries are given in Evans and Honkapohja (2009a) and in Section 2.5 of Evans and Honkapohja (2013).

Remark 3 *(i) Assume $\gamma \to 0$ and that agents' inflation forecast is given by (19). If $\psi_p > 0$ under the PLT rule (12), the target steady state is expectationally stable.*
(ii) The ZLB-constrained steady state is not expectationally stable under PLT without guidance.

4.2 Robustness of Policy with Opacity

We now compare performance of regimes of IT and PLT with opacity using the domains of attraction of the targeted steady state. This situation could happen if after a shift from IT to PLT agents stick with their earlier forecasting practice or because the target path is not made known. The latter is the case of opacity mentioned in Section 2.2.2.

The calibration for a quarterly framework $\pi^* = 1.005$, $\beta = 0.99$, $\alpha = 0.7$, $\gamma = 128.21$, $\nu = 21$, $\varepsilon = 1$, $\sigma_1 = \sigma_2 = 1$ and $g = 0.2$ is adopted. The calibrations of $\beta; \alpha;$ and g are standard. The chosen value of π^* corresponds to two percent annual inflation rate. We set the labor supply elasticity $\varepsilon = 1.$ The value for γ is based on a 15% markup of prices over marginal cost suggested in Leeper, Traum, and Walker (2011) (see their Table 2) and the price adjustment costs are estimated from the average frequency of price reoptimization at intervals of 15 months (see Table 1 in Keen and Wang (2007)). It is also assumed that interest rate expectations $r^e_{t+j} = R_{t+j-1}/\pi^e_{t+j}$ revert to the steady state value β^{-1} for $j \geq T$.[19] We use $T = 28$. To facilitate the numerical analysis the lower bound on the interest rate R is sometimes set slightly above 1 at value 1.0001. The gain parameter is set at $\omega = 0.002$, which is a low value. Sensitivity of this choice is discussed below.

The targeted steady state is $y^* = 0.943254$, $\pi^* = 1.005$ and the low steady state is $y_L = 0.943026$, $\pi_L = 0.99$. For policy parameters in the PLT regime we adopt the values

$$\psi_p = 0.25 \text{ and } \psi_y = 1;$$

which are also used by Williams (2010). For the IT rule (10) the policy parameter values are assumed to be the usual values $\psi_\pi = 1.5$ and $\psi_y = 0.5/4$.

We focus on sensitivity with respect to initial inflation and output expectations π^e_0 and y^e_0. (One could also study sensitivity with respect to initial conditions of other state variables.) Initial conditions on the interest rate R_0

[19] The truncation is done to avoid the possibility of infinite consumption levels for some values of the expectations. See Evans and Honkapohja (2010) for more details.

and its expectations R_0^e are set at the target value, while initial conditions on actual inflation and output are set at $y_0 = y_0^e + 0.0001$ and $\pi_0 = \pi_0^e + 0.0001$. We also set $X_0 = 1.003$ under PLT and comment on sensitivity of this below. For generating Figure 1, we simulate the model for various values of initial inflation and output expectations, π_0^e and y_0^e. π_0^e ranges from 0.935 to 1.065 at steps of 0.002 while y_0^e varies from 0.923254 to 0.963254 at steps of 0.0005. We say convergence has been attained when mean actual inflation over the last ten quarters is within 1% annually around the target inflation rate (i.e. between 1.0025 and 1.0075) and similarly mean output over the last ten quarters is 0.02% around the target steady state (i.e. between 0.943065 and 0.943443 so that this interval excludes the low steady state); otherwise we say the dynamics does not converge.[20]

From the numerical analysis we have the result:

Result 1: *PLT without guidance is less robust than IT.*

FIGURE 1 ABOUT HERE

Figure 1 illustrates the result by showing numerical computed domains of attraction for the two rules. We remark that the choice of the initial target price level $\bar{P}_0$ has only a minor effect on the domain of attraction of PLT. A smaller value of X_0 enlarges the domain of attraction but only a little bit in the case under consideration. (Precise results are available upon request.)

5 PLT with Full Credibility

5.1 Basic Considerations

Above it was assumed that the target path $\{\bar{P}_t\}$ for the price level does not influence the formation of inflation expectations due to an opaque move from a preceding IT regime. A public announcement of a target price level path includes useful information for forecasting inflation and thus can change the dynamics of the economy via expectations. We now describe a very simple

[20] For PLT we use the baseline gain while for IT we use a higher gain of 0.01 to speed up convergence since otherwise convergence is slow.

formulation of inflation forecasting that uses data of the gap between actual and target paths in forecasting of inflation. One introduces the variable

$$X_t = P_t/\bar{P}_t, \tag{22}$$

and so we have a further equation

$$X_t \equiv X_{t-1} \times (\pi_t/\pi^*). \tag{23}$$

Identity (23) is obtained using the definition of the gap (22) and evolution of the target path (11).

Future values of gap (22) between the actual and targeted price levels are a natural variable for agents to forecast. Agents can infer the associated expectations of inflation from the forecasted gap as follows. Moving (23) one period forward, agents can compute the implied inflation forecast from the equation

$${}_tX_t^e \pi_t^e = (X_t^e \times \pi^*) \tag{24}$$

assuming as before that information on current values of endogenous variables is not available at the time of forecasting. Here X_t^e denotes the forecasted value of the gap for the future periods and ${}_tX_t^e$ refers to the forecast of the current gap X_t in the beginning of period t.[21] The inflation forecasts π_t^e from (24) are then substituted into the aggregate demand function (14).

It remains to specify how the expectations X_t^e and ${}_tX_t^e$ are formed. It is assumed that agents update the forecasts X_t^e about the future by using steady-state learning, so that

$$X_t^e = X_{t-1}^e + \omega(X_{t-1} - X_{t-1}^e). \tag{25}$$

It is also assumed that the forecast ${}_tX_t^e$ for period t made at the end of $t-1$ is a weighted average of the most recent observation X_{t-1} and the previous forecast X_{t-1}^e of the gap for period t. Formally,

$${}_tX_t^e = \omega_1 X_{t-1} + (1-\omega_1)X_{t-1}^e, \text{ where } \omega_1 > 0. \tag{26}$$

For specifying the values of ω and ω_1 the following considerations seem pertinent. Forecasts X_t^e are forecasts for the entire future and then the usual assumption in learning models of a quite small ω seems natural. In contrast, the forecast ${}_tX_t^e$ is only about the immediate future and then a high

[21] Note that ${}_tX_{t+j}^e = X_t^e$ in more detailed notation.

weight for the most recent data point X_{t-1} is natural, so that the specification $\omega_1 \approx 1$ is adopted. In the numerics we use the assumption $\omega_1 = 1$ but analogous results hold for other values for ω_1.[22]

Output and interest rate expectations are assumed to be formed as before, see equations (18) and (20). The temporary equilibrium is then given by equations (24), (14), (15), (12) and the actual relative price is given by (23). We remark that Proposition 4 continues to hold when agents use information about the target path under PLT regime.[23] If the target path $\{\bar{P}_t\}$ is known, private agents have two ways of making inflation forecasts, one of them is given by equations (23)-(26) and while the other one is usual steady state learning (19).

5.2 Extreme Case: Full Credibility

We begin by discussing the extreme case of full credibility which is a steady state for the evolution of credibility. In this case private agents are assumed fully incorporate knowledge of the price-level target path in their forecasting (as described in the preceding section).[24] Output and interest rate expectations follow (18) and (20), while the temporary equilibrium is given by equations (14), (15), (12) and (23). Under full credibility of PLT the price gap expectations follow (25) and inflation expectations are given by[25]

$$\pi^e_t = (X^e_t \times \pi^*)/X_{t-1}. \tag{27}$$

Given the potential importance of the initial value of the target price path $\bar{P}_0$, it is necessary to specify carefully the introduction of the PLT regime in the form of the target path $\{\bar{P}_t\}_{t=0}^{\infty}$, where $\bar{P}_t/\bar{P}_{t-1} = \pi^*$ and the timing in the initial period. PLT is introduced in the beginning of period 0 as a surprise and the announcement is made after agents have formed their expectations π^e_0, y^e_0 and R^e_0. It is told that the policy maker aims to reach the target path

[22] We discuss the choice of the gain parameter and the formulation (26) further in Section C.3.

[23] In the PLT case, equation (24) becomes $0 = 0$ in the limit as $_tX^e_t; X^e_t \to 0$, so that inflation expectations are not defined by the equation. They are instead given by the steady state condition $\pi^e_t = \bar{\beta}$.

[24] In the case of full credibility agents put a zero weight on the use of steady state learning for inflation (19).

[25] Note that output or interest rate expectations cannot employ information about the target price level path unless agents have more sructural information than is assumed here.

in the medium term but no information is given about the interest rate rule. From period 1 onward agents take the regime to be fully credible and use the target price path in their inflation forecasting as described in Section 5.1.

We now continue to analyze robustness of PLT policy regime by computing the domain of attraction for the targeted steady state. We focus on sensitivity with respect to displacements of initial output expectations y_0^e and relative price level X_0 by computing the (partial) domain of attraction for the targeted steady state. This kind of analysis is necessarily numerical, so values for structural and policy parameters must be specified.

Full credibility of PLT has dramatic consequences.

Result 2: *The domain of attraction of the target steady state is very large under the PLT rule with full credibility and contains even values for y_0^e well below the low steady state.*

In comparison to IT, the domain of attraction for PLT with full credibility is much larger (Compare Figure 1, top panel and Figure 2 below).

The calibration and most assumptions about the numerical values are as before. In the computation, the set of possible initial conditions for X_0 and y_0^e is made large and we set the initial values of the other variables at the deflationary steady state $\hat{R} = 1$, $\hat{\pi} = \beta$, and $y = \hat{y}$. Also set $R_0 = \hat{R} = R_0^e$ and $\pi_0 = \hat{\pi} = \pi_0^e$. The system is high-dimensional, so only partial domains of attraction can be illustrated in the two-dimensional space. Figure 2 presents the partial domain of attraction for the PLT policy rule with these initial conditions and wide grids for y_0^e and X_0. The horizontal axis gives the initial output expectations y_0^e and vertical axis gives the initial relative prices $X_0.$ The grid search for y_0^e was over the range 0.94 to 1 at intervals of 0.0005 and that for X_0 over the range 0.1 to 2 at intervals of 0.02 with the baseline gain. (Recall that for equation (26) it is assumed that $\omega_1 = 1$ for simplicity.)

FIGURE 2 ABOUT HERE

It is seen that the domain of attraction covers the whole area above values $y_0^e = 0.94$, except the unstable low steady state where $X_0 = X_0^e = 0$.[26]

It must be emphasized that the preceding set of initial conditions includes cases of large pessimistic shocks that have taken the economy to a situation

[26] Other simulations have been run for a shock to interest rate expectations R_0^e with analogous results (details are not reported for reasons of space).

where the ZLB is binding. Figure 2 shows that incorporating fully credible guidance from the PLT path in agents' forecasting can play a key role in moving the economy out of the liquidity trap toward the targeted steady state. The robustness in terms of initial X_0 also indicates that PLT with full credibility will lead to asymptotic convergence to the target steady state from far away initial conditions. Naturally, the dynamic adjustment paths depend on the value of X_0 and this will be discussed below.

The mechanism works through resulting deviations of the price level from the target path, i.e., the gap variable X_t influences formation of inflation expectations. The details are discussed in Appendix C.1. A key observation is that if agents have fully incorporated guidance from PLT into their expectations formation, the price level target path continues to influence the economy through inflation expectations even when ZLB is binding.

6 Evolving Credibility

6.1 Learning through Reinforcement

The result about huge impact of full credibility on the performance of PLT is only an extreme case. Assuming full credibility as soon as the PLT policy is announced is not plausible. It usually takes time for agents to learn that the new policy performs better than IT. It is, therefore, very important to extend the analysis to cover evolving limited credibility where private agents initially put only some, possibly small weight on the target price path $\{\bar{P}_t\}$ when forecasting inflation and that the weight increases in accordance with relative performance.

This idea is modeled as follows. It is assumed that agents' forecast of inflation is a weighted average of forecasts $\pi^e_{C,t}$ and $\pi^e_{N,t}$, where $\pi^e_{C,t}$ refers to the forecast under full credibility defined by (55) (or equivalently (25)-(27)) and $\pi^e_{N,t}$ refers to the forecast as a constant-gain weighted average of past inflation (19) to capture the no credibility scenario. The weights on the two forecasts are assumed to evolve in accordance with reinforcement learning based on forecast accuracy of $\pi^e_{C,t}$ and $\pi^e_{N,t}$.

Intuitively, reinforcement is an empirical principle such that the higher the payoff (utility) from taking an action in the past, the higher likelihood that the action will be taken in the future. We make use of a very standard and simple model of reinforcement learning.[27] Our analysis is very much a first approach to model evolving credibility and we acknowledge that alternative formulations could be developed.

Formally, the propensity of each way of forecasting is updated as

$$\theta_t^C = \lambda\theta_{t-1}^C + u_{t-1}^C \tag{28}$$
$$\theta_t^N = \lambda\theta_{t-1}^N + u_{t-1}^N, \tag{29}$$

where $\lambda \in (0,1]$. The innovation term is constructed as follows. Define an auxiliary innovation variable in terms of the accuracy of forecasting

$$\tilde{u}_t^C = \begin{cases} 1 \text{ if } \left|\pi_t - \pi^e_{C,t-1}\right| < \left|\pi_t - \pi^e_{N,t-1}\right| \\ 0 \text{ otherwise} \end{cases}. \tag{30}$$

The innovation terms in (28) - (29) are utility weighted, so that

$$u_t^C = Max[0,\upsilon_t]\tilde{u}_t^C \text{ and } u_t^N = Max[0,\upsilon_t](1 - \tilde{u}_t^C), \tag{31}$$

where the realized utility (with assumption $\sigma_1 = \sigma_2 = 1$) for period t is used, i.e.,

$$\upsilon_t = \ln[y_t - \bar{g}] + \chi\ln[\frac{\beta R_{t-1}(y_{t-1} - g)}{(R_{t-1} - 1)\pi_t}] - \frac{y_t^{\frac{1+\varepsilon}{\alpha}}}{1+\varepsilon} - \frac{\gamma}{2}(\pi_t - 1)^2. \tag{32}$$

The Max operator in (31) is used to keep utility non-negative.[28] Note that if $\upsilon_t < 0$ for some t then propensities θ_t^C are not updated and in fact decline somewhat as $\lambda < 1$. Intuitively, the propensities, θ_t^C and θ_t^N, evolve as a function of the realized utilities obtained from the two forecasting schemes.

We then define the weight for computing the average forecast

$$q_t^C = \frac{\theta_t^C}{\theta_t^C + \theta_t^N} \text{ and } q_t^N = 1 - q_t^C,$$

[27] This is a standard formulation of reinforcement learning in game theory, see e.g. p.13 of Young (2004). See also Chapter 6 of Camerer (2003) for a review of different learning models in game theory.

[28] Standard models of reinforcement learning assume that payoffs in each period are non-negative, see e.g. Young (2004). We have run many of the simulations without the non-negativity constraints and have found that negative υ_t very seldom occur and in those cases the convergence properties are not affected. See Appendix C.3 below.

so the inflation forecast of the private agents is a weighted average

$$\pi_t^e = q_t^C \pi_{C,t}^e + (1 - q_t^C)\pi_{N,t}^e. \tag{33}$$

Note that the agents' probability of choosing the forecasting scheme corresponding to full credibility, q_t^C, is increasing in the propensity θ_t^C. Other expectation variables y_t^e and R_t^e are updated according to the earlier rules (18) and (20). Given these specifications for expectations, the model is the same as before. The equations are (14), (15), (16), (17), (11) and (12).

In order to run simulations of our model, a numerical value for the decay (or discount) parameter λ must be specified. (Other parameters are as above.) Estimates for λ can be found in game-theoretic literature where reinforcement learning models are fitted to data from a variety of experimental games. These estimates vary a lot depending on type of experimental games used to obtain the data and the precise specification of reinforcement learning.[29] We mostly employ the midpoint $\lambda = 0.85$ of the range $[0.8, 0.9]$ which seems reasonable given our simplified specification and the various estimates in the literature.

6.2 Robustness with Evolving Credibility

Given the very good robustness properties of the PLT policy regime in the extreme case of full credibility (shown in Figure 2), we ask whether the same kind of results can hold in the more realistic setting of evolving limited credibility described above. Inflation forecasts are assumed to be given by the combination forecasts (33). Forecast weights are updated in accordance with reinforcement learning.

Since the state space is high dimensional, we study properties of the domain of attraction by fixing some initial values during the process. In particular, the three variables of interest are y_0^e, π_0^e and q_0^C, where we reduce the dimension of initial inflation expectations by assuming $\pi_0^e = \pi_{C,0}^e = \pi_{N,0}^e$.[30] Rather than showing a three dimensional figure, we present the domain of attraction results by fixing one of these variables and varying the remaining

[29] See Camerer and Ho (1999) for analysis and Chapter 6 of Camerer (2003) for an overview.

[30] The three measures of inflation expectations are set to be equal initially, so that the number of degrees of freedom remains manageable in the simulations. For a few simulations noted below we do allow them to be different for robustness sake.

two. We feel that domain of attraction in two dimensional figures is much more revealing than three dimensional figures. For the first exercise, we fix y_0^e and vary π_0^e and q_0^C to plot the partial domain of attraction (see Figure 3). In the next exercise we fix q_0^C and vary π_0^e and y_0^e to plot aspects of the domain of attraction (see top panel of Figure 3).

6.2.1 Role of initial credibility

We now fix output expectations y_0^e at three different values and vary the initial inflation expectations $\pi_0^e = \pi_{C,t}^e = \pi_{N,t}^e$ along with the initial degree of credibility of the PLT policy regime, q_0^C. For each combination (y_0^e, π_0^e), we compute numerically the lowest value for q_0^C of the initial degree of credibility such that the dynamics of learning from this starting point converge to the target steady state. Our interest is to consider the possibility to escape from a state of the economy where ZLB and recession prevail. With this in mind, we fix initial output expectations y_0^e at three alternative values: one slightly above the targeted steady state output, the second one at this target level and the third one at the output level corresponding to the low steady state i.e. $y_0^e = \hat{y}$. A grid of points (q_0^C, π_0^e) is then done where the relation between the degree of initial credibility q_0^C and π_0^e is shown for the different values of y_0^e indicated above.

FIGURE 3 ABOUT HERE

Figure 3 shows the domain of attraction in (q_0^C, π_0^e) space when initial output expectation y_0^e is fixed at the three different initial levels just described. In all of these panels, interest rate expectations are fixed at 1.0002 i.e. marginally above the zero lower bound to capture the scenario that the economy is stuck in the vicinity of the ZLB. The gain parameter is set at 0.0008 in all the panels. We also set other initial conditions as follows: $\mu_0^N = 1 - \mu_0^C$, $u_0^C = 0.5$, $u_0^N = 0.5$, $y_0 = y_{-1} = y_0^e$, $\pi_0 = \pi_0^e$, $R_0^e = R_0 = 1.0002$ and $X_0 = 0.99$. We say convergence is obtained when mean actual inflation over the last ten quarters is within 1% annually around the target inflation rate (i.e. between 1.0025 and 1.0075) and similarly mean output over the last ten quarters is 0.02% around the target steady state (i.e. between 0.943065 and 0.943443 so that this interval excludes the low steady state).

In the top panel of Figure 3 $y^e_0 = y^* + 0.00045 = 0.943704,$ which is slightly higher than the level of output at the target steady state. As this figure shows, even deflationary expectations close to $\pi^e_0 = 0.9$ (more than 40% deflation in annual terms!) yield stability with high enough initial credibility. More generally, π^e_0 well below the low steady state value $\approx \pi_L$ are stable even with low credibility. The mere announcement of PLT even with low credibility suffices to increase the domain of attraction significantly to below zero net inflation levels. (Compare with the domain of attraction shown in the bottom panel of Figure 1 for the PLT regime with opacity.)

The middle panel of Figure 3 shows the domain of attraction in (q^C_0, π^e_0) space when initial output expectations, $y^e_0 = y^*,$ i.e. at the target level of output. Note that convergence to the target steady state continues to obtain for deflationary expectations though the values can't be as low as in the top panel. Nevertheless, deflationary expectations (approximately 0.98 which is somewhat below the low steady state value $\approx \pi_L$) continue to be convergent.

Finally, the bottom panel shows the domain of attraction when initial output expectations, $y^e_0 = \hat{y}$ i.e. at the low output steady state. As before convergence to target from deflationary expectations can take place. In particular, π^e_0 values down to the low steady state value $\approx \pi_L$ continue to give convergence for all values of initial credibility between 0 and 1! This is particularly striking since initial output expectations are very pessimistic (at the low output steady state) in this figure.

The main message from Figure 3 is that even a low degree of credibility can be enough to have big benefits in terms of the size of the domain of attraction.

Result 3: *The announcement of the PLT regime coupled with a very small degree of initial credibility among agents can make the economy converge back to the targeted steady state from initial conditions with inflation expectations well below the target steady state.*[31]

This result is in sharp contrast with the case of the PLT regime with no guidance. The general intuition for the result can be understood by looking at Figure A.2 in the Appendix indicating the dynamics of expectations in the constrained region in the case of full credibility. At first sight limited credibility might be thought as a weighted average of the cases of no and

[31] Figure 3 shows that the size of the domain of attraction depends the magnitude of y^e_0. If $y^e_0 < y^*$, then π^e_0 cannot be much below π_L.

full credibility analyzed previously. The full intuition is, however, more complex as the existence of the possibility of forecasting using partly the target price level path influences the actual inflation path which in turn affects also the outcome from simple statistical forecasting. These *self-referential* and *feedback* effects are the key to the results. We discuss this intuition further in Appendix C.2. The very small degree of initial credibility is illustrated further in Appendix C.3.

Another observation is that higher the initial credibility, $q_0^C;$ larger is the size of the domain of attraction; in particular lower and lower deflation rates may be supported in terms of convergence to the targeted steady state. This is illustrated by the downward sloping lines of the convergence boundary in the top two panels in Figure 3 in $(q_0^C; \pi_0^e)$ space. This boundary is especially pronounced in the top panel illustrating that as initial credibility is higher, lower and lower deflationary expectations may be supported under PLT. Higher credibility has beneficial effects in this sense. These beneficial effects are less evident when initial output expectations are very pessimistic as shown by the nearly horizontal line in the bottom panel. The message here is that if the policy maker contemplates a move to PLT during a liquidity trap scenario, it should not wait too long for output expectations to become very pessimistic since it is then more difficult to get out of this situation.

FIGURE 4 ABOUT HERE

One can also illustrate the results by computing domains of attraction in $(y_0^e; \pi_0^e)$ -space for different values of initial credibility q_0^C. The top panel in Figure 4 illustrates the case of high credibility where $q_0^C = 0.9$ and relatively high values of π_0^e while (to facilitate comparison) the bottom panel shows the domain of attraction for IT.[32] One can show that a higher level of initial credibility implies a bigger partial domain of attraction in $(y_0^e; \pi_0^e)$ -space. In particular, the upward sloping line shifts outward, higher is the level of initial credibility i.e. higher values of initial output and inflation expectations imply convergence with high credibility. Moreover, the domain of attraction for PLT with opacity is smaller than for PLT with even low credibility (for

[32] To plot the top panel, we use the gain parameter $0.001.$ We also set other initial conditions as follows: $\mu_0^N = 1 - \mu_0^C$, $u_0^C = 0.5$, $u_0^N = 0.5$, $y_0 = y_0^e$, $\pi_0 = \pi_0^e$, $R_0^e = R_0 = R_{-1.} = R^*$, $X_0 = 0.99$. Also $y_0 = y_{-1}$ and $\pi_0 = \pi_{-1}$. $\pi_0^e = \pi_{C,0}^e = \pi_{N,0}^e$ as before. For IT we use a gain of 0.01 to speed up convergence as before.

brevity we do not depict these figures). The message is that higher credibility enhances the size of the domain of attraction but even low credibility is beneficial.[33]

However, a very significant message emerges when the partial domain of attraction for IT is shown in the bottom panel of Figure 4 for the same region of the state space. It is seen that with IT there is convergence to target equilibrium for all starting points in this part of the state space. For this domain IT is a better policy than PLT even when credibility of the latter is very high. Thus IT is more robust than PLT when initial inflation expectations are at a high value. This superiority of IT with high inflation and output expectations is an important message which comes out of our analysis.

This observation is distinct from the usual criticism of PLT saying that if inflation (and output) are above target and a negative shock hits the economy, the history-dependence of PLT delays the adjustment. PLT dictates restrictive policy whenever inflation is above the target. In contrast, IT is free from history dependence and can respond in an easing fashion to a negative shock right away.

On the other hand, PLT even with low degree of credibility is superior to IT when initial expectations of inflation are in the deflationary domain (as shown by Figure 3). With deflationary expectations IT always leads to a deflationary spiral no matter what output expectations are whereas convergence to the target steady state can take place with PLT.[34] PLT is superior in times when binding ZLB and/or initial inflation and output expectations are pessimistic.

This analysis lends strong support to the suggestions of Evans (2012), Williams (2017) and Bernanke (2017) that guidance from price-level targeting can be helpful in a liquidity trap. Monetary policy alone is able to pull the economy out of the liquidity trap if PLT can be implemented so that from the beginning the newly introduced PLT regime has at least some credibility.[35]

[33] We have also looked at dynamics of IT and PLT with different degrees of initial credibility in terms of volatility of dynamics near the target steady state. For economy of space we do not present these results which are available upon request.

[34] The deflationary spiral mentioned could lead to a stagnation steady state under further assumptions on the economy, see Evans, Honkapohja, and Mitra (2016). This kind of analysis need not be introduced for our purposes.

[35] This result is in contrast to the case of inflation targeting studied in Evans, Guse, and Honkapohja (2008) and Evans and Honkapohja (2010).

From a more general perspective it is, however, important to add the qualification that PLT a good policy only during a liquidity trap scenario. During normal times IT is a better policy:

Result 4: *In terms of the robustness criterion (i.e. domain of attraction) PLT is globally a better policy than IT during a deflationary/liquidity trap scenario while IT is the better policy globally during normal times.*

6.2.2 Role of the initial target price level

The initial target price level is an important element for the results. Figures 5A-B show the partial domains of attraction in $(q_0^C; y_0^e)$ space for PLT with guidance for two different values of the initial target price level $\bar{P}_0$ (with other initial conditions set close to the low steady state). The formal analysis is presented in terms of $X_0 = P_0/\bar{P}_0$, where the initial price level is normalized at $1.$ The initial target price level $\bar{P}_0$ is such that $X_0 = 0.95$ and $X_0 = 1.05$ in the two cases. The numerical results are in line with our hypothesis that a relatively high value of $\bar{P}_0$ (i.e. a low value of X_0) is conducive to convergence to the target steady state when the economy is initially near the low steady state. In economic terms the results state that when introducing a PLT regime the initial value for the target price level path should be made relatively high, so that monetary policy is kept loose for longer.

FIGURES 5A-C ABOUT HERE

Similar exercise has been done in the $(q_0^C; \pi_0^e)$ space though here the results are roughly similar when $X_0 = 0.95$ and $X_0 = 1.05.$ However, with higher credibility lower inflation expectations yield convergence to target with both $X_0 = 0.95$ and $X_0 = 1.05$. (Details are available upon request.)

We also consider a situation when expectations of inflation, output and interest rate are above the targeted steady state to capture a boom-like scenario. Figure 5C depicts the domain of attraction in such a situation when $X_0 = 0.95$. Here $\pi_0^e = 1.015;$ $R_0^e = 1.02;$ $\pi_0 = \pi_0^e;$ $y_0 = y_0^e + 0.00001$ and $R_0 = R_0^e$. We also set $\pi_{C,0}^e = \pi_0^e + 0.0001$ and $\pi_{N,0}^e = \pi_0^e - 0.0001$ as in Figures 5A-B. As before, with higher credibility, higher output expectations are conducive to convergence to target. With $X_0 = 1.05;$ a similar figure obtains.

6.3 Setting the initial target price $\bar{P}(0)$

As already discussed, the choice of the initial value for the target price level can be an important issue if a move to PLT is contemplated. It turns out that simultaneously the current state of the economy has an impact on the dynamics of the economy and we now analyze how the nature and volatility of the dynamics depends on these two factors. We consider two sets of initial conditions: initial conditions are (i) near the low steady state or (ii) above the high steady state.[36] In Appendix C.4 two further interim cases are briefly reported.

As mentioned, the robustness property considered now is volatility: how big are the fluctuations during the adjustment path? Volatility in inflation, output and interest rate during the learning adjustment is computed in terms of median unconditional variances of inflation, output and interest rate (called $var(\pi); var(y)$ and $var(R)$ in the table below). We calculate the value of a quadratic loss function (called $LOSS$ in the table) in terms of the weights 0.5 for output, 0.1 for the interest rate and 1 for the inflation rate (the weights are taken from Williams (2010)) and also the median ex post utility of the representative consumer (called $Utility$).

In each case X_0 takes on the values 0.96 or 1.04, i.e. the target price level deviates approximately four percent in either direction. Table 1 below gives the volatility results for inflation, output and interest rates dynamics in the first three columns. The next two columns show the loss ($LOSS$) and ex-post intertemporal utility ($Utility$)

$$\sum_{t=0}^{Tend} \beta^{t} \mho_t ;$$

where $\mho_t$ is given by (32) and $Tend = 500$. The sixth column (ZLB) shows the frequency of the interest rate R to hit the ZLB (defined as a situation when $R < 1.001$) and the final column (Def) the frequency of entering deflation (defined as a situation when $\pi < 1$). In these final two columns the percentage of times in all simulations when the ZLB or deflation is encountered is reported.

Case (i): initial inflation and output expectations are around the low steady state and interest rate is at ZLB.[37]

[36] R_0 is at ZLB in cases (i) and (ii). $q_0^C = 0.9$ is assumed in all cases.

[37] Inflation expectations are from slightly below the low steady state to 1 and output

	$var(\pi)$	$var(y)$	$var(R)$	$LOSS$	$Utility$	ZLB	Def
$X_0 = 0.96$	0.58624	1.57473	5.49622	1.92323	327.714	0.31	0.32
$X_0 = 1.04$	1.19457	0.67805	35.3015	5.06374	290.572	0	0.28

Table 1: Volatility of inflation, output and interest rate for PLT with different values of X_0. The final two columns show the frequency of R to hit the ZLB and the frequency of deflation.
Note: the numbers for $var(\pi); var(y)$, $var(R)$ and $LOSS$ should be multiplied by 10^{-6}.

It is seen that $X_0 = 0.96$ is better than $X_0 = 1.04$ in terms of inflation, interest rate volatility, loss and utility criteria (except for output volatility and ZLB and Def criteria).

FIGURE 6 ABOUT HERE

Figure 6 shows the mean dynamics of inflation, output and the interest rate from the simulations used for Table 1. It is seen that the low value $X_0 = 0.96$ results in faster recovery from the recessionary current state of the economy. This is in line with the idea that a in recessionary situation a relatively high value of the initial target price $\tilde{P}_0$ can contribute to recovery by maintaining a less restrictive monetary policy (as indicated in the bottom panel of Figure 6).

The next case is designed to capture a boom scenario.

Case (ii): initial inflation and output expectations are above the targeted steady states and interest rate is at targeted steady state.

	$var(\pi)$	$var(y)$	$var(R)$	$LOSS$	$Utility$	ZLB	Def
$X_0 = 0.96$	1.4026	0.371856	5.5539	2.14392	301.906	0.0873	0.1447
$X_0 = 1.04$	0.0906416	0.36427	4.67443	0.740219	298.768	0	0.0012

Table 2: Volatility of inflation, output and interest rate for PLT with different values of X_0. The final two columns show the frequency of R to hit the ZLB and the frequency of deflation.

expectations symmetric around the low steady state. It turns out that too low inflation expectations leads to instability with $X_0 = 1.04$.

Note: the numbers for $var(\pi); var(y)$, $var(R)$ and $LOSS$ should be multiplied by 10^{-6}.

In this case the volatility results are as follows. Volatilities for inflation, output and the interest rate are somewhat higher when a relatively high initial value for the target price level is set. This result is also visible in Figure 7 for the initial periods of the dynamic paths using data from the simulations used for Table 2. It is also seen that a relatively low value for the initial target price avoids an initial recession episode.

FIGURE 7 ABOUT HERE

7 Application: Swedish Experience with Price Stabilization

The analysis in Section 6.3 demonstrates that when introducing PLT as a new policy regime, the decision on the initial level for the target price has major effects on the short and intermediate run dynamics of the economy. As noted in the Introduction, Sweden is the only country which has experimented with monetary policy geared to price level stabilization which is arguably akin to PLT. Here we review the Swedish experience in the 1920's and 1930's episodes, focusing on the choice of initial target price level and the resulting macroeconomic dynamics.[38] It should be noted that the former episode was not a move to a system resembling PLT but rather a return to gold standard in order to stabilize the price level. The episode is nevertheless informative about how to set the initial price level as part of a new policy regime as the choice of parity is an integral part of the gold standard.

The outbreak of the war in 1914 lead to collapse of the gold standard system which meant that a firm anchor for monetary policy was lost. World War I gave rise to large fluctuations in the Swedish price level and the Swedish money stock: between 1914-20, the price level increased by 165% and the money stock by 195% (see Figure 1 in Jonung (1979)). This was followed by a period of restrictive monetary policy as the Swedish government and

[38] For the full details of the Swedish experience the reader is referred to Jonung (1979) and Berg and Jonung (1999).

central bank decided to return to the gold standard at prewar parity. This meant that the target price level was much lower than the current level. The decision to go back to prewar parity was in line with the thinking of Knut Wicksell, the original proponent of price level stabilization and PLT. Wicksell had proposed a return to the price level of 1914 and a stabilization of prices at this level even if the return to gold standard was inconsistent with independent domestic control of the money stock.

The macroeconomic consequences of price level stabilization at the prewar level were not favorable, for the data see Figure 1 of Jonung (1979). The Swedish money stock was reduced by 29 % between 1920 and 1925, and the price level fell by 35 % during the same period. At the same time the international economy plunged into a deep depression which had an additional negative impact on Sweden. During these years unemployment reached the highest level ever recorded in Sweden. The inflation of 1914-1920 and the ensuing deflation of 1920-1923 led to a lively discussion on monetary policy.

The second episode of price level stabilization in Sweden occurred in the 1930's. As background we note that the Swedish economy was influenced relatively late by the world-wide depression which started in the U.S.A. at the end of the 1920's. The price level in those countries with currency tied to the gold standard dropped sharply during the last two years of the 1920's and the beginning of the 1930's. Swedish wholesale prices followed this pattern in 1929, 1930, and in the first three quarters of 1931, see Figure 2 in Jonung (1979). In the middle of September 1931, England left the gold standard due to speculation against the pound. The Riksbank and the Swedish government took the same step one week later (due to the huge outflow of foreign exchange reserves from the Riksbank). At the same time as Sweden left the gold standard and adopted a paper standard, the Minister of Finance declared that the aim of Swedish monetary policy should be to preserve the domestic purchasing power of the krona using 'all means available'. This statement became the core of the monetary policy program of 1931 and in fact the September 1931 price level was adopted as the starting point, see Berg and Jonung (1999), p.540.

The norm of price level stabilization that Wicksell presented in 1898 thus became, some thirty years later, the o¢cial foundation for Swedish monetary policy.[39] This episode of price level stabilization was successful as the pre-

[39] It should be noted that the the period of a paper standard was relatively short-lived as the Swedish krona was pegged to Pound Sterling in June and July 1933.

ceding deflation was stopped and unemployment and industrial production gradually started to move in favorable directions, see Figure 3 of Berg and Jonung (1999).

According to our analysis, the success of PLT depends to a large extent on the initial relative level of actual and target prices (see section 6.3) and/or whether the economy is in an inflationary or deflationary situation. Our results in Sections 5 and 6 are consistent with the circumstances surrounding the success or failure of price level stabilization experienced in Sweden during these two episodes. We remark that the role of the initial target price has not been much discussed after the Swedish debates in 1920's and 30's.

Price level stabilization policy during the first episode in Sweden took place during a situation of rising prices. Figure 4 in Section 6.2 shows that IT is a much better policy than PLT when agents have high (i.e. positive) inflation expectations. Agents are more likely to have such expectations during times of high inflation and rising prices. Wicksell had suggested a price level target at the 1914 level in Sweden during the 1920s. This can be interpreted to mean a situation when $X_0 >> 1$ in our analysis. Our Figures 5 and 6 and Table 1 demonstrate that when $X_0 >> 1$ the outcome with PLT is worse in comparison to when $X_0 < 1$. Many countries tried to achieve price stabilization via re-establishment of gold standard after World War I and the economic consequences were mixed, see e.g. Chapter 3 of Eichengreen (1996). We omit detailed discussion but note that the British return to gold standard in 1925 is a much cited problematic case: "The proper object of dear money is to check an incipient boom. Woe to those whose faith leads them to use it to aggravate depression!", p.19 of Keynes (1925).

The circumstances during the adoption of PLT in Sweden in the 1930s were very di¤erent to that in the 1920's. Prices (especially wholesale prices but also the consumer price index; see Figure 1 of Berg and Jonung (1999)) were declining from 1928 to 1931 so that the economy was characterized by generally a deflationary scenario in the run up to the adoption of PLT. Our results show that PLT is a good policy during these circumstances: see e.g. our Figure 2 for full credibility or Figures 3 and 5 with imperfect credibility, the latter being probably the more realistic scenario for agents to encounter during this time. Moreover, the Swedish initial price level target can be interpreted as corresponding to a scenario when $X_0 < 1$: Figure 5A, for instance, along with Figure 6 (and Table 1) show that PLT is a good policy in these circumstances.

8 Discussion of Learning and Related Literature

As noted in the introduction, RE is a very strong assumption about the agents' knowledge of the economy. A major starting point in this paper is to relax the RE hypothesis and instead use the assumption that private agents operate under imperfect knowledge and learning. The learning approach is increasingly used in the literature. For discussion and analytical results concerning adaptive learning in a wide range of macroeconomic models, see for example Sargent (1993), Evans and Honkapohja (2001), Sargent (2008), and Evans and Honkapohja (2009b). Recent papers that relax the RE assumption in the context of macroeconomic policy analysis include Taylor and Williams (2010) and Woodford (2013).

Gradual adjustment of expectations is a central part in the description of economic dynamics with adaptive learning. In this approach agents maximize in each period their anticipated utility or profit subject to expectations that are derived from an econometric forecasting model given the data available at the time of forecasting and the model is updated over time with arrival of new information. Agents know their own structural characteristics but not those of other agents. Thus individual agents have much less information than under RE. The learning approach contrasts with the existing literature on PLT that, as mentioned, is largely based on the RE hypothesis.[40] In the earlier literature Orphanides and Williams (2002), Orphanides and Williams (2007) and Orphanides and Williams (2013) argue that PLT can be effective when there is structural change and uncertainty.[41] We note that all of the cited studies use linearized models for their analysis.

This paper instead uses a nonlinear micro-founded New Keynesian (NK) model when private agents learn adaptively using infinite horizon forecasts advocated by Preston (2005) and Preston (2006), used in Evans and Honkapohja (2010) and Benhabib, Evans, and Honkapohja (2014) to study the properties of a liquidity trap.[42] The nonlinear framework is needed to assess the

[40] There is also a literature that incorporates imperfect information, credibility and optimal filtering about some limited aspects of the economy, but RE is otherwise maintained, see e.g. Faust and Svensson (2001) and Erceg and Levin (2003) for applications to monetary policy.

[41] Aspects of imperfect knowledge are also included in the discussion of price-level targeting by Gaspar, Smets, and Vestin (2007) and Williams (2010).

[42] The forecasting horizon is one modeling choice in the learning approach. See Honkapo-

global properties of the policy targeting regimes, including the possibility of multiple equilibria created by the ZLB.

In the model learning is about how to forecast future inflation, output and the interest rate. The model is purely forward-looking while the observable exogenous shock $\tilde{g}_t$ is an AR(1) process. Then the appropriate PLM is a linear projection of $(y_{t+1}, \pi_{t+1}, R_{t+1})$ onto an intercept and the exogenous shock. So the agents estimate

$$
\begin{aligned}
y_s &= a_y + b_y \tilde{g}_{s-1} + \varepsilon_{y,s}, \\
\pi_s &= a_\pi + b_\pi \tilde{g}_{s-1} + \varepsilon_{\pi,s}, \\
R_s &= a_R + b_R \tilde{g}_{s-1} + \varepsilon_{R,s}
\end{aligned}
$$

by using a version of least squares and data for periods $s = 1, \ldots, t-1$. The latter is a common timing assumption in the learning literature; at the end of period $t-1$ agents estimate the parameters using data on the variables through to period $t-1$. This gives estimates $a_{y,t-1}$, $b_{y,t-1}$, $a_{\pi,t-1}$, $b_{\pi,t-1}$, $a_{R,t-1}$, $b_{R,t-1}$ and using these estimates and data at time t the forecasts are given by

$$
\begin{aligned}
y^e_{t+j} &= a_{y,t-1} + b_{y,t-1}\rho^j \tilde{g}_t, \\
\pi^e_{t+j} &= a_{\pi,t-1} + b_{\pi,t-1}\rho^j \tilde{g}_t, \\
R^e_{t+j} &= a_{R,t-1} + b_{R,t-1}\rho^j \tilde{g}_t
\end{aligned}
\qquad (34)
$$

for future periods $t+j$. These forecasts are then substituted into the system to determine a temporary equilibrium of the economy in periods $t+j$. This determines a new data point and for the next period the estimates are updated accordingly and the process continues.

In this setting convergence of learning is fully governed by the dynamics of intercepts $a_{y,t-1}$, $a_{\pi,t-1}$, $a_{R,t-1}$ and not by the coefficients $b_{y,t-1}$, $b_{\pi,t-1}$, $b_{R,t-1}$ because the regressor is exogenous. For this reason the analysis of convergence and computation of the domains of attraction can be fully studied in the special case where the shock $\tilde{g}_t$ is taken to be zero identically. In this situation the agents just estimate the mean values of y_t, π_t and R_t. This is is often called "steady state learning" in the literature. We therefore assume in the analysis of domains of attraction that agents form expectations using so-called steady state learning with point expectations formalized as equation

hja, Mitra, and Evans (2013) for a discussion of infinite-horizon and short-horizon learning in contexts of monetary policy.

(13). It should be noted that in this notation expectations s_t^e refer to future periods (and not the current one).[43] As before, when forming s_t^e the newest available data point is s_{t-1}, i.e. expectations are formed in the beginning of the current period.

γ_t is called the "gain sequence" and measures the extent of adjustment of the estimates to the most recent forecast error. In stochastic systems one often sets $\gamma_t = t^{-1}$ and this "decreasing gain" learning corresponds to classic least-squares updating. Steady state learning then corresponds to least-squares regression on an intercept. Also widely used is the case $\gamma_t = \gamma$, for $0 < \gamma \leq 1$, called "constant gain" learning. In this case it is assumed that γ is small.

Under decreasing gains possible convergence to a fixed point is asymptotically to an REE. Under constant gain convergence is toward a random variable centered near the equilibrium. If the model is non-stochastic, then constant gain may converge exactly to the (non-stochastic) steady state.

The study of evolving credibility by means of reinforcement learning in Section 6 adds a new layer to the learning processes of agents. Here reinforcement learning is formally a way of describing evolution of weights in averaging of the forecasting models used by agents.[44]

9 Conclusions

Our study considers using the domain of attraction of the target steady state as a new way of assessing of price-level targeting that has been recently suggested as a possible improvement over inflation targeting monetary policy for the current environment with low inflation and low output growth. The results indicate that the performance of price-level targeting is clearly better than performance of inflation targeting, provided that private agents' learning has incorporated the guidance from the price level target path. With perfect credibility the domain of attraction of the target steady state under price-level targeting is very large with basically global convergence (except from the deflationary steady state). Moreover, the good convergence properties largely hold even with small degrees of initial credibility and evolutionary

[43] Note that (34) implies $s_{t+j}^e = s_t^e$ for all $j \geq 1$ when $\rho = 0$.

[44] A small literature on implications of model averaging for adaptive learning can be noted, see Evans, Honkapohja, Sargent, and Williams (2013), Gibbs (2015), Cho and Kasa (2017) and Gibbs and Kulish (2017).

adjustment of credibility based on forecasting performance.

If instead private agents' learning does not use the guidance at all, IT has a clearly bigger domain of attraction than PLT. Thus, if a move to price-level targeting is contemplated, it is important to try to influence the way private agents form inflation expectations, so that the guidance from PLT has some credibility and is thus incorporated into their learning.

Our analysis has two important starting points. It is assumed that agents have imperfect knowledge and therefore their expectations are not rational during a transition after a shock. Agents are assumed to make their forecasts using an econometric model that is updated over time. We have carefully introduced the nonlinear global aspects of a standard framework, so that the implications of the interest rate lower bound can be studied. As is well-known, inflation targeting with a Taylor rule su¤ers from global indeterminacy and the same problem exists for standard versions of price-level targeting.

The current results are a first step in this kind of analysis. Several extensions can be considered. We have used standard policy rules and standard values for the policy parameters, but these do not represent optimal policies. Deriving globally optimal rules in a nonlinear setting like ours is extremely demanding, but one could consider optimal simple rules, e.g., optimization of the parameter values of these instrument rules. One could also do away with the instrument rule formulations used in this paper and instead postulate that the central bank employs a target rule whereby in each period the policy instrument is set to meet the target exactly unless the ZLB binds.

It should also be noted that these results about the key roles of credibility and guidance have been obtained by comparing the properties of the di¤erent regimes when dynamics arise from learning. Our comparisons of di¤erent regimes are limited by the assumption that the economy starts in a given IT regime. Analysis of how and why private agents might change their forecasting practice after the introduction of a new regime would be well worth studying systematically. Central bank policies can probably influence this change in forecasting.

There are numerous more applied concerns about PLT that should be investigated before any final assessment. We just mention the issues connected with measurement and fluctuations of output and productivity. Orphanides (2003) and Orphanides and Williams (2007) discuss the measurement problems in output and output gap. Our non-stochastic model does not address these concerns. Another issue is the choice of the index for the target price level. We plan to analyze some of these extensions in the future.

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A Model Derivations

A.1 Optimal Decisions for Private Sector

In period t each household s is assumed to maximize its anticipated utility (1) under given expectations. As in Evans, Guse, and Honkapohja (2008), the first-order conditions for an optimum yield

$$\begin{aligned} 0 &= -h_{t,s}^{\varepsilon} + \frac{\alpha\gamma}{\nu}(\pi_{t,s}-1)\pi_{t,s}\frac{1}{h_{t,s}} \\ &\quad + \alpha\left(1-\frac{1}{\nu}\right) y_t^{1/\nu}\frac{y_{t,s}^{(1-1/\nu)}}{h_{t,s}} c_{t,s}^{-\sigma_1} - \frac{\alpha\gamma\beta}{\nu}\frac{1}{h_{t,s}} E_{t,s}(\pi_{t+1,s}-1)\pi_{t+1,s}, \end{aligned} \tag{35}$$

$$c_{t,s}^{-\sigma_1} = \beta R_t E_{t,s}\left(\pi_{t+1}^{-1} c_{t+1,s}^{-\sigma_1}\right) \text{ and} \tag{36}$$

$$m_{t,s} = (\chi\beta)^{1/\sigma_2}\left(\frac{\left(1-R_t^{-1}\right)c_{t,s}^{-\sigma_1}}{E_{t,s}\pi_{t+1}^{\sigma_2-1}}\right)^{-1/\sigma_2}, \tag{37}$$

where $\pi_{t+1,s} = P_{t+1,s}/P_{t,s}$ and $E_{t,s}(.)$ denotes the (not necessarily rational) expectations of agents s formed in period t.

Equation (35) is one form of the nonlinear New Keynesian Phillips curve describing the optimal price-setting by firms. The term $(\pi_{t,s}-1)\pi_{t,s}$ arises from the quadratic form of the adjustment costs, and this expression is increasing in $\pi_{t,s}$ over the allowable range $\pi_{t,s} \geq 1/2.$ To interpret this equation, note that the first term on the right-hand side is the marginal disutility of labor while the third term can be viewed as the product of the marginal revenue from an extra unit of labor with the marginal utility of consumption. The terms involving current and future inflation arise from the price-adjustment costs. Equation (36) is the standard Euler equation giving the intertemporal first-order condition for the consumption path. Equation (37) is the money demand function resulting from the presence of real balances in the utility function.

We now proceed to rewrite the decision rules for consumption and inflation so that they depend on forecasts of key variables over the infinite horizon (IH).

A.2 The Infinite-horizon Phillips Curve

Starting with (35), let

$$Q_{t,s} = (\pi_{t,s}-1)\pi_{t,s}. \tag{38}$$

The appropriate root for given Q is $\pi \geq \frac{1}{2}$ and so we need to impose $Q \geq -\frac{1}{4}$ to have a meaningful model. Using the production function $h_{t,s} = y_{t,s}^{1/\alpha}$ we can rewrite (35) as

$$Q_{t,s} = \frac{\gamma}{\alpha\gamma} y_{t,s}^{(1+\varepsilon)/\alpha} - \frac{\gamma - 1}{\gamma} y_t^{1/\gamma} y_{t,s}^{(\gamma-1)/\gamma} c_{t,s}^{-\sigma_1} + \beta E_{t,s} Q_{t+1,s}, \tag{39}$$

and using the demand curve $y_{t,s}/y_t = (P_{t,s}/P_t)^{-\gamma}$ gives

$$Q_{t,s} = \frac{\gamma}{\alpha\gamma}(P_{t,s}/P_t)^{-(1+\varepsilon)\gamma/\alpha} y_t^{(1+\varepsilon)/\alpha} - \frac{\gamma - 1}{\gamma} y_t (P_{t,s}/P_t)^{-(\gamma-1)} c_{t,s}^{-\sigma_1} + \beta E_{t,s} Q_{t+1,s}.$$

Defining

$$x_{t,s} \equiv \frac{\gamma}{\alpha\gamma}(P_{t,s}/P_t)^{-(1+\varepsilon)\gamma/\alpha} Y_t^{(1+\varepsilon)/\alpha} - \frac{\gamma - 1}{\gamma} y_t (P_{t,s}/P_t)^{-(\gamma-1)} c_{t,s}^{-\sigma_1}$$

and iterating the Euler equation[45] yields

$$Q_{t,s} = x_{t,s} + \sum_{j=1}^{\infty} \beta^j E_{t,s} x_{t+j,s}, \tag{40}$$

provided that the transversality condition

$$\beta^j E_{t,s} x_{t+j,s} \to 0 \text{ as } j \to \infty \tag{41}$$

holds. It can be shown that the condition (41) is an implication of the necessary transversality condition for optimal price setting.[46]

The variable $x_{t+j,s}$ is a mixture of aggregate variables and the agent's own future decisions. Thus it provides only a "conditional decision rule".[47] This equation for $Q_{t,s}$ can be the basis for decision-making as follows. So far we have only used the agent's price-setting Euler equation and the above limiting condition (41). We now make some further adaptive learning assumptions.

First, agents are assumed to have point expectations, so that their decisions depend only on the mean of their subjective forecasts. Second, we assume that agents have learned from experience that in fact, in temporary equilibrium, it is always the case that $P_{t,s}/P_t = 1$. Therefore we

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[45] Thus it is assumed that expectations satisfy the law of iterated expectations.

[46] For further details see Benhabib, Evans, and Honkapohja (2014).

[47] Conditional demand and supply functions are well known concepts in microeconomic theory.

assume that agents impose this in their forecasts in (40), i.e. they set $(P_{t+j,s}/P_{t+j})^e = 1$. Third, agents have learned from experience that in fact, in temporary equilibrium, it is always the case that $c_{t,s} = y_t - g_t$ in per capita terms. Therefore, agents impose in their forecasts that $c^e_{t+j,s} = y^e_{t,t+j} - g^e_{t,t+j}$, where $g^e_{t,t+j} = \bar{g} + \rho^j \hat{g}_t$. In the case of constant fiscal policy this becomes $c^e_{t+j,s} = y^e_{t+j} - \bar{g}$.

We now make use of the representative agent assumption, so that all agents s have the same utility functions, initial money and debt holdings, and prices. We assume also that they make the same forecasts $c^e_{t+1,s}$ $\pi^e_{t+1,s}$, as well as forecasts of other variables that will become relevant below. Under these assumptions all agents make the same decisions at each point in time, so that $h_{t,s} = h_t$, $y_{t,s} = y_t$, $c_{t,s} = c_t$ and $\pi_{t,s} = \pi_t$, and all agents make the same forecasts. For convenience, the utility of consumption and of money is also taken to be logarithmic ($\sigma_1 = \sigma_2 = 1$).

For optimal price setting (40) we get the infinite Phillips curve (7).

A.3 The Consumption Function

To derive the consumption function from (36) we use the flow budget constraint and the NPG condition to obtain an intertemporal budget constraint. First, we define the asset wealth

$$a_t = b_t + m_t$$

as the sum of holdings of real bonds and real money balances and write the flow budget constraint as

$$a_t + c_t = y_t - \tau_t + r_t a_{t-1} + \pi_t^{-1}(1 - R_{t-1}) m_{t-1}, \tag{42}$$

where $r_t = R_{t-1}/\pi_t$. Note that we assume $(P_{jt}/P_t) y_{jt} = y_t$, i.e. the representative agent assumption is being invoked. Iterating (42) forward and imposing

$$\lim_{j \to \infty} (D^e_{t,t+j})^{-1} a^e_{t+j} = 0, \tag{43}$$

where

$$D^e_{t,t+j} = \frac{R_t}{\pi^e_{t+1}} \prod_{i=2}^{j} \frac{R^e_{t+i-1}}{\pi^e_{t+i}}$$

with $r^e_{t+i} = R^e_{t+i-1}/\pi^e_{t+i}$, we obtain the life-time budget constraint of the household

$$0 = r_t a_{t-1} + \Phi_t + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} \Phi^e_{t+j} \tag{44}$$

$$= r_t a_{t-1} + \varphi_t - c_t + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} (\varphi^e_{t+j} - c^e_{t+j}), \tag{45}$$

where

$$\begin{aligned} \Phi^e_{t+j} &= y^e_{t+j} - \Upsilon^e_{t+j} - c^e_{t+j} + (\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1}) m^e_{t+j-1}, \\ \varphi^e_{t+j} &= \Phi^e_{t+j} + c^e_{t+j} = y^e_{t+j} - \Upsilon^e_{t+j} + (\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1}) m^e_{t+j-1}. \end{aligned} \tag{46}$$

Here all expectations are formed in period t, which is indicated in the notation for $D^e_{t,t+j}$ but is omitted from the other expectational variables.

Invoking the relations

$$c^e_{t+j} = c_t \beta^j D^e_{t,t+j}, \tag{47}$$

which is an implication of the consumption Euler equation (36), we obtain

$$c_t (1 - \beta)^{-1} = r_t a_{t-1} + y_t - \Upsilon_t + \pi_t^{-1}(1 - R_{t-1}) m_{t-1} + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} \varphi^e_{t+j}. \tag{48}$$

As we have $\varphi^e_{t+j} = y^e_{t+j} - \Upsilon^e_{t+j} + (\pi^e_{t+j})^{-1}(1 - R^e_{t+j-1}) m^e_{t+j-1}$, the final term in (48) is

$$\sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} (y^e_{t+j} - \Upsilon^e_{t+j}) + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} (\pi^e_{t+j})^{-1} (1 - R^e_{t+j-1}) m^e_{t+j-1}$$

and using (37) we have

$$\begin{aligned} &\sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} (\pi^e_{t+j})^{-1} (1 - R^e_{t+j-1}) m^e_{t+j-1} \\ = &\sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} (\pi^e_{t+j})^{-1} (-\chi\beta R^e_{t+j-1} c^e_{t+j-1}) = -\frac{\chi\beta}{1-\beta} c_t. \end{aligned}$$

We obtain the consumption function

$$c_t \frac{1+\chi\beta}{1-\beta} = r_t b_{t-1} + \frac{m_{t-1}}{\pi_t} + y_t - \Upsilon_t + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1}(y^e_{t+j} - \Upsilon^e_{t+j}).$$

So far it is not assumed that households act in a Ricardian way, i.e. they have not imposed the intertemporal budget constraint (IBC) of the government. To simplify the analysis, we assume that consumers are Ricardian, which allows us to modify the consumption function as in Evans and Honkapohja (2010).[48] From (4) ones has

$$\begin{aligned} b_t + m_t + \Upsilon_t &= \bar{g} + g_t + m_{t-1}\pi_t^{-1} + r_t b_{t-1} \text{ or} \\ b_t &= \Delta_t + r_t b_{t-1} \text{ where} \\ \Delta_t &= \bar{g} + g_t - \Upsilon_t - m_t + m_{t-1}\pi_t^{-1}. \end{aligned}$$

By forward substitution, and assuming

$$\lim_{T\to\infty} D_{t,t+T} b_{t+T} = 0, \tag{49}$$

we get

$$0 = r_t b_{t-1} + \Delta_t + \sum_{j=1}^{\infty} D^{-1}_{t,t+j} \Delta_{t+j}. \tag{50}$$

Note that Δ_{t+j} is the primary government deficit in $t+j$, measured as government purchases less lump-sum taxes and less seigniorage. Under the Ricardian Equivalence assumption, agents at each time t expect this constraint to be satisfied, i.e.

$$\begin{aligned} 0 &= r_t b_{t-1} + \Delta_t + \sum_{j=1}^{\infty} (D^e_{t,t+j})^{-1} \Delta^e_{t+j}, \text{ where} \\ \Delta^e_{t+j} &= \bar{g} + \rho^j g_t - \Upsilon^e_{t+j} - m^e_{t+j} + m^e_{t+j-1}(\pi^e_{t+j})^{-1} \text{ for } j = 1, 2, 3, \ldots . \end{aligned}$$

A Ricardian consumer assumes that (49) holds. His flow budget constraint (42) can be written as:

$$\begin{aligned} b_t &= r_t b_{t-1} + \psi_t, \text{ where} \\ \psi_t &= y_t - \Upsilon_t - m_t - c_t + \pi_t^{-1} m_{t-1}. \end{aligned}$$

[48] Evans, Honkapohja, and Mitra (2012) state the assumptions under which Ricardian Equivalence holds along a path of temporary equilibria with learning if agents have an infinite decision horizon.

The relevant transversality condition is now (49). Iterating forward and using (47) together with (49) yields the consumption function (8).

B Stability Results

Proposition 4 *Assume that $\beta^{-1}\pi^* - 1 < \psi_p$. Under the Wicksellian PLT rule (12), there exists a ZLB-constrained steady state in which $\hat{R} = 1$, $\hat{\pi} = \beta$, and $\hat{y}$ solves the equation*

$$\hat{\pi} = \Pi(Y(\hat{y}; \hat{\pi}; 1; 1); \hat{y}).$$

Proof of Proposition 4: (a) Consider the interest rate rule (12). Imposing $\hat{\pi} = \beta < 1$ implies that $P_t \to 0$ while $\bar{P}_t \to \infty$ (or $\bar{P}$ if $\pi^* = 1$) as $t \to \infty$. It follows that $\bar{R} - 1 + \psi_p[(P_t - \bar{P}_t)/\bar{P}_t] + \psi_y[(y_t - y^*)/y^*] < 0$ for t sufficiently large when $y_t \to \hat{y} < y^*$, so that $R_t = 1$ in the interest rate rule. A unique steady state satisfying (21) is obtained. Thus, $\hat{y}$, $\hat{\pi}$ and $\hat{R}$ constitute a ZLB-constrained steady state. □

The lemma states that, like IT with a Taylor rule, a commonly used formulation of price-level targeting suffers from global indeterminacy as the economy has two steady states under that monetary policy regime. We remark that the sufficient condition $\beta^{-1}\pi^* - 1 < \psi_p$ is not restrictive as for a quarterly calibration used below with $\beta = 0.99$ and $\pi^* = 1.005$ one has $\beta^{-1}\pi^* - 1 = -0.00505$.[49]

We now start to consider dynamics of the economy in these regimes under the hypothesis that agents form expectations of the future using adaptive learning. We remark that in the IT regime, knowledge of the target inflation rate π^* does not add to guidance in expectations formation as π^* is a constant and forecasting the gap between actual π and π^* is equivalent to forecasting future π. In contrast, the PLT regime can include different amounts of guidance as discussed in Section 2.2.2.

The first step in the analysis is to consider local stability or instability of the steady states. We begin with the IT regime. In our model expectations of output, inflation and the interest rate influence their behavior as is evident from equations (14) and (15). Then agents' expectations are given by equations (18)-(20) in accordance with steady-state learning. The local

[49] For PLT a weaker sufficient condition is $\beta^{-1}\pi^* - 1 - \psi_p + \psi_y(\hat{y}/y^* - 1) < 0$, in which the term $\hat{y}/y^*$ is complicated function of all model parameters.

stability conditions under learning for the IT regime (10) are given by the well-known Taylor principle for various versions of the model and formulations of learning.[50]

We derive expectational stability and instability results for the steady states for IT and PLT without guidance. Some of the results rely on the E-stability method discussed in Evans and Honkapohja (2001) while other results are based on the direct analysis of system (53) and (54).[51]

B.1 Stability Results for the IT Regime

Under IT the temporary equilibrium system is (14), (15), and (10). In an abstract form

$$F(x_t, x_t^e, x_{t-1}) = 0, \tag{51}$$

where the vector x_t contains the dynamic variables. The vector of state variables is $x_t = (y_t, \pi_t, R_t)^T$. The learning rules (18)-(20) can be written in vector form as

$$x_t^e = (1-\omega)x_{t-1}^e + \omega x_{t-1}. \tag{52}$$

We first consider local stability properties of steady states under the rule (10). Linearizing around a steady state we obtain the system

$$x_t = (-DF_x)^{-1}(DF_{x^e}x_t^e + DF_{x_{-1}}x_{t-1}) \equiv Mx_t^e + Nx_{t-1}, \tag{53}$$

where for brevity we use the unchanged notation for the deviations from the steady state. Recall that x_t^e refers to the expected future values of x_t and not the current one. Combining (53) and (52) we get the system

$$\begin{pmatrix} x_t \\ x_t^e \end{pmatrix} = \begin{pmatrix} N+\omega M & (1-\omega)M \\ \omega I & (1-\omega)I \end{pmatrix} \begin{pmatrix} x_{t-1} \\ x_{t-1}^e \end{pmatrix}. \tag{54}$$

We are interested in "small gain" results, i.e. stability obtains for all ω sufficiently close to zero.

Definition. The steady state is said to be **expectationally stable** or **(locally) stable under learning** if it is a locally stable fixed point of the system (53) and (52) for all $0 \le \omega < \bar{\omega}$ for some $\bar{\omega} > 0$.

[50] The seminal paper is Bullard and Mitra (2002) and recent summaries are given in Evans and Honkapohja (2009a) and in Section 2.5 of Evans and Honkapohja (2013).

[51] The stability condition from the definition above and the differential equation approach are identical. Mathematica routines for some computations in the proofs are available upon request.

Conditions for this can be directly obtained by analyzing (54) in a standard way as a system of linear difference equations. Alternatively, so-called expectational (E-stability) techniques based on an associated differential equation in virtual time can be applied, see for example Evans and Honkapohja (2001). Both methods are used in the Appendix in the proofs of the Propositions.

The local stability conditions under learning for the target steady state in the IT regime (10) are given by the well-known Taylor principle:

Proposition 5 *In the limit $\gamma \to 0$ the targeted steady state is expectationally stable if $\psi_\pi > \beta^{-1}$ under IT.*

By continuity of eigenvalues the result implies a corresponding condition for γ sufficiently small. In the text we carry out numerical simulations for other parameter configurations in the different policy regimes. The learning dynamics converge locally to the targeted steady state for π and y for many cases with non-zero value of γ.

Proof of Proposition 5: In the limit $\gamma \to 0$ the coefficient matrices take the form $N = 0$ and

$$M = \begin{pmatrix} \frac{\beta}{\beta - 1} & 0 & 0 \\ \frac{\pi^*(\pi^* y^* + (y^* - \vartheta)\beta^{-2}\psi_y)}{y^*(\vartheta - y^*)(\beta - 1)\beta\psi_\pi} & \frac{1}{(1-\beta)\beta\psi_\pi} & \frac{\beta}{(\beta - 1)\psi_\pi} \\ \frac{\pi^*}{(\vartheta - y^*)(\beta - 1)\beta} & \frac{1}{\beta(1-\beta)} & \frac{\beta}{\beta - 1} \end{pmatrix};$$

so that the system is forward-looking. The equation for y_t has the form

$$y_t = \frac{\beta}{\beta - 1} y_t^e;$$

which is E-stable and does not contribute to possible instability of the remaining 2×2 system for which the coefficient matrix $\bar{M}$ denotes the bottom right corner of M. It is easily verified that the both eigenvalues of matrix $\bar{M} - I$ have negative real parts. Its determinant is

$$Det(\bar{M} - I) = \frac{\beta\psi_\pi - \pi^*}{(1 - \beta)\beta\psi_\pi};$$

so the determinant is positive if and only if $\psi_\pi > \pi^*/\beta > \beta^{-1}$. Its trace is

$$Tr(\bar{M} - I) = -Det(\bar{M} - I) - 1.$$

The result follows. □

For the low steady state we have instability:

Proposition 6 *The ZLB-constrained steady state is not expectationally stable under IT.*

Proof of Proposition 6: When the ZLB binds, the interest rate R_t is constant and R^e_t converges to this value independently of the other equations. Moreover, with R_t constant, X_t has no influence on y_t and π_t. The temporary equilibrium system and learning dynamics then reduce to two variables y_t and π_t together with their expectations. Moreover, no lags of these variables are present, so that the abstract system (53) has only two state variables $x_t = (y_t, \pi_t)^T$ and with $N = 0$ it can be made two dimensional. We analyze this by usual E-stability method.

It can be shown that

$$Det(M - I) = \frac{\hat{y}^{(1+\varepsilon)/\alpha}(1+\varepsilon)^{o}(\bar{g}-\hat{y})^2 + \bar{g}\hat{y}\alpha^2(o-1)}{(\bar{g}-\hat{y})\hat{y}\alpha^2(\beta-1)^2\beta(2\beta-1)\gamma}.$$

The numerator is positive whereas the denominator is negative. Thus, $Det(M - I) < 0$, which implies E-instability (in fact the steady state is saddle path stable as shown in Evans and Honkapohja (2010)). $\square$

For later purposes we illustrate the learning dynamics under the ZLB-constraint (and assuming $R_t = R^e_t = 1$) under a phase diagram. The dynamics are clearly identical under the ZLB constraint and they are illustrated in Figure A.1 using the calibration below.[52] Formally, the dynamics are given by

$$\begin{aligned}
\Delta y^e_t &= \omega\,(Y(y^e_{t-1}, \pi^e_{t-1}, 1, 1) - y^e_{t-1}) \\
\Delta \pi^e_t &= \omega\,(\Pi(Y(y^e_{t-1}, \pi^e_{t-1}, 1, 1), y^e_{t-1}) - \pi^e_{t-1}).
\end{aligned}$$

In Figure A.1 the vertical isocline comes from the equation $\Delta y^e_t = 0$ and the downward-sloping curve is from equation $\Delta \pi^e_t = 0$. It is seen that in the ZLB region, which is south-west part of the state space bound by the isoclines $\Delta \pi^e_t = 0$ and $\Delta y^e_t = 0$ (shown by the two curves in the figure), the dynamics imply a deflation trap, i.e. expectations of inflation and output slowly decline under unchanged policies.

FIGURE A.1 HERE

[52] Mathematica routines for the numerical analysis and for technical derivations in the theoretical proofs are available upon request from the authors.

B.2 Price-Level Targeting with Opacity

To analyze learning dynamics under PLT the vector of state variables needs to augmented in view of the interest rate rule. Recall the variable defined in (22) which makes it possible to analyze also the situation where the actual price level is explosive. One also has (23) and the state variables are $x_t = (y_t, \pi_t, R_t, X_t)^T$ in system (51)-(52).

We start with the local stability result for PLT when there is no guidance. The system under PLT without guidance consists of equations (14), (15), (12) and (23), together with the adjustment of output, inflation and interest rate expectations given by (18), (19) and (20). Theoretical learning stability conditions for the PLT regime are available in the limiting case $\gamma \to 0$ of small price adjustment costs.[53]

Proposition 7 *Assume $\gamma \to 0$ and that agents' inflation forecast is given by (19). If $\psi_p > 0$ under the PLT rule (12), the targeted steady state $\pi = \pi^* \geq 1$ and $R = \beta^{-1}\pi^*$ is expectationally stable.*

Proof of Proposition 7: In the limit $\gamma \to 0$ for (53) we have the coefficient matrices

$$M = \begin{pmatrix} \frac{\beta}{\beta-1} & 0 & 0 & 0 \\ \frac{\pi^*(\pi^* y^* + (y^* - \vartheta)\beta^{-2}\psi_y)}{y^*(\vartheta - y^*)(\beta-1)\beta\psi_p} & \frac{\pi^*}{(1-\beta)\beta\psi_p} & \frac{\pi^*\beta}{(\beta-1)\psi_p} & 0 \\ \frac{\pi^*}{(\vartheta - y^*)(\beta-1)\beta} & \frac{1}{\beta(1-\beta)} & \frac{\beta}{\beta-1} & 0 \\ \frac{\pi^* y^* + (y^* - \vartheta)\beta^{-2}\psi_y}{y^*(\vartheta - y^*)(\beta-1)\beta\psi_p} & \frac{1}{(1-\beta)\beta\psi_p} & \frac{\beta}{(\beta-1)\psi_p} & 0 \end{pmatrix}, \quad N = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\pi^* \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

It is seen that in the limit $\gamma \to 0$ the equation for y_t is simply

$$y_t = \frac{\beta}{\beta - 1} y_t^e,$$

so that the movement of y_t under learning influences other variables but not *vice versa*. With learning rule (18) there is convergence to the steady state when ω is sufficiently small.

[53] Preston (2008) discusses local learnability of the targeted steady state with IH learning when the central bank employs PLT. In the earlier literature Evans and Honkapohja (2006) and Evans and Honkapohja (2013) consider E-stability of the targeted steady state under Eular equation learning for versions of PLT.

We can eliminate the sub-system for y_t and y^e_t from (54). We can also eliminate the equation for expectations of X_t since they do not appear in the system. This makes the system five-dimensional. Computing the characteristic polynomial it can be seen that it two roots equal to 0 and one root equal to $1 - \omega$. The roots of the remaining quadratic equation, written symbolically as $\lambda^2 + a_1\lambda + a_0 = 0;$ are inside the unit circle provided that

$$SC0 = 1 - |a_0| > 0;$$
$$SC1 = 1 + a_0 - |a_1| > 0.$$

It can be computed that $a_0 = -\pi^* \omega/[(\beta - 1)\beta\psi_p]$ and so $SC0 > 0$ for sufficiently small $\omega > 0$. For the second condition, it turns out that $SC1 = 0$ when $\omega = 0$ and $\partial SC1/\partial\omega = 1/(1 - \beta)$, which is positive. $\square$

Proposition 8 *The ZLB-constrained steady state under the Wicksellian PLT rule (12) is not expectationally stable*

This follows because under ZLB constraint the dynamics for IT and PLT without credibility are identical in view of the form of interest rate rules.

C Further Issues

C.1 Intuition for Robustness of PLT with Full Credibility

The simulations below are formally specified in terms of the system incorporating (55) that describes the evolution of inflation expectations π^e_t (and not X^e_t). The system for π^e_t is equivalent to that specified above using (25) and (27). This new system helps to understand the surprising result.

Equation (55) is obtained by noting that the dynamics of X^e_t translate into dynamics of π^e_t taking the form

$$\pi^e_t = \pi^e_{t-1}(\pi^*/\Upsilon(y^e_{t-1}, \pi^e_{t-1}, R_{t-1}, R^e_{t-1}))(1 - \omega) + \omega\pi^*; \qquad (55)$$

where $\Upsilon(y^e_{t-1}, \pi^e_{t-1}, R_{t-1}, R^e_{t-1}) = \Pi(Y(y^e_{t-1}, \pi^e_{t-1}, R_{t-1}, R^e_{t-1}), y^e_{t-1})$ by (15). (55) results from combining (25) and (24) and assuming that $\omega_1 = 1$.

To interpret the dynamics in the ZLB region, we first note that identity (23) can be written as $X_t/X_{t-1} = \pi_t/\pi^*$, so that the price gap variable X_t decreases whenever inflation is below the target value. In the region where ZLB is binding (and $R_t = R_t^e = 1$ imposed) the price gap $\pi^*/\Upsilon(y_{t-1}^e, \pi_{t-1}^e, 1, 1) = \pi^*/\pi_{t-1}$ widens (i.e. X_t declines) and the gap term raises inflation expectations, *ceteris paribus*. The dynamics of π_t^e and y_t^e for the deflation region resulting from equations (55) and (18) with $R_t = R_t^e = 1$ are illustrated in Figure A.2 by means of a phase diagram. In the figure the vertical line is again obtained from equation $\Delta y_t^e = 0$ and the downward-sloping curve from equation $\Delta \pi_t^e = 0$. (Recall that derivation of (55) assumes that X_t and X_t^e are not zero, so that the intersection of the isoclines in Figure A.2 is undefined.)

FIGURE A.2 HERE

Figure A.2 shows that guidance from PLT path leads to increasing inflation expectations in the constrained region defined by the intersections of area to the left of isocline $\Delta y_t^e = 0$ and area below the isocline $\Delta \pi_t^e = 0$. This adjustment eventually takes the economy out of the constrained region. Eventually the interest rate and its expectations also start to move away from the ZLB and there is convergence toward the targeted steady state.

This effect is absent from the dynamics for π_t^e under opacity, as inflation expectations then evolve according to (19). Recall Figure A.1 showing the deflation trap dynamics of π_t^e and y_t^e in the constrained region when agents do not incorporate the target price level path into their expectations formation. The contrast is very evident by comparing Figure A.2 to Figure A.1.

C.2 Intuition for Dynamics with Limited Credibility

We now develop the intuition for the result stated in Section 6.2 that the economy can converge to the target steady state with even small amount of initial credibility. We consider an example where the economy starts from initial conditions a little bit above the low steady state. The basic parameters are set at usual values specified earlier. The initial conditions are $y_0^e = y_0 = y_L + 0.00005$, $\pi_0^e = \pi_0 = \pi_{C,0}^e = \pi_{N,0}^e = \pi_L + 0.001$, $R_0 = R_0^e = 1.0001$, $X_0^e = X_0 = 1$ and $q_0^C = 0$. The last equality means that the initial weight for forecasting with use of target price level has zero weight.

If the dynamics starts with π_0 and π_0^e a little bit above π_L, there is an increase in π_t and an increase in π_t^e. This is in part because for $t = 1$, $\pi_{C,t}^e$ increases in view of the relation $X_{t-1}\pi_{C,t}^e = (X_t^e \times \pi^*)$ as the weight q_t^C becomes initially positive.[54] There is also an increase in y_t but y_t^e remains initially unchanged before it begins to rise. The increase in actual inflation π_t leads to an increase in $\pi_{N,t}^e$ because for the latter actual data point π_t is higher than earlier value of $\pi_{N,0}^e$. This is a crucial observation: the mechanism via an increase of $\pi_{C,t}^e$ to an increase in inflation that is above the statistical forecast $\pi_{N,0}^e$ raises the expectations $\pi_{N,t}^e$ as well. This is in contrast to the dynamics when the agents solely rely on statistical forecasting (as then there is no $\pi_{C,t}^e$ variable).

These movements are illustrated in Figures A.3-A.5. The two panels of Figure A.3 show, respectively, the movements of $\pi_{C,t}^e$ and $\pi_{N,t}^e$ for the first 10 periods. It is seen that $\pi_{C,t}^e$ increases quite strongly while the rise in $\pi_{N,t}^e$ is very gradual. The two panels of Figures A.4 show the movement of average inflation expectation π_t^e (computed from (33) with the weights q_t^C) and the time path of q_t^C (for a long time period $t = 120$). It can be seen after the initial rise the weight $q_{C,t}$ falls for over 40 periods as the forecast $\pi_{N,t}^e$ is more accurate than $\pi_{C,t}^e$ (the longer term movement of $q_{C,t}$ is commented later).

FIGURES A.3 - A.5 HERE

The slow monotonic increase in $\pi_{N,t}^e$ persists while initially the movements of $\pi_{C,t}^e$ are not monotonic, and initially this leads to fluctuating dynamics of the average inflation expectations π_t^e and output expectations y_t^e. In the longer run the economy moves toward the target steady state. This is illustrated in Figure A.5. The left and right panels show the paths of π_t and y_t, respectively. In the longer run and after some fluctuations the weight q_t^C reaches level 1 and PLT becomes fully credible as shown in the right panel of Figure A.4. As part of this process the economy enters the domain of attraction of PLT without guidance shown in Figure 1. In this region both $\pi_{C,t}^e$ and $\pi_{N,t}^e$ have tendency to converge. Both forecasting models are correctly specified in equilibrium but forecasts based on guidance from PLT have smaller forecast errors as $q_t^C = 1$ for large t.

[54] The early increase in $\pi_{C,t}^e$ is due to setting the initial impulse ϑ_0^C at 0.5. In addition, there is an increase in π_t as π_0 is slightly above π_L. Setting it at $\vartheta_0^C = 0$ would lead a delay in the initial adjustments with no change in the long run outcome.

C.3 Robustness Issues

In this section we consider some robustness exercises with respect to the values of the gain parameters, in particular ω_1 for which there is no earlier analyses in the learning literature. We also consider some variation for the value of ω. We also comment on the question of non-negative utility in reinforcement learning.

We start by checking the performance of combination forecasting near the targeted steady state. We set $\omega = 0.002$ which corresponds to the value used in the main text. It turns out that the results are not sensitive to the value of the gain parameter ω_1. The weight q_t^C will eventually converge to 1, though in transition $q_t^C = 0$ for a significant period of time even after the economy has approximately converged to the targeted steady state up to high degree of numerical precision in terms of key macro variables π_t, y_t. However, the forecast error $\left|\pi_t - \pi^e_{C,t-1}\right|$ eventually becomes slightly smaller than $\left|\pi_t - \pi^e_{N,t-1}\right|$ causing the switch $q_t^C \to 1$. Both ways of forecasting are asymptotically correctly specified and are doing a very good job very near the target steady state.

These observations are robust to values $0 < \omega_1 \leq 1$ of the second gain parameter. They are also robust to the initial weights of the two ways of forecasting. The qualitative results seem to be unaffected by the value of λ with even values as low as $\lambda = 0.01.$ Note that realized utility is positive for all these replications i.e. utility is never negative which is what reinforcement learning requires. For higher value $\omega = 0.005$ we have similar results about convergence.

Next, we assess the performance of combination forecasting near the low steady state. We continue to assume $\lambda = 0.85.$ If $\omega = 0.002$, then we have convergence $q_t^C \to 1$ for all values of $\omega_1 \in (0.003, 1].$ The transition again involves convergence to $q_t^C = 0$ in transition while the macro variables are converging to the target steady state. Eventually the system begins to converge to $q_t^C = 1$. The qualitative results seem to be unaffected by the value of λ as we tried $\lambda = 0.01$ too. Realized utility is (again) positive for all these replications.

For $\omega = 0.005$ there is convergence $q_t^C \to 1$ when $\omega_1 \geq 0.007$. When $\lambda = 0.01$ the qualitative results seem to be unaffected when $\omega_1 \geq 0.3$. The realized utility maybe negative in some cases with the baseline utility function. To conform with reinforcement learning, we modify the utility function by adding a large constant 15 (note that this does not affect agent behavior).

This makes realized utility positive in all cases.

We have also analyzed robustness of the results with respect to a reformulation of the learning rule (26) when $\omega_1 < 1$. One could argue that agents update ${}_tX_t^e$ using its previous forecast ${}_{t-1}X_{t-1}^e$ and the most recent available data point X_{t-1}. For $\omega = 0.002$ the numerical results about convergence to the target steady state are unaffected by this change. But the outcome is sensitive to the value of ω. In the case of dynamics near the high steady state and a higher value $\omega = 0.005$ we have convergence $q_t^C \to 1$ when ω_1 equal 1 or 0.9, but convergence to $q_t^C \to 0$ for $\omega_1 = 0.8, \ldots, 0.5.$ (for smaller values of ω_1 the system diverges.) For dynamics near the low steady state we get that for higher value $\omega = 0.005$ there is convergence $q_t^C \to 1$ when $\omega_1 = 1, 0.9$ or 0.8. For values $\omega_1 \leq 0.7$ the system diverges.

We now examine further how much initial credibility is required for PLT and its guidance to achieve convergence of the economy to the target steady state in the long run. The economy has very low initial condition ($y_{-1} = y_0 = y_0^e = y_L + 0.00005$, $\pi_0 = \pi_0^e = \pi_{C,0}^e = \pi_{N,0}^e = \pi_L - 0.0001$, $R_0 = R_0^e = 1.0001$, $X_0^e = 0.1$, $X_0 = X_0^e - 0.0001$) and we shut off the PLT with guidance completely by setting initial conditions $\theta_0^C = u_0^C = 0$, setting $\lambda = 0$ in (28) and $u_t^C = 0$ for all t. This last condition means that private agents do not do any forecast combination and in particular ignore comparisons of forecast errors as in (30). In this case the economy diverges (with realized utility positive for all t even with no additive constant in utility function).

If the initial conditions are modified so that there is either some initial credibility but no updating of the weight on $\pi_{C,t}^e$ ($\theta_0^C = 0$ and $u_0^C = 0.5$ but $\lambda = 0$) or there is no initial credibility and very slow updating of the weight on $\pi_{C,t}^e$ ($\theta_0^C = 0; u_0^C = 0$ and $\lambda = 0.001$), then the preceding divergence result is overturned. The economy converges to the target steady state. In the latter two examples any positive weight $q_t^C > 0$ comes purely or mostly as an impulse from the forecast comparison in a single period. (The preceding appendix discusses the intuition for this outcome.) Convergence takes place with the baseline gain $\omega = 0.002$ and for all $\omega_1 \geq 0.003$ (and even with $\lambda = 0.01$). Realized utility is also positive in all cases. With the gain $\omega = 0.005$ and for all $\omega_1 \geq 0.3$ with $\lambda = 0.01$ (and also even smaller $\lambda = 0.001$) there is convergence too. Realized utility occasionally becomes negative but adding the constant of 15 in the utility function gives the same qualitative results.

C.4 Setting the Initial Target Price: further cases

Here we report the two other cases mentioned in Section 6.3. As in that section, the robustness property considered is the volatility i.e. the magnitude of fluctuations during the adjustment path, *LOSS*, *Utility* along with the frequency of the interest rate R to hit the ZLB (*ZLB*) and the frequency of entering deflation (*Def*). Explanations for the various columns are as in Section 6.3.

Case (iii): initial inflation and output expectations between the low steady state and targeted steady state and interest rate is at ZLB.

	$var(\pi)$	$var(y)$	$var(R)$	*LOSS*	*Utility*	*ZLB*	*Def*
$X_0 = 0.96$	2.53793	1.35535	12.4	4.46	304.1	0.228	1.338
$X_0 = 1.04$	1.03	1.057	38.2	5.38	288.2	0.298	0.922

Table A.1: Volatility of inflation, output and interest rate for PLT with different values of X_0. The final two columns show the frequency of R to hit the ZLB and the frequency of deflation.
Note: the numbers for $var(\pi); var(y)$, $var(R)$ and *LOSS* should be multiplied by $10^{-6}.$

FIGURE A.6 ABOUT HERE

It is seen that the results in terms of different indicators are now more mixed in comparison to case (i) in Section 6.3. For the mean dynamics a relatively low initial target price , i.e., $X_0 = 1.04$ results in a sharp recession which does not appear when $X_0 = 0.96$. Figure A.6 shows the dynamic paths generated in the simulations for Table A.1 from the simulations used for Table A.1. The dynamic paths tend to show more volatility when $X_0 = 0.96$.

Case (iv): initial inflation and output expectations are around the targeted steady state and interest rate is at targeted steady state.[55]

[55] The initial values in the grid are distributed symmetrically around the high steady state.

	$var(\pi)$	$var(y)$	$var(R)$	$LOSS$	$Utility$	ZLB	Def
$X_0 = 0.96$	0.183554	0.117685	1.55562	0.397958	319.41	0.001	0
$X_0 = 1.04$	0.167092	0.101464	1.68069	0.385893	315.72	0	0.28

Table A.2: Volatility of inflation, output and interest rate for PLT with different values of X_0. The final two columns show the frequency of R to hit the ZLB and the frequency of deflation.
Note: the numbers for $var(\pi); var(y)$, $var(R)$ and $LOSS$ should be multiplied by 10^{-6}.

FIGURE A.7 ABOUT HERE

In this case the comparison of volatility results for different values of X_0 yields mixed results and the differences in the values of the indicators are close to each other. However, the dynamic paths show that a relatively low value of X_0 results in an initial boom whereas a high value for X_0 leads to an initial recession. (Data is from the simulations used for Table A.2.) Thus, a low value of X_0 avoids the initial recession.

C.5 Transparency of Monetary Policy Rule

We next consider the role of transparency about the policy instrument rule. The model is as summarized in Section 3, except that in the aggregate demand equation (14) private agents now know the interest rate rule. Under IT the interest rate R_t is given by (10) and expectations R^e_t are

$$R^e_t = 1 + \max[\bar{R} - 1 + \psi_\pi(\pi^e_t - \pi^*) + \psi_y[(y^e_t - y^*)/y^*]; 0].$$

Likewise, under PLT and transparency the interest rate rule is (12) and expectations are given by

$$R^e_t = 1 + \max[\bar{R} - 1 + \psi_p(X^e_t - 1) + \psi_y(relY^e_t - 1); 0];$$

where $X^e_t \equiv (P_{t+1}/\bar{P}_{t+1})^e$ and $relY^e_t \equiv (y_{t+1}/y^*)^e$ are the forecasts X^e_t and $relY^e_t$ that agents need to make. The evolution of expectations is assumed to be given by (25) for X^e_t and

$$relY^e_t = relY^e_{t-1} + \omega(relY_{t-1} - relY^e_{t-1})$$

for $relY^e_t$.

FIGURES A.8 ABOUT HERE

We focus on comparisons of PLT with guidance and limited credibility under opacity and transparency. The qualitative results are preserved under transparency to corresponding results under opacity. For example, higher initial credibility is conducive to convergence to the target equilibrium (see top panel of Figure 3 for the opacity case) and the domain of attraction is smaller when initial output expectations are lower (see other panels of Figure 3 for the opacity case).

The results corresponding to the top panel of Figure 4 and of Figures 5A-5C under opacity are qualitatively similar to those under transparency. It also emerges that the comparison between opacity and transparency depends on the current state of the economy. In general, opacity is better than transparency in the sense that the domain of attraction is larger under opacity when current state of the economy (in terms of y_0^e) is at recession or at most target level of activity. In a boom situation, the comparison goes the other way. Interestingly, transparency then dominates opacity at all degrees of initial credibility. These latter results are illustrated in Figures A.8 A-C which should be compared to Figures 5A-C.

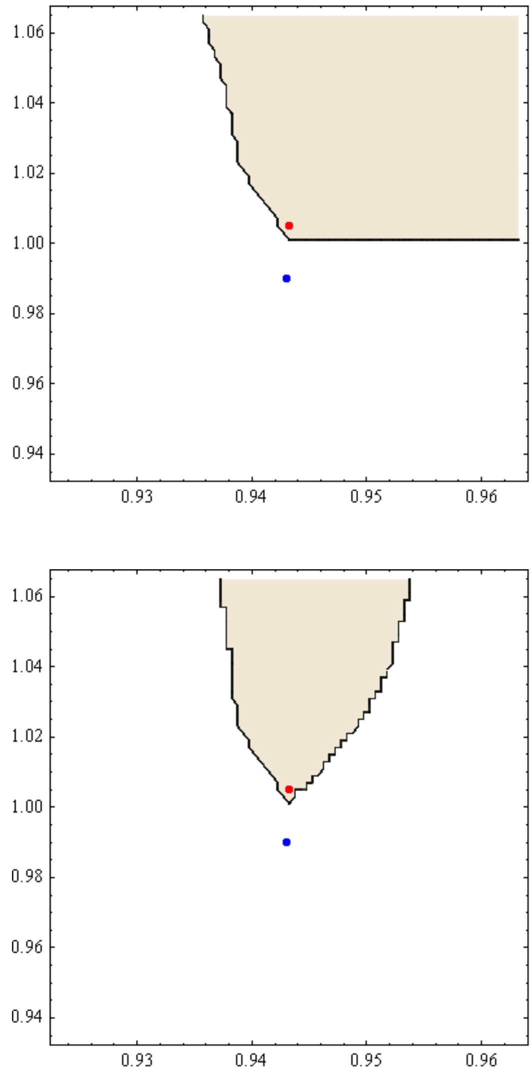

Figure 1: Domain of attraction for IT (top panel) and that for PLT without guidance (bottom panel). Horizontal axis gives y_0^e and vertical axis ¼$_0^e$. Shaded area indicates convergence. The circle in the shaded region denotes the intended steady state and the circle outside the shaded region denotes the unintended steady state.

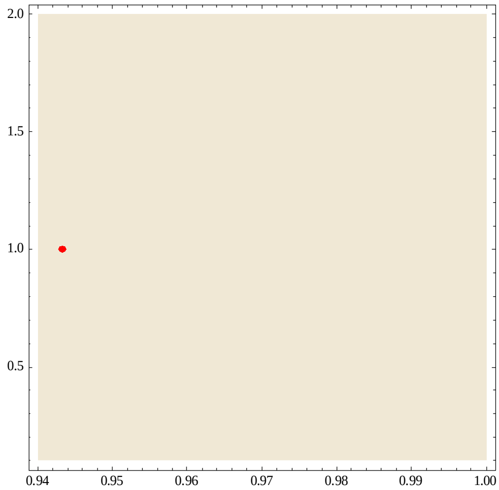

Figure 2: Domain of attraction for PLT with forecasting of gaps when initial conditions are close to the low steady state. Horizontal axis gives y_0^e and vertical axis X_0. The circle represents the targeted steady state. Shaded area indicates convergence with convergence criterion same as in Figure 1.

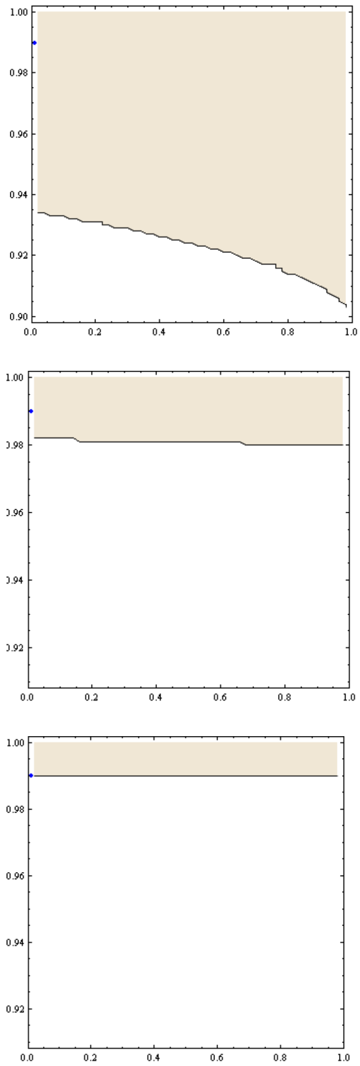

Figure 3: Domain of attraction for PLT with imperfect credibility corresponding to di¤erent levels of initial output expectations: Di¤erent degrees of initial credibility q_0^C are along the horizontal axis and inflation expectations are along the vertical axis. Output expectations are fixed just above the target steady state (at 0:943704) in the top panel, at the target steady state in the middle panel and at the low steady state in the bottom panel.

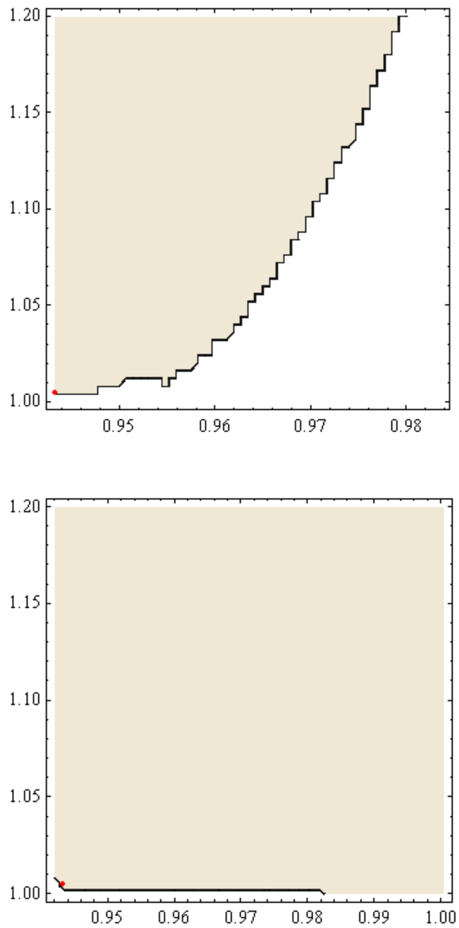

Figure 4: The top panel shows the domain of attraction for PLT with high credibility ($q_0^C = 0.9$) and the bottom panels shows the domain for IT. Output expectations are along the along horizontal axis and inflation expectations along vertical axis. Note that output expectations above 0.98 are not stable with PLT whereas even output expectations as high as one are stable with IT.

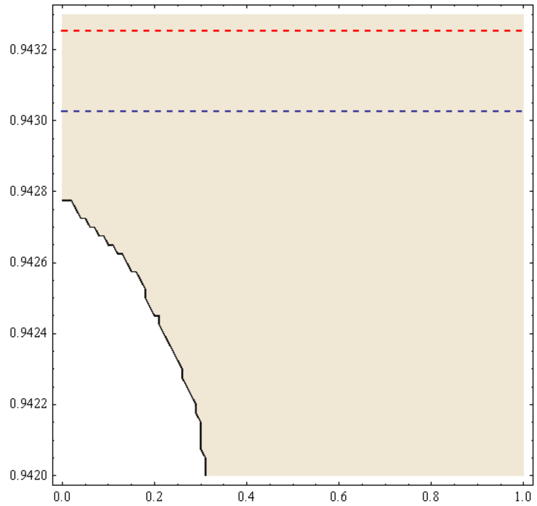

Figure 5A: Domain of attraction with X_0 = 0:95. Credibility along horizontal axis and output expectations along vertical axis.

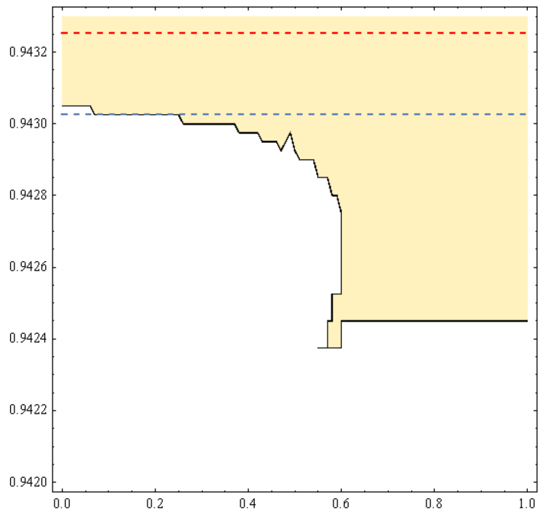

Figure 5B: Domain of attraction with X_0 = 1:05. Credibility along horizontal axis and output expectations along vertical axis.

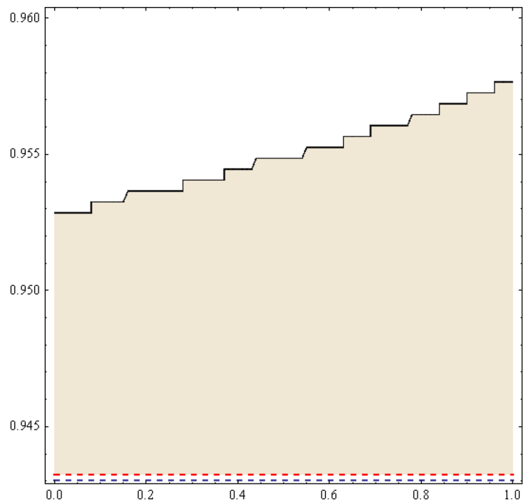

Figure 5C: Domain of attraction with X_0 = 0:95 in a boom like scenario. Credibility along horizontal axis and output expectations along vertical axis.

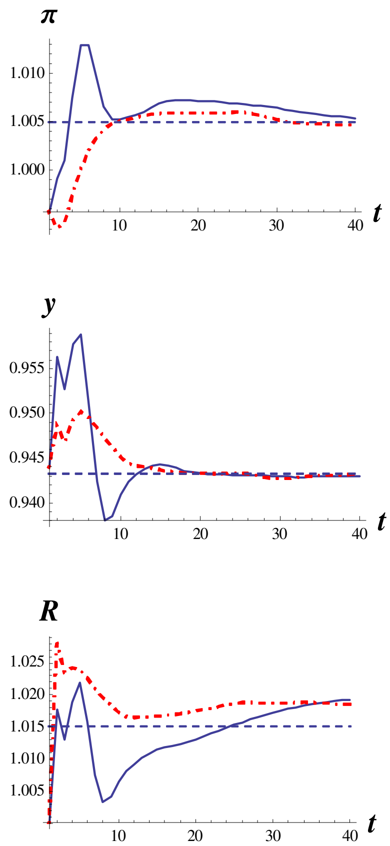

Figure 6: Inflation (top panel), output (middle panel) and interest rate mean dynamics (bottom panel) under PLT for different starting values of X_0 with initial conditions near deflationary steady state. Plots for $X_0 = 0.96$ are in solid line and for $X_0 = 1.04$ are in mixed dashed (red) line.

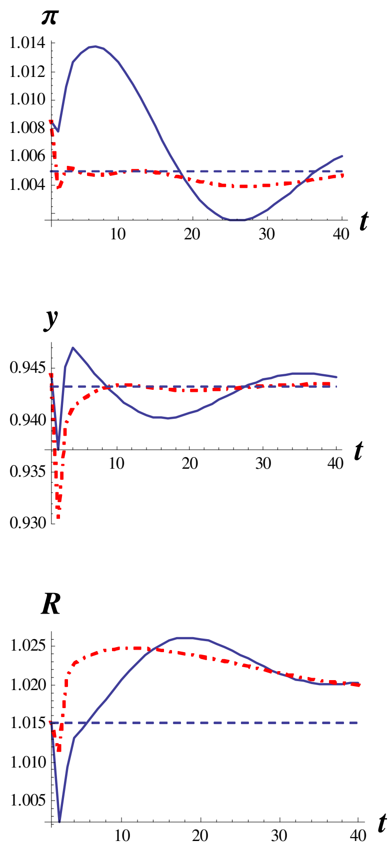

Figure 7: Inflation (top panel), output (middle panel) and interest rate mean dynamics (bottom panel) under PLT for different starting values of X_0 with initial conditions above the targeted steady state to capture a boom like scenario. Plots for $X_0 = 0.96$ are in solid line and for $X_0 = 1.04$ are in mixed dashed (red) line.

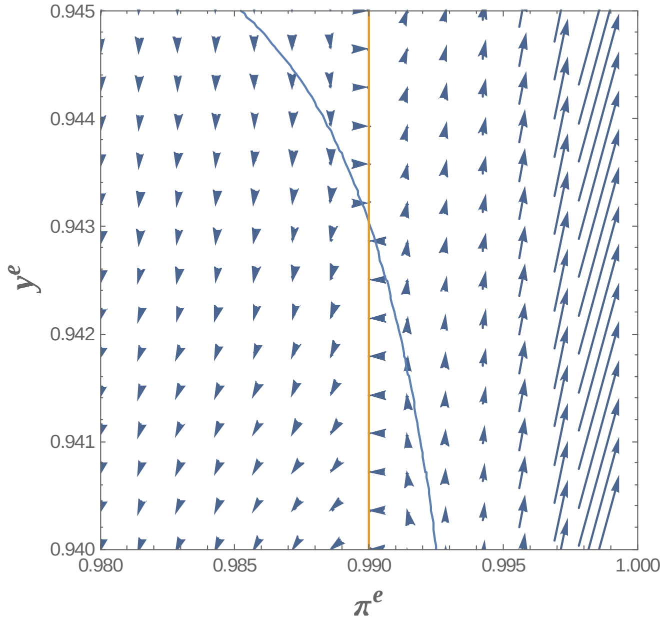

Figure A.1: Dynamics of inflation and output expectations in the constrained region when there is no guidance.

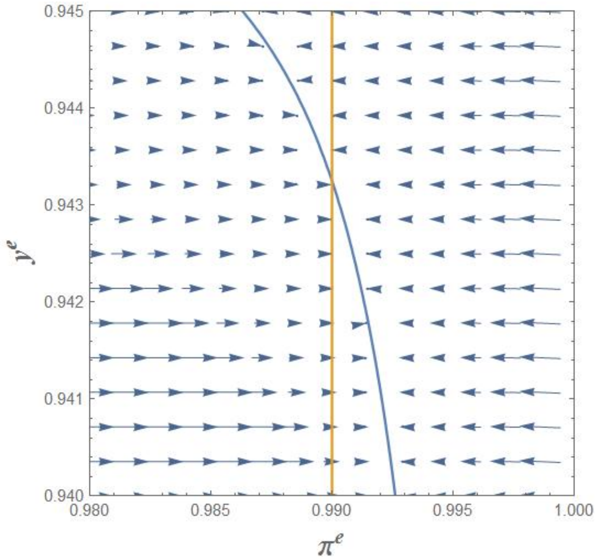

Figure A.2: Dynamics of inflation and output expectations in the constrained region with guidance.

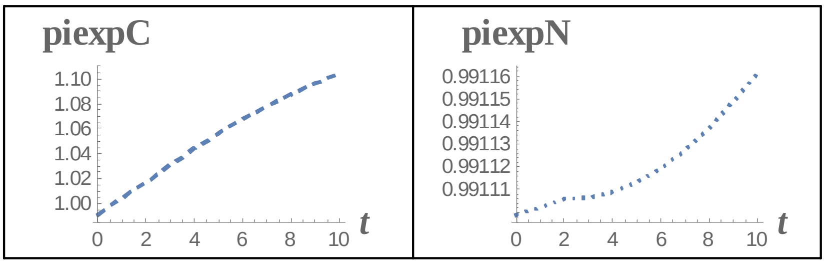

Figure A.3: Inflation forecasts $\pi^e_{C;t}$ and $\pi^e_{N;t}$ with and without guidance from PLT.

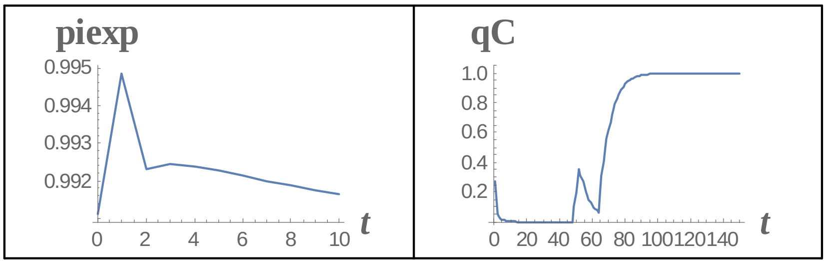

Figure A.4: Average inflation expectations ¼$_t^e$ and the weight $q_{C;t}$ of forecast based guidance.

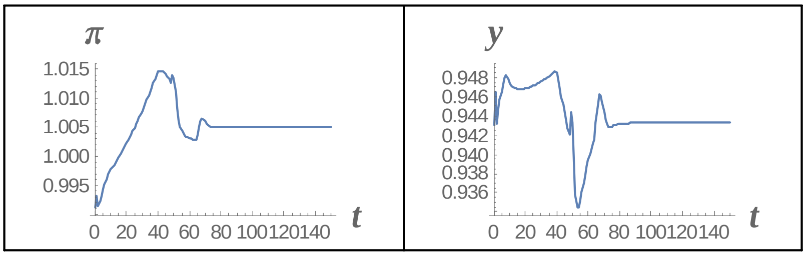

Figure A.5: Convergent dynamics of inflation and output.

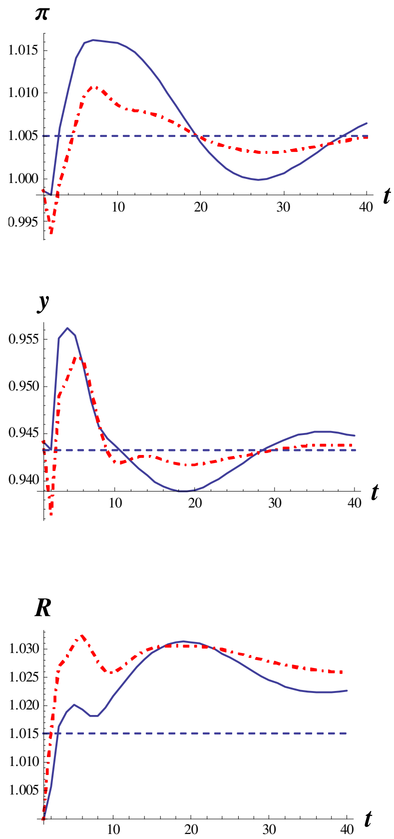

Figure A.6: Inflation (top panel), output (middle panel) and interest rate mean dynamics (bottom panel) under PLT for different starting values of X_0 with initial conditions between the deflationary and targeted steady state. Plots for $X_0 = 0.96$ are in solid line and for $X_0 = 1.04$ are in mixed dashed (red) line.

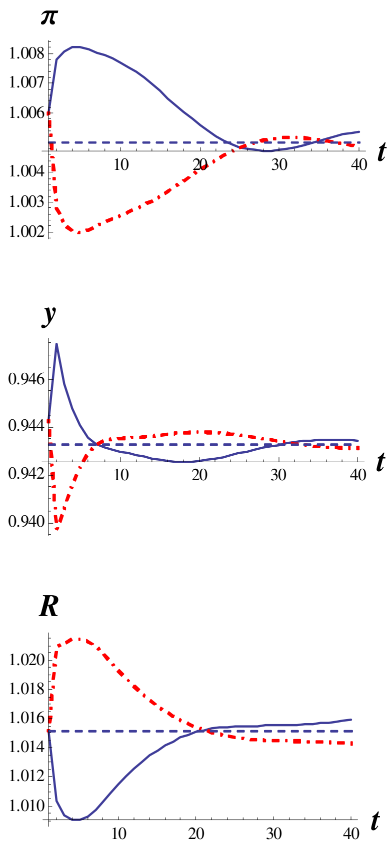

Figure A.7: Inflation (top panel), output (middle panel) and interest rate mean dynamics (bottom panel) under PLT for different starting values of X_0 with initial conditions close to the targeted steady state. Plots for $X_0 = 0.96$ are in solid line and for $X_0 = 1.04$ are in mixed dashed (red) line.

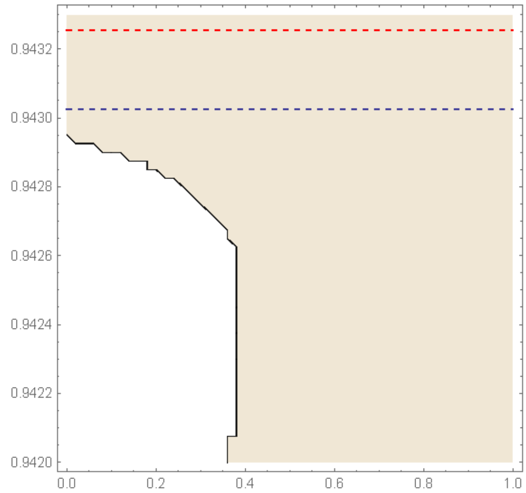

Figure A.8.A: Domain of attraction with X_0 = 0:95 and gain= 0:005 under transparency. Note that the domain is smaller compared to opacity (compare with Figure 5A).

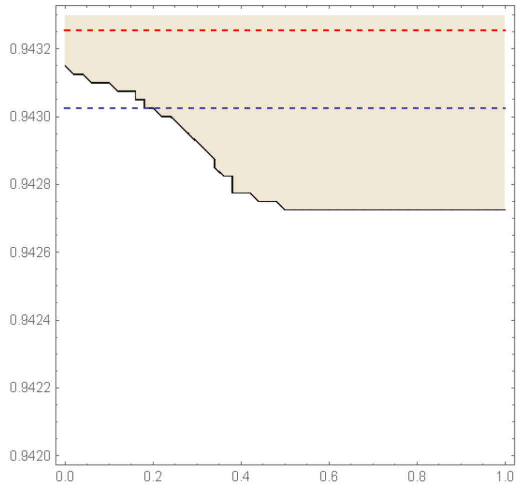

Figure A.8.B: Domain of attraction with X_0 = 1:01 and gain = 0:005 under transparency. When X_0 = 1:05 none of the points in this domain are stable. The domain is smaller compared to opacity (compare with Figure 5B).

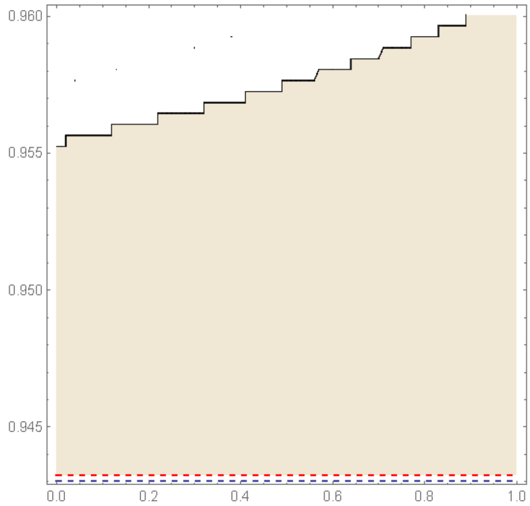

Figure A.8.C: The domain of attraction is larger under transparency than in Figure 5C.