Potential Output Pessimism and Austerity in the European Union*

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Abstract:

The paper documents important evidence on policy makers' perceived evolution of potential output, output gap and structural balance for EU countries since 2004 and builds a model incorporating policy makers' learning about potential output replicating this evidence. The Great Recession led to over-pessimism about potential output which amplified adverse shocks and underestimation of structural balances triggering fiscal austerity. The mutual reinforcement between pessimism and austerity may have contributed to the prolonged recession. Placing less weight to new data (compared to actual EU policy makers) may have reduced real-time mis-measurement of output gap and led to a less severe recession.

JEL classification: E62, D84.

Key words: Structural fiscal balances, learning, debt brake, pessimism, potential output.

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I. Introduction

In the wake of the 2008 Global Financial Crisis, the observed persistent and large declines in aggregate output led to large downward revisions to the estimates of potential output in the Euro Area, the UK and the US by national and international institutions. According to the European Commission (EC), between 2008 and 2011, the estimate of the average growth rate of real potential output over $2008-2011$ for the eurozone as a whole had been reduced from 2% to 1.1%.[1] Some observers express skepticism about the standard methodology for estimating potential output and suspect that estimates of potential output have over-reacted to the declines in output. For instance, Krugman (2013) notes "*the standard methods for estimating economic potential are working very badly in this slump; they are, all too often, causing officials to interpret the slump as 'structural'...*"

Recent research tends to support this evidence: policy makers have been overly pessimistic about potential output following the Great Recession. Coibion, Gorodnichenko, Ulate (2018, henceforth CGU) show that estimates of potential output in the US and other OECD countries made by a wide range of public institutions and private forecasters respond sensitively to shocks which have only transitory effects on GDP. They propose a new method to estimate potential output which suggests a more limited decline in potential output following the Great Recession.[2] This has implications for estimates of output gaps. For example, their estimate of 2017 output gap in the US is close to -10%, while the corresponding estimate made by the US Congressional Budget Office is close to zero.

Estimate of potential output is key for assessing the macroeconomic stance of different countries in real time and put at work in a wealth of monetary and fiscal policy evaluation exercises. In the European Union (EU), structural budget balance plays a central role in the surveillance of fiscal policies of Member States since the 2005 reform of the Stability and Growth Pact. The role of this indicator has increased with the introduction of the EU Fiscal Compact, which entered into force on 1 January 2013. It requires the structural deficit ratio – the budget deficit-to-GDP ratio that would prevail if the economy was at potential – for EU member states to be at most 1% (or 0.5%) if the public debt-to-GDP ratio is at most (or above) 60%; see European Parliament (2017). This is usually called 'debt brake' in policy circles and the media.

The EC uses the estimate of structural balance, computed using the output gap, to recommend if and how much fiscal austerity is needed in a member country. The Great Recession led to a large increase in the estimates of structural deficits made by the EC for almost all EU countries which required them to implement fiscal consolidation programmes to reduce structural deficits. Over-pessimistic estimates of potential output will lead to underestimation of the size of output gaps and structural balances, which may in turn translate into unnecessary fiscal austerity for the member country concerned; see the informal discussions of Wren-Lewis (2013), Cohen-Setton and Valla (2010), Cohen-Setton (2013), Hers and Suyker (2014). In a March 2016 letter to the EC, the ministers of finance of eight EU member states expressed concerns about the implications of mis-measuring potential output on structural balances and called for intensification of the technical work on this issue, see Letter (2016). Hence, policy makers have strong incentive to search for a reliable method to estimate output gap with the view to minimize its real time mis-measurement.

The paper provides a formal model that can capture the essence of commentators' arguments: it analyzes the role for (mis-)measuring potential output and the "debt brake" policy in the EU recession following the financial crisis, incorporating policy makers' uncertainty in estimating potential output in real time. The model also provides recommendations for minimizing real time mis-measurement of output gap and structural balance.

One of the major contributions of the paper is to provide a new set of evidence on policy makers' perceived evolution of potential output, output gaps and structural balances present in the EC data of different vintages covering 2004 to 2016 and show that the model replicates this evidence. We use this dataset because it is used to formulate fiscal policy in the EU. Studying the properties of not just the forecasts but also historical *vintages* of estimates of these unobserved variables and their revisions is particularly important since these estimates crucially determine the real time evolution of fiscal policy and macroeconomic outcomes.[3] We summarize *four* important pieces of evidence below; see Section II. for the details.

Evidence 1: The Great Recession has led the EC to revise massively downward not only the forecasts but also historical estimates of potential output growth rates in the eurozone (and the UK). As an example, from 2008 to 2011, the growth rate of Italian potential output in year 2006 and 2007 had been revised from 1.5% and 1.5% to 0.6% and 0.8%, respectively. Moreover, across the EU countries, the size of the downward revisions are heterogeneous and there is a strong positive correlation between the size of downward revisions and the severity of recession. A country which has a larger (or smaller) recession during $2008-10$ tends to have a larger (or smaller) downward revision to average growth rate of real potential output over a pre-crisis period.

The downward revisions to historical estimates of potential output growth rates have led to upward revisions to estimates of output gaps and downward revisions to estimates of structural fiscal balances for almost all countries during a pre-crisis period. For example, from 2008 to 2011, Italian output gaps of year 2006 and 2007 had been revised from -0.3% and -0.3% to 2.2% and 2.9%; structural balance of year 2006 and 2007 had been revised from -3.3% and -1.7% of GDP to -4.5% and -3.0%.

Evidence 2: Using very recent data vintage (i.e., Autumn 2016 data vintage), we find that for most countries in the sample, the 2012 potential output growth rates formed in 2013 Autumn have been revised upward during the recovery phase, while for the remaining countries they have been revised downward. The heterogeneity in belief revision is strongly positively correlated with heterogeneous economic performance of EU economies during the recovery phase. A country which has higher output growth during $2013-2016$ (relative to a previous period) tends to have a larger upward revision to 2012 potential output growth rates from 2013 to 2016.

Evidence 3: For all countries, we find that surprises to the growth rates of real output are strongly positively correlated with revision to the forecast of the growth rate of real potential output.

Evidence 4: Following Blanchard and Leigh (2013) but using EC data, the paper finds a strong negative cross-country correlation between the forecast of fiscal consolidation and subsequent growth forecast errors during 2010-2011. A country which planned a larger fiscal consolidation subsequently tended to have lower growth than expected.

The evidence above appears incompatible with full information RE models. For example, these models do not produce systematic revisions to the historical estimates of endogenous variables (potential output, output gaps and structural balances). In addition, exisiting business cycle models usually produce constant forecasts of potential output growth rates and hence zero correlation between revisions to these forecasts and surprises in output growth, as opposed to the strong positive correlation observed in the data.

CGU (2018) too provide evidence which we think is incompatible with full information RE models. In particular, they find estimates of potential output for the US and other OECD countries made by policy makers and private forecasters increase (or decrease) *persistently* following an identified positive (or negative) i.i.d total factor productivity (TFP) innovation (and respond sensitively to monetary shocks and government expenditure shocks). Our learning model, in contrast, is able to replicate this evidence, as explained later.

Policy makers typically employ the production function approach and/or statistical filtering methods to form estimates of potential output; see eg. D'Auria et al (2010). These practical methods fail to adequately distinguish the underlying sources of changes in aggregate output. A change in the level of output is partially and incorrectly interpreted as a shift in the trend growth of output. Similarly, a change in output due to a purely cyclical fluctuation is partially and incorrectly attributed to a shift in the trend output; see CGU (2018) for a discussion. We believe this way of forming estimates of potential output is better captured by the spirit of adaptive learning (AL) models rather than RE models (see eg. Evans and Honkapohja (2001) for a treatise on AL, Eusepi and Preston (2011, 2018) and Adam, Marcet and Beutel (2017) as applications). Our model captures these aspects of policy maker behavior.

Policy makers form subjective estimates of potential output, output gaps and structural balances under uncertainty in real time. They make policy decisions (e.g., fiscal policy) based on these estimates, which in turn influences the macro economy. Using new data observations, they continually revise these estimates. There is dynamic interaction between policy maker beliefs, fiscal policy and market outcomes. The learning model is able to capture this interaction.

The dynamic interaction in our AL model is in stark contrast with existing business cycle models, including RE models with full or incomplete information and non-RE models (e.g., adaptive learning models). These models typically imply that agents have exact knowledge of the evolution of trend growth rate of aggregate output, see Kuang and Mitra (2016), sections 6 and 7 for a discussion. As a consequence, there is no feedback from macroeconomic outcomes to policy makers' beliefs about the growth rate of trend output unlike our model. In addition, these models typically do not make use of data on the estimates of important unobserved macro variables (e.g., potential output) of different vintages.

The AL model is shown to replicate the evidence provided in CGU (2018). In response to a positive (or negative) TFP innovation, policy makers use the rising (or declining) output data to revise potential output estimates and the subsequent dynamic interaction between potential output beliefs and equilibrium outcomes helps to produce the persistent response of potential output estimates to a transitory productivity shock.

Our analysis suggests that policy makers over-reacted to sustained declines in aggregate output following the Great Recession with too aggressive downward revisions to future and past trend output. This mechanism is supportive of Krugman's (2013) quote in the first paragraph. As a consequence, estimates of pre-crisis output gaps (structural balances) were revised upward (downward). Pessimistic beliefs feed back to the economy and amplify the decline in aggregate activities which in turn reinforces the pessimism. Pessimism about potential output also leads to underestimation of the size of output gaps and structural fiscal balances which in turn triggers undue fiscal austerity due to structural balance targeting in the EU fiscal policy framework (the actual size of the underestimation is discussed in Section V.B.).[4] This mutual reinforcement between potential output pessimism and undue fiscal austerity contributes significantly to the recession in the model.

Since policy makers have imperfect knowledge, mis-measurement of output gaps (and structural balances) in real time is inevitable. However, policy makers have strong motivation to minimize these mis-measurements since they may otherwise magnify the adverse consequences for the economy. The question arises as to how policy makers may minimize these real-time mis-measurements. In the AL model, the size of bias in estimation of output gap and structural balances depends on the extent to which the policy maker discounts past data when forming beliefs of the trend output (this is captured by what is called *gain* parameter in the AL literature).

The policy makers' gain parameter is calibrated to match the (large) revisions to estimates of potential output growth rates following the Great Recession and hence is relatively large (i.e., 0.034). In practical terms, the magnitude of the gain parameter indicates the weight attached to historical data so a large gain means more weight is attached to recent data. For example, the gain parameter of 0.034 means the weight received by output growth data 20 years ago, is $(1-0.034)^{80} \simeq 0.063$. In contrast, the model suggests the policy makers should place a more modest weight on recent data points: this gain parameter is approximately 0.0045 and the weight received by output growth data 20 years ago is 0.70 (much larger number).

If the gain parameter is large, the bias in measuring output gap tends to be large because the trend output estimates are sensitive to random innovations leading to frequent and substantial deviations from its true value. On the other hand, if the gain parameter is close to zero, policy makers' trend output estimates may remain far from the true value for long periods of time producing a substantial bias. In particular, the model implies that if policy makers had placed less weight to the data during the Great Recession period, this would have led to higher estimates of structural balances, less fiscal austerity and a less severe recession.

The learning model is able to explain the other pieces of evidence discussed earlier. It reproduces the positive relation between the severity of recession and the size of downward revisions to potential growth rates across the EU countries and a positive correlation between surprises in output growth rates and revisions to forecasts of potential output growth rates observed in the data. Interestingly, the AL model also sheds some light to the evidence on the relation between planned fiscal consolidations and subsequent forecast errors of output growth rates across the EU countries.

The paper is related to a literature examining the unreliability of real-time mis-measurement of output gaps and its implications for monetary policy. Orphanides and van Norden (2002) illustrate real-time output gap estimates are unreliable and depend on the filtering methods used. Orphanides (2001, 2003, 2004) argues that the Federal Reserve's mis-measurement of the output gap in the 1970s was one of the primary reasons for the big inflation in 1970s.

The rest of the paper is organized as follows. Section II. discusses properties of policy makers' estimates of potential output, output gaps and structural balances. The model setup is presented in Section III.. Agents' beliefs and learning are introduced in Section IV.. Section V. simulates the Great Recession scenario. Section VI. illustrates the consequences of undue fiscal consolidation. Section VII. examines a way to minimize real-time mis-measurement of output gaps and conducts a counterfactual analysis. Section VIII. concludes.

II. Evidence

This section presents properties of policy makers' real-time estimates which we seek to replicate. The data is taken from the annual macroeconomic database of the EC and covers data vintages from 2004 Autumn to 2016 Autumn. This dateset is used for policy making purposes in the context of the EU fiscal compact and hence is particularly relevant. The dataset has been computed consistently across countries by the EC using a uniform estimation methodology for potential output.

A. Evidence 1: how did the Great Recession change policy makers' view of the past and future?

We document how the Great Recession changed policy makers' view on the evolution of three unobserved variables (potential output, ouput gaps, and structural budget balances) for 13 EU countries. Figure 1 plots the revision to estimate of the growth rate of real potential GDP in $2003-2012$ from 2008 Spring to 2011 Spring. The potential growth rates had been revised downward for almost all countries and all years. Taking Ireland as an example, the estimate of annual potential output growth rate had been revised downward by 3.1%, 5.6%, 4.7%, 4.4%, 3.7% for the year $2008-2012$ and by 0.14%, 0.69%, 1.15%, 1.56%, 2.03%

for the years $2003-2007$.[5] Figure 1 also reveals heterogeneity in the size of downward revision to pre-crisis growth rates of real potential output across countries. For example, the downward revision to average annual growth rate of real potential output in Ireland over $2005-07$ is 1.6%, while the corresponding revision for Germany is 0.1%.

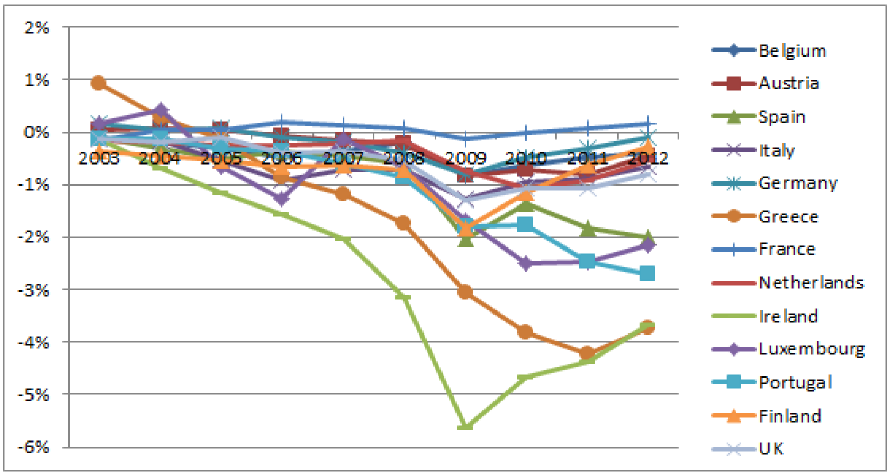

Figure 1: Revision to estimates of potential GDP growth rates: from 2008 Spring to 2011 Spring

Figure 2 shows that the size of the revision to the average growth rate of real potential output of a country during the pre-crisis period (e.g., 2005-07) is strongly positively correlated with the severity of the recession during 2008-10 measured by the average growth rate of real output over 2008-10 minus that over a previous period (i.e., 2003-07); the correlation coefficient is 0.77 and statistically significant at 1% level. A country which has a larger recession during 2008-10 tends to have a larger downward revision to average growth rate of real potential output over the pre-crisis years 2005-07.

Figure 3 plots the revision to 2003-07 output gaps from 2008 Spring to 2011 Spring. For almost all countries and years, the downward revision to 2003-07 potential growth is associated with an upward revision to 2003-07 output gaps. For example, the estimate of 2006 and 2007 Italian output gaps made in 2008 Spring are -0.3% and -0.3%, which suggests policy makers thought the Italian economy had been growing slightly under potential. They were revised up to 2.2%

and 2.9% respectively in 2011 Spring; policy makers then perceive that Italian economy during 2006-2007 was much above the potential level. Consistent with our evidence, in a European Central Bank occasional paper, Anderton et al (2014) find that the Great Recession led to a sign change to estimates of 2007 output gaps for a large number of countries: "*For all countries for which the 2007 output gap was estimated at the time to be negative – Portugal, Cyprus, Italy, Ireland, Malta, Spain, the Netherlands, France and Belgium – it was subsequently, in 2014, estimated to have been positive.*" Bundesbank (2014) warned of the high uncertainty of output gap estimates and expressed doubts on the suitability of such estimates in economic policy.

Figure 4 shows the revision to 2005-2007 estimates of structural balance to GDP ratio. For almost all countries and years, associated with downward revisions to the estimate of potential output and upward revisions to output gaps, the Great Recession has led to downward revision to pre-crisis estimates of structural balance ratio. For example, the estimate of 2005-07 structural balance ratio for Italy made in 2008 Spring are −3.9%, −3.2% and −1.7%. In 2011 Spring, they were revised to −4.7%, −4.5% and −3.0%.

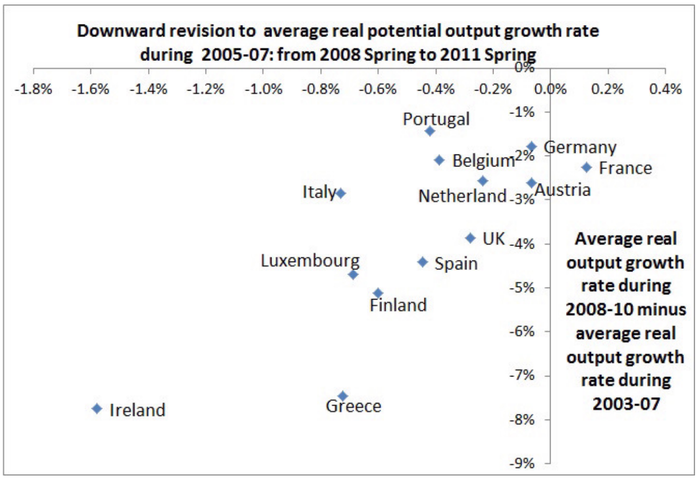

Figure 2: Relation between severity of the recession and revisions to the growth rate of pre-crisis real potential output across countries

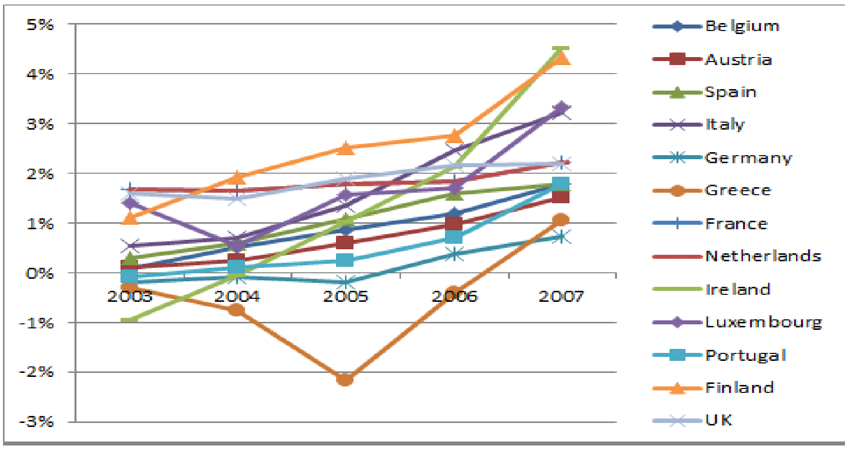

Figure 3: Revision to estimates of pre-crisis (2003-2007) output gap: from 2008 Spring to 2011 Spring

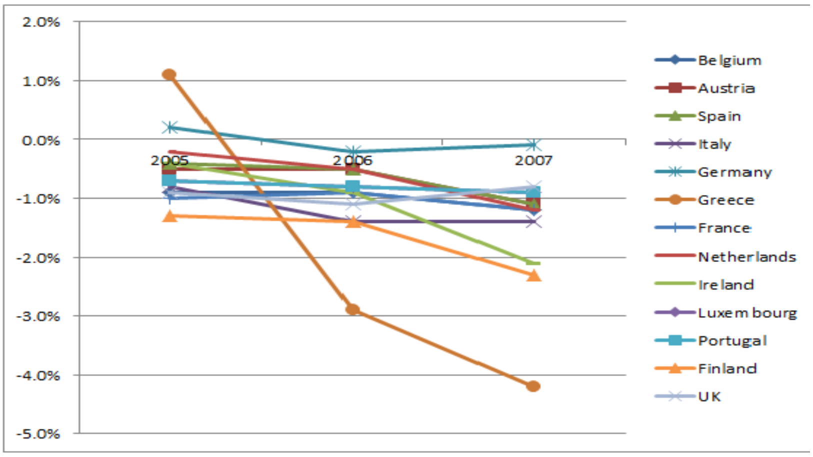

Figure 4: Revision to estimates of pre-crisis (2005-2007) structural budget balance: from 2008 Spring to 2011 Spring

B. Evidence 2: how did policy makers' view evolve during the recovery phase?

In the wake of the financial crisis, the eurozone economy was hit by two waves of recessions and started to recover around 2013. This section examines how policy makers' beliefs about potential output growth rate of different countries in the aftermath of the recessions (i.e., beliefs about 2012 potential output growth rate) evolve during the recovery phase. Figure 5 displays a heterogeneity in the pattern of revisions to 2012 potential growth rates made in 2013 Autumn, comparing to a very recent data vintage (2016 Autumn). For eight countries (Ireland, Greece, Spain, Luxembourg, Portugal, Netherlands, UK and Belgium), the growth rate of 2012 potential output had been revised upward. For Germany, Italy, France, Finland and Austria, the growth rate of potential output had been revised downward.

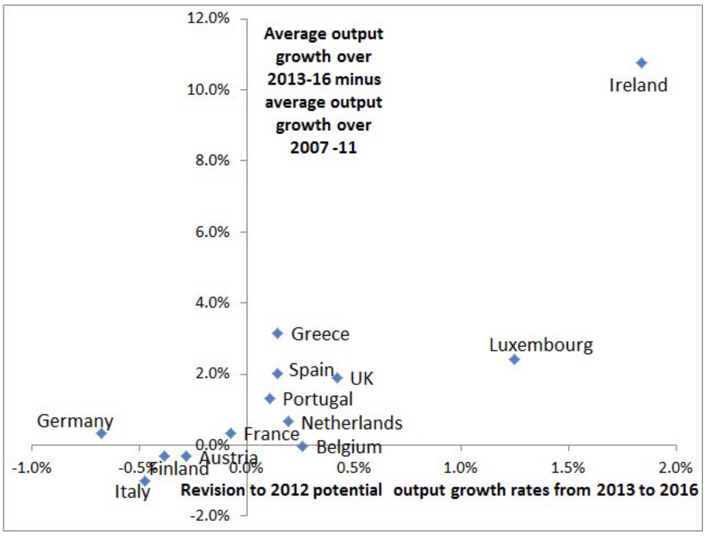

Figure 5: Relation between revision to 2012 potential output growth rates and the pace of recovery from 2013 to 2016.

Interestingly, Figure 5 reveals a strong positive cross-country correlation (i.e., correlation coefficient of 0.85) between revision to 2012 growth rate of potential output and the pace of recovery. The pace of recovery is measured by average GDP growth rate during 2013-16 minus that during 2007-11. When the average growth rate of real GDP of an economy during 2013-16 is higher (or lower) than its average growth rate during 2007-12, this economy tends to have an upward (or downward) revision to 2012 potential output growth rate from 2013 to 2016. Moreover, a country which has a larger increase (or decline) in the average growth rate from the period 2007-12 to the period 2013-16 tends to have a larger upward (or downward) revision to the growth rate of 2012 potential output.

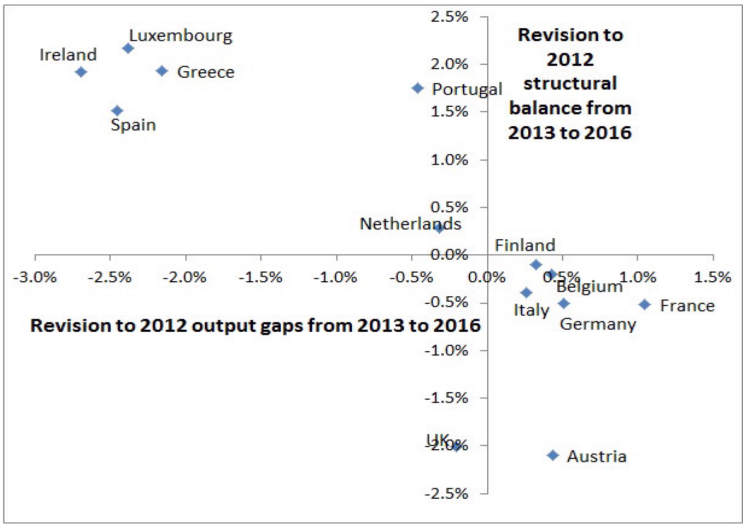

Figure 6: Revision to estimate of 2012 output gaps and structural balance during the recovery: from 2013 Autumn to 2016 Autumn

Figure A.1 in the Appendix displays the revision to the estimate of 2012 potential growth rates and output gaps from 2013 Autumn and 2016 Autumn. A heterogeneity in the pattern of revision appears. Associated with upward (or downward) revision to potential growth rates is downward (upward) revision to output gaps (with Belgium as an exception). For Ireland, Spain, Luxembourg and Greece, the revision to 2012 output gaps is sizable and more than 2%.

Figure 6 plots the revision to 2012 structural balance ratio and output gap from 2013 Autumn to 2016 Autumn. There appears a strong cross-country negative relation between output gaps revisions and structural balance ratio revisions. Associated with upward (or downward) revision to output gaps is downward (or upward) revision to structural balance ratio (with the UK as an exception). For six countries (Ireland, Luxembourg, Spain, Portugal, Greece and Netherlands), the 2012 structural budget balance ratio is revised upward from 2013 to 2016. For Ireland, Luxembourg, Spain, Portugal and Greece, the revision is sizable and ranges from 1.5% to 2.2%.

C. *Evidence 3: how were surprises to output growth translated to revisions to potential output growth forecasts?*

Table 1: Correlation coefficient between surprises to estimates of output growth rates and revisions to potential output growth forecasts

	Belgium	Germany	Greece	Spain	France	Ireland	Italy
corr.	0.52	0.61	0.75	0.51	0.77	0.62	0.58
p-value	0.013	0.003	0.000	0.016	0.000	0.002	0.005
	Luxembourg	Netherlands	Austria	Portugal	Finland	UK	
corr.	0.69	0.63	0.46	0.44	0.65	0.52	
p-value	0.000	0.003	0.049	0.042	0.001	0.013	

From Spring of year t to Autumn of year t (or from Autumn of year t to Spring of year $t+1$), policy makers receive new information about real output growth in year t and revise their estimate of year t real output growth rate. We investigate to what extent the surprises to actual growth rate of real output (or the arrival of new information) is translated into the revision to their forecast of growth rate of real potential output in year $t+1$. Table 1 shows that for all countries, there is a strong positive correlation between revisions to the forecast of the growth rate of real potential output and revisions to the estimate of the growth rate of real output using all data vintages covering $2004-2015$. An upward (or downward) revision to the forecast of the potential output growth rate tends to associate with an upward (or downward) revision to the estimate of real output growth rate. All correlation coefficients are significant at 5% level and most are significant at 1% level.

Evidence 4: Following Blanchard and Leigh (2013) (but using EC data), Appendix A shows that there is a strong negative cross-country correlation between the forecast of fiscal consolidation and subsequent growth forecast errors during 2010-2011. A country which plans a larger fiscal consolidation subsequently tends to have lower growth than expected.

III. Model setup

This section presents an extension of Kuang and Mitra (2016, henceforth KM). KM develop a standard Real Business Cycle (RBC) model but assuming households face uncertainty about the trend and cycle of endogenous variables and learn continually about the long-run growth rate. The model replicates procyclical forecasts of the growth rate of potential output observed in the data and suggests a crucial role for optimism and pessimism about the growth rate of potential output in business cycle fluctuations. Four main features of the KM model are King-Plosser-Rebelo (KPR) non-separable preferences between consumption and leisure, constant returns to scale technology, variable capital utilization, and a random walk with drift productivity process. In contrast to KM, the current model incorporates a fiscal policy maker and distortionary taxes to be able to sensibly analyze the relevant issues.[6]

The representative household maximizes

$$\widehat{E}_t \sum_{t=0}^{\infty} \beta^t u(C_t, L_t);\ u(C_t, L_t) = \frac{C_t^{1-\sigma} v(1-L_t)}{1-\sigma}$$

subject to the flow budget constraint

$$C_t + K_{t+1} + B_{t+1} + T_t = \left(R_t^K U_t - \delta(U_t)\right)\left(1-\tau^K\right) K_t + W_t H_t \left(1-\tau^H\right) + K_t + R_t B_t.$$

$\widehat{E}_t$ denotes the subjective expectations of agents for the future, which agents hold in the absence of RE. RE analysis is standard. $C_t, L_t, H_t, K_t, U_t,\ B_t, T_t$ are consumption, leisure, hours, capital, capacity utilization rate, bond holdings and lump-sum taxes. τ^K and τ^H are capital and labor income tax rate, respectively. W_t, R_t^K and R_t are the wage rate, rental rate for capital services and interest rate. β is the discount rate between 0 and 1. $\sigma > 1$ and $v', v'' > 0$. Capacity utilization in the data displays pronounced procyclical variability (see King and Rebelo (1999)) and is used to improve the fit of the model. Capital depreciation is assumed to increase with capacity utilization U_t according to the function $\delta(U_t) = \theta^{-1} U_t^{\theta}$ where $\theta > 0$.

There are a continuum of identical competitive firms of mass one. Each produces the economy's only good Y_t using capital K_t and labor H_t as inputs according to the production function $Y_t = (U_t K_t)^{\alpha}(X_t H_t)^{1-\alpha}$,where $0 < \alpha < 1$. Each firm maximizes profits, $\Pi_t = Y_t - R_t^K U_t K_t - W_t H_t$, choosing labor and capital inputs and taking factor prices as given. Stochastic variations in the technology factor are the source of aggregate fluctuations and we assume that the technology factor X_t is a random walk with drift, i.e.

$$\log(X_t/X_{t-1}) = \gamma_t = \log(\overline{\gamma}) + \widehat{\gamma}_t, \tag{1}$$

where $\widehat{\gamma}_t$ is an independently and identically distributed (i.i.d) random variable with zero mean and standard deviation σ_γ and $\overline{\gamma} > 0$.

The government budget constraint is

$$B_{t+1} = B_t R_t - \left(R_t^K U_t - \delta(U_t)\right) K_t \tau^K - W_t H_t \tau^H + G_t - T_t, \tag{2}$$

Denote by Y_t^P and OG_t^S policy makers' *real-time* estimate of potential output and output gaps under subjective beliefs. Government expenditure (G_t) is assumed to follow a fiscal rule

$$G_t/Y_t = \lambda_0 + \lambda_1 OG_t^S + \lambda_2 B_t/Y_t, \tag{3}$$

$$\text{where } OG_t^S = Y_t/Y_t^P - 1. \tag{4}$$

Thus, government expenditure to GDP ratio (G_t/Y_t) responds to debt-to-GDP ratio (B_t/Y_t) and output gaps, following Leeper, Plante and Traum (2010). A similar fiscal rule for government expenditure is used in e.g. Corsetti, Meier and Mueller (2012). Both λ_1 and λ_2 are assumed to be negative. $\lambda_1 < 0$ means G_t/Y_t increases (or decreases) in response to a decline (or increase) in output gaps. $\lambda_2 < 0$ implies that the policy maker cuts G_t/Y_t in response to a rise in B_t/Y_t.

Balanced growth requires consumption, investment, output, the capital stock, and real wages to grow at the rate of the stochastic trend so that $k_t = K_t/X_{t-1}, y_t = Y_t/X_t, c_t = C_t/X_t$ etc are stationary. Hours and the rental rate of capital are stationary. Hatted variables are percentage deviations from the steady state. Details of the first order conditions of the household and firm, steady state and the log-linear approximation are presented in the Appendix D. Utilizing the consumption

Euler equation and the intertemporal budget constraint of households, we obtain the linearized consumption decision rule

$$\widehat{c}_t+\sigma^{-1}\psi(1-\sigma)\widehat{H}_t = (1-\chi)(1-\widetilde{\beta})\varepsilon_c^{-1}\left[\widetilde{\beta}^{-1}\widehat{a}_t+\widetilde{R}\widehat{R}_t^K-\widetilde{\beta}^{-1}\widehat{\gamma}_t+(\varepsilon_w+\varepsilon_c\chi\,(1-\chi)^{-1})\widehat{w}_t\right]$$
$$+\widetilde{\beta}\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\Gamma_1\widehat{R}_{T+1}^K+\widetilde{\beta}\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\Gamma_2\widehat{w}_{T+1}, \quad (5)$$

$\widetilde{\beta}$ is growth-adjusted discount factor and $\widehat{a}_t$ household's wealth (i.e., the sum of bond and capital holdings). The composite parameters ε_w, $\varepsilon_c, \psi, \Gamma_1, \Gamma_2, \chi$ are explained in the Appendix. Equation (5) says that the linear combination of consumption and labor supply depends on the forecast of the discounted sum of future rental rates, wage rates, and productivity as well as current wealth, productivity and market prices. We do not assume that households have knowledge of the government budget constraint. Note the forecast of taxes are functions of forecasts of wage rates and rental rates and do not explicitly appear in the decision rule. We consider a symmetric equilibrium in what follows. To determine equilibrium prices and quantities, the learning model needs to be augmented by belief specification and updating presented in Section IV..

IV. Learning by households and the policy maker

This section presents beliefs and learning by households and the policy maker and discusses the connection of the learning behavior of the policy maker in the model with policy makers in reality. It then discusses estimates of output gaps and cyclically-adjusted budget balances (CABs) under imperfect knowledge and the potential bias in their estimation.[7]

A. Households' beliefs and learning

Following KM, households are assumed to have imperfect knowledge of the trend and cycle and the long-run growth rates of endogenous variables. This is in contrast to the belief specification in existing business cycle models with AL that agents learn about detrended variables. The latter implies that agents have exact knowledge of the stochastic and deterministic component of the trend growth rate of endogenous variables, as elaborated in KM, section 7.

Households need to forecast wage rates and rental rates up to the indefinite future to make consumption and other decisions; see (5). They are assumed to have a simple econometric model, relating wages and the capital rental rate to the aggregate stock of capital and bond holdings

$$\Delta \log R_t^K = \omega_0^r + \omega_1^r \Delta \log K_t + \omega_2^r \Delta \log B_t + e_t^r, \tag{6}$$

$$\Delta \log W_t = \omega_0^w + \omega_1^w \Delta \log K_t + \omega_2^w \Delta \log B_t + e_t^w, \tag{7}$$

$$\Delta \log K_{t+1} = \omega_0^k + \omega_1^k \Delta \log K_t + \omega_2^k \Delta \log B_t + e_t^k, \tag{8}$$

$$\Delta \log B_{t+1} = \omega_0^b + \omega_1^b \Delta \log K_t + \omega_2^b \Delta \log B_t + e_t^b, \tag{9}$$

where e_t^r, e_t^w, e_t^k, and e_t^b are regression errors.[8] The beliefs have the same functional form as the linearized minimum-state-variable RE solution to the model (reformulated in levels).

Let $\omega^{i'} = (\omega_0^i, \omega_1^i, \omega_2^i)$ for $i = r, w, k, b$. $z_t^r = \Delta \log R_t^K, z_t^w = \Delta \log W_t, z_t^k = \Delta \log K_{t+1}, z_t^b = \Delta \log B_{t+1}$ and $q_{t-1}^{'} = (1, \Delta \log K_t, \Delta \log B_t)$. Beliefs at period t, ω_t^i are updated recursively by the constant-gain Generalized Stochastic Gradient (GSG) learning algorithm as in Evans, Honkapohja and Williams (2010, henceforth EHW)

$$\widehat{\omega}_t^i = \widehat{\omega}_{t-1}^i + g^i \Gamma q_{t-1} \left(z_t^i - \omega_{t-1}^{i'} q_{t-1} \right), \tag{10}$$

where $\widehat{\omega}_t^i$ denotes the current-period's coefficient estimate.[9] Γ controls the direction of belief updating and $g^i \in (0,1)$, the constant gain, determining the rate at which older observations are discounted. Bayesian and robustness justification for the GSG algorithm are provided in EHW. In particular, they show that (1) GSG learning algorithm asymptotically approximates the Bayesian optimal estimator when agents allow for drifting coefficients models, and (2) it is also the "maximally robust" estimator when agents allow for model uncertainty. As is standard in the literature, beliefs at t are updated using data up to period $t-1$.

B. Policy maker's learning

The policy maker needs to estimate output gaps to determine government expenditures. We follow Holston, Laubach and Williams (2017) and Stock and Watson (1998) by specifying the policy maker's perceived log potential output as a random walk with a stochastic drift that itself follows a random walk. Denote by μ_t^y the policy maker's beliefs about the growth rate of potential output. μ_t^y is updated optimally according to

$$\mu_t^y = \mu_{t-1}^y + \widetilde{g}\left(\Delta \log Y_{t-1} - \mu_{t-1}^y\right), \tag{11}$$

where, for generality, we allow $\widetilde{g} > 0$ to be different from the gain g^i used by households. A positive (or negative) surprise to output growth will lead to an upward (or downward) revision to the trend growth of output.[10]

We make two remarks about the choice of $\widetilde{g}$. First, EC estimates of potential output of different vintages are used to discipline the choice of $\widetilde{g}$ which in our model becomes the actual gain parameter used by the policy maker. Second, Section VII. explores an alternative choice of $\widetilde{g}$ by the policy maker which minimizes real-time mis-measurement of output gaps (and hence structural balance); we call this the optimal gain parameter there. This optimal gain is smaller than the actual gain used by the policy maker and we use this optimal gain to provide a counterfactual analysis.

Relation to practical methods of estimating potential output .—We provide a discussion on how learning by the policy maker in our model relates to the methods of estimating potential output used by actual policy makers. First, we note that this belief specification is consistent with the empirical findings in CGU (2018). They find that the estimates of potential output growth rates for the US and a large number of industrialized countries made by a wide range of public institutions and private forecasters (and produced by the methods discussed below) are well approximated by a weighted moving-average of recent GDP changes.

A popular method to estimate potential output is the production function approach used by e.g. EC, IMF, OECD and the US CBO; see D'Auria et al (2010) for details. An aggregate production function is used. Statistical filtering methods (e.g., HP filter by the EC) are applied to historical data on factor inputs such as working hours, unemployment, and total factor productivity to obtain the estimate of their trend components. The trend level of the factor inputs are then substituted into the production function to obtain the estimate of potential output. Another popular method is applying filtering methods (like the HP filter for EC) to aggregate output. The EC publishes the estimation results produced by both methods.

A main problem of these methods is that they fail to adequately distinguish the underlying sources of output changes. For example, a decline in aggregate output which is due to purely cyclical changes will be partly and incorrectly attributed to a change in the trend. Similarly, a change in the level of aggregate output will be partly and incorrectly interpreted as a shift in the trend growth of output. CGU provide evidence on these implications of the main practical methods. Our model can capture these aspects of learning behavior by policy makers.

Our learning model is able to capture another aspect of behavior by actual policy makers. For example, in the production function approach, applying statistical filtering methods to individual factor inputs inevitably yields different trend growth rates for different inputs and output. That is, this approach does not impose any cointegration relation among macroeconomic variables or the "great ratios" (constant long-run capital output ratio or capital hours ratio in e.g. Klein and Kosobud (1961)). This is in contrast to RE models (including incomplete information RE models) which usually impose such cointegration relation and have "great ratios". Therefore, in line with the practical methods used, households and the policy maker in our model are assumed to not possess knowledge of the balanced growth path and not make use of this knowledge to form forecasts about the future; see (6)-(9) and (11).

C. Output gaps and cyclically-adjusted budget balances

The policy maker's imperfect knowledge of the economy gives rise to a potential bias in estimating output gaps and structural balances unlike full-information RE models. We make a distinction between true output gaps (OG_t^T) and subjective output gaps (OG_t^S). If the policy maker had full information, she would know the evolution of true potential output $\log Y_t^{P,T}$ is determined by the productivity process $\log Y_t^{P,T} = \log Y_{t-1}^{P,T} + \log(\overline{\gamma}) + \widehat{\gamma}_t$. OG_t^T is calculated using the true trend output and given by $OG_t^T = \log Y_t - \log Y_t^{P,T}$. Subjective trend output ($\log Y_t^{P,S}$) and OG_t^S are $\log Y_t^{P,S} = \log Y_{t-1}^{P,S} + \mu_t^y + \widehat{\gamma}_t$ and $OG_t^S = \log Y_t - \log Y_t^{P,S}$. The bias in the estimation of output gap is

$$OG_t^S - OG_t^T = \log Y_t^{P,T} - \log Y_t^{P,S}. \tag{12}$$

The EC makes use of output gaps to compute the structural fiscal balances of EU member states. The EC's approach is elaborated in Mourre et al (2013) where CAB is computed as

$$CAB_t = BB_t/Y_t - \varepsilon * OG_t. \tag{13}$$

BB_t/Y_t is budget balance (BB_t) to GDP ratio and the cyclical component, $\varepsilon * OG_t$, is the product of the cyclical sensitivity of the budget balance to the cycle (ε) and the output gap (OG_t). The budgetary semi-elasticity, ε, for EU countries is on average 0.53 (ranging from 0.3 to 0.61); see Mourre et al (2013). CAB is lower (higher) than actual balance during expansions (recessions).

Mis-measurement of output gaps may cause bias in estimated CAB. Given BB_t/Y_t, this bias in estimating CAB is given by

$$Bias_{CAB}(t) = -\varepsilon \left(OG_t^S - OG_t^T \right). \tag{14}$$

so that this bias is proportional to the bias in output gaps. If the policy maker is too pessimistic about potential output (during a recession), she will underestimate the size of output gaps and structural budget balance, which may in turn lead to undue fiscal austerity, i.e., excessive requirement on reducing structural deficits. For example, the CPB Netherlands Bureau for Economic Policy Analysis (in Hers and Suyker (2014)) noted that "*The volatility of the structural balance is especially problematic because the indicator is used in the EU as a basis for the recommendations for a country. Volatile estimations could, for example, cause a government to be obliged to undertake significant additional fiscal consolidation* *in order to meet the structural balance requirement – only to be confronted with a structural balance that does not change at all, because of revisions in potential growth."*

V. Quantitative Results

This section examines the role of subjective beliefs of households and the policy maker in the Great Recession with a view to capturing the evidence presented in Section II..

A. Calibration

Parameters are calibrated for the euro area. Capital share α is set to 0.38 and σ in the utility function 2, as in Trabandt and Uhlig (2011). The discount factor $\tilde{\beta}$ is set to 0.995, which corresponds to a 2% real interest rate at the steady state. The unconditional mean of the growth rate of productivity is 2% per year (i.e., $\overline{\gamma} = 1.005$). The depreciation rate δ is set to 2.5%. The inverse of Frisch elasticity of labor supply is set to 0.1.[11] The steady state of government expenditure-to-output ratio $\overline{g}/\overline{y}$ is 0.18. The debt-to-GDP ratio is set to 75%. Capital income tax rate τ^K is set to 0.4 and labor income tax rate $\tau^H = 0.42$. The budgetary semi-elasticity ϵ for computing CAB is set to the EU average value 0.53; see Mourre et al (2013). In the fiscal rule, debt sensitivity of government spending λ_2 is set to -0.02 taken from Corsetti, Meier, and Mueller (2012) and output gap sensitivity of spending λ_1 is set to -0.002.[12]

Using data from the European Central Bank's Survey of Professional Forecasters, Andrade and Le Bihan (2013) provide evidence on strong autocorrelation in the (average) 1−year ahead forecast errors of euro area real GDP growth rates, inflation rate, and unemployment rate across forecasters; see their Table 1. In particular, the autocorrelation coefficient of the forecast errors of real GDP growth rate is 0.849. This evidence is used to impose discipline on the choice of the gain for households. The gain parameter g is chosen as $g^w = g^K = g^b = 0.004$ and $g^R = 0.015$, which leads to the corresponding autocorrelation coefficient of 0.92 in the learning model.[13] Our choice of gain is consistent with values found in the literature, which range from $0.002 - 0.05$; see eg. Eusepi and Preston (2011).

The moment matrix in (10) is set to the identity matrix, corresponding to the classical Stochastic Gradient (SG) learning.[14]

We make the following assumption on the policy maker's initial beliefs. In reality policy makers usually have large uncertainty in measuring output gaps in the current period and recent past. However, when the past is sufficiently distant from current period, they are likely to have much less uncertainty in decomposing historical aggregate output. Therefore, for simplicity, we assume that the policy maker's belief about the trend output N periods ago is fixed at the true value where N is relatively large. N is set to 16 below which means that the policy maker does not update the trend output level four years ago. The choice of $N = 16$ is motivated by Figure 1 which shows that after the Great Recession and for almost all countries, the revision to potential growth rates in 2003 (i.e., 16 periods ago) from 2008 Spring to 2011 Spring is very small. We also assume that before the shock, the economy was at the long-run growth path (and hence at the steady state) and the policy maker's initial belief about the growth rate of trend output is at the RE value $log(\overline{\gamma})$.[15]

We take a broad interpretation of productivity shocks and model the Great Recession in terms of a sequence of negative productivity shocks in our RBC economy with distortionary taxes.[16] While we do not deny that financial frictions may be a cause of the current recession, we think we may nevertheless plausibly represent the aggregate consequences of these frictions by a productivity shock. This belief is backed by Chari, Kehoe and McGrattan (2007) who show that suitable input financing frictions are observationally equivalent to negative productivity shocks. In particular, they set up a detailed economy with input-financing frictions across firms leading to inefficient utilization of factor inputs. They show that an economy in which technology is constant but input financing frictions vary over time is observationally equivalent to a neoclassical model with productivity shocks in the sense that the economies display the same aggregate properties. We are not interested in the causes of the recession *per se.* One may thus view the reduced productivity as a convenient short-cut for modelling distortions associated with the financial crisis by appealing to the equivalence result of Chari, Kehoe and McGrattan (2007).[17]

The cumulative decline in aggregate output during 2009 - 2010 for the eurozone is 3.8%. With a 2% annual growth rate of trend output, the 3.8% decline in aggregate output means a 7.8% (i.e., 3.8% + 2% × 2) decline in output relative to the pre-crisis trend. The Great Recession is modeled as a sequence of negative productivity shocks for six periods, the size of which is chosen to match this decline in aggregate output in the AL model; see top left plot of Figure 7.[18] From 2008 to 2011, the estimate of the average growth rate of potential output in 2008 - 2011 for eurozone was revised downward from 2% to 1.1%. The gain parameter for the policy maker is chosen to match the size of this revision, which yields $\widetilde{g} = 0.034$.

B. The Great Recession scenario

We discuss the analysis of impulse response to a productivity shock in Appendix C where the intuition is provided (this is similar to that in KM). We note that our learning model is able to reproduce the evidence from CGU (2018) that the estimates of potential output made by a range of public institutions decline (or rise) persistently in response to a i.i.d negative (or positive) TFP shock. In the learning model, a negative productivity shock leads to a fall in output and the policy maker uses the new observation of output to revise downward their potential output estimate. In addition, the interaction between belief revision and equilibrium outcomes helps to produce this persistent response; see the top left plot of Figure A.5. By contrast, the RE version of the model is inconsistent with this evidence because it does not produce a *persistent* response of potential output estimate to the i.i.d shock.

In Figure 7, percentage deviations from the unshocked balanced growth path are plotted for output (Y), consumption (C) and investment (I). For hours (H), deviations from the steady state are plotted. For G/Y ratio and debt-to-GDP ratio, percentage changes are plotted. This figure shows that in the AL model, the shocks lead to a large decline in Y, C, I, H and a rise in debt-to-GDP ratio. G/Y ratio declines initially in response to rising debt-to-GDP ratios and rise gradually later. Relative to the RE model, the learning model generates large amplification arising mainly from the downward revision to households' wage (and income) forecasts and mutual reinforcement between households' pessimism and market outcomes. For example, the maximum response of output (i.e. at the trough) in the learning model is about twice as large as that in the RE model.

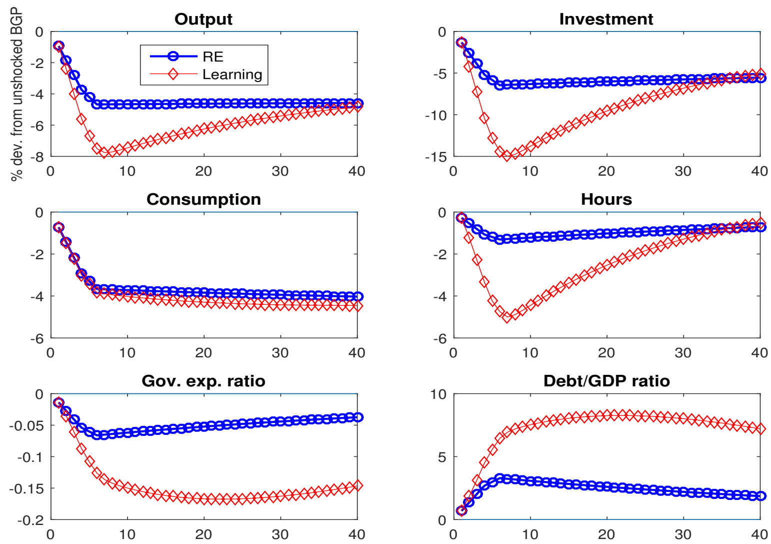

Figure 7: Response: the Great Recession scenario

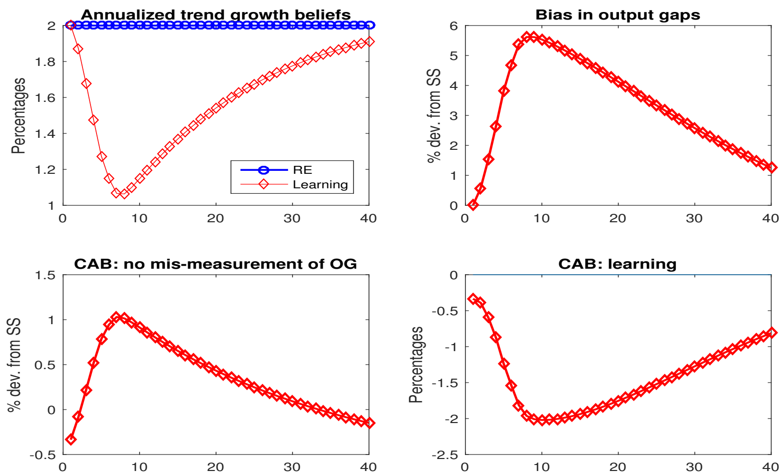

Figure 8: Policy maker's beliefs: the Great Recession scenario. Note the lower two panels shows the change in the CAB *vis-a-vis* the steady state.

The recession leads to a large downward revision to estimates of potential output growth made by policy makers (top left plot of Figure 8) as in the eurozone data while the (deterministic component of) true trend growth remains at 2% per year in the model. In addition, the recession produces a 2% decline in the subjective estimate of CAB at the trough (the bottom right plot) which is computed using the EC's formula (13). This is because the recession reduces tax revenues and generates government budget deficits as the decline in revenues dominates the decline in government expenditures. After cyclical adjustment with subjective estimates of output gaps, CAB continues to be negative and declining. Note the size of the decline in CAB matches the data as the structural balance ratio for euro area in 2010 minus the average structural balance ratio over 2003-2007 is 2%.[19] However, pessimism about potential output leads to a large underestimation of the size of output gap and structural balances which at the trough is 5.7% and 3%, respectively (3% is computed using equation (14)).

Due to structural balance targeting in the EU fiscal policy framework, the underestimation of CABs will in turn lead to excessive requirement to reduce structural deficits and output losses, which is analyzed in Section VI.. The bottom left plot of Figure 8 displays estimates of CAB when there is no mismeasurement of output gaps, i.e., when the true (instead of subjective) output gaps are used for computing CABs. This measure of CABs declines a bit initially and then rises for some periods. They are much higher than the subjective estimates of CAB (on the bottom right plot).

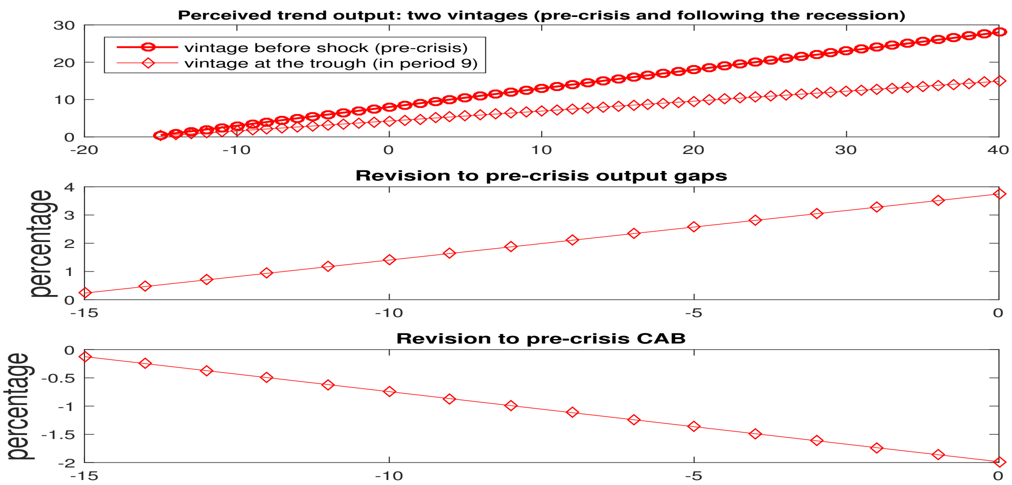

Figure 9: Changes to policy makers' view following the Great Recession.

Figure 9 displays changes to the policy maker's view following the recession in the model. The top plot displays the policy maker's perceived evolution of (the deterministic component of) potential output of two vintages: one before the shocks and the other when aggregate output reaches its trough (two years since the start of the recession). In this figure, period $-n$ corresponds to $(n+1)$ quarter before the crisis period for $n = 0, 1, 2, ...$ and the trend output in period -15 is normalized to zero. Before the shocks, trend output (in the past and future) is perceived to grow at the rate of $log(\overline{\gamma})$. Following the recession, the policy maker perceives a flatter path for future and past trend output, consistent with Figure 1. The middle and bottom plot of Figure 9 shows that the lower perceived potential output leads to upward revision to estimates of pre-crisis output gaps and downward revisions to estimates of pre-crisis structural budget balances, consistent with the evidence in Figure 3 and 4. Note the general pattern in Figure 3 and 4: the size of the revision to output gaps and structural balance for a year is larger, closer it is to the crisis year (2008). For example, the revision to the output gap in 2007 is larger than in 2006 for all countries (similarly, for structural balances).[20] Interestingly, our learning model is consistent with this feature of the revisions. For instance, the revision to the output gap is 2.6% in period -5 while it is 1.5% in period -10.

C. Additional properties of European Commission's estimates

We discuss how the learning model can replicate several additional properties of the EC estimates. First, the learning model helps to produce the heterogeneity in the revisions to potential output growth rates and the strong positive correlation between the severity of recession and the size of revision to potential growth across EU countries documented in Section II.A.. For illustration, the learning model is simulated for two countries with the same parameterizations except the shocks: one is hit by the same sequence of productivity shocks as in Section V.B. and the other by a sequence of smaller shocks (the shocks are half of the size for every period). The country which is hit by larger (or smaller) shocks experiences a larger (or smaller) decline in output. This yields heterogeneity in downward revision to potential output growth (bottom plot of Figure 10). Moreover, consistent with empirical observation, Figure 10 shows that potential output growth rates are revised downward by more (or less) for the country with a larger (or smaller) recession as the policy maker uses new observation of output to update beliefs.[21]

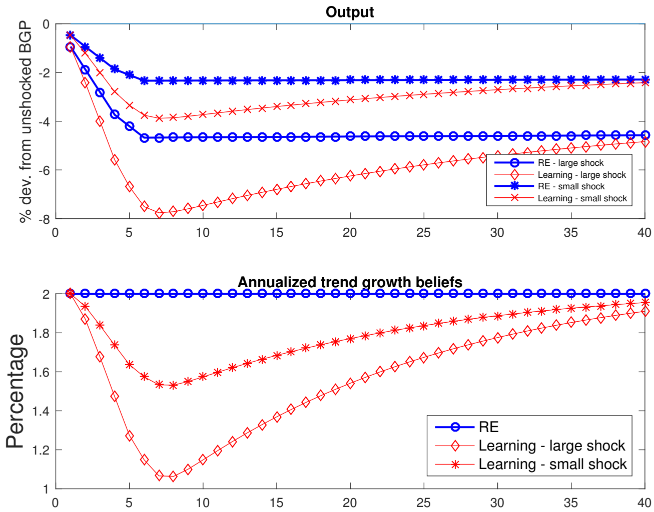

Figure 10: Relation between the severity of recession and the size of downward revisions to estimates of potential output growth rates

Second, the learning model helps to reproduce Evidence 2 discussed in Section II.B.: (a) heterogeneity in revisions to potential growth rates during the EU recovery phase and the strong positive correlation between the pace of recovery and the revisions to potential growth and (b) the pattern of revisions to output gaps and CABs. We do not provide these details here due to space limitation (which are available upon request) but the intuition is straightforward. We first comment on the evidence in Figure 5. Recall for eight countries, the average growth rate of output during $2013-2016$ is higher than the average growth during $2007-2011$. In the model, the policy maker uses output growth data during $2013-2016$ to revise estimate of 2012 potential output growth rates according to equation (11) which tends to yield an upward revision to the 2012 potential output growth rate made from 2013 to 2016. Similarly, for the remaining countries, the learning model yields a downward revision to the 2012 potential output growth rate because the average growth rate of output during $2013-2016$ is lower than that during $2007-2012$. Moreover, in the AL model, upward (or downward) revisions to potential output tends to associate with downward (or upward) revisions to output gaps and upward (or downward) revisions to CABs, consistent with the evidence in Figure 6 and 7.

Third, the learning model can reproduce a strong positive correlation between surprises to output growth rates and revisions to forecasts of potential output growth rates documented in II.C. (Evidence 3). In the model, surprises in output growth for year t are measured by output growth rate of year t minus forecast of the year t output growth rate made in year $t-1$. Revisions to forecasts of potential output growth rates are measured by forecasts of estimated year $t+1$ potential output growth rates after output data of year t is available minus that before output data of year t is available. The stationary distribution of agents' and the policy maker's beliefs is obtained by simulating the learning model for T periods and N repetitions. We set $T = 2000$ and $N = 500$. After obtaining the stationary distribution, we simulate the learning model for 48 periods, which corresponds to the length of our sample (12 years). We calculate the median correlation coefficient over repetitions. The learning model produces a correlation coefficient of 0.88, which is a bit higher than the corresponding number in the data of countries like France and Greece. Consider a positive (or negative) surprise to output growth rate, i.e., outturn higher (or lower) than prediction. This tends to lead to more optimistic (or pessimistic) forecast of the growth rate of potential output in the following year, i.e., an upward (downward) revision to the forecast of potential output growth rate.

Standard full-information RE models cannot explain why the arrival of new data observations (e.g., sustained declines in output) will lead to a revision of the *historical* estimate of the trend and cycle of *endogenous* variables (e.g., aggregate output and budget balances). This is because agents can deduce the evolution of endogenous variables and have exact knowledge of the trend and cycle of endogenous variables. Moreover, the correlation between surprises to output growth rates and revisions to forecasts of potential growth rates is usually zero, as opposed to a strong positive correlation in the data. This is because the forecasts of potential output growth rates are usually constant, given the (trend) productivity process (1).

VI. Effects of "undue" fiscal consolidation

The Great Recession led to a large fall in the subjective estimates of structural budget balances made by policy makers and debt-to-GDP ratio increased very rapidly for almost all countries in the sample. For example, p. 15 of the 2010 public finance report, EC (2010) states that none of the EU countries in our sample are expected to achieve their medium-term objective of structural budget balances in 2010 and 2011. Targeting structural balance in the context of the Stability and Growth Pact requires fiscal actions and EU policy makers indeed responded by adopting consolidation programmes.

The learning model in Section V.B. reproduces a large fall in subjective estimates of CABs by policy makers (see the lower right panel of Figure 8) and a large increase in debt-to-GDP ratio (see the lower right panel of Figure 7) following the recession. The AL model suggests that the recession leads to potential output pessimism which translates as underestimation of structural balances and undue requirement in reducing the structural deficits due to targeting structural balance. We call the fiscal consolidation programmes undue because the AL model suggests the recession would only reduce CABs a bit initially and then improve CABs if there was no mis-measurement of output gaps (see the bottom left plot of Figure 8). We now examine the macroeconomic consequences of fiscal consolidation programmes which respond to the deterioration of estimates of CABs and debt ratios in an effort to capture the actual scenarios in EU countries. The model suggests the mutual reinforcement between growth expectations and undue fiscal consolidation leads to further substantial output declines.

Undue fiscal consolidation is modeled as unexpected declines in the steady state of government expenditures for two and a half years from 2010 onwards (i.e., period 9 onwards). The size of the expenditure shocks is chosen to produce about 0.5% improvement in structural budget balance ratio from period 8 to the end of the consolidation period in the AL model.[22] Figure 11 shows that under RE, the undue fiscal consolidation does not produce much difference in output and working hours (comparing the line labeled "RE - austerity" and the line labeled "RE"). Note increases in private investment are associated with reductions in government expenditures. Moreover, fiscal consolidation leads to immediate declines in debt-to-GDP ratio.

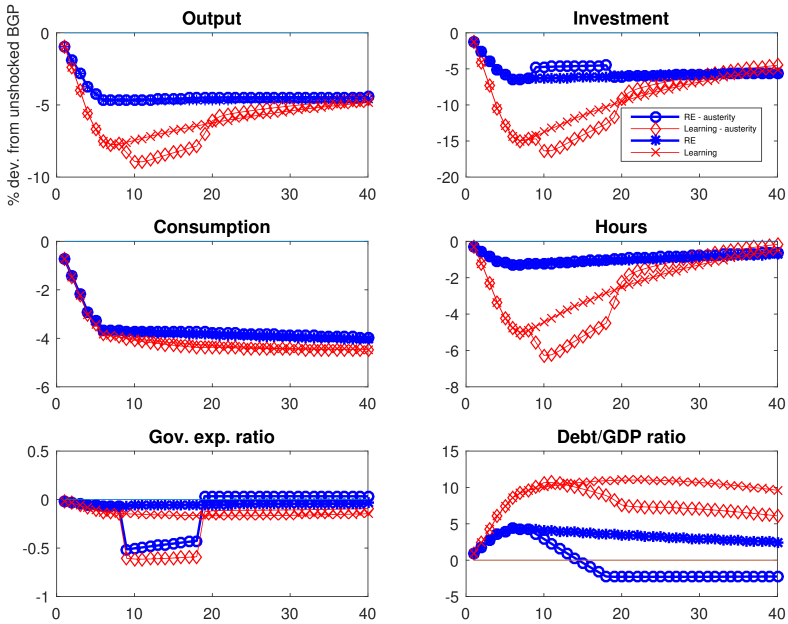

Figure 11: Impact of undue fiscal consolidation

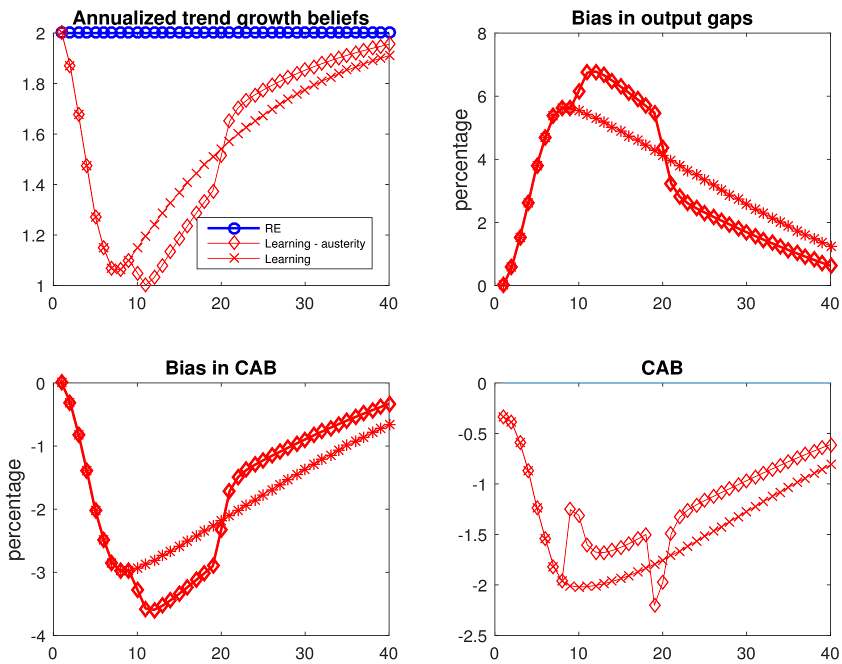

Figure 12: Impact of undue fiscal consolidation (continued)

In contrast, the same austerity programme in the AL model has a much larger impact and leads to further declines in output, investment, hours and consumption (comparing the line labeled "Learning - austerity" with the line labeled "Learning").[23] Note private investment declines are associated with declining government expenditures unlike the RE model. During the consolidation period, the average output loss per year is 1.44%. The large amplification of the effect of fiscal austerity under learning is because the consolidation is implemented when households have pessimistic beliefs about future wage rates (and income) and the subsequent interaction between households' pessimistic beliefs and equilibrium outcomes. Based on the pessimism, households reduce I, H, and C following the consolidation, which leads to lower aggregate output. The latter reinforces households' pessimistic beliefs about future wages (and income).

Lower aggregate output due to consolidation yields significantly lower estimates of potential output growth rates made by the policy maker, see the top left plot of Figure 12. During the consolidation period, lower estimates of potential output produce a substantially larger bias in estimation of output gaps and CABs (top right and bottom left plots of Figure 12).

In the AL model, declines in G/Y ratio lead to lower aggregate output and tax revenues relative to RE. The decline in tax revenues dominates the reduction in government expenditures, which leads to a (small) rise in debt/GDP ratio for some periods (in contrast with declines in debt ratios in the RE model). This is consistent with the observation in the data. In the eurozone crisis, most EU countries, such as Spain and Ireland, have cut government spending, in order to reduce their budget deficits. However, these spending cuts were associated with a decline in economic growth, leading to lower tax revenues and rising debt to GDP ratio.

In reality, the over-pessimistic estimate of potential output and structural balances are likely to play a role in amplifying the increase in government bond risk premium and output decline for countries concerned following the recession. This mechanism is not captured by the current model but can be incorporated in an extension which is left for future research.

VII. Minimizing real-time mis-measurement of output gaps (and CAB) and counterfactual analysis

Estimates of potential output are put to work in macroeconomic policy decisions which in turn influence aggregate economic outcomes. We explore how policy makers can minimize real-time mis-measurement of output gaps (and CABs) which is of interest to EU fiscal policy makers. Standard full information RE models do not have room to analyze this issue given the assumption of policy makers' exact knowledge of the economy.

A. Minimizing mis-measurement

Real-time mis-measurement in output gaps and CABs is inevitable due to the uncertainty in measuring cyclical conditions. How can the policy maker minimize the real-time bias? This section explores whether there exists a value of the gain parameter that minimizes this bias. The gain parameter determines the weight placed on recent data observations relative to more distant observations (or degree of extrapolation). We focus on minimizing the bias in output gaps (which then implies minimization of the bias in CABs). We assume that the policy maker wishes to minimize the distance between the real-time estimates of the output gap and the true output gap.

Two issues deserve some discussion before we spell out the exact objective function of the policy maker. First, the true output gap is, of course, not observed. However, to implement this exercise in practice, policy makers may proxy this by the *ex post* estimate of output gap from the distant past. For example, the policy maker in 2018 has much less uncertainty about the output gap in 1990s; thus the *ex post* estimate of output gap in the distant past may be close to the true output gap. Second, as the policy maker learns using a constant gain parameter, the beliefs of trend growth do not converge pointwise to the RE value. In the exercise below, initial beliefs are drawn from a distribution.[24] Given initial beliefs, the policy maker considers alternative choices of the value of the gain parameter which in turn will yield different estimates of output gaps, fiscal policy decisions and aggregate outcomes.

We assume that this (alternative) choice of the policy maker's gain parameter does not influence households' gain parameter in their belief formation (i.e. household gain parameter is fixed in this exercise).[25] We think this is a plausible assumption. Consider the following scenario: around 2002, the EC switched from using HP filtering to the production function approach as the main method for estimating potential output (conceptually this is similar to alternative choices of gain parameter). Realistically, households would not understand the implications of this change by the EC and are unlikely to discount past data differently as a result. Nevertheless, there is an indirect impact from this alternative choice of the policy maker's gain parameter on households' belief formation in the AL model: this alternative gain parameter leads to different macroeconomic outcomes which in turn is used by households to form their beliefs.

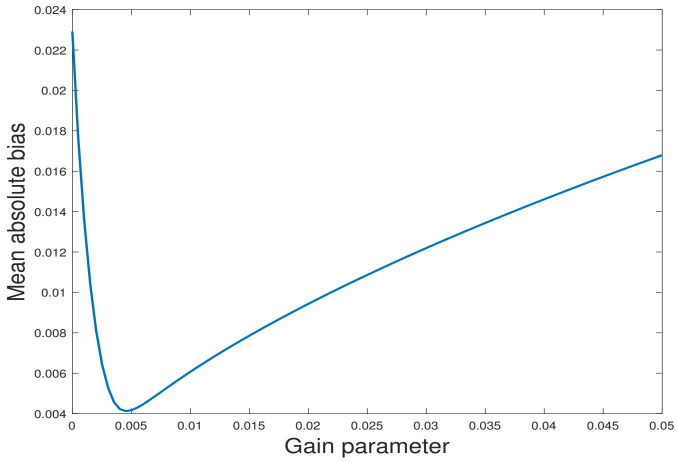

Figure 13: The mean absolute bias as a function of the gain parameter chosen by the policy maker

Specifically, we calculate the gain parameter which minimizes the mean absolute bias over a number of periods (or an evaluation span T_1) and across different initial beliefs as follows

(15) $$Bias = T_1^{-1}T_2^{-1}\sum_{t=1}^{T_1}\sum_{i=1}^{T_2}\left|OG_t^S(i) - OG_t^T(i)\right|$$

where $OG_t^S(i)$ stands for the output gap under subjective beliefs in replication i and period t; similarly $OG_t^T(i)$ the true output gap. T_2 is the number of replications. The size of the bias in estimating output gap and the derived CAB depends on the gain parameter used by the policy maker.

Figure 13 displays the bias measured by (15) as a function of the policy maker's gain parameter in the interval $[0, 0.05]$ with $T_1 = 40$ periods (10 years) and $T_2 = 500$. The mean absolute bias is decreasing in the gain parameter initially and then increasing. The optimal gain parameter yielding the minimum bias is 0.45% which is relatively small (appearing robust to the choice of T_1); the associated mean bias in estimation of output gaps is 0.42%.

There are two things to note here. First, the gain parameter which minimizes the bias (0.0045) is much less than the calibrated gain parameter (0.034). This suggests actual policy makers may have over-reacted to the sustained output declines during the Great Recession by placing too large a weight on this recession data; note that this observation is consistent with Krugman's (2013) quote discussed in the Introduction. In ordinary least squares regressions, all data points receive equal weights and we can normalize the weight received by each data point to 1. A calibrated gain parameter of 0.034 for the policy maker implies that the weight received by the output growth data of (say) 20 years ago is $(1-0.034)^{80} \simeq 0.063$. In contrast, the optimal gain parameter of 0.0045 implies the weight received by the output growth data 20 years ago is much larger and equals $(1 - 0.0045)^{80} \simeq 0.70$.

Second, a moderate speed of learning (or not discounting past data too much) by the policy maker minimizes the bias when forming potential output estimates. On the one hand, a large gain parameter (like 0.034) leads to a large bias because potential output estimates will be quite sensitive to random innovations which leads to more frequent and larger deviations from the true value. On the other hand, a small gain (close to zero) is not optimal because potential output estimates could remain far from the true value for long periods of time since beliefs are revised slowly; this produces a substantial bias.[26]

B. Counterfactual analysis

This section provides a counterfactual analysis when the policy maker adopts the optimal gain parameter of 0.0045 from the previous section (rather than the calibrated gain parameter 0.034). With the optimal and smaller gain parameter, the learning model is simulated using the same sequence of negative productivity shocks as in Section V.. A smaller gain parameter implies placing a smaller weight to data during the Great Recession period, which yields less pessimistic estimates of trend output and CAB following the Great Recession. The latter is above the -0.5% threshold, as can be seen in the left plot of Figure 14. Since the CAB is above the -0.5% threshold, fiscal austerity due to the EU debt brake (considered in Section VI.) is not necessary. Figure 14 shows the counterfactual path for CAB and aggregate output (along with the simulated learning paths from Section VI.). The real output, without implementing the fiscal consolidation plans, would be substantially higher for the consolidation period (i.e., period $9-18$) relative to the case where the gain parameter 0.034 is adopted.

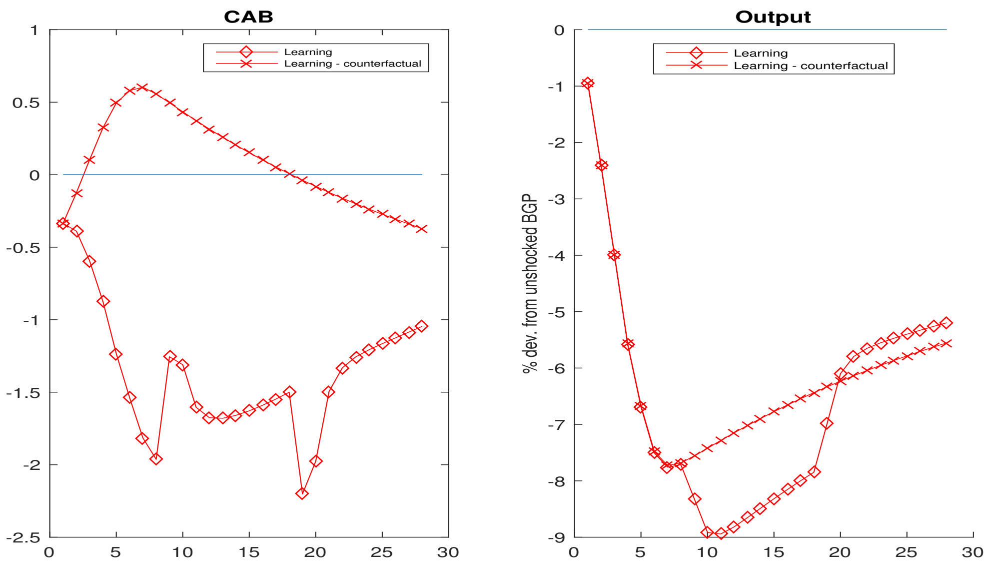

Figure 14: Couterfactual path for CAB and output (line with "x"). The lines with diamonds illustrate the learning path from section VI..

VIII. Conclusion

The paper documents a new set of evidence on policy makers' perceived evolution of potential output, output gap and structural budget balance for 13

EU countries since 2004. Another evidence is that potential output estimates respond persistently to transitory TFP shocks found by CGU (2018). We build a model with policy makers' learning of potential output in real time which replicates the evidence.

The learning model captures realistic dynamic interaction between policy makers' beliefs, fiscal policy and market outcomes. Unlike our model, existing business cycles model typically do not have the feedback from market outcomes to policy makers' beliefs about the trend growth of endogenous variables and do not utilize data on policy makers' estimates of endogenous variables (e.g., potential output and structural balance) of different vintages.

Our analysis suggests that the initial shocks following the financial crisis led to over-pessimism about potential output. The dynamic interaction between pessimism and market outcomes amplified the shocks and led to large and sustained declines in output. The pessimism generated large underestimation of structural fiscal balances which then triggered undue austerity due to structural balance targeting in the EU fiscal policy framework. The mutual reinforcement between potential output pessimism and undue austerity contributed significantly to further recession in the learning model.

We explored how policy makers can reduce real-time mis-measurement of output gap and structural balance which is of interest for EU fiscal policy making. The model suggests that when forming beliefs about potential output, policy makers should place a moderate weight on recent data, which is much smaller than that used by policy makers in reality (calibrated to match the massive downward revisions to the potential output estimates in response to the Great Recession). If policy makers placed a much smaller weight to the data during the Great Recession period, there would be higher estimate of structural budget balances, less fiscal austerity and a less severe economic recession.

Notes

[1]Similar revisions have been made by other institutions, such as the IMF. For the US, projected potential GDP for 2017 made by the Congressional Budget Office (CBO) in 2014 is 7.3% lower than its projection in 2007. The large downward revisions to

potential output for OECD countries is also documented in Ball (2014).

[2]See also Jarocinski and Lenza (2018) which has a similar finding for the euro zone. Kuang and Mitra (2016) find that estimates (forecasts) of the growth rate of US potential output are highly pro-cyclical.

[3]A frequent objection to models using data on subjective beliefs is that agents may not act based on the *subjective* data. Our paper provides an example where such a concern is not an issue because the EC's real-time estimates of output gap and structural balance are used to formulate fiscal policy.

[4]By undue fiscal austerity we do not mean relative to an optimal benchmark but rather relative to a situation when there is no mis-measurement of output gap and in the presence of a "debt brake".

[5]In the dataset, from 2008 Spring to 2011 Spring, potential growth rates are often revised without any revision to actual GDP growth data. For some years, there is data revision in actual growth but the size of the revision is much smaller than the revision in potential growth rates. This suggests only a small role for data revision in explaining the revision in potential growth rates in this sample. We, therefore, focus on the role for policy makers' uncertainty in estimating potential output in explaining the pattern of revisions in the three unobserved variabels. Orphanides and Van Norden (2002) also find data revision is not the primary source of revision in measured output gap.

[6]An alternative model to analyze these issues would be the New Keynesian (NK) model; see Woodford (2003). However, interest rates have been stuck at low levels since the Great Recession and monetary policy has been relatively powerless in this scenario (notwithstanding measures like quantitative easing). Our focus, on the other hand, is on potential output beliefs and its implications for fiscal balances. Moreover, issues relating to trend growth (emphasized in our paper) are not discussed in the context of the benchmark NK model whereas the stochastic growth model (like the RBC model with distortionary taxes and fiscal policy) is perhaps the natural way to study these issues.

[7]In practice, structural budget balance ratio is measured as cyclically-adjusted budget balance ratio net of one-off and temporary measures. The latter is usually small and hence we omit it in our analysis and use structural balance and cyclically-adjusted budget balance interchangeably in the AL model.

[8]We note that under RE the residuals in the perceived law of motion are correlated but that agents ignore possible correlations when estimating the forecast functions. This approach under AL is consistent with the standard approach in the literature; see

eg. Eusepi and Preston (2011, 2018) and Sargent et al. (2009).

[9]An alternative learning rule is the constant-gain recursive least squares (CG-RLS) algorithm. The impulse response functions of our model with CG-RLS learning are similar to the results with GSG learning. However, CG-RLS learning often imposes a projection facility on beliefs and/or generates singularity problem in inverting the moment matrix; this is perhaps not ideal and we prefer presenting our results with GSG learning where the projection facility is not invoked.

[10]Learning about growth rates of asset prices (stock and house prices) turns out to be important to match survey expectations and asset boom-bust cycles in Adam, Marcet and Nicolini (2016), Adam, Marcet and Beutel (2017) and Adam, Kuang and Marcet (2012).

[11]This is the benchmark value in KM. If a smaller Frish elasticity of labor supply is used in the current model, e.g. 5 or 7, our results are quantitatively similar. If we use a Frish elasticity as low as say 0.5 or 1, our main results are still preserved except that the learning model does not produce the comovement between forecasted changes in consumption and working hours discussed in Rotemberg and Woodford (1996), see Section C2 of the Online Appendix of KM for a discussion.

[12]Similar tax rates have been used in Trabandt and Uhlig (2011). Note the 75% debt ratio corresponds to the average of eurozone countries during 1993 - 2012. Detrended lump-sum taxes are assumed to be constant and chosen to yield this value of the debt ratio. The results are not sensitive to the choice of λ_1.

[13]This way of disciplining gain parameters has similarly been used in Eusepi and Preston (2011), Kuang and Mitra (2016) and Adam, Marcet and Beutel (2017). Heterogenous gain parameters have been used in Branch and Evans (2011) and Kuang and Mitra (2016).

[14]Note even if the SG learning algorithm is adopted here, the quantitative results of our learning model are scale invariant and remain exactly the same in response to a change in the units of regressors; see Footnote 13 of KM. The quantitative results of the AL model are robust to alternative moment matrices and to labor elasticities but we do not report them here for economy of space. The Online Appendix of KM contains similar robustness exercises.

[15]A difference of our learning model from most existing adaptive learning models is that we also seek to replicate the data when policy makers look back, i.e., revisions to historical estimates.

[16]A similar technique has been used in Mitra, Evans and Honkapohja (2016).

[17]In particular, Chari, Kehoe and McGrattan (2007) consider a special case where on average financing frictions across firms are constant but relative distortions (interest rate spreads) fluctuate so that capital and labor taxes are constant while only the technology fluctuates. Periods in which relative distortions increase would show up as periods of technological regress (see their discussion on page 790-791).

[18]The 3.8% is taken from Table 1 of the Statistical Annex of EC's publication European Economic Forecast, Autumn 2011. The sequence of six productivity shocks are −0.89%, −0.89%, −0.89%, −0.89%, −0.45% and −0.45%.

[19]See Table 42, Statistical Annex of the European Commission's Economic Forecast, Autumn 2012. Under RE, log of budget balances and output share a common stochastic trend. So the productivity shocks will move the trend of log of budget and output by the same amount. Thus, the resulting trend component of budget balance to GDP ratio (or CAB) will not change in response to the shocks and is not plotted here.

[20]Greece is the only exception for output gaps (though not for structural balances).

[21]Note the maximum size of response of output under the AL model is 3.1% larger than that under the RE model following the larger sequence of shocks while it is 1.5% for the smaller shock sequence.

[22]In the data, the size of improvement in the structural budget balance due to fiscal consolidation from 2010-11 varies across countries; eg. Germany 0.2%, Belgium 0.5%, Italy 0.6%, Spain 1.2% and the average for the Euro area is approximately 1% (see Table 41 of the Statistical Annex of Autumn 2012 European Economic Forecast published by the EC). If we take 1% instead of 0.5% as the benchmark, our results are strengthened. Note the length of fiscal consolidation period differs across countries. Our results are also strengthened if the length of fiscal consolidation is longer.

[23]It seems after the consolidation period, output recovers quicker than without consolidation.

[24]The distribution is the stationary distribution of beliefs about trend output growth rates using the actual gain parameter of the policy maker.

[25]Similar assumptions are widespread in the AL literature eg. Adam, Beutel, Marcet and Merkel (2015) and Mitra, Evans and Honkapohja (2016).

[26]Our model does not feature shocks to trend productivity growth rates for two reasons. First, large shocks to the trend growth rate are rarely observed in industrialized economies. Second, while data on estimates of the trend growth rate of output and output per hour may display high positive autocorrelation, this autocorrelation can be *endogenously* obtained in learning models with *zero* correlation in trend productivity

growth; see the discussion at the end of Section 6 in Kuang and Mitra (2016).

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Appendix (For Online Publication)

This is the figure referred to in Section II.B..

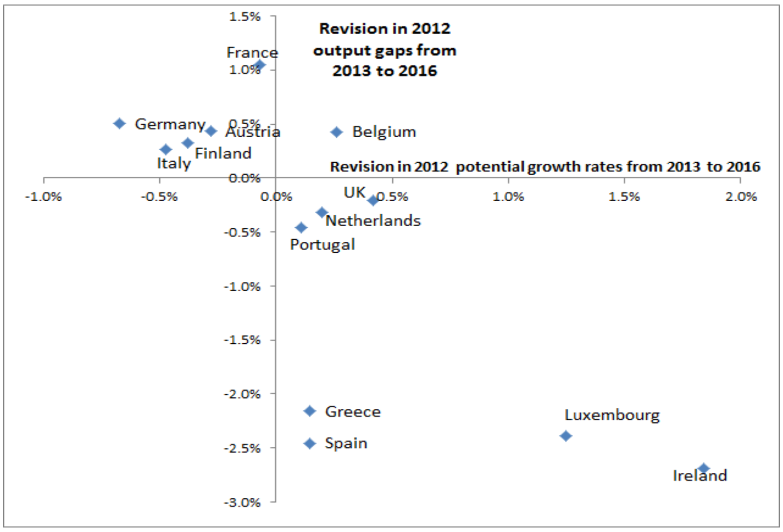


Figure A.1: Revision to the estimate of 2012 potential growth rates and output gaps: from 2013 Autumn to 2016 Autumn

A Evidence 4: fiscal consolidation and growth forecast errors

Blanchard and Leigh (2013) is an influential paper in the fiscal multiplier debate. Under RE, fiscal consolidation forecasts should be unrelated to subsequent growth forecast errors. However, they find larger planned fiscal consolidation has been associated with lower growth than expected, using data from IMF World Economic Outlook for 26 (EU and other advanced) economies. They interpret the result as substantial underestimation of fiscal multiplier by forecasters (at the IMF). Similar to Blanchard and Leigh (2013), House, Proebsting, and Tesar (2017) examine the relation between forecast errors in government purchases and forecast errors in GDP, inflation, consumption and investment for 29 advanced economies. They find that austerity policies in the wake of the Great Recession negatively affect economic performance by reducing GDP, inflation, consumption and investment and that variation in austerity policies can account for the differences in economic performance.

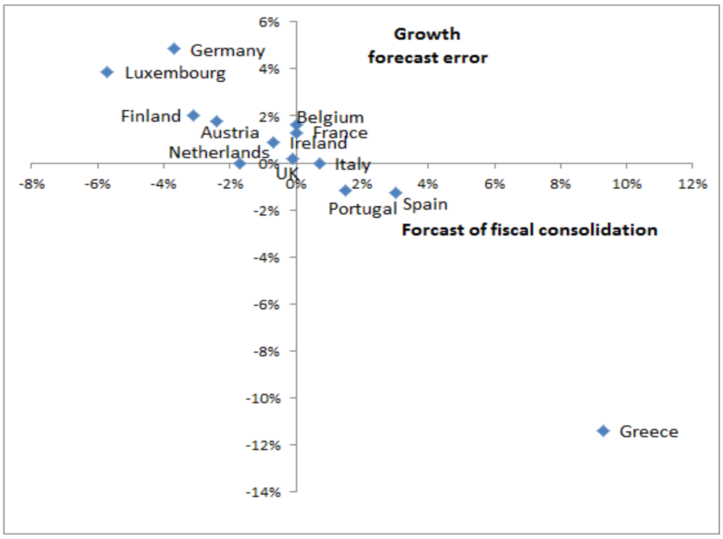


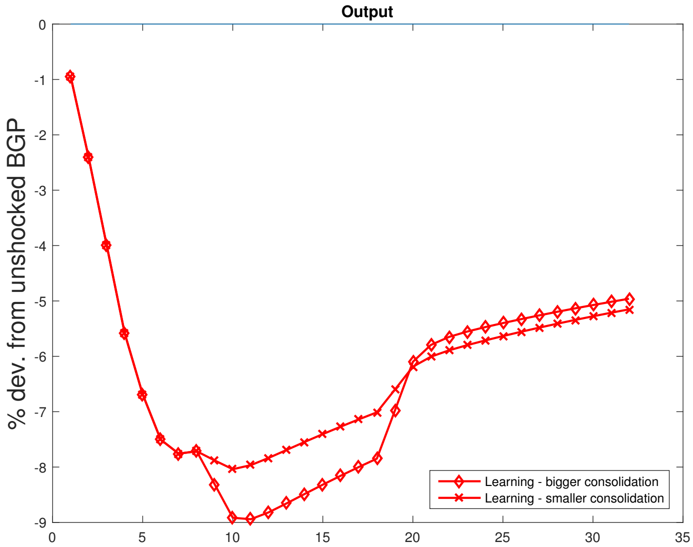
Figure A.2: Growth forecast errors versus fiscal consolidation forecasts

Notes: Figure plots forecast error for real GDP growth in 2010 and 2011 relative to forecasts made in the spring of 2010 on forecasts of fiscal consolidation for 2010 and 2011 made in Spring of year 2010.

Following Blanchard and Leigh (2013) but using the EC data, Figure A.2 illustrates a strong negative correlation (correlation coefficient of -0.95) between forecast of fiscal consolidation and subsequent growth forecast errors for our sample of countries. Growth forecast errors measure the difference between actual cumulative real GDP (year-over-year) growth during 2010-2011, minus the forecast made in Spring 2010. The forecast of fiscal consolidation is the forecast of the change in the structural balance ratio during 2010-2011 made in Spring 2010. In section B below, our learning model provides an illustration of the negative correlation between forecast of fiscal consolidation and growth forecast errors.

B Relation between fiscal consolidation and growth forecast errors

This section illustrates that our learning model can produce the negative correlation between the forecast of fiscal consolidation and growth forecast errors (Evidence 4). Consider two countries A and B which are subject to the same sequence of negative productivity shocks as in Section II.B.. After the shocks, country A implements a larger consolidation modeled as larger reductions in the steady state of government expenditures which is the same size as the consolidation implemented in Section VI.. Country B implements a smaller fiscal consolidation (e.g., 40% of the shocks to country A for every period). The two countries are simulated with the same parameterization except the size of fiscal consolidation.



A.3: Bigger vs smaller fiscal consolidation.

In the AL model, policy makers (and households) have imperfect knowledge and do not fully understand the general equilibrium consequences of fiscal consolidations. They extrapolate historical patterns (i.e. the estimated perceived law of motion) in output to forecast the growth rate of output. Thus, the forecasts will typically fail to be adequately dependent on the size of fiscal consolidation. The learning model in this way helps to explain the correlation between the size of fiscal consolidations and growth forecast errors. For country A (or B), the realized cumulative growth is lower (or higher) and hence the size of growth forecast errors is larger (or smaller). In this simulation, the size of the forecast error of output growth for country A is 0.97% larger than country B.

C Impulse response analysis

Figure A.4 depicts the impulse response functions under RE and learning for 40 quarters in response to an unexpected negative productivity shock of 0.45% in period 1. Percentage deviations from the unshocked balanced growth path are plotted for output, consumption and investment. Therefore, they will not return to zero in this figure. For hours, deviations from the steady state are plotted. For G/Y ratio and debt-to-GDP ratio, percentage changes are plotted.

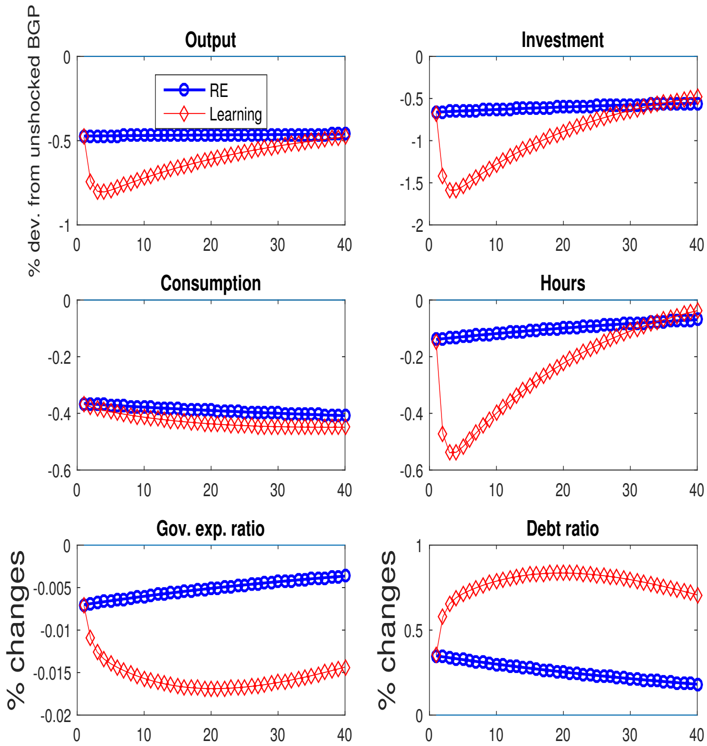


Figure A.4: Response to a 0.45% unexpected negative i.i.d productivity shock.

We first describe the dynamic effects under RE. There is an abundance of capital after the shock which decreases the marginal product of capital and hence the real interest rate. This implies the marginal utility of consumption must increase over time. Capacity utilization declines on impact due to the decline in rental rates. Output decreases initially because working hours, capacity utilization and productivity decrease initially. The low interest rates induce falling consumption and leisure over time. Output, on the other hand, increases slightly over time: this is because capacity utilization and working hours increase over time and this is sufficiently pronounced to offset the effect on output from the decline in capital stock. Debt-to-GDP ratio increases initially and then decreases gradually. Government expenditures to GDP ratio responds negatively to a rising debt-to-GDP ratio and positively to a negative output gap. However, the former effect dominates so that government expenditures to GDP ratio falls initially. Subsequently, in response to the fall in the debt ratio, government expenditures to GDP ratio increases gradually.

We now turn to the dynamics under AL. After the impact period, the low realization of rental rates and wage rates leads to downward revisions to the forecasts of the discounted sum of wages and rental rates. The dominance of the pessimism about future wage rates produces a further decline in consumption. The response of these forecasts following a productivity shock in our model is similar to those in the model of KM; see Figure 3 of KM which considers a *positive* shock (in contrast to the negative shock here). We suppress the plots of the forecast of discounted sum of wage rates and rental rates here. Working hours decrease further as firms' labor demand decreases and the constant-consumption labor supply shifts inward. The decline in the return to capital due to declining productivity and working hours induces an decline in investment. Output also decreases due to decreases in productivity, working hours, investment and capacity utilization. The decrease in output reduces tax revenues, which offsets the decline in government expenditures and leads to a persistent rise in debt-to-GDP ratio. At some point, investment and hours revert and move towards the steady state. During the initial periods of the reversion of output, the effect of lower tax revenues dominates so that the debt ratio continue to increase. Overall, the amplification in the response of output, hours and investment under learning arises mainly from households' belief revisions and the mutual reinforecement between households' wages (and income) forecasts and equilibrium outcomes.

CGU (2018) find that the estimates of potential output made by a range of public institutions decline (or rise) persistently in response to a negative (or positive) TFP shock. The RE version of the model is inconsistent with this finding because it does not produce a *persistent* response of estimate of potential output made by the policy maker to the i.i.d shock (due to her exact knowledge of the evoluation of potential output). In contrast, our learning model is consistent with this finding. The negative productivity shock leads to a fall in aggregate output and the policy maker uses the new observation of output to revise downward her estimate of potential output; see equation (11) and the top left plot of Figure A.5.

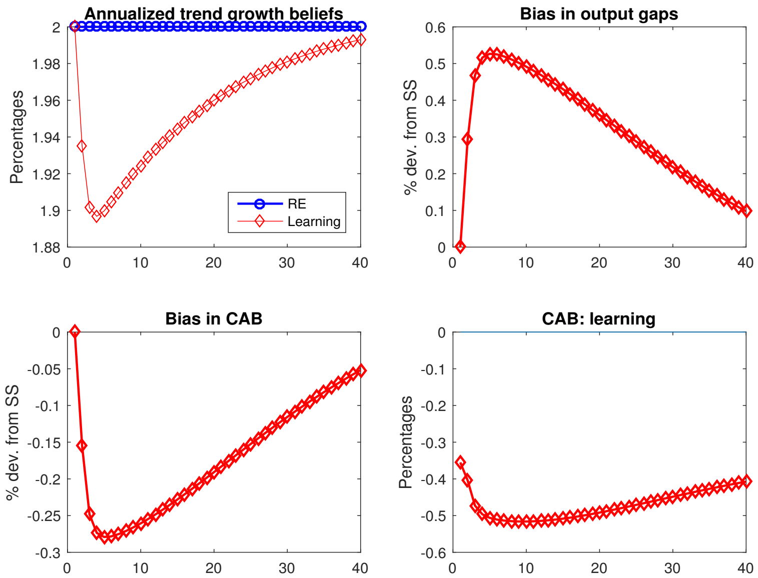


Figure A.5: Policy maker's beliefs: following a 0.45% unexpected negative i.i.d productivity shock.

The downward revision to subjective estimates of trend output made by the policy maker is associated with an underestimation of the size of output gaps, so the bias in the estimation of output gaps is positive (recall this bias is given by subjective minus true output gaps, see equation (12)). The underestimation of output gaps in turn generates an underestimation of CAB, as can be seen from the lower left plot. The size of the bias in estimating CAB is calculated by equation (14). Subjective estimates of CAB made by the policy maker become negative following the shock and decline further for several periods and then increase towards the steady state value, see the bottom right plot.

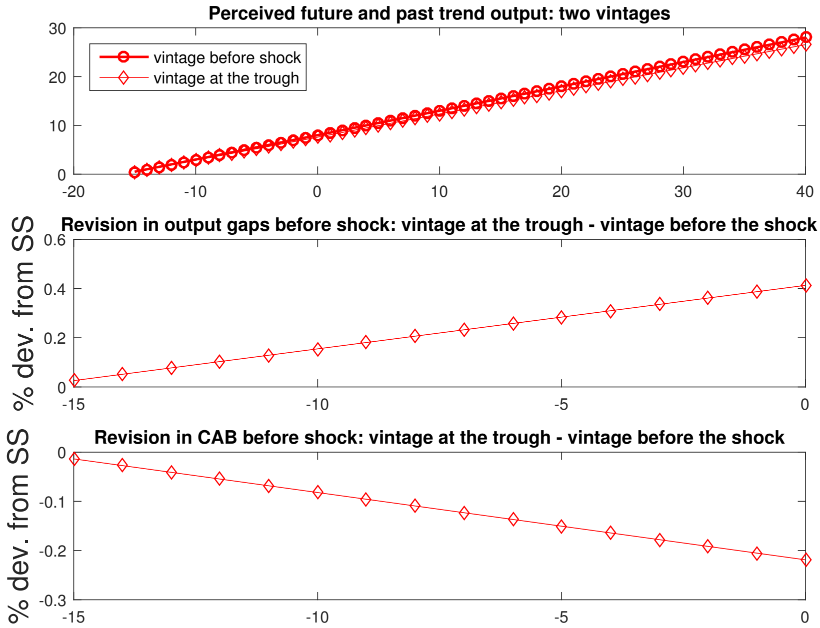


Figure A.6: Changes to policy maker's view following a 0.45% unexpected negative i.i.d productivity shock.

Note: Period $-n$ in Figure A.6 corresponds to $(n+1)$ quarters before the shock for $n = 0, 1, 2...$

We now study how the negative shock changes policy makers' beliefs about the evolution of trend output and historical estimates of output gaps and structural fiscal balances. We consider two vintages: one before the shock and the other when aggregate output reaches its trough (which is period 4 in this simulation). The top plot of Figure A.6 displays the evolution of the deterministic component of trend output before and after the shock (with initial period trend output normalized to zero). The quantitative difference is not that apparent in the top plot since the shock is only one-off. Before the shock, trend output (in the past and future) are perceived to grow at the rate of $log(\overline{\gamma})$. When output reaches its trough, policy makers perceive a lower growth rate of trend output for future and past (since policymakers use lower than expected growth rates to revise estimates of growth rates of trend output by equation (11)). Trend output up to 4 years backward (recall $N = 16$) and in the future are revised downward.

The downward revision to the historical estimate of trend output is associated with an upward revision to historical estimate of output gaps (the middle plot) and a downward revision to historical estimate of CABs (the lower plot).

Note under RE, the negative productivity shock will not lead to revisions to historical estimate of potential output, output gaps and CABs due to the policy maker's exact knowledge of the economy.

D The Model

The variables with a bar are the non-stochastic steady state values while the variables with a hat denote log-linearized variables around the non-stochastic steady state i.e. $x_t = \log \frac{X_t}{\bar{X}}$. Capital letters denote levels while small case letters denote their stationary counterpart.

A. Model Setup

Household's problem

The representative household maximizes

$$\begin{aligned} & \widehat{E}_t \sum_{t=0}^{\infty} \beta^t u(C_t, L_t); \\ u(C_t, L_t) &= \frac{C_t^{1-\sigma} v(1-L_t)}{1-\sigma} \end{aligned}$$

subject to the flow budget constraint

$$\begin{aligned} C_t + K_{t+1} + B_{t+1} &= \left(R_t^K U_t - \delta(U_t)\right)\left(1-\tau^K\right) K_t \\ &\quad + W_t H_t \left(1-\tau^H\right) + K_t + R_t B_t - T_t \end{aligned} \tag{D1}$$

$$L_t = 1 - H_t \tag{D2}$$

$\delta(U_t) = \frac{1}{\theta} U_t^{\theta}$. Define $A_t = B_t + K_t$.The arbitrage condition implies

$$R_t = 1 + \left(R_t^K U_t - \delta(U_t)\right)\left(1-\tau^K\right). \tag{D3}$$

The budget constraint can be simplified as

(D4) $$C_t + A_{t+1} = R_t A_t + W_t H_t \left(1 - \tau^H\right) - T_t$$

Evolution of capital $K_{t+1} = (1 - \delta(U_t))K_t + I_t$.

Firm's problem

The firm's problem is

(D5) $$\max_{U_t K_t, H_t} Y_t - W_t H_t - R_t^K (U_t K_t)$$

subject to the production technology

(D6) $$Y_t = (U_t K_t)^\alpha (X_t H_t)^{1-\alpha}$$

Resource constraint

(D7) $$Y_t = C_t + I_t + G_t$$

Fiscal rule

Government budget constraint

(D8) $$B_{t+1} = B_t R_t - \left(R_t^K U_t - \delta(U_t)\right) K_t \tau^K - W_t H_t \tau^H + G_t - T_t$$

Government expenditures follow

(D9) $$\begin{aligned} G_t/Y_t &= \lambda_0 + \lambda_1 OG_t + \lambda_2 B_t/Y_t + u_t^G, \\ \text{where } OG_t &= Y_t/Y_t^P - 1 \end{aligned}$$

Y_t^P policy makers' estimate of trend output.

B. First Order Condition

Households' first-order conditions

The Lagrangian for the household problem is

(D10)
$$\widehat{E}_t \sum_{t=0}^{\infty} \beta^t \left(u(C_t, L_t) + \Lambda_t \left(\begin{array}{c} \left(R_t^K U_t - \delta(U_t)\right)\left(1-\tau^K\right) K_t + W_t H_t \left(1-\tau^H\right) + \\ K_t + R_t B_t - C_t - K_{t+1} - B_{t+1} - T_t \end{array} \right) \right)$$

Household optimization yields the following first-order conditions:

$$\begin{array}{llcl}
\text{(D11)} & C_t & : & u_C(C_t, L_t) = \Lambda_t \\
\text{(D12)} & K_{t+1} & : & \beta \widehat{E}_t \Lambda_{t+1} R_{t+1} - \Lambda_t = 0 \\
\text{(D13)} & L_t & : & u_L(C_t, L_t) = \Lambda_t W_t \left(1-\tau^H\right) \\
\text{(D14)} & U_t & : & R_t^K = \delta'(U_t) \\
\text{(D15)} & B_{t+1} & : & \beta \widehat{E}_t \Lambda_{t+1} R_{t+1} = \Lambda_t
\end{array}$$

where Λ_t is the Lagrangian multiplier associated with period t budget constraint.

Firm's first-order conditions

The firm's first order condition with respect to hours is $W_t = (1-\alpha)\left(U_t K_t\right)^{\alpha} (X_t)^{1-\alpha} H_t^{-\alpha}$. The capital input decision gives $R_t^K = \alpha(U_t \frac{K_t}{X_t})^{\alpha-1}(H_t)^{1-\alpha}$.

C. Detrended Equations

For consumption,

$$\lambda_t \equiv X_t^{\sigma} \Lambda_t = X_t^{\sigma} u_c\left(C_t, L_t\right) = X_t^{\sigma} C_t^{-\sigma} \upsilon\left(H_t\right) = c_t^{-\sigma} \upsilon\left(H_t\right)$$

For capital,

$$\begin{aligned}
1 &= \beta \widehat{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left(1 + \left(R_{t+1}^K U_{t+1} - \delta(U_{t+1})\right)\left(1-\tau^K\right)\right) \right] \\
&= \beta \widehat{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{X_{t+1}^{\sigma}}{X_{t+1}^{\sigma}} \frac{X_t^{\sigma}}{X_t^{\sigma}} \left(1 + \left(R_{t+1}^K U_{t+1} - \delta(U_{t+1})\right)\left(1-\tau^K\right)\right) \right] \\
\text{(D16)} \qquad &= \beta \widehat{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{\gamma_{t+1}^{\sigma}} \left(1 + \left(R_{t+1}^K U_{t+1} - \delta(U_{t+1})\right)\left(1-\tau^K\right)\right) \right]
\end{aligned}$$

For leisure,

$$\begin{aligned}
\lambda_t w_t \left(1-\tau^H\right) &= X_t^{\sigma-1}\Lambda_t W_t \left(1-\tau^H\right) = X_t^{\sigma-1} u_L\left(C_t, L_t\right) \\
&= -X_t^{\sigma-1}\frac{C_t^{1-\sigma}}{1-\sigma}\upsilon'\left(H_t\right) = -\frac{c_t^{1-\sigma}}{1-\sigma}\upsilon'\left(H_t\right)
\end{aligned}$$

Evolution of capital

$$\begin{aligned}
\frac{K_{t+1}}{X_t} &= (1-\delta(U_t))\frac{K_t}{X_{t-1}}\frac{X_{t-1}}{X_t} + \frac{I_t}{X_t} \\
k_{t+1} &= (1-\delta(U_t))\frac{k_t}{\gamma_t} + i_t \qquad \text{(D17)}
\end{aligned}$$

The household budget constraint becomes

$$\begin{aligned}
\frac{C_t}{X_t} + \frac{K_{t+1}}{X_t} + \frac{B_{t+1}}{X_t} &= \left[1 + \left(R_t^K U_t - \delta(U_t)\right)\left(1-\tau^K\right)\right]\frac{K_t}{X_{t-1}}\frac{X_{t-1}}{X_t} \\
&\quad + \frac{W_t}{X_t}H_t\left(1-\tau^H\right) + R_t\frac{B_t}{X_{t-1}}\frac{X_{t-1}}{X_t} - \tau \\
c_t + k_{t+1} + b_{t+1} &= \left[1 + \left(R_t^K U_t - \delta(U_t)\right)\left(1-\tau^K\right)\right]\frac{k_t}{\gamma_t} \\
&\quad + w_t H_t\left(1-\tau^H\right) + R_t\frac{b_t}{\gamma_t} - \tau
\end{aligned}$$

where we assume T_t/X_t is a constant τ.

The simplified household budget constraint ($D4$) becomes

$$\begin{aligned}
\frac{C_t}{X_t} + \frac{A_{t+1}}{X_t} &= R_t\frac{A_t}{X_{t-1}}\frac{X_{t-1}}{X_t} + \frac{W_t H_t}{X_t}\left(1-\tau^H\right) - \tau \qquad \text{(D18)} \\
c_t + a_{t+1} &= R_t a_t \gamma_t^{-1} + w_t H_t\left(1-\tau^H\right) - \tau \qquad \text{(D19)}
\end{aligned}$$

The production function

$$\begin{aligned} Y_t &= (U_t K_t)^\alpha (X_t H_t)^{1-\alpha} \\ \frac{Y_t}{X_t} &= (U_t \frac{K_t}{X_{t-1}} \frac{X_{t-1}}{X_t})^\alpha (H_t)^{1-\alpha} \\ y_t &= (U_t k_t \gamma_t^{-1})^\alpha (H_t)^{1-\alpha} \end{aligned}$$

For capital demand,

$$\begin{aligned} R_t^K &= \alpha (U_t \frac{K_t}{X_t})^{\alpha-1} (H_t)^{1-\alpha} \\ &= \alpha (U_t \frac{K_t}{X_{t-1}} \frac{X_{t-1}}{X_t})^{\alpha-1} (H_t)^{1-\alpha} \\ &= \alpha (\frac{U_t k_t}{\gamma_t})^{\alpha-1} (H_t)^{1-\alpha} \qquad \text{(D20)} \end{aligned}$$

Using the definition of output yields

$$R_t^K = \alpha \gamma_t \frac{y_t}{U_t k_t} \qquad \text{(D21)}$$

For labor demand,

$$\begin{aligned} W_t &= (1-\alpha) (U_t K_t)^\alpha (X_t)^{1-\alpha} H_t^{-\alpha} \\ \frac{W_t}{X_t} &= (1-\alpha) (U_t \frac{K_t}{X_{t-1}} \frac{X_{t-1}}{X_t})^\alpha H_t^{-\alpha} \\ w_t &= (1-\alpha) \gamma_t^{-\alpha} (U_t k_t)^\alpha H_t^{-\alpha} \end{aligned}$$

Using the definition of output gives

$$w_t = (1-\alpha) \frac{y_t}{H_t} \qquad \text{(D22)}$$

Government budget constraint

$$
\begin{aligned}
B_{t+1} &= B_t R_t - \left(R_t^K U_t - \delta(U_t)\right) K_t \tau^K - W_t H_t \tau^H + G_t - T_t \\
\frac{B_{t+1}}{X_t} &= \frac{B_t}{X_{t-1}} \frac{X_{t-1}}{X_t} R_t - \left(R_t^K U_t - \delta(U_t)\right) \frac{K_t}{X_{t-1}} \frac{X_{t-1}}{X_t} \tau^K - \frac{W_t}{X_t} H_t \tau^H + \frac{G_t}{X_t} - \frac{T_t}{X_t} \\
b_{t+1} &= \frac{b_t}{\gamma_t} R_t - \left(R_t^K U_t - \delta(U_t)\right) \frac{k_t}{\gamma_t} \tau^K - w_t H_t \tau^H + g_t - \tau
\end{aligned}
$$

Resource constraint (D7)

$$
\begin{aligned}
\frac{Y_t}{X_t} &= \frac{C_t}{X_t} + \frac{I_t}{X_t} + \frac{G_t}{X_t} \\
y_t &= c_t + i_t + g_t \qquad \text{(D23)}
\end{aligned}
$$

Government expenditure

$$
\begin{aligned}
\frac{G_t/X_t}{Y_t/X_t} &= \lambda_0 + \lambda_1 OG_t + \lambda_2 \frac{\frac{B_t}{X_{t-1}} \frac{X_{t-1}}{X_t}}{\frac{Y_t}{X_t}} + u_t^G \qquad \text{(D24)} \\
\frac{g_t}{y_t} &= \lambda_0 + \lambda_1 \frac{y_t - y_t^P}{y_t^P} + \lambda_2 \frac{b_t}{y_t} \gamma_t^{-1} + u_t^G \qquad \text{(D25)} \\
OG_t &= \frac{y_t - y_t^P}{y_t^P} \qquad \text{(D26)}
\end{aligned}
$$

The definition of A_t

$$
\begin{aligned}
A_t &= B_t + K_t \\
\frac{A_t}{X_{t-1}} &= \frac{B_t}{X_{t-1}} + \frac{K_t}{X_{t-1}} \\
a_t &= b_t + k_t \qquad \text{(D27)}
\end{aligned}
$$

D. Steady State

The definition of interest rates (D3) yields

(D28) $$\overline{R} = \left[1 + \left(\overline{R}^K\overline{U} - \overline{\delta}\right)\left(1 - \tau^K\right)\right]$$

The Euler equation (D16) implies

$$1 = \beta \frac{1}{\overline{\gamma}^{\sigma}} \overline{R}$$

and

$$\overline{R} = \frac{\overline{\gamma}^{\sigma}}{\beta}$$

Define $\widetilde{\beta} \equiv \overline{R}^{-1}\overline{\gamma}$. So

$$\overline{R} = \frac{\overline{\gamma}}{\widetilde{\beta}}$$

and

$$\beta = \widetilde{\beta}\overline{\gamma}^{\sigma-1}$$

(D28) gives

(D29) $$\overline{R}^K\overline{U} = \frac{\overline{R} - 1}{1 - \tau^K} + \overline{\delta}$$

The capacity utilization equation (D14) implies $\overline{R}^K = \delta'(\overline{U}) = \overline{U}^{\theta-1} = \frac{\theta\overline{\delta}}{\overline{U}}$ or equivalently

(D30) $$\theta = \frac{\overline{R}^K\overline{U}}{\overline{\delta}} = \frac{\overline{R} - 1}{\overline{\delta}\left(1 - \tau^K\right)} + 1$$

The capital demand equation (D21) implies

$$\begin{aligned} R_t^K &= \alpha\gamma_t \frac{y_t}{U_t k_t} \\ \frac{\overline{R}^K\overline{Uk}}{\overline{y\gamma}} &= \alpha \end{aligned}$$

Note

(D31)
$$\frac{\overline{k}}{\overline{y}} = \frac{\alpha\overline{\gamma}}{\overline{R}^K\overline{U}} = \frac{\alpha\overline{\gamma}}{\frac{\overline{R}-1}{1-\tau^K} + \overline{\delta}}$$

The labor demand equation (D22) implies

$$\frac{\overline{w}\overline{H}}{\overline{y}} = 1 - \alpha$$

The household budget constraint (D19) implies

(D32)
$$\overline{c} + \overline{a} = \overline{R}\overline{a}\overline{\gamma}^{-1} + \overline{w}\overline{H}\left(1 - \tau^H\right) - \tau$$

(D33)
$$\frac{\overline{c}}{\overline{a}} + 1 = \overline{R}\overline{\gamma}^{-1} + \frac{\overline{w}\overline{H}}{\overline{a}}\left(1 - \tau^H\right) - \frac{\tau}{\overline{a}}$$

The capital accumulation equation (D17) yields

(D34)
$$k_{t+1} = (1 - \delta(U_t))\frac{k_t}{\gamma_t} + i_t$$

$$\overline{k} = (1 - \overline{\delta})\frac{\overline{k}}{\overline{\gamma}} + \overline{i}$$

(D35)
$$\frac{\overline{i}}{\overline{k}} = 1 - \frac{(1 - \overline{\delta})}{\overline{\gamma}}$$

Investment to output ratio is determined by combining (D31) and (D35)

$$\frac{\overline{i}}{\overline{y}} = \frac{\overline{i}/\overline{k}}{\overline{y}/\overline{k}}$$

(D36)
$$= \frac{1 - \frac{(1-\overline{\delta})}{\overline{\gamma}}}{\frac{\overline{R}^K\overline{U}}{\alpha\overline{\gamma}}}$$

The resource constraint (D23) yields

(D37)
$$1=\frac{\overline{c}}{\overline{y}}+\frac{\overline{i}}{\overline{y}}+\frac{\overline{g}}{\overline{y}}$$

So

(D38)
$$\frac{\overline{c}}{\overline{y}}=1-\frac{\overline{i}}{\overline{y}}-\frac{\overline{g}}{\overline{y}}$$

The government budget constraint yields

$$\begin{aligned}
b_{t+1} &= \frac{b_t}{\gamma_t}R_t-\left(R_t^K U_t-\delta\left(U_t\right)\right)\frac{k_t}{\gamma_t}\tau^K-w_t H_t\tau^H+g_t-\tau \\
\overline{b} &= \frac{\overline{b}}{\overline{\gamma}}\overline{R}-\left(\overline{R}^K\overline{U}-\overline{\delta}\right)\frac{\overline{k}}{\overline{\gamma}}\tau^K-\overline{w}\overline{H}\tau^H+\overline{g}-\tau \\
\text{(D39)}\quad \frac{\overline{b}}{\overline{y}}\left(1-\frac{\overline{R}}{\overline{\gamma}}\right) &= -\frac{\overline{R}^K\overline{U}-\overline{\delta}}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\tau^K-\frac{\overline{w}\overline{H}}{\overline{y}}\tau^H+\frac{\overline{g}}{\overline{y}}-\tau \\
\text{(D40)}\quad &= -\alpha\tau^K-(1-\alpha)\tau^H+\frac{\overline{g}}{\overline{y}}+\frac{\overline{\delta}}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\tau^K-\tau
\end{aligned}$$

The fiscal rule (D25) yields

(D41)
$$\frac{\overline{g}}{\overline{y}}=\lambda_0+\lambda_2\frac{\overline{b}}{\overline{y}}\overline{\gamma}^{-1}$$

The definition of A_t (D27) yields

(D42)
$$\frac{\overline{a}}{\overline{y}}=\frac{\overline{b}}{\overline{y}}+\frac{\overline{k}}{\overline{y}}$$

The marginal utility of consumption gives

$$\overline{\lambda}=\overline{c}^{-\sigma}\upsilon(\overline{H})$$

and the first order condition for labor supply yields

$$\overline{\lambda}\overline{w}(1-\tau^H) = -\frac{\overline{c}^{1-\sigma}}{1-\sigma}\upsilon'(\overline{H})$$

Combining the above two equations delivers

$$\begin{aligned}
\overline{c}^{-\sigma}\upsilon(\overline{H})\overline{w}(1-\tau^H) &= -\frac{\overline{c}^{1-\sigma}}{1-\sigma}\upsilon'(\overline{H}) \\
\frac{\overline{w}\overline{H}(1-\tau^H)}{\overline{c}} &= -\frac{\upsilon'(\overline{H})\overline{H}}{\upsilon(\overline{H})}(1-\sigma)^{-1}
\end{aligned}$$

Define

$$\begin{aligned}
\psi &\equiv -\frac{\overline{H}\upsilon'(\overline{H})}{\upsilon(\overline{H})}(1-\sigma)^{-1} = \frac{\overline{w}\overline{H}(1-\tau^H)}{\overline{c}} \\
&= \frac{\overline{w}\overline{H}}{\overline{y}}\frac{\overline{y}}{\overline{c}}(1-\tau^H) = (1-\alpha)\frac{\overline{y}}{\overline{c}}(1-\tau^H)
\end{aligned}$$

E. Log-linearization

Households

1. Marginal utility of consumption

(D43) $$\widehat{\lambda}_t = -\sigma\widehat{c}_t - \psi(1-\sigma)\widehat{H}_t$$

where in steady state

$$\psi \equiv -\frac{\overline{H}\upsilon'(\overline{H})}{\upsilon(\overline{H})}(1-\sigma)^{-1} = \frac{\overline{w}\overline{H}}{\overline{c}}\left(1-\tau^H\right)$$

2. Euler equation. Starting from

$$
\begin{aligned}
1 &= \beta \widehat{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\gamma_{t+1}^{\sigma}}\left(1+\left(R_{t+1}^K U_{t+1}-\delta(U_{t+1})\right)\left(1-\tau^K\right)\right)\right] \\
0 &= \frac{\beta}{\overline{\gamma}^{\sigma}}\widehat{E}_t\left[\overline{R}\left(\widehat{\lambda}_{t+1}-\widehat{\lambda}_t-\sigma\widehat{\gamma}_{t+1}\right)+\overline{R}^K\overline{U}\left(1-\tau^K\right)\left(\widehat{R}_{t+1}^K+\widehat{U}_{t+1}\right)-\overline{\delta}\theta\left(1-\tau^K\right)\widehat{U}_{t+1}\right] \\
0 &= \widehat{E}_t\left[\overline{R}\left(\widehat{\lambda}_{t+1}-\widehat{\lambda}_t-\sigma\widehat{\gamma}_{t+1}\right)+\overline{R}^K\overline{U}\left(1-\tau^K\right)\widehat{R}_{t+1}^K\right] \\
0 &= \widehat{E}_t\left[\left(\widehat{\lambda}_{t+1}-\widehat{\lambda}_t-\sigma\widehat{\gamma}_{t+1}\right)+\frac{\beta}{\overline{\gamma}^{\sigma}}\overline{R}^K\overline{U}\left(1-\tau^K\right)\widehat{R}_{t+1}^K\right]
\end{aligned}
$$

Note $\beta\frac{\overline{R}}{\overline{\gamma}^{\sigma}}=1$, $\delta^{'}(\overline{U})\overline{U}=\delta\theta$.

3. Labor-leisure choice

$$
\begin{aligned}
\lambda_t w_t\left(1-\tau^H\right) &= -\frac{c_t^{1-\sigma}}{1-\sigma}\upsilon^{'}\left(H_t\right) \\
\overline{\lambda}\overline{w}\left(\widehat{\lambda}_t+\widehat{w}_t\right) &= -\frac{\overline{c}^{1-\sigma}}{(1-\sigma)\left(1-\tau^H\right)}\upsilon^{'}\left(\overline{H}\right)\left((1-\sigma)\widehat{c}_t+\frac{\upsilon^{''}\left(\overline{H}\right)\overline{H}}{\upsilon^{'}\left(\overline{H}\right)}\widehat{H}_t\right) \\
(1-\sigma)\widehat{c}_t+\epsilon_v\widehat{H}_t &= \widehat{\lambda}_t+\widehat{w}_t \\
\epsilon_v &= \frac{\upsilon^{''}\left(\overline{H}\right)\overline{H}}{\upsilon^{'}\left(\overline{H}\right)}>0
\end{aligned}
$$

Combining the second equation above with the expression for marginal utility, gives

(D44) $$\sigma^{-1}\widehat{\lambda}_t+\widehat{w}_t=\epsilon_H\widehat{H}_t$$

where

$$\epsilon_H=\epsilon_v-\frac{(\sigma-1)^2}{\sigma}\psi>0$$

is the inverse Frisch elasticity of labor supply.

4. Capital utilization

$$
\begin{aligned}
R_t^K &= \delta'(U_t) \\
\widehat{R}_t^K \overline{R}^K &= \delta' \frac{\delta'' \overline{U}}{\delta'} \widehat{U}_t \\
\widehat{U}_t &= \frac{1}{\theta - 1} \widehat{R}_t^K
\end{aligned}
$$

Note $\frac{\delta'' \overline{U}}{\delta'} = \theta - 1$.

Using the expression for marginal utility of consumption, the Euler equation yields

$$
\begin{aligned}
0 &= \widehat{E}_t \left[\left(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t - \sigma \widehat{\gamma}_{t+1} \right) + \beta \frac{\overline{R}^k \overline{U} \left(1 - \tau^K\right)}{\overline{\gamma}^\sigma} \widehat{R}_{t+1}^K \right] \\
\widehat{\lambda}_t &= \widehat{E}_t \left[\left(\widehat{\lambda}_{t+1} - \sigma \widehat{\gamma}_{t+1} \right) + \beta \frac{\overline{R}^k \overline{U} \left(1 - \tau^K\right)}{\overline{\gamma}^\sigma} \widehat{R}_{t+1}^K \right] \\
-\sigma \widehat{c}_t - \psi(1-\sigma) \widehat{H}_t &= \widehat{E}_t \left[-\sigma \widehat{c}_{t+1} - \psi(1-\sigma) \widehat{H}_{t+1} \right] - \sigma \widehat{E}_t \widehat{\gamma}_{t+1} + \widehat{E}_t \beta \frac{\overline{R}^k \overline{U} \left(1 - \tau^K\right)}{\overline{\gamma}^\sigma} \widehat{R}_{t+1}^K \\
\widehat{Q}_t &= -\widehat{E}_t \beta \frac{\overline{R}^k \overline{U} \left(1 - \tau^K\right)}{\overline{\gamma}^\sigma} \widehat{R}_{t+1}^K + \widehat{E}_t \widehat{Q}_{t+1} + \sigma \widehat{E}_t \widehat{\gamma}_{t+1}
\end{aligned}
$$

where

$$
\widehat{Q}_t = \sigma \widehat{c}_t + \psi(1-\sigma) \widehat{H}_t.
$$

Combining the marginal utility of consumption and labor-leisure choice yields

(D45)
$$
\left(\epsilon_H - \frac{\sigma - 1}{\sigma} \psi \right) \widehat{H}_t = \widehat{w}_t - \widehat{c}_t
$$

Linearizing the interest rate

$$
\begin{aligned}
R_t &= 1+\left(R_t^K U_t-\delta(U_t)\right)\left(1-\tau^K\right) \\
\overline{R}\widehat{R}_t &= \left(\overline{R}^K\overline{U}\left(\widehat{R}_t^K+\widehat{U}_t\right)-\overline{\delta}\theta\widehat{U}_t\right)\left(1-\tau^K\right) \\
&= \overline{R}^K\overline{U}\left(1-\tau^K\right)\widehat{R}_t^K \\
\widehat{R}_t &= \frac{\overline{R}^K\overline{U}}{\overline{R}}\left(1-\tau^K\right)\widehat{R}_t^K
\end{aligned}
$$

Firms

Labor demand equation becomes

(D46) $$\widehat{w}_t=\widehat{y}_t-\widehat{H}_t$$

Using the definition of output yields

$$R_t^K=\alpha\gamma_t\frac{y_t}{U_t k_t}$$

which in log-linear form is

$$\widehat{R}_t^K=\widehat{\gamma}_t+\widehat{y}_t-\widehat{U}_t-\widehat{k}_t$$

The evolution of capital is

$$\widehat{k}_{t+1}=\frac{\overline{i}}{\overline{k}}\widehat{i}_t+\frac{(1-\delta)}{\overline{\gamma}}\left(\widehat{k}_t-\widehat{\gamma}_t\right)-\frac{\delta\theta}{\overline{\gamma}}\widehat{U}_t$$

The production function

$$
\begin{aligned}
y_t &= (U_t k_t\gamma_t^{-1})^{\alpha}(H_t)^{1-\alpha} \\
\widehat{y}_t &= \alpha\widehat{U}_t+\alpha\widehat{k}_t-\alpha\widehat{\gamma}_t+(1-\alpha)\widehat{H}_t
\end{aligned}
$$

Good market clearing condition

$$
\begin{aligned}
y_t &= c_t + i_t + g_t \\
\widehat{y}_t &= \frac{\overline{c}}{\overline{y}}\widehat{c}_t + \frac{\overline{i}}{\overline{y}}\widehat{i}_t + \frac{\overline{g}}{\overline{y}}\widehat{g}_t
\end{aligned}
$$

The definition of a_t

$$
\begin{aligned}
a_t &= b_t + k_t \\
\frac{\overline{a}}{\overline{y}}\widehat{a}_t &= \frac{\overline{b}}{\overline{y}}\widehat{b}_t + \frac{\overline{k}}{\overline{y}}\widehat{k}_t
\end{aligned}
$$

Government budget constraint

$$
\begin{aligned}
b_{t+1} &= \frac{b_t}{\gamma_t}R_t - \left(R_t^K U_t - \delta\left(U_t\right)\right)\frac{k_t}{\gamma_t}\tau^K - w_t H_t \tau^H + g_t - \tau \\
\overline{b}\widehat{b}_{t+1} &= \overline{b}\frac{\overline{R}}{\overline{\gamma}}\left(\widehat{b}_t - \widehat{\gamma}_t + \widehat{R}_t\right) - \left(\overline{R}^K\overline{U} - \overline{\delta}\right)\tau^K\frac{\overline{k}}{\overline{\gamma}}\left(\widehat{k}_t - \widehat{\gamma}_t\right) \\
&\quad - \left(\overline{R}^K\overline{U}\left(\widehat{R}_t^K + \widehat{U}_t\right) - \overline{\delta}\theta\widehat{U}_t\right)\tau^K\frac{\overline{k}}{\overline{\gamma}} - \overline{w}\overline{H}\tau^H\left(\widehat{w}_t + \widehat{H}_t\right) + \overline{g}\widehat{g}_t \\
\overline{b}\widehat{b}_{t+1} &= \overline{b}\frac{\overline{R}}{\overline{\gamma}}\left(\widehat{b}_t - \widehat{\gamma}_t + \widehat{R}_t\right) - \left(\overline{R}^K\overline{U} - \overline{\delta}\right)\tau^K\frac{\overline{k}}{\overline{\gamma}}\left(\widehat{k}_t - \widehat{\gamma}_t\right) \\
&\quad - \tau^K\frac{\overline{k}}{\overline{\gamma}}\overline{R}^K\overline{U}\widehat{R}_t^K - \overline{w}\overline{H}\tau^H\left(\widehat{w}_t + \widehat{H}_t\right) + \overline{g}\widehat{g}_t \\
\frac{\overline{b}}{\overline{y}}\widehat{b}_{t+1} &= \frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\left(\widehat{b}_t - \widehat{\gamma}_t + \widehat{R}_t\right) - \left(\overline{R}^K\overline{U} - \overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\left(\widehat{k}_t - \widehat{\gamma}_t\right) \\
&\quad - \tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\overline{R}^K\overline{U}\widehat{R}_t^K - \frac{\overline{w}\overline{H}}{\overline{y}}\tau^H\left(\widehat{w}_t + \widehat{H}_t\right) + \frac{\overline{g}}{\overline{y}}\widehat{g}_t \\
\frac{\overline{b}}{\overline{y}}\widehat{b}_{t+1} &= \frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\left(\widehat{b}_t + \widehat{R}_t\right) - \left(\overline{R}^K\overline{U} - \overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\widehat{k}_t \\
&\quad - \tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\overline{R}^K\overline{U}\widehat{R}_t^K - \left(1-\alpha\right)\tau^H\left(\widehat{w}_t + \widehat{H}_t\right) + \frac{\overline{g}}{\overline{y}}\widehat{g}_t \\
&\quad + \left(\left(\overline{R}^K\overline{U} - \overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}} - \frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\right)\widehat{\gamma}_t
\end{aligned}
$$

Under RE, government expenditure (fiscal rule)

$$\begin{aligned}
\frac{g_t}{y_t} &= \lambda_0 + \lambda_1 \frac{y_t - y_t^P}{y_t^P} + \lambda_2 \frac{b_t}{y_t} \gamma_t^{-1} \\
\frac{\overline{g}}{\overline{y}} \left(\widehat{g}_t - \widehat{y}_t \right) &= \lambda_1 \widehat{y}_t + \lambda_2 \frac{\overline{b}}{\overline{y}} \overline{\gamma}^{-1} \left(\widehat{b}_t - \widehat{y}_t - \widehat{\gamma}_t \right) \\
\frac{\overline{g}}{\overline{y}} \widehat{g}_t &= \left(\frac{\overline{g}}{\overline{y}} + \lambda_1 - \lambda_2 \frac{\overline{b}}{\overline{y}} \overline{\gamma}^{-1} \right) \widehat{y}_t + \lambda_2 \frac{\overline{b}}{\overline{y}} \overline{\gamma}^{-1} \left(\widehat{b}_t - \widehat{\gamma}_t \right)
\end{aligned}$$

F. Consumption decision rule

Linearizing the budget constraint

$$c_t + a_{t+1} = R_t a_t \gamma_t^{-1} + w_t H_t \left(1 - \tau^H\right) - \tau$$

yields

$$\begin{aligned}
\overline{c}\widehat{c}_t + \overline{a}\widehat{a}_{t+1} &= \overline{R}\overline{a}\overline{\gamma}^{-1} \left(\widehat{R}_t + \widehat{a}_t - \widehat{\gamma}_t \right) + \overline{w}\overline{H} \left(1 - \tau^H\right) \left(\widehat{w}_t + \widehat{H}_t \right) \\
\text{(D47)} \quad \frac{\overline{c}}{\overline{a}} \widehat{c}_t + \widehat{a}_{t+1} &= \overline{R}\overline{\gamma}^{-1} \left(\widehat{R}_t + \widehat{a}_t - \widehat{\gamma}_t \right) + \frac{\overline{w}\overline{H}}{\overline{a}} \left(1 - \tau^H\right) \left(\widehat{w}_t + \widehat{H}_t \right)
\end{aligned}$$

where $\frac{\overline{w}\overline{H}}{\overline{a}} = \frac{\overline{w}\overline{H}}{\overline{y}} \div \frac{\overline{a}}{\overline{y}}$.

Combining (D45) and (D47) yields

$$\begin{aligned}
\frac{\overline{c}}{\overline{a}} \widehat{c}_t + \widehat{a}_{t+1} &= \overline{R}\overline{\gamma}^{-1} \left(\widehat{R}_t + \widehat{a}_t - \widehat{\gamma}_t \right) + \frac{\overline{w}\overline{H}}{\overline{a}} \left(1 - \tau^H\right) \left(\widehat{w}_t + \left(\epsilon_H - \frac{\sigma - 1}{\sigma} \psi \right)^{-1} \left(\widehat{w}_t - \widehat{c}_t \right) \right) \\
\frac{\overline{c}}{\overline{a}} \widehat{c}_t + \widehat{a}_{t+1} &= \overline{R}\overline{\gamma}^{-1} \left(\widehat{R}_t + \widehat{a}_t - \widehat{\gamma}_t \right) \\
&\quad + \frac{\overline{w}\overline{H}}{\overline{a}} \left(1 - \tau^H\right) \left(\left(1 + \left(\epsilon_H - \frac{\sigma - 1}{\sigma} \psi \right)^{-1} \right) \widehat{w}_t - \left(\epsilon_H - \frac{\sigma - 1}{\sigma} \psi \right)^{-1} \widehat{c}_t \right)
\end{aligned}$$

$$
\begin{aligned}
\widehat{a}_t &= \widehat{\gamma}_t - \widehat{R}_t + \overline{R}^{-1}\overline{\gamma}\left(\frac{\overline{c}}{\overline{a}}\widehat{c}_t + \widehat{a}_{t+1}\right) \\
&\quad -\overline{R}^{-1}\overline{\gamma}\frac{\overline{w}\overline{H}}{\overline{a}}\left(1-\tau^H\right)\left(\left(1+\left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\right)\widehat{w}_t - \left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\widehat{c}_t\right) \\
&= \widehat{\gamma}_t - \widehat{R}_t + \overline{R}^{-1}\overline{\gamma}\widehat{a}_{t+1} + \overline{R}^{-1}\overline{\gamma}\left(\frac{\overline{c}}{\overline{a}} + \frac{\overline{w}\overline{H}}{\overline{a}}\left(1-\tau^H\right)\left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\right)\widehat{c}_t \\
&\quad -\overline{R}^{-1}\overline{\gamma}\frac{\overline{w}\overline{H}}{\overline{a}}\left(1-\tau^H\right)\left(1+\left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\right)\widehat{w}_t \\
&= \widehat{\gamma}_t - \widehat{R}_t + \widetilde{\beta}\widehat{a}_{t+1} + \widetilde{\beta}\epsilon_c\widehat{c}_t - \widetilde{\beta}\epsilon_w\widehat{w}_t
\end{aligned}
$$

where $\widetilde{\beta} \equiv \overline{R}^{-1}\overline{\gamma}$, $\epsilon_c \equiv \frac{\overline{c}}{\overline{a}} + \frac{\overline{w}\overline{H}}{\overline{a}}\left(1-\tau^H\right)\left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1} = \frac{\overline{c}/\overline{y}}{\overline{a}/\overline{y}} + \frac{(1-\alpha)}{\overline{a}/\overline{y}}\left(1-\tau^H\right)\left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}$, and $\epsilon_w \equiv \frac{\overline{w}\overline{H}}{\overline{a}}\left(1-\tau^H\right)\left(1+\left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\right) = \frac{1-\alpha}{\overline{a}/\overline{y}}\left(1-\tau^H\right)\left(1+\left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}\right)$.

Iterating forward and taking expectations

$$
\begin{aligned}
\widehat{a}_t &= \widehat{\gamma}_t - \widehat{R}_t + \widetilde{\beta}\widehat{a}_{t+1} + \widetilde{\beta}\epsilon_c\widehat{c}_t - \widetilde{\beta}\epsilon_w\widehat{w}_t \\
&= \widehat{\gamma}_t - \widehat{R}_t + \widetilde{\beta}\left(\widehat{\gamma}_{t+1} - \widehat{R}_{t+1} + \widetilde{\beta}\widehat{a}_{t+2} + \widetilde{\beta}\epsilon_c\widehat{c}_{t+1} - \widetilde{\beta}\epsilon_w\widehat{w}_{t+1}\right) \\
&\quad +\widetilde{\beta}\epsilon_c\widehat{c}_t - \widetilde{\beta}\epsilon_w\widehat{w}_t \\
&= \ldots \\
&= \sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\left(\widehat{\gamma}_T - \widehat{R}_T\right) + \sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\widetilde{\beta}\epsilon_c\widehat{c}_T - \sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\widetilde{\beta}\epsilon_w\widehat{w}_T \\
\widehat{a}_t &= \widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\left(\widehat{\gamma}_T - \widehat{R}_T\right) + \widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\widetilde{\beta}\left(\epsilon_c\widehat{c}_T - \epsilon_w\widehat{w}_T\right) \\
\epsilon_c\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\widehat{c}_T &= \widetilde{\beta}^{-1}\widehat{a}_t + \widetilde{\beta}^{-1}\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\left(\widehat{R}_T - \widehat{\gamma}_T\right) + \widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\epsilon_w\widehat{w}_T \\
&= \widetilde{\beta}^{-1}\widehat{a}_t + \widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\left(\widetilde{\beta}^{-1}\widehat{R}_T - \widetilde{\beta}^{-1}\widehat{\gamma}_T + \epsilon_w\widehat{w}_T\right)
\end{aligned}
$$

Recall the Euler equation

$$
\widehat{Q}_t = -\widehat{E}_t\beta\frac{\overline{R}^k\overline{U}\left(1-\tau^K\right)}{\overline{\gamma}^{\sigma}}\widehat{R}^K_{t+1} + \widehat{E}_t\widehat{Q}_{t+1} + \sigma\widehat{E}_t\widehat{\gamma}_{t+1}
$$

Define $\widetilde{R}$ such that $\beta\frac{\overline{R}^k\overline{U}\left(1-\tau^K\right)}{\overline{\gamma}^\sigma} = \widetilde{\beta}\widetilde{R}$. Solving backward from time T gives

$$\begin{aligned}
\widehat{E}_t\widehat{Q}_T &= \widehat{Q}_t + \widehat{E}_t\left(\sum_{i=t}^{T-1}\left(\widetilde{\beta}\widetilde{R}\widehat{R}_{i+1}^K - \sigma\widehat{\gamma}_{i+1}\right)\right) \\
\widehat{E}_t\left(\sigma\widehat{c}_T + \psi(1-\sigma)\widehat{H}_T\right) &= \sigma\widehat{c}_t + (1-\sigma)\psi\widehat{H}_t + \widehat{E}_t\left(\sum_{i=t}^{T-1}\left(\widetilde{\beta}\widetilde{R}\widehat{R}_{i+1}^K - \sigma\widehat{\gamma}_{i+1}\right)\right)
\end{aligned}$$

Substituting for the constant-consumption labor supply yields

$$\begin{aligned}
&\widehat{E}_t\left(\sigma\widehat{c}_T + \psi(1-\sigma)\left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}(\widehat{w}_T - \widehat{c}_T)\right) \\
= \; &\sigma\widehat{c}_t + (1-\sigma)\psi\widehat{H}_t + \widehat{E}_t\left(\sum_{i=t}^{T-1}\left(\widetilde{\beta}\widetilde{R}\widehat{R}_{i+1}^K - \sigma\widehat{\gamma}_{i+1}\right)\right)
\end{aligned}$$

which can be simplified to

$$\begin{aligned}
&\widehat{E}_t\left(\left(1-\chi\right)\sigma\widehat{c}_T + \chi\sigma\widehat{w}_T\right) \\
= \; &\sigma\widehat{c}_t + (1-\sigma)\psi\widehat{H}_t + \widehat{E}_t\left(\sum_{i=t}^{T-1}\left(\widetilde{\beta}\widetilde{R}\widehat{R}_{i+1}^K - \sigma\widehat{\gamma}_{i+1}\right)\right)
\end{aligned}$$

where

$$\chi \equiv \sigma^{-1}\psi(1-\sigma)\left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)^{-1}$$

Rearranging in terms of expected consumption

$$\widehat{E}_t\widehat{c}_T = \frac{1}{1-\chi}\left[\widehat{c}_t + \sigma^{-1}\psi(1-\sigma)\widehat{H}_t + \widehat{E}_t\left(\sum_{i=t}^{T-1}\left(\sigma^{-1}\widetilde{\beta}\widetilde{R}\widehat{R}_{i+1}^K - \widehat{\gamma}_{i+1}\right)\right) - \chi\widehat{w}_T\right]$$

We now substitute the above equation into the intertemporal budget constraint

$$\text{(D48)} \qquad \epsilon_c\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\widehat{c}_T = \widetilde{\beta}^{-1}\widehat{a}_t + \widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\left(\widetilde{\beta}^{-1}\widehat{R}_T - \widetilde{\beta}^{-1}\widehat{\gamma}_T + \epsilon_w\widehat{w}_T\right)$$

Note $\widetilde{\beta}\widetilde{R}\widehat{R}_T^K = \beta\frac{\overline{R}^k\overline{U}\left(1-\tau^K\right)}{\overline{\gamma}^\sigma}\widehat{R}_T^K = \widehat{R}_T$. So (D48) becomes

$$\epsilon_c\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\widehat{c}_T = \widetilde{\beta}^{-1}\widehat{a}_t + \widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\left(\widetilde{R}\widehat{R}_T^K - \widetilde{\beta}^{-1}\widehat{\gamma}_T + \epsilon_w\widehat{w}_T\right)$$

Based on the derivation of Eusepi and Preston (2011), we get

$$\begin{aligned}\widehat{c}_t + \sigma^{-1}\psi(1-\sigma)\widehat{H}_t &= \frac{(1-\chi)(1-\widetilde{\beta})}{\varepsilon_c}\left[\widetilde{\beta}^{-1}\widehat{a}_t + \widetilde{R}\widehat{R}_t^K - \widetilde{\beta}^{-1}\widehat{\gamma}_t + \left(\varepsilon_w + \varepsilon_c\frac{\chi}{1-\chi}\right)\widehat{w}_t\right] \\ \text{(D49)} \qquad & + \widetilde{\beta}\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\Gamma_1\widehat{R}_{T+1}^K + \widetilde{\beta}\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\Gamma_2\widehat{w}_{T+1},\end{aligned}$$

where $\Gamma_1 = \left[\frac{(1-\chi)\left(1-\widetilde{\beta}\right)}{\varepsilon_c} - \widetilde{\beta}\sigma^{-1}\right]\widetilde{R}$ and $\Gamma_2 = \frac{(1-\chi)\left(1-\widetilde{\beta}\right)}{\varepsilon_c}\left(\varepsilon_w + \varepsilon_c\frac{\chi}{1-\chi}\right)$.

G. RE system of equations

$$\widehat{w}_t - \left(\epsilon_H - \frac{\sigma-1}{\sigma}\psi\right)\widehat{H}_t - \widehat{c}_t = 0$$
$$-\widehat{w}_t + \widehat{y}_t - \widehat{H}_t = 0$$
$$-\widehat{R}_t^K + \widehat{y}_t - \widehat{U}_t - \left(\widehat{k}_t - \widehat{\gamma}_t\right) = 0$$
$$-\widehat{y}_t + \frac{\overline{i}}{\overline{y}}\widehat{i}_t + \left(1 - \frac{\overline{i}}{\overline{y}} - \frac{\overline{g}}{\overline{y}}\right)\widehat{c}_t + \frac{\overline{g}}{\overline{y}}\widehat{g}_t = 0$$
$$-\widehat{y}_t + (1-\alpha)\widehat{H}_t + \alpha\widehat{U}_t + \alpha\left(\widehat{k}_t - \widehat{\gamma}_t\right) = 0$$
$$-\frac{\delta\theta}{\overline{\gamma}}\widehat{U}_t - \widehat{k}_{t+1} + \frac{\overline{i}}{\overline{k}}\widehat{i}_t + \frac{(1-\delta)}{\overline{\gamma}}\left(\widehat{k}_t - \widehat{\gamma}_t\right) = 0$$

$$-\frac{\widehat{R}_t^K}{\theta-1}+\widehat{U}_t=0$$

$$(\sigma\widehat{c}_t+\psi(1-\sigma)\widehat{H}_t)-\hat{E}_t[((\sigma\widehat{c}_{t+1}+\psi(1-\sigma)\widehat{H}_{t+1})+\sigma\widehat{\gamma}_{t+1})]+\beta\frac{\overline{R}^k\overline{U}\left(1-\tau^K\right)}{\overline{\gamma}^\sigma}\hat{E}_t\widehat{R}_{t+1}^K=0$$

$$\widehat{R}_t-\frac{\overline{R}^K\overline{U}\left(1-\tau^K\right)}{\overline{R}}\widehat{R}_t^K=0$$

$$\frac{\overline{a}}{\overline{y}}\widehat{a}_t-\frac{\overline{b}}{\overline{y}}\widehat{b}_t-\frac{\overline{k}}{\overline{y}}\widehat{k}_t=0$$

$$\begin{aligned}&\frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\left(\widehat{b}_t+\widehat{R}_t\right)-\left(\overline{R}^K\overline{U}-\overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\widehat{k}_t-\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\overline{R}^K\overline{U}\widehat{R}_t^K\\&-(1-\alpha)\tau^H\left(\widehat{w}_t+\widehat{H}_t\right)+\frac{\overline{g}}{\overline{y}}\widehat{g}_t\\&+\left(\left(\overline{R}^K\overline{U}-\overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}-\frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\right)\widehat{\gamma}_t-\frac{\overline{b}}{\overline{y}}\widehat{b}_{t+1}=0\end{aligned}\tag{D50}$$

$$\left(\frac{\overline{g}}{\overline{y}}+\lambda_1-\lambda_2\frac{\overline{b}}{\overline{y}}\overline{\gamma}^{-1}\right)\widehat{y}_t+\lambda_2\frac{\overline{b}}{\overline{y}}\overline{\gamma}^{-1}\left(\widehat{b}_t-\widehat{\gamma}_t\right)+u_t^G-\frac{\overline{g}}{\overline{y}}\widehat{g}_t=0$$

We now eliminate $\widehat{R}_t$ and $\widehat{a}_t$ from the system of equations for the learning model. Equation (D50) becomes

$$\begin{aligned}&\frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\left(\widehat{b}_t+\frac{\overline{R}^K\overline{U}\left(1-\tau^K\right)}{\overline{R}}\widehat{R}_t^K\right)-\left(\overline{R}^K\overline{U}-\overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\widehat{k}_t\\&-\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\overline{R}^K\overline{U}\widehat{R}_t^K-(1-\alpha)\tau^H\left(\widehat{w}_t+\widehat{H}_t\right)\\&+\frac{\overline{g}}{\overline{y}}\widehat{g}_t+\left(\left(\overline{R}^K\overline{U}-\overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}-\frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\right)\widehat{\gamma}_t-\frac{\overline{b}}{\overline{y}}\widehat{b}_{t+1}=0\end{aligned}$$

and

$$\frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\widehat{b}_t + \left(\frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\frac{\overline{R}^K\overline{U}\left(1-\tau^K\right)}{\overline{R}} - \tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\overline{R}^K\overline{U}\right)\widehat{R}_t^K - \left(\overline{R}^K\overline{U} - \overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}}\widehat{k}_t$$
$$-\left(1-\alpha\right)\tau^H\left(\widehat{w}_t + \widehat{H}_t\right)$$
$$+\frac{\overline{g}}{\overline{y}}\widehat{g}_t + \left(\left(\overline{R}^K\overline{U} - \overline{\delta}\right)\tau^K\frac{1}{\overline{\gamma}}\frac{\overline{k}}{\overline{y}} - \frac{\overline{b}}{\overline{y}}\frac{\overline{R}}{\overline{\gamma}}\right)\widehat{\gamma}_t - \frac{\overline{b}}{\overline{y}}\widehat{b}_{t+1} = 0$$

Equation (D49) becomes

$$\begin{aligned} & \widehat{c}_t + \sigma^{-1}\psi(1-\sigma)\widehat{H}_t \\ = \; & \frac{(1-\chi)(1-\widetilde{\beta})}{\varepsilon_c}\left[\widetilde{\beta}^{-1}\left(\frac{\frac{\overline{b}}{\overline{y}}}{\frac{\overline{a}}{\overline{y}}}\widehat{b}_t + \frac{\frac{\overline{k}}{\overline{y}}}{\frac{\overline{a}}{\overline{y}}}\widehat{k}_t\right) + \widetilde{R}\widehat{R}_t^K - \widetilde{\beta}^{-1}\widehat{\gamma}_t + \left(\varepsilon_w + \varepsilon_c\frac{\chi}{1-\chi}\right)\widehat{w}_t\right] \\ & +\widetilde{\beta}\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\Gamma_1\widehat{R}_{T+1}^K + \widetilde{\beta}\widehat{E}_t\sum_{T=t}^{\infty}\widetilde{\beta}^{T-t}\Gamma_2\widehat{w}_{T+1} \end{aligned}$$

Additional References for Online Appendix

House, C., C. Proebsting, L. Tesar (2017). Austerity in the Aftermath of the Great Recession. NBER working paper no. 23147.