Ambiguity, Monetary Policy and Trend Inflation*

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Abstract

Allowing for ambiguity about the behavior of the policymaker in a simple new-Keynesian model gives rise to wedges between long-run inflation expectations, trend inflation, and the inflation target. The degree of ambiguity we measure in *Blue Chip* survey data helps explain the dynamics of long-run inflation expectations and the inflation trend measured in the US data. Ambiguity also has implications for monetary policy. We show that it is optimal for policymakers to lean against the households' pessimistic expectations, but also document the limits to the extent the adverse effects of ambiguity can be undone.

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1 Introduction

The private sector's expectations about monetary policy are key determinants of economic outcomes. Private agents make the economic decisions that determine inflation and output. Their confidence in the conduct of monetary policy is thus instrumental to the success of the policy itself. We are now far removed from the times in which monetary policy was perceived as an arcane undertaking best practiced out of public view,[1] and transparency has become a key tenet of modern central banking. And yet, as the quote below notes, it is practically impossible to completely eliminate uncertainty about the conduct of monetary policy.

... the faulty estimate was largely attributable to misapprehensions about the Fed's intentions. [...] Such misapprehensions can never be eliminated, but they can be reduced by a central bank that offers markets a clearer vision of its goals, its 'model' of the economy, and its general strategy.
Blinder (1998)

In this paper we study how these misgivings can impact the effectiveness of monetary policy in anchoring inflation expectations at the target level. And, in particular, we show how the evolution of long-run inflation expectations over the last 30 plus years can be explained by the increased degree of confidence the private sector has regarding the conduct of monetary policy.

To do so, we build a model that explicitly allows for multiple priors about the monetary policy rule, as a way of formalizing the *misapprehensions* discussed above in Blinder (1998). In particular, we augment a prototypical new-Keynesian model by introducing ambiguity about the monetary policy rule and assuming that agents are averse to ambiguity. Agents in our model lack the confidence to assign probabilities to all relevant events. Rather, they entertain as possible a set of multiple beliefs. Together with their strict preference for knowing probabilities, this implies that they act as if they evaluated

[1] As Bernanke pointed out in a 2007 speech: Montagu Norman, the Governor of the Bank of England from 1921 to 1944, reputedly took as his personal motto, "Never explain, never excuse."

plans based on the worst in their set of beliefs. We use a recursive version of the multiple prior preferences (see Gilboa and Schmeidler, 1989, and Epstein and Schneider, 2003), pioneered in business cycle models by Ilut and Schneider (2014) and used, for example, by Baqaee (2015) to model how asymmetric responses to news can explain downward wage rigidity.

The assumption that agents are not fully confident about the conduct of monetary policy helps explain features of the data that are otherwise difficult to make sense of. We focus on three. First, our model provides a rationale for the observed low-frequency component in the series for inflation (inflation trend) and its relationship with the inflation target. Second, it is able to capture the dynamics of long-run inflation expectations and to provide a rationale for why, as Chan, Clark and Koop (2017) suggest, they differ from both the target and the inflation trend. Third, we show that, in this class of models, policymakers can do better than simply tracking the natural rate of interest.

Trend inflation and inflation expectations. The dynamics of inflation and, in particular, its persistence are driven in large part by a low-frequency, or trend, component, as documented for example in Stock and Watson (2007) and Cogley and Sbordone (2008). Most of the macroeconomic literature relies, however, on models that are approximated around a zero-inflation steady state[2] and that, consequently, cannot capture the persistent dynamic properties of inflation and long-run inflation expectations, nor their relationship. In these models, the target, trend inflation and inflation expectations all coincide.

A strand of the macroeconomic literature, summarized in Ascari and Sbordone (2014), studies the effects of a trend in inflation by allowing for a time-varying steady state level of inflation. This results in what they refer to as a Generalized New-Keynesian Phillips and helps make sense of inflation persistence. The model studied by Ascari and Sbordone (2014) treats inflation trend as a primitive (taken from time-series estimates) and implies that long-run inflation expectations will equal the inflation trend. Chan, Clark and Koop (2017), however, show that long-run inflation expectations and trend

[2] Or alternatively, and equivalently from this perspective, a full indexation steady-state.

are separate concepts that cannot be simply equated.

Our model builds on that in Ascari and Sbordone (2014) but provides a microfoundation for the level of long-run inflation expectations and that of the measured inflation trend. Ambiguity causes long-run inflation expectations under worst-case beliefs to differ from the inflation target as well as from a statistical measure of trend – which compares to the ergodic steady state of the model. The key driver of the deviations of trend inflation from target, as well as of the wedge between trend inflation and long-run inflation expectations, is the time-varying degree of confidence the private sector has in the conduct of monetary policy – *i.e.* how large the private sector's set of beliefs about the interest rate is.

To measure ambiguity, we follow Ilut and Schneider (2014) and use data on forecasters' disagreement about their nowcasts of the policy rate. Using this measure, our model helps make sense of the dynamics of trend inflation and inflation expectations in the US since the early 1980s.[3]

Our model implies that, when the set of conditional interest rate expectation means over which agents are ambiguous is symmetric, long-run inflation expectations will exceed a statistical measure of trend inflation, and both will be larger than the inflation target. Proximity of rates to the ZLB tends to make the aforementioned set asymmetric – in a way we will detail and test for below – which results in our model implying long-run inflation expectations will be lower than both trend and target under these circumstances. Both these implications are in line with measures of long-run inflation expectations and our TVP-VAR estimate of trend inflation, which builds on Cogley and Sbordone (2008).

At a more quantitative level, our model can explain how it is that high long-run inflation expectations and inflation trend measures in the early 1980s fell through most of the 1990s and 2000s and settled to around the target level, before falling in the aftermath of the Great Recession. The observed fall in trend inflation since the early 1980s, and the progressive alignment of long-run inflation expectations to trend, is explained by an increase in private

[3]Data limitations prevent us from going back further in time.

sector confidence, which in turn can be traced back to the great increase in transparency and communications enacted by the Fed, as highlighted by Lindsey (2003).

Work by Swanson (2006) supports this claim by showing that, since the late 1980s, the cross-sectional dispersion of interest rate forecasts by U.S. financial markets and private sector forecasters – *i.e.* their disagreement – shrank and, importantly, by providing evidence that these phenomena can be traced back to increases in central bank transparency. Similarly, Ehrmann, Eijffinger and Fratzscher (2012) find that increased central bank transparency lowers disagreement among professional forecasters. This seems natural. For a given degree of uncertainty about the state of the economy, improved knowledge about the policymakers' objectives and model will make forecasters more confident about monetary policy and their predictions more homogeneous. As confidence in the conduct of monetary policy increases, our model implies that long-run inflation expectations will approach the target.

Monetary Policy Implications. If the policymaker could dispel ambiguity completely, it would be optimal to do so. It is however natural to imagine that there is a limit to the extent to which Knightian uncertainty can be reduced. For example, the changing composition of the policymaking committee might introduce some uncertainty about the conduct of monetary policy in the future.

Our model generates interesting prescriptions for monetary policy in the presence of ambiguity and can help make sense of policy actions, such as the perceived excessive tightening early in the Volker tenure (Goodfriend, 2005). In a situation like that of the early 1980s, characterized by high degree of ambiguity and inflation above target, our model implies that the policymaker should be more *hawkish* than what would be optimal in the absence of ambiguity. In particular, the policymaker should increase its responsiveness to inflation and, on top of that, should set the intercept of its policy rule above the natural rate of interest. On the contrary, in situations in which ambiguity drives long-run inflation expectations because of the proximity of the ZLB, it is optimal for policymakers to aim for a rate below the natural rate of interest.

Our paper to work by Benigno and Paciello (2014), who consider optimal monetary policy in a model in which agents have a desire for robustness to misspecification about the state of the economy, in the spirit of Hansen and Sargent (2007). The advantage of the multiple-priors approach is that we can characterize the effects of ambiguity on the steady state.

The rest of the paper is organized as follows. Section 2 provides a description of the model we use for our analysis and characterizes the steady state of our economy as a function of the degree of ambiguity. In Section 3 we show how our simple model can match the dynamics of trend inflation, while in Section 4 we discuss the implications for monetary policy of the presence of some unavoidable Knightian uncertainty about monetary policy. Section 5 concludes.

2 The Model

We augment a simple New-Keynesian model (see Yun, 2005 or Galí, 2008) by assuming that private agents face ambiguity about the expected future policy rate. To isolate the effects of ambiguity, we set up our model so that, absent ambiguity, the first-best steady state would be attained, thanks to a government subsidy that corrects the distortion introduced by monopolistic competition. Ambiguity, however, will cause worst-case steady-state inflation to deviate from its target. For expositional simplicity, the derivation of the model is carried out assuming the inflation target is zero, so the steady-state level of inflation we find should be interpreted as a deviation from the target. However, the model is equivalent to one in which the central bank targets a positive constant level of inflation, to which firms index their prices.

We present the model's building blocks starting with a description of the monetary policy rule, which is critical for our analysis.

Monetary policy. In our economy the only disturbance unrelated to the policymaker's behavior is a technology shock A_t. A policy rule responding more than one for one to inflation and including the natural rate of interest, $R_t^n = \mathbb{E}_t \left(\frac{A_{t+1}}{\beta A_t} \right)$, would thus be optimal (Galí, 2008). Indeed, together with the optimal production subsidy, this policy rule would attain the first-best allocation at all times.

However, we augment the policy rule to account for the possibility that the policymaker might deviate from the rule or might follow a poorly measured estimate of the natural rate of interest:

$$R_t = \left(R_t^n e^{\zeta_{t+1}} \right) (\Pi_t)^{\phi}, \tag{1}$$

where ζ_{t+1} is characterized by the following law of motion:

$$\zeta_{t+1} = \rho^{\zeta} \zeta_t + u_{t+1}^{\zeta} + \mu_t \qquad 0 < \rho^{\zeta} < 1 \tag{2}$$

which would be a standard AR(1) process if it wasn't for the presence of μ_t. This formulation of the disturbance process serves multiple purposes. First, it captures the idea that a deviation from the optimal rule today also represents a signal of future likely deviations from the rule, which, for example, can be thought of as capturing serial correlation in the mismeasurement of the natural rate. Second, it captures autocorrelation in the policy rate, while not breaking the result that, in the absence of this disturbance, the policy rule is such that the economy attains first-best.[4]

Finally, it is important to note that the realization of ζ_{t+1} is not known at the time decisions are made a time t, and therefore agents are required to compute expectations regarding the conduct of monetary policy. This is consistent with the assumption adopted when identifying monetary policy shocks with timing restrictions as in Christiano, Eichenbaum and Evans (2005). Also, if monetary policy committees meet several times a quarter (twice a quarter in the US), agents have to make predictions about the policymakers' decisions

[4] A literature going back to Rudebush (2002) has discussed whether this formulation is preferable to one in which the lagged policy rate enters the right-hand side of the policy rule. The optimality of the response to technological shocks and tractability considerations - this formulation allows us to compute the solution to the log-linear approximation analytically - make us opt for this specification.

even within a quarter. In practice, so long as any relevant economic decision has to be made prior to the last policy meeting of the quarter, our specification captures a relevant economic situation faced by the private sector. And, indeed, this uncertainty is evident in the survey data we use to measure ambiguity, namely the nowcast (forecast for the current quarter) of the Fed Funds rate reported in the second month of the quarter in the *Blue Chip* survey. From this perspective, ρ^{ζ} captures the predictability of future deviations of rates from the underlying optimal rule.

When it comes to the interpretation of μ, we follow Ilut and Schneider (2014) and posit that the two terms that make up the innovations to ζ_t, $z_{t+1} \equiv \zeta_{t+1} - \rho^{\zeta}\zeta_t$, are a sequence of i.i.d. Gaussian innovations $u^{\zeta}_{t+1} \sim \mathcal{N}(0, \sigma_u)$, and a deterministic sequence μ_t. By assumption, the empirical moments of μ_t converge in the long run to those of an i.i.d. zero-mean Gaussian process, independent of u^{ζ}_{t+1}, and with variance $\sigma_z^2 - \sigma_u^2 > 0$, which result in μ_t being extremely hard to recover.

Agents thus treat that term as ambiguous. The information at their disposal only enables them to put bounds on their conditional expectations for policy rates,[5] which we parametrize with the interval $[\underline{\mu}_t, \overline{\mu}_t]$.[6] The width of the interval $[\underline{\mu}_t, \overline{\mu}_t]$ measures the agents' confidence, a smaller interval capturing the idea that agents are more confident in their prediction of the policy rate.

Households. The representative household's felicity function is a function of consumption C_t and hours worked N_t:

$$u(C_t, N_t) = \log(C_t) - \frac{N_t^{1+\psi}}{1+\psi},$$

[5]Given our timing convention, the realization of ζ_{t+1} only becomes known after decisions are made.

[6]Throughout the paper, we will maintain the assumption that $\underline{\mu}_t < 0$ and $\overline{\mu}_t > 0$ so that the rational expectations model ($\mu_t = 0$) is nested.

and utility can be written recursively as a function of a consumption-hours plan (C, N) :

$$U_t(C, N; s^t) = u(C_t, N_t) + \beta \min_{p \in \mathcal{P}_t(s^t)} \mathbb{E}^p \left[U_{t+1}(C, N; s_t,\ s_{t+1}) \right] \tag{3}$$

where $\mathcal{P}_t(s^t)$ is a set of conditional probabilities about next period's state of the economy $s_{t+1} \in S$. The standard rational expectations model is a special case of (3) in which the conditional probability set $\mathcal{P}_t(s^t)$ contains only the one correct belief. The conditional probability p is selected to minimize expected continuation utility, subject to the constraint that p must lie in the set $\mathcal{P}_t(s^t)$. The minimization of the expected continuation utility captures a strict preference for knowing probabilities, as detailed in Gilboa and Schmeidler (1989) and Epstein and Schneider (2003).

We parametrize the belief set with an interval $\left[\underline{\mu}_t,\ \overline{\mu}_t\right]$ of conditional mean distortions for ζ_{t+1}, so we can rewrite the utility function as:

$$U_t(C, N; s^t) = u(C_t, N_t) + \beta \min_{\mu_t \in \left[\underline{\mu}_t,\ \overline{\mu}_t\right]} \mathbb{E}^{\mu_t} \left[U_{t+1}(C, N; s_t,\ s_{t+1}) \right]. \tag{4}$$

Households maximize (4) subject to their budget constraint:

$$P_t C_t + B_t = R_{t-1} B_{t-1} + W_t N_t + T_t, \tag{5}$$

where P_t is the price of the final good, W_t is the nominal wage, B_t are bonds with a one-period nominal return R_t – which are in zero net supply – and T_t includes government transfers as well as profit payouts. There is no heterogeneity across households, because they all earn the same wage in the competitive labor market, they own a diversified portfolio of firms, they consume the same Dixit-Stiglitz consumption bundle and face the same level of ambiguity.

As mentioned above, we assume that households make their decisions *before* the current value of R_t is realized, along the lines of Christiano, Eichenbaum and Evans (2005). This timing assumption and the zero-net supply of bonds implies that:

1. At the beginning of time t, when decisions are made, the realization of ζ_{t+1} is not yet known, so the household's expected policy rate (substituting the expectation into (1) and taking logs) is:

$$\mathbb{E}_t^{\mu_t} r_t = r_t^n + \rho^\zeta \zeta_t + \mu_t + \phi \pi_t.$$

2. Consumption will be pinned down so that desired savings are zero, given this expectation for the policy rate (which is common across all households).
3. When the actual policy rate is set, it will not affect households wealth, because bond holdings are zero.

The household's intertemporal and intratemporal Euler equations are thus:

$$\frac{1}{C_t} = \mathbb{E}_t^{\mu_t} \left[\frac{\beta R_t}{C_{t+1} \Pi_{t+1}} \right] \quad (6)$$

$$N_t^\psi C_t = \frac{W_t}{P_t}. \quad (7)$$

We can rewrite the intertemporal Euler equation substituting in the distorted expectations for the policy rate:

$$\frac{1}{C_t} = \mathbb{E}_t \left[\frac{\beta R_t^n e^{\rho^\zeta \zeta_t + \mu_t} (\Pi_t)^\phi}{C_{t+1} \Pi_{t+1}} \right], \quad (8)$$

where $\mathbb{E}_t$ is the rational expectations operator. Equations (7) and (8) characterize the maximization problem of the households. We will turn to finding the level of μ_t that solves the minimization problem in the next section.

Firms. The final good Y_t is produced by perfectly competitive producers using a continuum of intermediate goods $Y_t(i)$ and the standard CES production function

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}. \quad (9)$$

Taking prices as given, the final good producers choose intermediate good quantities $Y_t(i)$ to maximize profits, resulting in the usual Dixit-Stiglitz demand function for intermediate goods

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t \tag{10}$$

and in the aggregate price index

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}.$$

Intermediate goods are produced by a continuum of monopolistically competitive firms using the following linear technology:

$$Y_t(i) = A_t N_t(i), \tag{11}$$

where A_t is a stationary technology process. Prices are sticky in the sense of Calvo (1983): only a random fraction $(1-\theta)$ of firms can re-optimize their price in any given period. Whenever a firm can re-optimize, it sets its price maximizing the expected presented discounted value of future profits

$$\max_{P_t^*(i)} \sum_{s=0}^{\infty} \theta^j \beta^j E\left[\frac{C_{t+j}^{-1}}{P_{t+j}}\left[P_t^* Y_{t+j}(i) - P_{t+j} MC_{t+j} Y_{t+j}(i)\right]\right], \tag{12}$$

where $MC_t = (1-\tau)\frac{W_t}{A_t P_t}$ is the real marginal cost, net of the subsidy $\tau = 1/\epsilon$.

The firm's price-setting decision, which is the same for all the firms setting prices in given period, can be characterized by the firms' first-order condition[7]

$$\frac{P_t^*(i)}{P_t} = \frac{\frac{\epsilon}{\epsilon-1}\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \theta^j MC_{t+j}\left(\frac{P_t}{P_{t+j}}\right)^{-\epsilon}}{\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \theta^j \left(\frac{P_t}{P_{t+j}}\right)^{1-\epsilon}} \tag{13}$$

[7]Log-preferences in consumption and the fact that $Y_t = C_t$ simplify this expression as the marginal utility of future consumption simplifies out with aggregate output in the profit function.

together with the following equation derived from the law of motion for the price index:

$$\frac{P_t^*(i)}{P_t} = \left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{1}{1-\epsilon}}. \tag{14}$$

Government and Market Clearing. The Government runs a balanced budget and finances the production subsidy with a lump-sum tax. Out of notational convenience, we include the firms' aggregate profits in the lump-sum transfer:

$$\begin{aligned} T_t &= P_t\left[-\tau\frac{W_t}{P_t}N_t + Y_t\left(1-(1-\tau)\frac{W_t\Delta_t}{P_tA_t}\right)\right] \\ &= P_tY_t\left(1-\frac{W_t\Delta_t}{P_tA_t}\right). \end{aligned}$$

where Δ_t is the price dispersion term, which we define below.

Market clearing in the goods markets requires that $Y_t(i) = C_t(i)$ for all firms $i \in [0,1]$ and all t, which implies

$$Y_t = C_t.$$

Market clearing on the labor market implies that

$$N_t = \int_0^1 N_t(i)di = \int_0^1 \frac{Y_t(i)}{A_t}di = \frac{Y_t}{A_t}\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}di,$$

where we define $\Delta_t \equiv \int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}di$ as the relative price dispersion across intermediate firms (Yun, 2005).

Finally, it can be established that the relative price dispersion evolves as follows:

$$\Delta_t = \theta\Pi_t^{\epsilon}\Delta_{t-1} + (1-\theta)\left(\frac{1-\theta\Pi_t^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{15}$$

2.1 Steady State

We study our model economy around the worst-case steady state, since agents act *as if* the economy converges there in the long run (see Ilut and Schneider, 2014). The agents' pessmistic expectations however are not fulfilled by the realization of the exogenous process, so in general the ergodic steady state will differ from its worst-case counterpart. We will discuss both steady states in turn.

We derive the steady state of the agents' first-order conditions as a function of a generic level of the belief distortion induced by ambiguity, μ, and we then rank the different steady states, indexed by μ, to characterize the worst-case steady state. To keep notation to a minimum, we will define worst-case steady-state variables as having no time subscript,[8] even though we will later on give an anticipated-utility (Kreps, 1998 and Cogley and Sargent, 2008) interpretation to these quantities, in the tradition of Cogley and Sbordone (2008).

Finally it is convenient to collect parameter values by defining $\omega = \left[\beta,\ \epsilon,\ \theta,\ \phi,\ \rho^{\zeta},\ \rho^{a},\ \psi\right]$. The set of admissible parameter values can then be characterized as:

$$\begin{aligned} \Omega &= \Big\{\omega : \left(\beta \in (0,1), \epsilon \in (1,\infty), \theta \in (0,1), \phi \in (1,\infty), \rho^{\zeta} \in (0,1), \rho^{a} \in (0,1), \psi \in [0,\infty)\right) \\ &\cap \left(\rho^{\zeta} + \frac{\epsilon\mu}{\log(\theta)(\phi-1)} < 1\right)\Big\}. \end{aligned} \tag{16}$$

The first part of the definition of Ω simply includes the standard economic restrictions on the discount factor, the demand elasticity, the probability of firms not being able to reset prices, the responsiveness of monetary policy to inflation deviations from target (assuming the Taylor principle is satisfied), the autocorrelation coefficients for the two exogenous processes and the inverse Frish elasticity, respectively. The second part of the definition restricts a combination of parameter values to ensure the steady state is well defined.[9] This condition does not restrict the set of admissible paramters at all for $\mu = 0$

[8] With the understanding that setting $\mu = 0$ in those expressions delivers the first-best steady state.

[9] As shown in Appendix A, this condition ensures that $N(\mu, \omega) > 0$ and real.

and becomes more restrictive as the degree of ambiguity increases. In practice, we will only use it in the context of our estimation exercise by checking that it is met by estimated parameters.

Worst-Case Steady State. We can divide the presentation of the worst-case steady state into two separate parts. We start by expressing inflation as a function of a generic perceived value of the disturbance ζ in steady state. All other steady state variables can be then computed as a function of steady-state inflation in a way that mimics that of the trend inflation literature (e.g. Ascari and Ropele, 2009).

Proposition 2.1. *In a steady state in which agents perceive the disturbance ζ_{t+1} to have non-zero mean, inflation, relative to target, takes value:*

$$\Pi(\mu, \omega) = e^{-\frac{\zeta}{\phi - 1}}. \tag{17}$$

where $\zeta = \frac{\mu}{1-\rho^{\zeta}}$.
As a result, for any $\omega \in \Omega$, $\mu > 0 \Rightarrow \Pi(\mu, \omega) < \Pi(0, \omega) = 1$, while the opposite is true for $\mu < 0$.

Proof. Proof in Appendix B. □

Proposition 2.1 has two key implications. First, it shows how the worst-case steady state level of inflation depends on two parameters only, which provides a tight parametrization to be brought to the data. Second, it illustrates how inflation is a decreasing function of the belief distortion μ, as long as the Taylor principle is satisfied. To build some intuition, let us consider the case in which household decisions are based on an expected level of the interest rate that is systematically lower than the true policy rate ($\mu < 0$). Other things being equal, agents will want to bring consumption forward, thus causing demand pressure and driving up inflation.

Proposition 2.1 also shows that the effects of ambiguity are decreasing in ϕ. For a given level of μ, a more aggressive response to inflation deviations will keep inflation closer to target thus reducing the adverse effects on welfare.[10]

Having worked out the value of steady state inflation for a generic value for μ, we now turn to determining the value of $\mu \in [\underline{\mu}, \overline{\mu}]$ that minimizes the agents' welfare.

In our simple model, the presence of the production subsidy ensures that monetary policy implements the first-best steady state in the absence of ambiguity ($\underline{\mu} = \overline{\mu} = 0$). Therefore, any belief distortion $\mu \neq 0$ will generate a welfare loss. However, it is not a priori clear whether a negative μ is worse than a positive one of the same magnitude, *i.e.* whether underestimating the interest rate is worse than overestimating it by the same amount.

The following proposition characterizes the properties of steady-state welfare, which we refer to as $\mathbb{V}(\mu, \omega)$, in detail.

Proposition 2.2. *For any $\omega \in \Omega$, $\mathbb{V}(\mu, \omega)$ is continuously differentiable around $\mu = 0$ and:*

i. attains a maximum at $\mu = 0$,

ii. is strictly concave in μ,

iii. under symmetry of the bounds ($\underline{\mu} = -\overline{\mu}$), for β sufficiently close to one, attains its minimum on $[-\overline{\mu}, \overline{\mu}]$ at $\mu = -\overline{\mu}$.

Proof. See Appendix B □

Proposition 2.2 states that for any economically viable calibration,[11] our economy attains its welfare maximum in the absence of ambiguity – indeed we know it attains first best – and that welfare is strictly concave. This corresponds to the intuition that any deviation of μ from zero reduces welfare and also immediately rules out interior minima, which leaves us with the two bounds $\underline{\mu}$ and $\overline{\mu}$ as candidate minima.

[10] This fact can be seen formally by noting that the second derivative of the welfare function with respect to μ (see equation (30)), governs the concavity of welfare as a function of μ, is negative but increasing in ϕ.

[11] The condition that β be close to one is required to derive the results analytically but it is easy to verify numerically that it is not restrictive in practice.

Figure 1: Steady-state welfare as a function of μ (measured in basis points of an annualized interest rate).

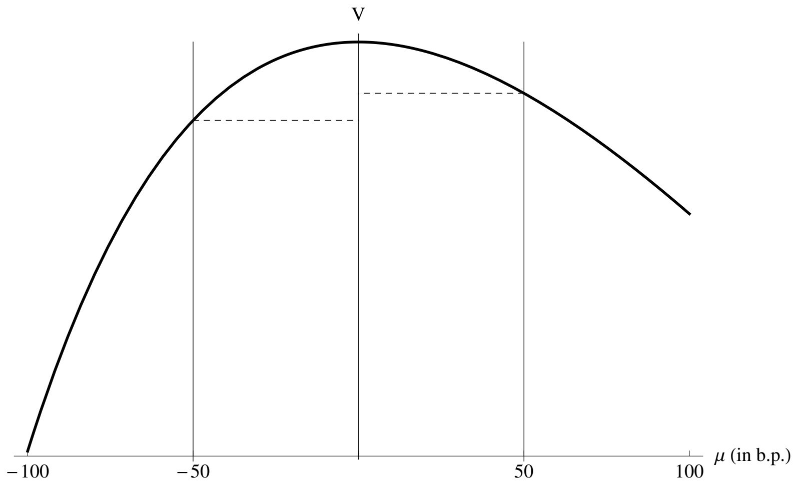


Figure 1 shows that our welfare function is asymmetric and that, if the bounds are symmetric, welfare is minimized when $\mu = \underline{\mu} = -\overline{\mu}$, as part *iii.* of Proposition 2.2 states in general terms. This corresponds to a situation in which agents act as if monetary policy will be too loose. As a consequence, as implied by Proposition 2.1, under symmetry, the worst-case steady state is characterized by inflation exceeding target.[12]

This asymmetry has an intuitive economic interpretation. Positive inflation tends to lower the relative price of firms that do not get a chance to re-optimize. These firms will face a very high demand, which, in turn, will push up their labor demand. In the limit, as their relative price goes to zero, these firms will incur huge production costs while their real unitary revenues shrink. On the other hand, negative trend inflation will reduce the demand for firms which do not re-optimize and this will reduce their demand for labor. In the limit, demand for their goods will tend to zero, but so will their production costs.

When the ambiguity bounds are not symmetric, the location of the worst case depends on the exact shape of the ambiguity interval. If the ambiguity

[12]In Appendix C.2 we verify that the worst-case we identify in Proposition 2.2 corresponds to the worst-case even when we approximated the equilibrium conditions around their worst-case steady state.

interval is capped from below, for example because the policy rate is close to the zero lower bound, welfare might be minimized at $\overline{\mu}$. In this situation, households fear that the policy rate is set higher than it should be and this will, ultimately, determine trend inflation and inflation expectations to be below target. We will test for asymmetry of our measure of ambiguity in the following section.

Ergodic Steady State. Agents act *as if* the process for ζ_{t+1} was governed by (2). However, the true law of motion for ζ_{t+1} is $\zeta_{t+1} = \rho^{\zeta}\zeta_t + u^{\zeta}_{t+1}$. This means that the agents pessimistic expectations are not, in general, validated by the realization of the exogenous process, which results in the ergodic steady state to differ from its worst-case counterpart.

Here we focus on the case in which hours worked enter the felicity function linearly because it allows us to maintain tractability and to explore the differences between the two steady states in detail. Formally we define $\Omega^0 = \Omega \cap \{\psi = 0\}$.

Proposition 2.3. *The ergodic steady state for inflation in deviation from target can be expressed in logs as:*

$$\overline{\pi} = \pi^W - \frac{\mu \lambda_{\pi\zeta}(\mu,\omega)}{1-\rho^{\zeta}} = -\frac{\mu}{1-\rho^{\zeta}}\left(\frac{1}{\phi-1} + \lambda_{\pi\zeta}(\mu,\omega)\right) \quad (18)$$

$$\lambda_{\pi\zeta}(\mu,\omega) \equiv -\frac{\kappa_0 \rho^{\zeta}}{(1-\rho^{\zeta})\left(1+\kappa_0 \frac{\phi-\rho^{\zeta}}{1-\rho^{\zeta}} - \rho^{\zeta}\left(\kappa_2 + \rho^{\zeta}\frac{\kappa_1\kappa_5}{1-\rho^{\zeta}\kappa_6}\right)\right)} \quad (19)$$

where π^W is the log of inflation in the worst-case steady state, $\lambda_{\pi\zeta}(\mu,\omega)$ is the coefficient governing the equilibrium response of inflation to ζ_t, and the κ's are functions of (μ,ω) which represent the coefficients in the log-linearized set of equilibrium conditions, described in Appendix C.

When bounds are symmetric ($\underline{\mu} = -\overline{\mu}$), $\pi^W = log(\Pi(-\overline{\mu},\omega)) = \frac{\overline{\mu}}{1-\rho^{\zeta}}\frac{1}{\phi-1}$ and $\overline{\pi} = \frac{\overline{\mu}}{1-\rho^{\zeta}}\left(\frac{1}{\phi-1} + \lambda_{\pi\zeta}(\mu,\omega)\right)$.

Proof. See Appendix C.3 □

The intuition behind Proposition 2.3 is better understood starting from the special case in which $\rho^{\zeta} \to 0$. If the degree of ambiguity increases unexpectedly, agents will act as if the interest rate will be lower, thus bringing consumption forward and pushing up inflation. At time $t+1$ agents will observe that the actual realizations of ζ_{t+1} and, thus, of r_t differ from the one they expected. Because bonds are in zero net supply, a surprise in the interest rate will not affect the agents' level of wealth. Moreover, if the autocorrelation in ζ is negligible, realizing $\zeta_{t+1} \neq \mathbb{E}_t \zeta_{t+1}$ has no bearing on their expectation for ζ_{t+2}. In sum, when $\rho^{\zeta} \to 0$, the fact that the expected bad news did not materialize has no impact on the economy, and the ergodic steady state tends to the worst-case steady state ($\lambda_{\pi\zeta}(\mu,\omega) \to 0$ and $\overline{\pi} \to \pi^W$).

For a generic positive ρ^{ζ}, however, the moment agents realize that the outcome for ζ_{t+1} is not as bad as they anticipated, they will revise their expectations for the future. In particular, they will determine their consumption level based on an interest rate profile that is not as low as the one they expected at time t. As a result, demand pressures will be reduced and inflation will not be as high as anticipated at time t. Equation (18) reflects this correction and implies that, in the ergodic steady state, inflation will be lower than in the worst-case.

At the same time $\frac{1}{\phi-1} + \lambda_{\pi\zeta}(\mu,\omega) > 0$, which implies that the level of ergodic steady state inflation exceeds the target for strictly positive levels of ambiguity (under the symmetry assumption), which is another readily testable implication of our model.

The following proposition, formalizes the properties of the ergodic steady-state inflation we just discussed.

Proposition 2.4. *For small μ, for any $\omega \in \Omega^0$:*

i. $-\frac{1}{\phi-1} < \lambda_{\pi\zeta}(\mu,\omega) < 0$

ii. π^W *and* $\overline{\pi}$ *are both decreasing in* μ

iii. *when the worst case corresponds to* $\mu = \underline{\mu} < 0$ *(*$\mu = \overline{\mu} > 0$ *),* $0 < \overline{\pi} < \pi^W$ *(*$0 > \overline{\pi} > \pi^W$*)*

Proof. See Appendix C.4. □

Figure 2: Worst-case steady state inflation (red), ergodic steady state inflation (orange), approximation to the ergodic steady state (blue), and inflation target (black dashed), as a function of μ (measured in basis points of annualized rate) and parameter values from the estimation in the first column of Table 1.

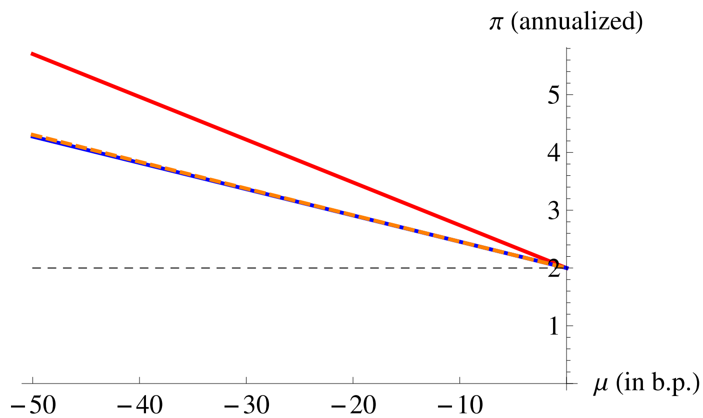


Figure 2 illustrates the main points implied by Proposition 2.4 for our baseline symmetric-bounds case. Both the worst-case and the ergodic levels of inflation in steady state exceed the target and both increase with the degree of ambiguity. However, the effect of ambiguity on the ergodic steady state is smaller, in line with the economic intuition presented above regarding how the ergodic steady state reflects the fact that the actual law of motion for ζ_{t+1} does not validate the pessimistic expectations underlying the worst-case steady state. Figure 2, which is drawn for the range for μ we measure in the data and for the parameter values we estimate in the next section, also shows that the results we derive in Proposition 2.4 under the assumption of μ being sufficiently small, apply for the entire range we consider. We can see that by noting that the blue line, which represents an approximation in a neighborhood of $\mu = 0$, and the orange dashed line, which accounts for the fact that $\lambda_{\pi\zeta}(\mu, \omega)$ varies with μ, are virtually indistinguishable.

Testable Implications. The theoretical results in this section have clear testable implications, which we will bring to the data. In particular, in keeping with the inflation trend literature (e.g. Ascari and Sbordone, 2014), we will give to our model an anticipated utility interpretation and test the following implications of our model. When the level of ambiguity that minimizes welfare is $\underline{\mu}$ $(\overline{\mu})$

i. both the worst-case steady-state inflation, which maps into our data on long-run inflation expectations, and the ergodic steady state inflation, which corresponds to statistical measures of trend inflation, are above (below) target;

ii. statistical measures of trend inflation should lie between long-run inflation expectations and the target;

iii. as the degree of ambiguity falls, all measures should tend to converge to the inflation target.

3 Trend Inflation and Long-run Inflation Expectations

In order to capture long-run inflation dynamics, Del Negro and Eusepi (2011) and Del Negro, Giannoni and Schofheide (2015) propose replacing the constant inflation target with a very persistent, exogenous, inflation target process. This assumption is very effective at matching inflation data, but seems at odds with evidence from sources such as the *Blue Book* – a document illustrating monetary policy alternatives, presented to the FOMC by Fed staff before each meeting. While the Federal Reserve officially did not have an explicit numerical target for inflation until 2012, *Blue Book* simulations have been produced assuming targets of 1.5% and 2% since at least 2000. Indeed, Lindsey (2003) states that, as early as July 1996, numerous FOMC committee members had indicated at least an informal preference for an inflation rate in the neighborhood of 2%, as indicated by FOMC transcripts of the July 2-3, 1996.

Recently, Carvalho *et al.* (2017) proposed a model of inflation in which changes in long-run inflation beliefs are a state-contingent function of short-run inflation surprises. Their model predicts observed measures of long-term inflation expectations, but cannot account for the observed wedge between the inflation trend and inflation expectations, because, as in most models, the inflation trend and long-run survey expectations coincide. However, Chan, Clark and Koop (2017) present evidence that these two quantities should not simply be equated; their relationship is more complicated and time-varying.

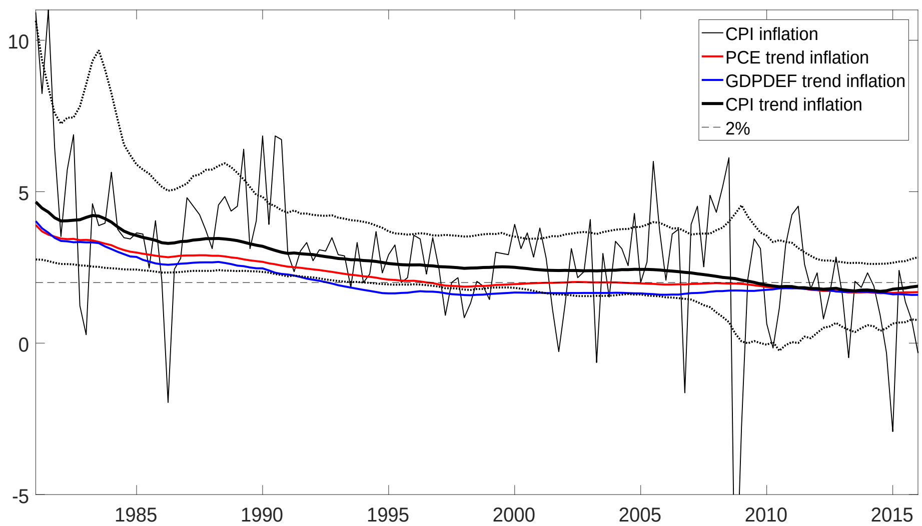
By introducing ambiguity in our simple New-Keynesian model, we can make progress in the quest for explaining why long-run inflation expectations do not correspond to measures of inflation trend, and both differ from the inflation target. From the perspective of our model, long-run inflation expectations would naturally correspond to the worst-case steady state inflation level. However, an econometrician working with realized inflation would estimate inflation to settle around a different value in the long run, which corresponds to the ergodic steady state.[13]

So, our model allows us to extend the analysis in Sbordone and Ascari (2014) and Cogley and Sbordone (2008) that studied inflation trend as a time-varying inflation steady state, as it allows us to study inflation trend and long-run inflation expectations as separate but related variables.

Inflation Trend. We estimate trend inflation with a Bayesian vector autoregression model with drifting coefficients (TVP-BVAR), along the lines of Cogley and Sargent (2002). The specification is taken from Cogley and Sbordone (2008), and comprises the following series: real GDP growth, unit labor cost, the federal funds rate, and a series for inflation. We alternate three different measures of inflation: the GDP deflator, the PCE deflator and CPI inflation. Figure 3 reports CPI inflation along with the trend inflation series implied by our model, when using as inflation measure the implicit GDP deflator (blue line), the PCE deflator (red line) and CPI price index (bold black line). For the CPI-based trend inflation we also show the 90% confidence bands (dotted lines). We focus our attention on the CPI inflation to facilitate the comparison between trend and long-run inflation expectations: the major-

[13]See Ilut and Schneider's (2014) discussion of the econometrician's data-generating process.

Figure 3: Trend inflation implied by different measures of inflation



CPI inflation trend inflation implied by a TVP-BVAR using GDP deflator (blue), CPI (bold black line), PCE deflator (red). The dotted lines indicate the 90% confidence bands for the trend inflation obtained using CPI as a measure on inflation.

ity of the existing measures of long-run inflation expectations concern CPI.[14] Clearly, inflation is characterized by a trend component, which has fallen since the early 1980s and is currently estimated to be slightly below the FOMC's 2% target for all three inflation measures listed above.

Inflation Expectations. We consider three alternative measures of long-run inflation expectations, two survey-based (Blue Chip and SPF) and one based on surveys as well as inflation swaps and other financial market data (the Cleveland Fed's measure of 10 years ahead inflation expectations, see Haubrich, Pennacchi and Ritchken (2011) for more details[15]). The Blue Chip CPI inflation forecast 5 to 10 years ahead is available biannually since 1986, the SPF 10 year-ahead CPI inflation forecast is available quarterly since 1991, while the

[14]In 2012 the FOMC announced that it was targeting core PCE inflation, but long-run inflation expectations for PCE are, to our knowledge, available only in the Survey of Professional Forecasters, and only from 2007 onwards.

[15]This is a monthly series. We make quarterly by taking the average of the 3 observations in each quarter.

Cleveland Fed produces estimates of inflation expectations at a monthly frequency starting from 1982. We will focus on the Cleveland Fed's and the Blue Chip expectations, mainly because of the longer sample, but we will present the results for SPF expectations as well.

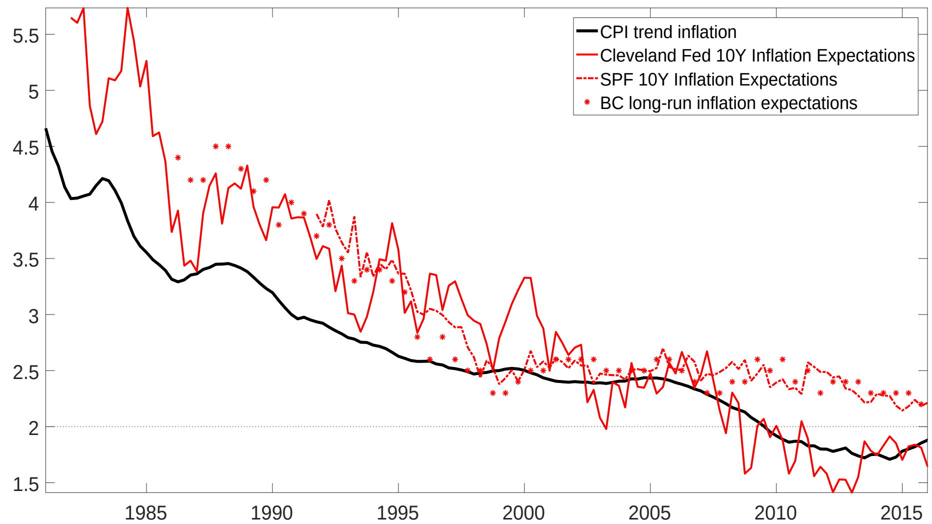
Figure 4 reports these three measures of long-run inflation expectations and compares them with the measure of trend we obtained with the TVP-BVAR. Inflation expectations exceed our measure of trend inflation through the early 2000s. In the aftermath of the Great Recession our preferred measure of inflation clearly falls below target and also below our declining measure of trend inflation. The survey-based measures of inflation expectations fall much less and remain above the 2 percent mark. We attribute the more evident fall in the Cleveland Fed's measure of inflation expectation to it including financial variables. However, it is also important to keep in mind that the 2 percent target is defined in terms of PCE inflation. CPI inflation has been on average .4 percentage points higher than PCE inflation over the last 20 years (see for example Bullard, 2013) as is also evident from our estimates for trend inflation trend in Figure 3. Taken literally, that would mean that expectations of CPI below 2.4 percent could be interpreted as below-target expectations.

Ambiguity. We follow Ilut and Schneider (2014) and use disagreement in survey nowcasts for the Fed Funds rate as our measure of ambiguity. Both Swanson (2006) and Ehrmann, Eijffinger and Fratzscher (2012) present evidence of a clear link between an increase in central bank transparency in the 1990s and 2000s and a decrease in forecasters' disagreement about the policy rate.

Our headline measure of ambiguity about the conduct of monetary policy is the interdecile dispersion in nowcasts of the current quarter's federal funds rate from the Blue Chip Financial Forecasts dataset, which are available from 1983 onwards.

The Blue Chip nowcasts of the federal funds rate are collected on a monthly basis. Since we want to isolate the uncertainty relating to monetary policy, rather than macro uncertainty in general, we compare the Blue Chip release

Figure 4: Trend inflation and various measures of inflation expectations

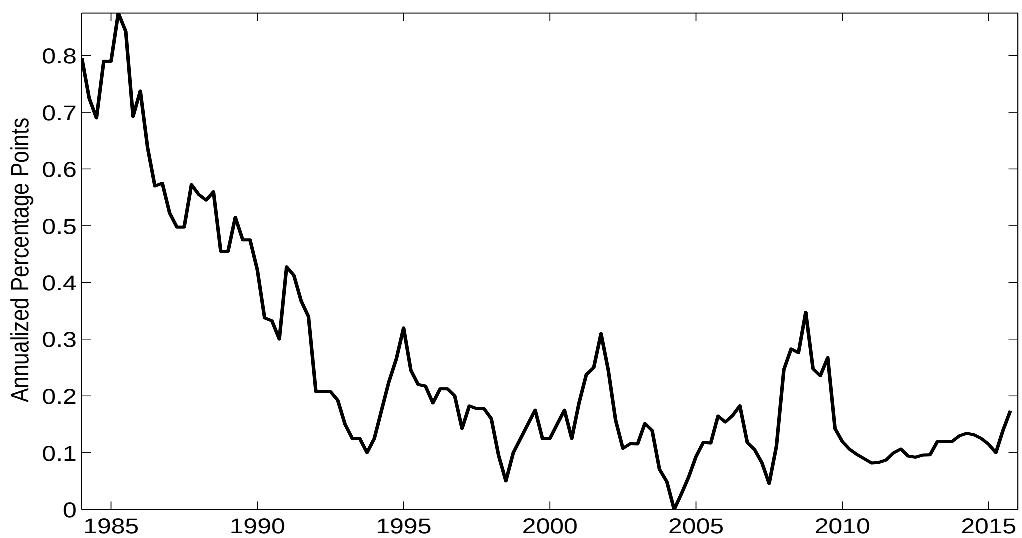


The Cleveland Fed's estimate is the solid red line, the SPF 10Y ahead inflation expectations is the dashed-dotted line and the stars are the Blue Chip measure of 5-10 years ahead inflation expectations. The black line is trend inflation estimated with the TVP-BVAR using CPI as a measure of inflation.

dates with the FOMC meeting dates. We find that the nowcasts produced on the second month of the quarter are the most likely to capture the type of uncertainty we are modelling. FOMC meetings mostly happened before the third month's survey was administered, dispelling most of the uncertainty relating to policy for those nowcasts. The nowcasts produced for the first month of the quarter, instead, also reflect uncertainty about the incoming macro data, while on the second month of the quarter most of the relevant information available to the FOMC at the time of their meeting has already been released. Therefore we choose the nowcasts produced for the second month. We take a 4-quarter moving average, smoothing out very high-frequency variations, which do not have much to say about trend, while leaving the scale unaffected.

Figure 5 shows the 4-quarter moving-average of our preferred measure of ambiguity. It is obvious that the degree of dispersion was much larger in the early 1980s than it is now. From the mid-1990s onwards, the dispersion is on average below 25 basis points, which means that the usual disagreement among

Figure 5: A measure of disagreement about the federal funds rate



The solid line is the interdecile dispersion of Blue Chip nowcasts of the federal funds rate in the second month of the quarter.

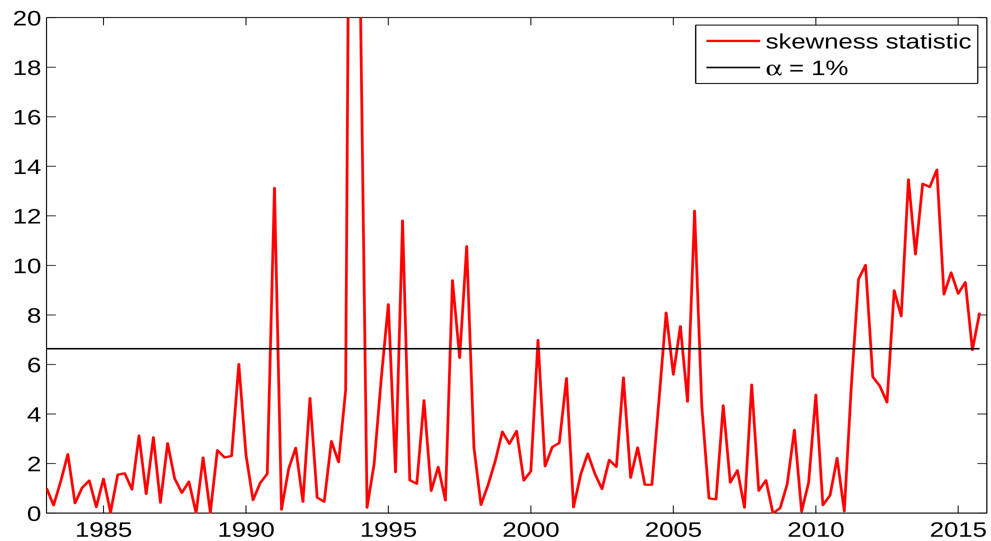
the *hawkish* and *dovish* ends of the professional forecasters' pool amounts to situations like the former expecting a 25bp tightening and the latter no change – a very reasonable scenario in the late 1990s and early 2000s. In the early 80s, however, that number exceeded three quarters of a percent.

The implications of our model depend crucially on whether we can think of the interval $[\underline{\mu}_t, \overline{\mu}_t]$ as being symmetric or not. If the interval is symmetric we can simply calibrate $\overline{\mu}_t$ to half our measure of dispersion and $\underline{\mu}_t = -\overline{\mu}_t$. To verify this is a sensible assumption, we test for the symmetry in the dispersion of short-term rate nowcasts using a test developed by Premaratne and Bera (2015).[16] Figure 6 shows that the null of symmetry of the distribution of individual Blue Chip nowcasts of the federal funds rate is only occasionally rejected, and never for several subsequent quarters, up to 2010.[17] After 2010/2011, however, the dispersion started to display a noticeable and per-

[16]This test adjusts the standard $\sqrt{b_1}$ test of symmetry of a distribution, which assumes no excess kurtosis, for possible distributional misspecifications.

[17]We perform the test on the cross-section of nowcasts at each date in our sample.

Figure 6: Test for the null that the distribution of the individual Blue Chip nowcasts is symmetric.



We test for the symmetry of the distribution of the individual Blue Chip nowcasts of the federal funds rate. We use the Premaratne and Bera (2015) test for symmetry of the distribution and perform it period per period from 1983 to 2015. The test is χ^2-distributed. The points above the black line are those in which the null of symmetry is rejected with 99% confidence.

sistent upward skew and this determines a persistent rejection of the null hypothesis of symmetry. Indeed, it is plausible that, the ZLB on policy rates limits disagreement on the downside precisely in this fashion. In these situations it is natural that agents would expect the worst-case scenario to be one in which rates are too high, resulting in a low level of inflation, as our theory predicts.

3.1 Estimation

Our measures of long-run inflation expectations and inflation trend are highly correlated (the sample correlation coefficient being .95) and are both negatively correlated with our measure of ambiguity (sample correlation of -.88 and -.91 for long-run inflation expectations and trend respectively).

Figure 7: Cleveland Fed Long-Run Inflation Expectations (red) and Inflation Trend (blue) as a function of our measure of ambiguity.

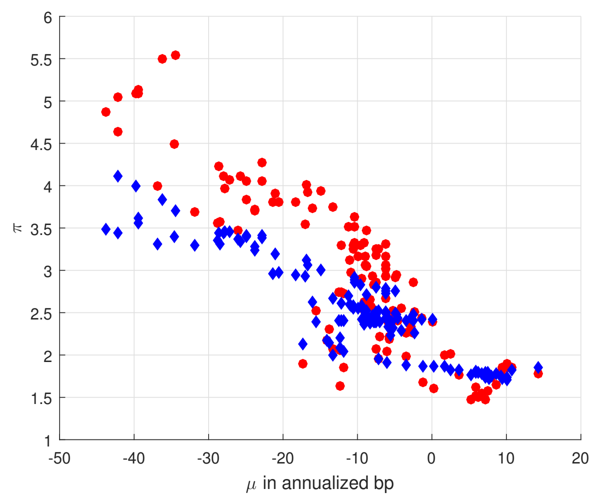


Figure 7 illustrates this by overlaying a scatter plot of our estimate of trend inflation vis-a-vis our measure of ambiguity (blue diamonds) to a scatter plot which relates long-run inflation expectations to ambiguity (red circles). Clearly, negative values for μ associate with inflation expectations (and trend) exceeding the target value, while the opposite is true for positive values of μ as our theory suggests. Moreover, Figure 7 shows how long-run inflation expectations appear to have been moving more with changes in ambiguity than the inflation trend, in line with the prediction of Proposition 2.4.

To substantiate this claim, we estimate the parameters of our model as a way to check that the observed levels of inflation trend can be attained for reasonable parameter values.

We adopt a minimum-distance procedure that bears similarities with Cogley and Sbordone (2008) two-step procedure. Cogely and Sbordone (2008) estimate the parameters of a simple New-Keynesian DSGE with trend inflation using the estimates for trend inflation obtained from their TVP-VAR.

While we estimate the same TVP-VAR, our approach features two important differences. First, trend inflation in Cogley and Sbordone (2008) is a parameter in their model so their moment conditions compare conditional expectations in the model and in the VAR and the relationship between inflation and marginal cost. In our case, however, because trend inflation is not exogenous but a function of the level of ambiguity and model parameters, we can directly minimize the distance between the VAR estimate for trend inflation and the model-implied measure.

The second difference is that our model provides separate restrictions for long-run inflation expectations and trend inflation so we can use both to estimate our parameter values.

In particular, we can define our data as $\underline{z}_t = \left[\begin{array}{c} z_t^{\mu}, z_t^{\pi^e}, z_t^{trend} \end{array} \right]'$, $t = 1, ..., T$ as including a measure of ambiguity[18], a measure of long-run inflation expectations and a measure of inflation trend.

We can then use the restrictions from our model to define:

$$\underline{m}_t\left(\omega, \underline{z}_t\right) = \left[\begin{array}{c} z_t^{\pi^e} - \left(\pi^* - \frac{z_t^{\mu}}{\left(1-\rho^{\zeta}\right)(\phi-1)}\right) \\ z_t^{trend} - \left(\pi^* - \frac{z_t^{\mu}}{1-\rho^{\zeta}}\left(\frac{1}{\phi-1} + \lambda_{\pi\zeta}\left(z_t^{\mu}, \omega\right)\right)\right) \end{array} \right] \tag{20}$$

so that we can estimate ω by solving the following minimization:

$$\min_{\omega \in \tilde{\Omega}^0} \sum_{t=1}^{T} \underline{m}_t\left(\omega, \underline{z}_t\right)' \mathcal{W} \underline{m}_t\left(\omega, \underline{z}_t\right) \tag{21}$$

where we use the identity matrix as a weighting matrix $\mathcal{W}$ in our baseline estimation. The admissible set $\tilde{\Omega}^0$ reflects a number of considerations. First, our restrictions have nothing to say about ρ^a so we do not estimate it at all. In keeping with most empirical macroeconomic literature, we simply calibrate $\beta = .995$. More interestingly, we face two identification challenges. Firstly, it is very difficult to separately identify θ and ϵ. These two parameters enter our moment restrictions via $\lambda_{\pi\zeta}\left(z_t^{\mu}, \omega\right)$ and they both tend to magnify the effects of steady state inflation,[19] so we calibrate $\epsilon = 11$ – which implies firms price

[18]We define z_t^{μ} based on disagreement and the test for symmetry of the interval as described above.

[19]The definitions of the log-linear coefficients in Appendix C show that the expression $\theta\Pi\left(\mu, \omega\right)^{\epsilon}$ is recurring. So when $\Pi\left(\mu, \omega\right) > 1$, as is the case for most of our sample observations, ϵ and θ have similar effects in that they both increase $\theta\Pi\left(\mu, \omega\right)^{\epsilon}$, which, in turn,

their goods at a ten percent markup over their marginal cost – and estimate θ.

Moreover, lower ρ^{ζ} and higher ϕ produce the same effect on the worst-case steady state level of inflation (our first model condition) and similar effects on the level of trend inflation – their values impact $\lambda_{\pi\zeta}\left(z_t^{\mu}, \omega\right)$ differently. As a result in our baseline estimation we set $\phi = 1.5$ (as advocated by Taylor (1993)) and focus on estimating ρ^{ζ}. We report the estimation results in which we estimate both ϕ and ρ^{ζ} in Appendix D.1– while the parameter estimates show more variation when we experiment with different series for long-run inflation expectations, they are all well within the acceptable range.[20]

Lastly, imposing that $\omega \in \tilde{\Omega}^0$ ensures that the steady state is well defined (see equation (16)).[21]

Table 1 reports our baseline estimates for three different series for inflation expectations ($z_t^{\pi^e}$). The estimates for θ are between .5 and .6, roughly in line with estimates by Cogely and Sbordone (2008) and Christiano, Eichenbaum and Evans (2005) who estimate the corresponding parameter with limited-information techniques at .588 and .60 respectively in different setups and not far from the .65 estimate in Smets and Wouters (2007). ρ^{ζ} is estimated to be around .7, not far from estimates of the autocorrelation term in the monetary policy rule[22] in Smets and Wouters (2007) and Clarida, Galí and Gertler (1999) which estimate it at .81 and .79 respectively.

We find these results encouraging, since the moments we are matching up to are not normally used in the estimation of DSGEs. And importantly, from

affects the value of the κ coefficients that enter the definition of $\lambda_{\pi\zeta}(\mu, \omega)$ in equation (19).

[20] In our estimations we set the inflation target π^* to 2%, the value announced by the FOMC in 2012 for PCE. This implies a rather conservative value for CPI inflation, which has been on average .4 points above PCE inflation in the last 20 years (see for example Bullard, 2013).

[21] In practice, we run an unconstrained minimization and then verify that the inequality in equation (16) is satisfied given the estimates.

[22] The autocorrelation is most normally modeled as the coefficient on the lagged interest rate. For tractability purposes, we elect to adopt this specification instead, which allows us to keep the derivation analytical as discussed above.

Table 1:

	Cleveland Fed 10Y	Blue Chip 5-10Y	SPF 10Y
θ	0.5275	0.5834	0.5231
ρ^{ζ}	0.7335	0.7395	0.6962
No of obs.	129	60	97
Sample period	1983Q1-2015Q4	1986H1-2015H2	1991Q4-2015Q4

Estimates of θ and ρ^{ζ} obtained using different measures of long-run inflation expectations

our perspective, our estimates imply that about half of the overall variance of ζ_{t+1} can be traced back to the predictable component (${\rho^{\zeta}}^2 \approx .5$) and half to the innovation u^{ζ}_{t+1}, which seems reasonable given that our dispersion measure is collected midway through the quarter, in particular between the two FOMC meetings usually taking place in each quarter.

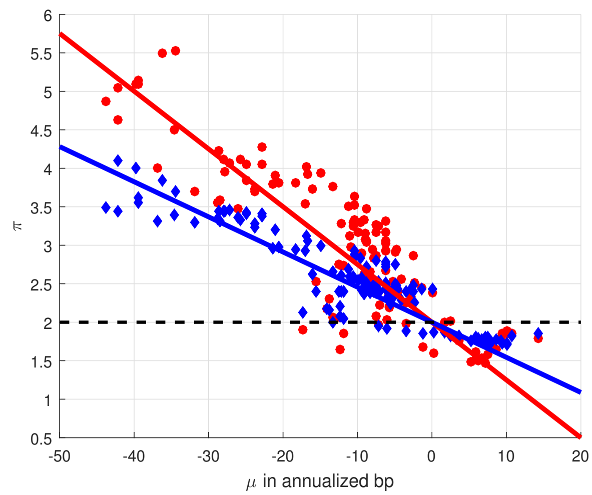
Figure 8 illustrates the fit of our model and estimation exercise by combining the data (Figure 7) and the model-implied values (Figure 2).[23] This simple scatter plot confirms our intuition that augmenting a simple New-Keynesian DSGE with ambiguity regarding the monetary policy rule can go a long way towards explaining why away from the ZLB inflation expecations tend to exceed both the target and an estimated trend, with the difference increasing with the degree of Knightian uncertainty. In the proximity of the lower bound on policy rates the opposite ranking applies.

The fit is overall better for the inflation trend series than for the inflation expectations which, in part, is a mechanical consequence of the fact that our model implies that, of the parameters we estimate, only ρ^{ζ} can affect the model counterpart to long-run inflation expectations while both ρ^{ζ} and θ can make the model fit the trend series.

In sum, we find that a very simple mininum-distance estimation exercise of our tightly parametrized model-implied long-run inflation expectations and inflation trend concepts can do a good job at explaining why with the observed reduction in our measure of Knightian uncertainty the differences between long-run expectations and trend fell and both approached the target, only to

[23]The blue line in Figure 8 corresponds to the blue line in Figure 2, i.e. it uses the approximation for μ clos to zero. The estimation uses the exact formulation for $\lambda_{\pi\zeta}(\mu, \omega)$ but, as shown above in Figure 2 the two are indistinguishable.

Figure 8: Cleveland Fed Long-Run Inflation Expectations (red circles) and Inflation Trend (blue diamonds) as a function of our measure of ambiguity. The solid lines represent our model-implied values for long-run inflation expecatations (red) and inflation trend (blue) given the esitmates in Table 1



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fall below the target level as interest rates approached their lower bound. In this respect, our model can offer a compelling explanation for why inflation expectations, while falling to below target, remained close to it. In particular, this pattern should be credited to the high degree of confidence in the conduct of monetary policy that built over the years.

4 Optimal Monetary Policy

In our simple economy the policy implemented by equation (1) is optimal if the policymaker measures the natural rate accurately ($\sigma_u \to 0$) and if the private sector has full confidence in this happening ($\underline{\mu} = \overline{\mu} = 0$).

It is thus out of question that optimizing policymakers would want to reduce the error component in their measure of the natural rate as much as possible as well as increase the degree of confidence to the extent possible. The interesting question is, what happens when the ambiguity regarding the policy rule cannot be totally dispelled. Here we explore optimal rules for this scenario.

Our results characterize policy rules that attain the best of possible *worst-case steady-state welfare* levels - a concept we will refer to as *steady-state* optimality. Steady-state optimality is very often disregarded in analysis of optimal policy set-ups because, in the absence of ambiguity, zero inflation and the optimal subsidy deliver the first-best allocation, independent of the values the other parameters. We will show how, in our setting, the degree of policy responsiveness to deviations of inflation from target plays a critical role instead.

We characterize the optimal monetary policy rule when there is a bound on the responsiveness of the policy rate to inflation. As can be seen in equation (17), as $\phi \to \infty$, steady state approaches first best. However, as Schmitt-Grohé and Uribe (2007) point out, values of ϕ above around 3 are impractical. Hence, we will work under the assumption that values of ϕ are bounded.

Proposition 4.1. *For any $\omega \in \Omega$, a small $\overline{\mu} > 0$, $\underline{\mu} = -\overline{\mu}$ and $\underline{\phi} \leq \phi \leq \overline{\phi}$, the following rule is steady-state optimal in its class:*

$$R_t \quad = \quad R_t^* \Pi_t^{\overline{\phi}} \tag{22}$$

where $R_t^* = R_t^n e^{\delta^*(\overline{\mu},\overline{\phi};\omega)}$ and $0 < \delta^*(\overline{\mu},\overline{\phi};\omega) < \frac{\overline{\mu}}{1-\rho^{\varsigma}}$.

Proof. See Appendix E.2 □

We can summarize the result by saying that the central bank needs to be more *hawkish* than in the abscence of ambiguity, because it is optimal to respond as strongly as possible to inflation and to increase the monetary policy rule's *intercept*. The overly tight policy stance that Chairman Volcker followed in early 1982 (see Goodfriend 2005, p. 248) can be better appreciated from the perspective of this result. In an economy in which ambiguity about policy was rampant, it was optimal to tighten above and beyond what the business cycle conditions would seem to dictate.

Both high ϕ and positive δ reduce the wedge between steady-state inflation and the target, thus increasing welfare. The slope coefficient ϕ reduces the effects of ambiguity on inflation because, even if the worst-case interest rate tends to drive inflation up, this effect is mitigated by a more forceful response by the policymaker to any deviation from target.

The optimal intercept (R_t^*) is higher than the natural rate, because, as inflation is still inefficiently high in the presence of ambiguity, the central bank would like to tighten more. Facing a bound on ϕ, it can do so only by increasing the intercept of its policy rule. However, increasing it too much can be detrimental. On average, the private sector is underestimating the policy rate by $\zeta = \frac{\overline{\mu}}{1-\rho^{\zeta}}$. A *naïve* policymaker could respond by systematically setting rates higher than its standard policy rule by the same amount (ζ). If agents did not evaluate welfare in the worst-case scenario when they make their decisions, this policy action would implement first best. In our setting, however, ambiguity-averse agents would realize that the worst-case scenario had become one in which interest rates are too high and steady state inflation would end up falling below target. The level of δ that maximizes worst-case welfare is positive but strictly smaller than $\frac{\overline{\mu}}{1-\rho^{\zeta}}$, capturing the idea that the policymaker can do better than blindly following the policy rule that would be optimal in the absence of ambiguity, yet he or she has to prevent the degree of extra-tightening ($\delta > 0$) to become so large as to make agents fear excessive tightening more than excessive loosening.

Another important aspect of Proposition 4.1 is that the rule in equation (22) is optimal in *its class*, *i.e.* the class of rules including inflation and a measure of the natural rate. Clearly, plenty of alternative specifications could legitimately be proposed. It is not in the spirit of this paper to try to review and numerically evaluate a great number of alternative rules, but it is immediate to show that our functional form can deliver as high a level of steady-state welfare as *any* other policy scheme, provided we are prepared to relax the constraint on ϕ. In other words, our functional form is only potentially restrictive in terms of practical implementability, because high values of ϕ can be hard to justify in practice as we mentioned above, but is otherwise as good as any.[24]

Finally, our previous result applies under the assumption that the interval over which the ambiguity is defined is symmetric ($\underline{\mu} = -\overline{\mu}$). When that is not the case, as in recent years, the policy prescription still features the highest possible value for ϕ but a negative value for δ as we formalize in the following Corollary.

Corollary 4.1. *Given the setup in Proposition 4.1 except for the fact that* $|\underline{\mu}| << |\overline{\mu}|$, *so that* $\mathbb{V}\left(\underline{\mu}, \omega\right) > \mathbb{V}\left(\overline{\mu}, \omega\right)$, *then*

$$R_t = R_t^* \Pi_t^{\overline{\phi}} \tag{23}$$

where $R_t^* = R_t^n e^{\delta^*(\underline{\mu}, \overline{\mu}, \overline{\phi}, \omega)}$ *and* $-\frac{\overline{\mu}}{1-\rho^{\zeta}} < \delta^*(\underline{\mu}, \overline{\mu}, \overline{\phi}; \omega) < 0$ *is steady-state optimal in its class.*

Proof. See Appendix E.3 □

This corollary highlights the different roles played by ϕ and δ. Higher ϕ always tends to bring inflation closer to target, so it is always optimal to increase ϕ as much as possible, irrespective of whether trend inflation is above or below target. As for the "policy-rule intercept," however, when the worst-case level of inflation is below target, it is optimal for R_t^* to be smaller than the natural rate to generate inflationary pressures that would push inflation up towards its target.

5 Conclusions

We augment a standard New-Keynesian DSGE to study the consequences of changes in confidence on the success of an inflationary targeting regime. When ambiguity-averse agents face Knightian uncertainty about the conduct of monetary policy long-run inflation expectations do not correspond to the inflation targeting. In particular in normal times inflation expectations will exceed the target, the distance between the two depending on the degree of ambiguity

[24]See Corollary in Appendix E.1 for a formal statement.

and the central bank's response to inflation deviations from target, while in the proximity of the ZLB our model predicts long-run inflation expectations will fall below the inflation target as the worst-case scenario becomes one in which policy will be too tight.

Our model also can explain why statistical measures of inflation trend tend to differ from both the target and long-run expectations, usually falling somewhere in between the two of them. From a modeling perspective this follows from the realized mean level of inflation reflecting the interaction between pessimistic expectations and the fact that the worst-case expectations about the policy rate do not actually materialize.

By keeping the solution of the model analytical we can easily estimate the key parameters governing inflation using disagreement on the policy rate as a measure of ambiguity, long-run inflation expectations from surveys and a TVP-BVAR estimate for the inflation trend. Our simple minimum distance procedure is able to fit US trend and inflation expectations data since the early 80's reasonably well, especially considering how tight the parametrization is – in our baseline estimation we only estimate two parameters.

Our results are consistent with the idea that the increased level of transparency can explain the transition from a situation in which long-run inflation expectations exceeded 5 percent in the early 80s to one in which they are remarkably well anchored around the target – the post Great Recession fall is much smaller in magnitude relative to the deviations observed in the early part of our sample, a testament to the much improved degree of anchoring.

We conclude our analysis but characterizing the optimal policy rule in an ambiguity-ridden economy. In particular, we show how in the presence of ambiguity a simple monetary policy rule that tracks movements in the natural rate of interest can be improved upon by one that tracks a higher interest rate (lower in the vicinity of the ZLB). This provides a novel insight in the seemingly overly tight policy stance pursued by chairman Volcker in the early 1980s and the ultra low rates observed in the aftermath of the Great Recession.

In sum, by building on the latest advances in the inflation trend literature (Ascari and Sbordone, 2014) and in the ambiguity literature (Ilut and Schneider, 2014) we can provide an economic rationale for observed low-frequency variations in inflation and inflation expectations which are normally treated as purely exogenous if not disregarded altogether.

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Appendix - For Online Publication

A Steady State

Pricing. In our model firms index their prices based on the first-best inflation, which corresponds to the inflation target and is zero in the case presented in the main body of the paper. Because of ambiguity, however, steady-state inflation will not be zero and therefore there will be price dispersion in steady state:

$$\Delta(\mu,\omega) = \frac{(1-\theta)\left(\frac{1-\theta\Pi(\mu,\omega)^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}}}{1-\theta\Pi(\mu,\omega)^{\epsilon}} \tag{24}$$

Δ is minimised for $\Pi = 1$ - or, equivalently, $\mu = 0$ - and is larger than unity for any other value of μ. As in Yun (2005), the presence of price dispersion reduces labour productivity and ultimately welfare.

Hours, Consumption and Welfare. In a steady state with no real growth, steady-state hours are the following function of μ:

$$N(\mu,\omega) = \left(\frac{\left(1-\theta\Pi(\mu,\omega)^{\epsilon-1}\right)\left(1-\beta\theta\Pi(\mu,\omega)^{\epsilon}\right)}{\left(1-\beta\theta\Pi(\mu,\omega)^{\epsilon-1}\right)\left(1-\theta\Pi(\mu,\omega)^{\epsilon}\right)}\right)^{\frac{1}{1+\psi}}, \tag{25}$$

while consumption is:

$$C(\mu,\omega) = \frac{A}{\Delta(\mu,\omega)}N(\mu,\omega) \tag{26}$$

Hence the steady state welfare function takes a very simple form:

$$\mathbb{V}(\mu,\omega) = \frac{1}{1-\beta}\left(\log\left(C(\mu,\omega)\right) - \frac{N(\mu,\omega)^{1+\psi}}{1+\psi}\right). \tag{27}$$

Bound on μ. Equation (25) delivers the upper bound on steady-state inflation that is commonly found in this class of models (e.g. Ascari and Sbordone (2014)). As inflation grows, the denominator goes to zero faster than the numerator, so it has to be that $\Pi(\mu, \omega) < \theta^{-\frac{1}{\epsilon}}$ for steady state hours to be finite[25]. Given the formula for steady-state inflation (Result 2.1), we can then derive the following restriction on admissible paramter values for given levels of μ :

$$\left(1 - \rho^{\zeta}\right)(\phi - 1)\log(\theta) < \epsilon\mu \tag{28}$$

which can be rearranged into the expression in equation (16). Notice that given natural paramter restriction the left-hand side of equation (28) is negative as $0 < \theta < 1$. Under symmetry ($\underline{\mu} = -\overline{\mu}$) the worst case level of μ is also negative and keeps decreasing as the degree of ambiguity increases. So, in practice, it is enough to verify that for the largest (in absolute value) level of ambiguity this restriction is satisfied for our calibration/estimation. For concreteness, in our estimation exercise we express this constraint as an upper bound on ρ^{ζ} and verify our estimate for ρ^{ϵ} satisfies it.

B Proofs of Steady State Results

Proof of Result 2.1. In steady state, equation 8 becomes:

$$\frac{1}{C} = \frac{\beta \frac{1}{\beta} e^{\zeta} \Pi^{\phi}}{C\Pi} \tag{29}$$

where $\frac{1}{\beta}$ is the steady state value for the natural rate of interest. Simplifying and rearranging delivers equation (17). The second part follows immediately. □

[25]Indeed, the same condition could be derived from the formula for price dispersion in equation (24).

Proof of Proposition 2.2. $\mathbb{V}(\mu,\omega)$, as defined in equation (27), is continuously differentiable around zero. Direct computation, or noting that the first-best allocation is attained in our model when $\mu = 0$, shows that $\frac{\partial \mathbb{V}(\mu,\omega)}{\partial \mu} = 0$. Direct computation also delivers:

$$\left.\frac{\partial^2 \mathbb{V}(\mu,\omega)}{\partial \mu^2}\right|_{\mu=0} = -\frac{\theta\left((\beta-1)^2\theta + \epsilon(\beta\theta-1)^2(1+\psi)\right)}{(1-\beta)(\theta-1)^2(\beta\theta-1)^2(\phi-1)^2(1+\psi)(-1+\rho^{\varsigma})^2} \tag{30}$$

All the terms are positive given the minimal theoretical restrictions we impose, hence the second derivative is strictly negative, which completes the proof of parts *i.* and *ii.*.

Direct computation shows that the third derivative evaluated at $\mu = 0$ can be expressed as:

$$\left.\frac{\partial^3 \mathbb{V}(\mu,\omega)}{\partial \mu^3}\right|_{\mu=0} = \frac{\epsilon(2\epsilon-1)\theta(1+\theta)}{(1-\beta)(1-\theta)^3(\phi-1)^3(1-\rho^{\varsigma})^3} + \mathcal{R}(\beta) \tag{31}$$

Where, given our parameter restrictions, the first term on the RHS is positive and $\mathcal{R}(\beta)$ is a term in β such that $\lim_{\beta\to 1^-}\mathcal{R}(\beta) = 0$. Hence, $\lim_{\beta\to 1^-} \left.\frac{\partial^3 \mathbb{V}(\mu,\omega)}{\partial \mu^3}\right|_{\mu=0} = +\infty$.

Moreover, $\partial\left(\left.\frac{\partial^3 \mathbb{V}(\mu,\omega)}{\partial \mu^3}\right|_{\mu=0}\right)/\partial\beta$ exists, which ensures continuity of the third derivative in β. Hence the third derivative is positive for any β sufficiently close to but below unity.

A third-order Taylor expansion around zero can be used to show that, for a generic small but positive μ_0:

$$\mathbb{V}(\mu_0,\omega) - \mathbb{V}(-\mu_0,\omega) = \left.\frac{\partial^3 \mathbb{V}(\mu,\omega)}{\partial \mu^3}\right|_{\mu=0} \frac{2\mu_0^3}{6} + o(\mu_0^4) > 0, \tag{32}$$

So the steady state value function attains a lower value at $-\mu_0$ than it does at at $+\mu_0$. This, combined with the absence of internal minima, delivers our result under symmetry ($\underline{\mu} = -\overline{\mu}$). □

C Model Dynamics

To study the dynamic properties of our model, we log-linearize the equlibrium conditions around the worst-case steady state. As explained in Ascari and Ropele (2007), having price dispersion in steady state essentially results in an additional term in the Phillips Curve. Appendix C.1 presents the log-linear approximation around a generic steady state indexed by μ. By setting $\mu = -\overline{\mu}$, we obtain the log-linear approximation to the worst-case steady state (under symmetry).
Armed with the solution of the log-linearized model, we can verify if the worst-case indeed corresponds to our conjecture not only in steady state but also around it.

Once we have verified our conjecture about the worst-case steady state, we turn our attention to the implications of changes in the agents' confidence in their understanding of the monetary policy rule on the determinacy region and we then study the effects of shocks to ambiguity.

C.1 Log-linearized Equations and Solution

The following equations describe the dynamics of the variables of interest around a generic steady state indexed by μ[26]. Setting μ to its worst-case steady state value, one obtains the log-linear approximation around the worst-

[26]Where we refer to the numerator of equation (13) as F1 and to the denominator as F2 and express both in recursive form as $F1_t = \frac{\epsilon}{\epsilon-1} MC_t + \beta\theta \mathbb{E}_t \Pi^{\epsilon}_{t+1} F1_{t+1}$ and $F2_t = 1 + \beta\theta \mathbb{E}_t \Pi^{\epsilon-1}_{t+1} F2_{t+1}$ respectively.

case steady state:

$$
\begin{aligned}
c_t &= \mathbb{E}_t \left[c_{t+1} - (r_t - \pi_{t+1})\right] && (33)\\
\pi_t &= \kappa_0 (\mu, \omega)\, mc_t + \kappa_1 (\mu, \omega)\, \mathbb{E}_t \widehat{F2}_{t+1} + \kappa_2 (\mu, \omega)\, \mathbb{E}_t \pi_{t+1} && (34)\\
r_t &= (r_t^n + \zeta_{t+1}) + \phi \pi_t && (35)\\
\hat{\Delta}_t &= \kappa_3 (\mu, \omega)\, \hat{\Delta}_{t-1} + \kappa_4 (\mu, \omega)\, \pi_t && (36)\\
\widehat{F2}_t &= \kappa_5 (\mu, \omega)\, \mathbb{E}_t \pi_{t+1} + \kappa_6 (\mu, \omega)\, \mathbb{E}_t \widehat{F2}_{t+1} && (37)\\
mc_t &= w_t - a_t && (38)\\
w_t &= c_t + \psi n_t && (39)\\
y_t &= a_t - \tilde{\Delta}_t + n_t && (40)\\
c_t &= y_t && (41)\\
r_t^n &= \mathbb{E}_t a_{t+1} - a_t && (42)\\
y_t^n &= a_t && (43)\\
a_t &= \rho^a a_{t-1} + u_t^a && (44)
\end{aligned}
$$

Where κ's are known convultions of deep parameters and the degree of ambiguity, defined as:

$$\kappa_0(\mu,\omega) \equiv \frac{\left(\left(\frac{1}{\Pi(\mu,\omega)}\right)^{\epsilon-1}-\theta\right)\left(1-\beta\theta\Pi(\mu,\omega)^{\epsilon}\right)}{\theta} \tag{45}$$

$$\kappa_1(\mu,\omega) \equiv \beta\left(\left(\frac{1}{\Pi(\mu,\omega)}\right)^{\epsilon-1}-\theta\right)\left(\Pi(\mu,\omega)-1\right)\Pi(\mu,\omega)^{\epsilon-1} \tag{46}$$

$$\kappa_2(\mu,\omega) \equiv \beta\Pi(\mu,\omega)^{\epsilon-1}\left(-\theta(\epsilon-1)(\Pi(\mu,\omega)-1)+(1-\epsilon+\epsilon\Pi(\mu,\omega))\left(\frac{1}{\Pi(\mu,\omega)}\right)^{\epsilon-1}\right) \tag{47}$$

$$\kappa_3(\mu,\omega) \equiv \Pi(\mu,\omega)^{\epsilon}\theta \tag{48}$$

$$\kappa_4(\mu,\omega) \equiv \frac{\theta\epsilon\left(\Pi(\mu,\omega)-1\right)}{\left(\frac{1}{\Pi(\mu,\omega)}\right)^{\epsilon-1}-\theta} \tag{49}$$

$$\kappa_5(\mu,\omega) \equiv (\epsilon-1)\beta\theta\Pi(\mu,\omega)^{\epsilon-1} \tag{50}$$

$$\kappa_6(\mu,\omega) \equiv \beta\theta\Pi(\mu,\omega)^{\epsilon-1} \tag{51}$$

The equations above can be summarized in the following system of four equations:

$$\tilde{y}_t = \mathbb{E}_t\left[\tilde{y}_{t+1}-(\phi\pi_t+\zeta_{t+1}-\pi_{t+1})\right] \tag{52}$$

$$\pi_t = \kappa_0(\mu,\omega)\left((1+\psi)\tilde{y}_t+\psi\hat{\Delta}_t\right)+\kappa_1(\mu,\omega)\,\mathbb{E}_t\widehat{F2}_{t+1}+\kappa_2(\mu,\omega)\,\mathbb{E}_t\pi_{t+1} \tag{53}$$

$$\hat{\Delta}_t = \kappa_3(\mu,\omega)\,\hat{\Delta}_{t-1}+\kappa_4(\mu,\omega)\,\pi_t \tag{54}$$

$$\widehat{F2}_t = \mathbb{E}_t\left(\kappa_5(\mu,\omega)\,\pi_{t+1}+\kappa_6(\mu,\omega)\,\widehat{F2}_{t+1}\right) \tag{55}$$

Where $\tilde{y}_t \equiv y_t - y_t^n = y_t - a_t$ is the flex-price output gap. Moreover, $\mathbb{E}_t[\zeta_{t+1}] = \rho^{\zeta}\zeta_t+\mu_t$ and, assuming symmetric bounds, $\mu_t = -\tilde{\overline{\mu}}_t$. $\tilde{\overline{\mu}}_t = \widehat{\overline{\mu}}_t-(1-\rho^{\mu})\overline{\mu}$ is the degree of ambiguity in deviation from its mean, and $\widehat{\overline{\mu}}_t = (1-\rho^{\mu})\overline{\mu}+\rho^{\mu}\widehat{\overline{\mu}}_{t-1}+u_t^{\mu}$ governs the cyclical variations of ambiguity around its steady state. In words, a positive shock u_t^{μ} implies that ambiguity will be higher than usual which, under our worst-case steady state results[27], means that agents will "increase their underestimation of the policy rate".

These equation make it transparent that technological shocks do not affect any of the four variables in this system and, in particular, the output gap and inflation. This is common in models in which the natural rate enters the policy rule but it useful to verify that this property extends to the case in which we log-linearize the model around the worst-case steady state. a_t will still play a role, though, in that consumption, and thus utility, respond to it.

C.2 Worst-Case around the Steady State

We now need to study welfare around the worst-case steady state. To preserve analytical tractability we focus on the case in which hours enter felicity linearly ($\omega \in \Omega^0$) and follow a similar approach to that in Ilut and Scheider (2014), section II. Namely:

i. Based on Result 2.2, we determine the worst-case steady state, i.e. $\mu = -\overline{\mu}$ under symmetry.

ii. We then solve for the linear policy functions, thus getting a mapping from the state of the economy $s_t = (\hat{\Delta}_{t-1}, \zeta_t, a_t, \tilde{\overline{\mu}}_t)$[28] onto all the endogenous variables of interest, most notably c_t and n_t, which determine welfare.

iii. We then verify whether welfare is increasing in ζ_t, consistent with the guess that the worst-case scenario is that in which its expectation takes the lower value on the range ($-\overline{\mu}$).

If hours enter felicity linearly ($\psi = 0$), the price dispersion term drops out of equation (53). As a result, while price dispersion still affects hours worked and thus welfare, it does not enter the policy functions for inflation, the output gap and the forward-looking term $\widehat{F2}_t$. Moreover, the presence of the natural

[27]The next section explores how it holds around the worst-case steady state.

[28]Under symmetry, we do not need to keep track of variations in $\underline{\mu}$ and $\overline{\mu}$ separately, hence the state of the economy is smaller.

rate in the policy rule, implies that technology also does not affect any of these variables (while affecting welfare via consumption and hours). So ultimately

Policy functions for these variables are thus linear functions in $(\zeta_t, \tilde{\overline{\mu}}_t)$, where we use λ's to denote the coefficients, which can be computed analytically in this case using the method of undetermined coefficients.

For this particular exercise, the only relevant coefficients are those pertaining to ζ_t so we only report the expressions for them:

$$\pi_t = \lambda_{\pi\zeta}\zeta_t + \lambda_{\pi\mu}\tilde{\overline{\mu}}_t \qquad \lambda_{\pi\zeta} \equiv \frac{-\kappa_0\rho^\zeta}{(1-\rho^\zeta)\left(1 + \frac{(\phi-\rho^\zeta)\kappa_0}{1-\rho^\zeta} - \rho^\zeta\left(\kappa_2 + \frac{\rho^\zeta\kappa_1\kappa_5}{1-\rho^\zeta\kappa_6}\right)\right)} \tag{56}$$

$$\tilde{y}_t = \lambda_{y\zeta}\zeta_t + \lambda_{y\mu}\tilde{\overline{\mu}}_t \qquad \lambda_{y\zeta} \equiv \frac{\rho^\zeta + \left(\phi - \rho^\zeta\right)\lambda_{\pi\zeta}}{1-\rho^\zeta} \tag{57}$$

$$\widehat{F2}_t = \lambda_{F\zeta}\zeta_t + \lambda_{F\mu}\tilde{\overline{\mu}}_t \qquad \lambda_{F\zeta} \equiv \frac{\rho^\zeta\kappa_5\lambda_{\pi\zeta}}{1-\rho^\zeta\kappa_6} \tag{58}$$

$$\hat{\Delta}_t = \kappa_3\hat{\Delta}_{t-1} + \kappa_4\left(\lambda_{\pi\zeta}\zeta_t + \lambda_{\pi\mu}\tilde{\overline{\mu}}_t\right) \tag{59}$$

where κ's are defined in equations (45)-(51).
Using the expressions in Section C.1 we can then out the policy functions for consumption and hours:

$$c_t = \tilde{y}_t + a_t = \lambda_{y\zeta}\zeta_t + +\lambda_{y\mu}\tilde{\overline{\mu}}_t + a_t \tag{60}$$

$$n_t = c_t - a_t + \hat{\Delta}_t = \tilde{y}_t + \hat{\Delta}_t = \lambda_{y\zeta}\zeta_t + \lambda_{y\mu}\tilde{\overline{\mu}}_t + \kappa_3\hat{\Delta}_{t-1} + \kappa_4\left(\lambda_{\pi\zeta}\zeta_t + \lambda_{\pi\mu}\tilde{\overline{\mu}}_t\right) \tag{61}$$

Turning to the value function, we can express it using the policy functions, as:

$$\mathbb{V}\left(\hat{\Delta}_{t-1}, \zeta_t, a_t, \tilde{\overline{\mu}}_t\right) = \log\left((1+c_t)C(-\overline{\mu},\omega)\right) - (1+n_t)N\left(-\overline{\mu},\omega\right) + \beta\mathbb{E}_t\mathbb{V}\left(\hat{\Delta}_t, \zeta_{t+1}, a_{t+1}, \tilde{\overline{\mu}}_{t+1}\right) \tag{62}$$

$$\mathbb{V}\left(\hat{\Delta}_{t-1}, \zeta_t, a_t, \tilde{\overline{\mu}}_t\right) = \left[\log\left(C(-\overline{\mu},\omega)\right) - N\left(-\overline{\mu},\omega\right)\right] + \log\left(1+c_t\right) - n_tN\left(-\overline{\mu},\omega\right) + \beta\mathbb{E}_t\mathbb{V}\left(\hat{\Delta}_t, \zeta_{t+1}, a_{t+1}, \tilde{\overline{\mu}}_{t+1}\right) \tag{63}$$

$$\mathbb{V}\left(\hat{\Delta}_{t-1}, \zeta_t, a_t, \tilde{\overline{\mu}}_t\right) = u\left(\overrightarrow{C}\left(-\overline{\mu},\omega\right)\right) + \log\left(1+c_t\right) - n_tN\left(-\overline{\mu},\omega\right) + \beta\mathbb{E}_t\mathbb{V}\left(\hat{\Delta}_t, \zeta_{t+1}, a_{t+1}, \tilde{\overline{\mu}}_{t+1}\right) \tag{64}$$

We can then define $d\mathbb{V}\left(\hat{\Delta}_{t-1}, \zeta_t, a_t, \tilde{\overline{\mu}}_t\right) \equiv \mathbb{V}\left(\hat{\Delta}_{t-1}, \zeta_t, a_t, \tilde{\overline{\mu}}_t\right) - \mathbb{V}\left(-\overline{\mu}, \omega\right)$, which can be expressed as:

$$d\mathbb{V}\left(\hat{\Delta}_{t-1}, \zeta_t, a_t, \tilde{\overline{\mu}}_t\right) = \log\left(1 + c_t\right) - n_t N\left(-\overline{\mu}, \omega\right) + \beta \mathbb{E}_t d\mathbb{V}\left(\hat{\Delta}_t, \zeta_{t+1}, a_{t+1}, \tilde{\overline{\mu}}_{t+1}\right) \quad (65)$$

$$d\mathbb{V}\left(\hat{\Delta}_{t-1}, \zeta_t, a_t, \tilde{\overline{\mu}}_t\right) = c_t - n_t + \beta \mathbb{E}_t d\mathbb{V}\left(\hat{\Delta}_t, \zeta_{t+1}, a_{t+1}, \tilde{\overline{\mu}}_{t+1}\right) \quad (66)$$

$$d\mathbb{V}\left(\hat{\Delta}_{t-1}, \zeta_t, a_t, \tilde{\overline{\mu}}_t\right) = a_t - \hat{\Delta}_t + \beta \mathbb{E}_t d\mathbb{V}\left(\hat{\Delta}_t, \zeta_{t+1}, a_{t+1}, \tilde{\overline{\mu}}_{t+1}\right) \quad (67)$$

$$d\mathbb{V}\left(\hat{\Delta}_{t-1}, \zeta_t, a_t, \tilde{\overline{\mu}}_t\right) = a_t - \kappa_3 \hat{\Delta}_{t-1} - \kappa_4 \lambda_{\pi\zeta} \zeta_t - \kappa_4 \lambda_{\pi\mu} \tilde{\overline{\mu}}_t + \beta \mathbb{E}_t d\mathbb{V}\left(\hat{\Delta}_t, \zeta_{t+1}, a_{t+1}, \tilde{\overline{\mu}}_{t+1}\right) \quad (68)$$

The result[29] clearly illustrates how, to a first-order approximation, welfare around the worst-case steady state moves with the difference between consumption and hours worked (equation (66)) which, in turn can be expressed as the difference between the level of technology and that of price dispersion (equation (67)). This is obvious if one considers the resource constraint: $C_t = \frac{A_t}{\Delta_t} N_t$. Both A_t and Δ_t act as wedges between the level of consumption and that of hours, though they work in opposite directions.
Moreover, while A_t is completely exogenous, Δ_t is a discounted sum of past deviations of inflation from steady state. As demonstrated in Proposition 2.4, $\lambda_{\pi\zeta} < 0$. On the other hand, $\kappa_4 > 0$ when $\Pi\left(\mu, \omega\right) > 1$, as is the case in the worst-case steady state under symmetry[30].

This means that a lower value for ζ_t will lead to an increase in inflation (it is a loosening in the monetary policy stance), which results in an increase in the price dispersion index for the foreseeable future ($\kappa_3 > 0$). As a result, lower ζ_t will result in a lower level of welfare, which illustrates how welfare is increasing in ζ_t, so that a pessimistic agent would act on the assumption that ζ_t will be lower in the future. Which is the conjecture we make when we assume that the expectation distortion μ_t equals $-\overline{\mu}_t$ at all times so long as

[29] We use the Taylor approximation for $\log(1+c_t)$ and the fact that $N\left(-\overline{\mu}, \omega\right)$, as defined in (25), is extremely close to one for reasonable values for $\overline{\mu}$.

[30] Notice that the condition presented in equation (28) implies that $\theta\Pi\left(\mu, \omega\right)^{\epsilon} < 1$. When $\Pi\left(\mu, \omega\right) > 1$, $\theta\Pi\left(\mu, \omega\right)^{\epsilon-1} < \theta\Pi\left(\mu, \omega\right)^{\epsilon} < 1$ so the condition in equation (28), which we is necessary for hours worked in steady state to be positive, implies that the denominator of equation (49) is positive.

the symmetry assumption ($\underline{\mu}_t = -\overline{\mu}_t$) holds.

Two more remarks are in order. First, if one was to approximate the economy around the ergodic steady state instead, the argument would still go through. The key parameter in equation (68) is κ_4. So long as the (gross) steady state level of inflation around which the model is approximated is larger than one, $\kappa_4 > 0$. Proposition 2.4 implies that the (gross) ergodic steady state level of inflation exceeds unity whenever the worst-case does. The sign of κ_4 would thus be positive even if the model was log-linearized around the ergodic steady state, thus implying that welfare would still be increasing in ζ_t.

Second, when $|\underline{\mu}| << |\overline{\mu}|$ so that worst case inflation is $\Pi(\overline{\mu}, \omega) < 1$, $\kappa_4 < 0$ and the welfare function is decreasing in ζ_t, verifying the maintained conjecture that $\mu_t = \overline{\mu}_t$ at all times under those circumstances.

C.3 Proof of Proposition 2.3

We report the proof for the case in which $\mu = \underline{\mu} = -\overline{\mu}$ (i.e. the symmetry case). But the same steps apply when in the worst-case $\mu = \overline{\mu}$.

We can collect the relevant set of variables in deviation from their worst-case steady state in a vector $x_t = \begin{bmatrix} \tilde{y}_t & \pi_t & \hat{\Delta}_t & F2_t & \zeta_t & a_t & \widehat{\overline{\mu}}_t \end{bmatrix}'$ and the innovations in a vector $\underline{u}_t = \begin{bmatrix} u_t^\zeta & u_t^a & u_t^\mu \end{bmatrix}'$.

The law of motion for x_t around the worst case steady state can be represented as:

$$x_t = \begin{bmatrix} 0 & 0 & 0 & 0 & \rho^\zeta \lambda_{y\zeta} & \rho^a \lambda_{ya} & \rho^\mu \lambda_{y\mu} \\ 0 & 0 & 0 & 0 & \rho^\zeta \lambda_{\pi\zeta} & \rho^a \lambda_{\pi a} & \rho^\mu \lambda_{\pi\mu} \\ 0 & 0 & \kappa_3 & 0 & \rho^\zeta \kappa_4 \lambda_{\pi\zeta} & \rho^a \kappa_4 \lambda_{\pi a} & \rho^\mu \kappa_4 \lambda_{\pi\mu} \\ 0 & 0 & 0 & 0 & \rho^\zeta \lambda_{F\zeta} & \rho^a \lambda_{Fa} & \rho^\mu \lambda_{F\mu} \\ 0 & 0 & 0 & 0 & \rho^\zeta & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & \rho^a & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho^\mu \end{bmatrix} x_{t-1} + \begin{bmatrix} \lambda_{y\zeta} & \lambda_{ya} & \lambda_{y\mu} \\ \lambda_{\pi\zeta} & \lambda_{\pi a} & \lambda_{\pi\mu} \\ \kappa_4 \lambda_{\pi\zeta} & \kappa_4 \lambda_{\pi a} & \kappa_4 \lambda_{\pi\mu} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underline{u}_t \tag{69}$$

Or, more compactly:

$$x_t = Ax_{t-1} + R\underline{u}_t \tag{70}$$

Equation (70) governs the dynamic around the worst-case steady state. We define the ergodic steady state starting from this representation of the economy, as in Ilut and Schneider (2014), and correcting for the fact that the realizazions for ζ_t differ from those expected in the previous period by a factor μ_{t-1}.

$$\left(\tilde{x}_t - x^W\right) = A\left(\tilde{x}_{t-1} - x^W\right) + R\underline{u}_t + R\begin{bmatrix} \mu_{t-1} \\ 0 \\ 0 \end{bmatrix} \tag{71}$$

So the ergodic steady state can be defined as:

$$\overline{x} = x^W + (I - A)^{-1} R \begin{bmatrix} \overline{\mu} \\ 0 \\ 0 \end{bmatrix} \tag{72}$$

which can be readily computed.
In particular, we can compute analytically the ergodic steady state for inflation by defining $B \equiv I - A$ and noting that we only need the second row of B^{-1} which we can define as a row vector $\underline{w}$ such that:

$$\underline{w}B = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{73}$$

Direct computation shows that:

$$\underline{w} = \begin{bmatrix} 0 & 1 & 0 & 0 & \frac{\rho^\zeta \lambda_{\pi\zeta}}{1-\rho^\zeta} & \frac{\rho^a \lambda_{\pi a}}{1-\rho^a} & \frac{\rho^\mu \lambda_{\pi\mu} - \frac{\rho^\zeta \lambda_{\pi\zeta}}{1-rho^\zeta}}{1-\rho^\mu} \end{bmatrix} \tag{74}$$

And finally:

$$\underline{w}R\begin{bmatrix}\overline{\mu}\\0\\0\end{bmatrix} = \overline{\mu}\lambda_{\pi\zeta}+\frac{\overline{\mu}\lambda_{\pi\zeta}\rho^{\zeta}}{1-\rho^{\zeta}}=\frac{\overline{\mu}\lambda_{\pi\zeta}}{1-\rho^{\zeta}} \tag{75}$$

Which defines the ergodic steady state realtive to its worst-case counterpart π^{W}. Since in the worst-case under symmetry $\mu=-\overline{\mu}$, $\frac{\overline{\mu}\lambda_{\pi\zeta}}{1-\rho^{\zeta}}=-\frac{\mu\lambda_{\pi\zeta}}{1-\rho^{\zeta}}$.

C.4 Proof of Proposition 2.4

In a neighborhood of $\mu=0$ we can approximate the expression for $\overline{\pi}$ in (18) with its first-order Taylor expansion:

$$\overline{\pi}(\mu,\omega)=\overline{\pi}(0,\omega)+\left.\frac{\partial\overline{\pi}}{\partial\mu}\right|_{\mu=0}\mu=\left.\frac{\partial\overline{\pi}}{\partial\mu}\right|_{\mu=0}\mu \tag{76}$$

Where:

$$\left.\frac{\partial\overline{\pi}}{\partial\mu}\right|_{\mu=0} = \left.\left(-\frac{1}{1-\rho^{\zeta}}\left[\frac{1}{\phi-1}+\lambda_{\pi\zeta}(\mu,\omega)\right]-\frac{\mu}{1-\rho^{\zeta}}\frac{\partial\lambda_{\pi\zeta}(\mu,\omega)}{\partial\mu}\right)\right|_{\mu=0} \tag{77}$$

$$= -\frac{1}{1-\rho^{\zeta}}\left[\frac{1}{\phi-1}+\lambda^{*}_{\pi\zeta}\right] \tag{78}$$

where we define $\lambda^{*}_{\pi\zeta}\equiv\lambda_{\pi\zeta}(0,\omega)$.
From equations (45) - (51) it is immediate to notice that as $\mu\to 0$:

$$\kappa_0 \to \frac{(1-\theta)(1-\beta\theta)}{\theta} \tag{79}$$

$$\kappa_1 \to 0 \tag{80}$$

$$\kappa_2 \to \beta \tag{81}$$

$$\kappa_6 \to \beta\theta \tag{82}$$

So $\lambda^*_{\pi\zeta} = \frac{1}{1-\frac{\phi}{\rho^\zeta}-\frac{\theta\left(1-\beta\rho^\zeta\right)\left(1-\rho^\zeta\right)}{\rho^\zeta(1-\beta\theta)(1-\theta)}}$.

Moreover:

$$\frac{\partial\lambda^*_{\pi\zeta}}{\partial\theta} = \frac{\rho^\zeta(1-\beta\theta)\left(1-\rho^\zeta\right)\left(1-\beta\rho^\zeta\right)}{\left((\theta-\rho^\zeta)(1-\beta\theta\rho^\zeta)-(1-\theta)(1-\beta\theta)\phi\right)^2} > 0 \quad \forall\omega\in\Omega^0 \tag{83}$$

$$\frac{\partial\lambda^*_{\pi\zeta}}{\partial\rho^\zeta} = -\frac{(1-\theta)(1-\beta\theta)\left(\theta\left(1-\beta(\rho^\zeta)^2\right)+(1-\theta)(1-\beta\theta)\phi\right)}{\left((\theta-\rho^\zeta)(1-\beta\theta\rho^\zeta)-(1-\theta)(1-\beta\theta)\phi\right)^2} < 0 \quad \forall\omega\in\Omega^0 \tag{84}$$

$$\frac{\partial\lambda^*_{\pi\zeta}}{\partial\beta} = \frac{\rho^\zeta\theta(1-\theta)\left(1-\rho^\zeta\right)\left(\theta-\rho^\zeta\right)}{\left((\theta-\rho^\zeta)(1-\beta\theta\rho^\zeta)-(1-\theta)(1-\beta\theta)\phi\right)^2} \tag{85}$$

So $\lambda^*_{\pi\zeta}$ is continuous and monotonic in θ and ρ^ζ. It is also weakly monotonic in β: increasing if $\theta>\rho^\zeta>0$, flat if $\theta=\rho^\zeta$ and decreasing otherwise. So we only need to check the extremes:

$$\lambda^*_{\pi\zeta} \to \begin{cases} -\frac{\rho^\zeta}{\phi-\rho^\zeta} & \text{as } \theta\to 0 \\ 0 & \text{as } \theta\to 1 \\ 0 & \text{as } \rho^\zeta\to 0 \\ -\frac{1}{\phi-1} & \text{as } \rho^\zeta\to 1 \end{cases} \tag{86}$$

$$\frac{1}{1-\frac{\phi}{\rho^\zeta}-\frac{\theta\left(1-\rho^\zeta\right)\left(1-\rho^\zeta\right)}{\rho^\zeta(1-\theta)(1-\theta)}} \to \begin{cases} -\frac{\rho^\zeta}{\phi-\rho^\zeta} & \text{as } \theta\to 0 \\ 0 & \text{as } \theta\to 1 \\ 0 & \text{as } \rho^\zeta\to 0 \\ -\frac{1}{\phi-1} & \text{as } \rho^\zeta\to 1 \end{cases} \tag{87}$$

$$\frac{1}{1-\frac{\phi}{\rho^\zeta}-\frac{\theta\left(1-\rho^\zeta\right)}{\rho^\zeta(1-\theta)}} \to \begin{cases} -\frac{\rho^\zeta}{\phi-\rho^\zeta} & \text{as } \theta\to 0 \\ 0 & \text{as } \theta\to 1 \\ 0 & \text{as } \rho^\zeta\to 0 \\ -\frac{1}{\phi-1} & \text{as } \rho^\zeta\to 1 \end{cases} \tag{88}$$

Where the second and third expressions are the limits for $\lambda^*_{\pi\zeta}$ as β tends to 1 and to 0, respectively.

So $-\frac{1}{\phi-1} < \lambda^*_{\pi\zeta} < 0$.

Given our definition for $\lambda^*_{\pi\zeta}$, this proves point $i.$.

Using these inequalities in equation (77) proves that $\frac{\partial\overline{\pi}}{\partial\mu}\Big|_{\mu=0} < 0$, i.e. the ergodic steady state level of inflation is decreasing in μ. $\pi^W = -\frac{\mu}{(1-\rho^\zeta)(\phi-1)}$ is also clearly decreasing in μ. Which proves point $ii.$.

Finally $-\frac{1}{\phi-1} < \lambda^*_{\pi\zeta} < 0$ implies that:

$$0 > \frac{\partial\overline{\pi}}{\partial\mu}\Big|_{\mu=0} = -\frac{1}{1-\rho^\zeta}\left[\frac{1}{\phi-1} + \lambda^*_{\pi\zeta}\right] > \frac{\partial\pi^W}{\partial\mu} = -\frac{1}{(1-\rho^\zeta)(\phi-1)} \tag{89}$$

Which means that while both π^W and $\overline{\pi}$ are negatively related to μ, the former responds more than the latter to changes in μ. Which proves point $iii.$ when we consider that cases $\mu = -\overline{\mu} < 0$ and $\mu = \overline{\mu} > 0$. □

D Estimation

D.1 Alternative estimation specification

Table 2 reports estimation results when we relax the restriction $\phi = 1.5$. Importantly, while estimates are more sensitive to the choice of the inflation-expectations series (and, as a consequence, of the sample length), all estimates are well within the acceptable range.

In particular, the estimates for ϕ range between 1.44 and 1.74, while those for ρ^ζ between .69 and .82. These estimates also highlight the identification issues mentioned in the main body. Namely, higher values for ϕ associate with higher values for ρ^ζ. That is because, for a given level of z^μ_t higher ϕ pushes down the model-implied value for worst-case steady state inflation while higher ρ^ζ pushes the same value up so that they can easily compensate thus making identification weaker (the identification relies on the differing effects these two parameters have on $\lambda_{\pi\zeta}(z^\mu_t, \omega)$).

The variation in the estimate for θ is at first more puzzling. However the differences in the samples cast some light on the differences. The early 80s were not only characterized by high inflation expectation and trend but also by a larger difference between the two series. In our model the difference between the two series depends primarily by $lambda_{\pi\zeta}$. To gain some intuition we can use the formula for $\lambda^*_{\pi\zeta}$ we derived in Appendix 2.4, the level to which $\lambda_{\pi\zeta}(\mu, \omega)$ converges to as $\mu \to 0$, and plug in the estimated values under the different scenarios. If we use the estimates in Table 2 the implied values for $\lambda^*_{\pi\zeta}$ range between -.88 (for the Cleveland Fed 10Y inflation expectations series, the series that goes back to the early 80s) and -.52 (for the SPF 10Y estimation, which starts in the 90's). If we consider the estimates, the corresponding values for $\lambda^*_{\pi\zeta}$ are -.78 and -.68. Restricting the value for ϕ, trivially, makes it harder to fit the two series and, as it seems, consequently reduces the effect that different samples have on the estimates (primarily that of θ).

Since even our more restrictive estimation seem to be fitting the data reasonably well we opt to use that as our baseline.

Table 2:

	Cleveland Fed 10Y	Blue Chip 5-10Y	SPF 10Y
ϕ	1.4372	1.7407	1.6737
θ	0.2661	0.7921	0.7462
ρ^ζ	0.6950	0.8242	0.7745
No of obs.	129	60	97
Sample period	1983Q1-2015Q4	1986H1-2015H2	1991Q4-2015Q4

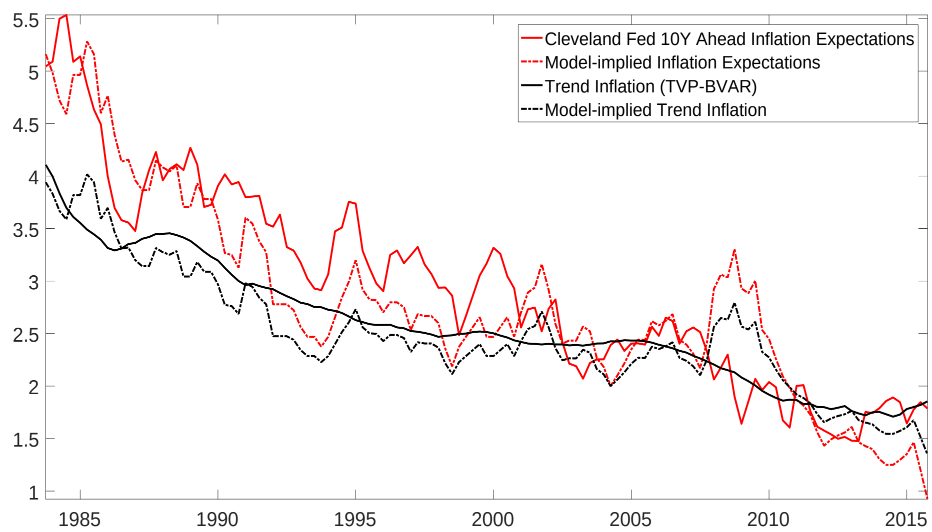
Estimates of θ and ρ^ζ obtained using different measures of long-run inflation expectations

D.2 Time-Series of the actual and fitted values for trend inflation and inflation expectations

Figure 9 overlays the measures of trend inflation implied by our model and our baseline estimate for the CPI inflation trend, along with the Cleveland Fed's measure of inflation expectations and the long-run inflation expectations implied by the model.

As mentioned in the main body of the text, our estimate for long-run inflation expectations undershoots the outturn for a number of years in the

Figure 9: Trend inflation and inflation expectations implied by our measure of disagreement



1990s, which is mostly attributable to the fact that our inflation target of 2 percent is defined in terms of PCE inflation, which is on average lower than CPI inflation.

Also, we can note a short-lived period around the onset of the Great Recession when our model read the increase in uncertainty as pushing up on inflation expectations until rates fell to the point when the $[\underline{\mu}, \overline{\mu}]$ became asymmetric thus changing the direction in which inflation moves in response to an in increase in Knightian uncertainty.

All things considered, though, we find that the model-implied trend inflation does a good job at capturing the decline in the secular component of inflation in the data, the fact that, over recent years, the inflation trend fell below the 2% mark, while also helping us to make sense of the progressive reduction of the wedge between inflation expectations and trend inflation.

E Optimal Policy

E.1 Useful Lemmas

With a slight abuse of notation we use ω to refer to all parameter values *but* ϕ in the following Lemma and in other similar circumstances in this section when we want to study the effects of ϕ while keeping the other parameters fixed.

Lemma E.1. *For any $\omega \in \Omega$ and $\overline{\mu}$, a small positive number, given any pair $(\mu, \phi) \in [-\overline{\mu}, 0) \times (1, \infty)$, for any $\mu' \in [\underline{\mu},\ 0)$ there exists $\phi' \in (1,\ \infty)$ such that:*

$$\mathbb{V}(\mu, \phi', \omega) = \mathbb{V}(\mu', \phi, \omega)$$

And $\phi' \geq \phi$ iff $\mu' \geq \mu$.
A corresponding equivalence holds for $\mu \in (0,\ \overline{\mu}]$.

Proof. Inspection reveals that μ and ϕ only enter steady-state welfare through the steady-state inflation term $\Pi(\mu, \omega) = e^{\overline{\frac{\mu}{(1-\phi)(1-\rho^{\zeta})}}}$. It follows immediately that, for a given μ', $\phi' = 1 + \frac{(\phi-1)\mu}{\mu'}$ implies that (μ, ϕ') is welfare equivalent to (μ', ϕ). $(\mu, \mu') \in [\underline{\mu}, 0) \times [\underline{\mu}, 0)$ ensures that $\mu' \cdot \mu > 0$ and so $\phi' \in (1, \infty)$ for any $\phi > 1$. The inequalities follow immediately from the definition of ϕ' given above and the fact that both μ and μ' are both negative.
A similar argument would go through for $(\mu, \mu') \in (0, \overline{\mu}] \times (0, \overline{\mu}]$. □

Lemma E.2. *Define $\mathbb{V}_{\delta}(\mu, \delta, \omega)$ to be the welfare function of an economy identical to that described in Section 2 except for the fact that the policy rule has a constant term δ entering as follows:*

$$R_t = \left(R_t^n e^{\zeta_{t+1}} e^{\delta}\right)(\Pi_t)^{\phi} \tag{90}$$

Then $\mathbb{V}_\delta()$, for a generic value μ, can be expressed as a function of welfare in the baseline economy for a different value of μ in the following way:

$$\mathbb{V}_\delta(\mu, \delta, \omega) = \mathbb{V}\left(\mu + \tilde{\delta}, \omega\right) \tag{91}$$

where $\tilde{\delta} \equiv \left(1 - \rho^\varsigma\right)\delta$.

Proof. The computation of the steady state of the model in Section A reveals that the steady state in this model can be thought of as expressing all the variables as a function of steady state inflation and steady state inflation as a function of μ, because μ only enters the model via the steady state level of inflation. The same is true of δ.
Substituting equation (90) for equation (1) and following the Proof of Result 2.1 yield the following steady state value for inflation:

$$\Pi(\mu, \delta, \omega) = e^{-\frac{\mu}{(\phi-1)(1-\rho^\varsigma)} - \frac{\delta}{\phi-1}} = e^{-\frac{\mu+\tilde{\delta}}{(\phi-1)(1-\rho^\varsigma)}} \tag{92}$$

This level of steady state inflation (and consequently of welfare) is equivalent to that in the original economy for a value μ', where $\mu' = \mu + \tilde{\delta}$. $\square$

Lemma E.3. *Assuming that $\mathbb{V}(\mu, \omega)$ takes only real values over some interval $(\underline{m}, \overline{m})$ such that $\overline{m} > \overline{\mu} > 0 > \underline{\mu} > -\overline{m}$, is continuously differentiable, strictly concave and attains a finite maximum at $\mu = \mu_0 \in (\underline{m}, \overline{m})$; then the level of $\tilde{\delta}$ that maximizes worst-case steady-state welfare is implicitly defined as:*

$$\tilde{\delta}^*(\underline{\mu}, \overline{\mu}): \qquad \mathbb{V}\left(\underline{\mu} + \tilde{\delta}^*(\underline{\mu}, \overline{\mu}), \omega\right) = \mathbb{V}\left(\overline{\mu} + \tilde{\delta}^*(\underline{\mu}, \overline{\mu}), \omega\right) \tag{93}$$

Proof. Given Lemma E.2 we can use the welfare function for the original economy and its properties derived in Proposition 2.2.
First, note that strict concavity ensures μ_0 is the unique maximum.
Second, consider the following cases:

1. $\mu_0 \in (\underline{\mu}, \overline{\mu})$: then $\mathbb{V}'(\underline{\mu}, \omega) > 0 > \mathbb{V}'(\overline{\mu}, \omega)$

a. $\mathbb{V}(\underline{\mu},\omega) < \mathbb{V}(\overline{\mu},\omega)$. Together with strict concavity this implies that the minimum (or worst-case) over $[\underline{\mu},\overline{\mu}]$ is $\underline{\mu}$. Then there exists a small $\tilde{\delta} > 0$ such that

$$\mathbb{V}(\underline{\mu},\omega) < \mathbb{V}(\underline{\mu}+\tilde{\delta},\omega) < \mathbb{V}(\overline{\mu}+\tilde{\delta},\omega) < \mathbb{V}(\overline{\mu},\omega).$$

So now the worst case $\underline{\mu}+\tilde{\delta}$ generates a higher level of welfare. The worst-case welfare can be improved until the second inequality above holds with equality. Continuity ensures such a level of $\tilde{\delta}^*$ exists. Our assumptions also ensure that $\mu_0 - \underline{\mu} > \tilde{\delta}^*$[31], because if $\underline{\mu}+\tilde{\delta} = \mu_0$, $\mathbb{V}(\underline{\mu}+\tilde{\delta}) = \mathbb{V}(\mu_0)$ which is a unique maximum so the equality cannot hold. This, in turn ensures that $\mathbb{V}'(\underline{\mu}+\tilde{\delta}^*) > 0 > \mathbb{V}'(\overline{\mu}+\tilde{\delta}^*)$. Hence, any further increase in $\tilde{\delta}$ would make $\overline{\mu}+\tilde{\delta}$ the worst case and welfare at the worst-case would fall.

b. $\mathbb{V}(\underline{\mu},\omega) > \mathbb{V}(\overline{\mu},\omega)$. Together with strict concavity, this implies that the minimum is attained at $\overline{\mu}$. Then there exists a small enough $\tilde{\delta} < 0$ such that:

$$\mathbb{V}(\overline{\mu},\omega) < \mathbb{V}(\overline{\mu}+\tilde{\delta},\omega) < \mathbb{V}(\underline{\mu}+\tilde{\delta},\omega) < \mathbb{V}(\underline{\mu},\omega)$$

By the same arguments as above $\mu_0 - \overline{\mu} < \tilde{\delta}^* < 0$ makes the second inequality hold with equality and attains the higher level of welfare.

c. $\mathbb{V}(\underline{\mu},\omega) = \mathbb{V}(\overline{\mu},\omega)$. Any $\tilde{\delta} \neq 0$ would lower the worst-case welfare: $\tilde{\delta}^* = 0$.

2. $\mu_0 \in [\overline{\mu},\overline{m})$. Strict concavity implies that welfare is strictly increasing over $[\underline{\mu},\overline{\mu}]$ and $\underline{\mu}$ mimizes welfare over that range. For all $0 \leq \tilde{\delta} \leq \mu_0 - \overline{\mu}$

$$\mathbb{V}(\underline{\mu},\omega) \leq \mathbb{V}(\underline{\mu}+\tilde{\delta},\omega) < \mathbb{V}(\overline{\mu}+\tilde{\delta},\omega) \leq \mathbb{V}(\mu_0,\omega)$$

The second inequality will always be strict. For $\tilde{\delta}$ just above $\mu_0 - \overline{\mu}$ the logic of case 1a above applies and $\tilde{\delta}^*$ can be determined accordingly.

[31] Clearly $\mu_0 - \underline{\mu} > 0$, since $\mu_0 > \underline{\mu}$ in this case.

3. $\mu_0 \in (\underline{m}, \underline{\mu}]$. Strict concavity implies that welfare is strictly decreasing over $[\underline{\mu}, \overline{\mu}]$ and $\overline{\mu}$ mimizes welfare over that range. For all $0 \geq \tilde{\delta} \geq \mu_0 - \underline{\mu}$

$$\mathbb{V}(\mu_0, \omega) \geq \mathbb{V}(\underline{\mu} + \tilde{\delta}, \omega) > \mathbb{V}(\overline{\mu} + \tilde{\delta}, \omega) \geq \mathbb{V}(\overline{\mu}, \omega)$$

The second inequality will always be strict. For $\tilde{\delta}$ just below $\mu_0 - \underline{\mu}$ the logic of case case 1b above applies and $\tilde{\delta}^*$ can be determined accordingly.

□

E.2 Proof of Proposition 4.1

Lemma E.2 applies so we will use the welfare function of the original economy. Then, the proof of static optimality proceeds in two steps by first findind the optimal value of ϕ given assumptions on the welfare function and on $\tilde{\delta}$ and then verifying that for $\overline{\phi}$, the conjectures made in the previous point hold.

i. *Suppose that* $\mathbb{V}(-\overline{\mu} + \tilde{\delta}, \omega)$ *corresponds to the worst-case steady-state welfare for some* $\tilde{\delta} \in (0, \overline{\mu})$*, then* $\phi = \overline{\phi}$ *maximizes worst-case welfare over* $[\underline{\phi}, \overline{\phi}]$.

Following the same logic as in Lemma E.1, but using the expression in equation (92) for inflation, it is easy to verify that for any $1 < \phi' < \overline{\phi}$, there exists a μ' s.t.:

$$\mathbb{V}\left(\mu' + \tilde{\delta}, \phi', \omega\right) = \mathbb{V}\left(-\overline{\mu} + \tilde{\delta}, \overline{\phi}, \omega\right) \tag{94}$$

In particular:

$$\mu' = -\overline{\mu}\left(\frac{\phi' - 1}{\overline{\phi} - 1}\right) - \tilde{\delta}\left(1 - \frac{\phi' - 1}{\overline{\phi} - 1}\right) \tag{95}$$

Given our parameter restrictions, this implies that $0 > -\tilde{\delta} > \mu' > -\overline{\mu}$. In our economy $\mathbb{V}(0, \omega)$ corresponds to the maximum, so the argmax $\mu = -\tilde{\delta}$. Together with strict concavity (Proposition 2.2), this implies that, in this case, $\mathbb{V}()$ strictly increasing for $\mu < -\tilde{\delta}$, hence:

$$\mathbb{V}\left(-\overline{\mu}+\tilde{\delta},\overline{\phi},\omega\right) = \mathbb{V}\left(\mu'+\tilde{\delta},\phi',\omega\right) > \mathbb{V}\left(-\overline{\mu}+\tilde{\delta},\phi',\omega\right) \tag{96}$$

which implies that $\overline{\phi}$ maximizes welfare over $[\underline{\phi},\overline{\phi}]$ for $\underline{\phi} > 1$ which we maintain throughout (equation (16)).

ii. *$0 < \tilde{\delta}^* < \overline{\mu}$ defined by $\mathbb{V}\left(-\overline{\mu}+\tilde{\delta}^*,\omega\right) = \mathbb{V}\left(\overline{\mu}+\tilde{\delta}^*,\omega\right)$ maximizes worst-case welfare for $\phi = \overline{\phi}$.*

Proposition 2.2 guarantees that our welfare function satisfies the assumptions of Lemma E.3 in a neighborhood of zero.
To find the bounds on $\tilde{\delta}^*$, note that for $\overline{\mu} > 0$ and $\underline{\mu} = -\overline{\mu}$, Proposition 2.2 also implies that the maximum $\mu_0 = 0$ is interior and that $\mathbb{V}(\underline{\mu},\omega) < \mathbb{V}(\overline{\mu},\omega)$. So, case 1a of the proof of Lemma E.3 applies, which implies that $\mu_0 - \underline{\mu} = \overline{\mu} > \tilde{\delta}^* > 0$. These considerations apply for any $\phi > 1$, thus also for $\overline{\phi}$. □

E.3 Proof of Corollary 4.1

Following the same approach as for Proposition 4.1

i. *Suppose that $\mathbb{V}(\overline{\mu}+\tilde{\delta},\omega)$ corresponds to the worst-case steady-state welfare for some $\tilde{\delta} \in (-\overline{\mu}, 0)$, then $\phi = \overline{\phi}$ maximizes worst-case welfare over $[\underline{\phi},\overline{\phi}]$.*

For any $1 < \phi' < \overline{\phi}$, there exists a μ' s.t.:

$$\mathbb{V}\left(\mu'+\tilde{\delta},\phi',\omega\right) = \mathbb{V}\left(\overline{\mu}+\tilde{\delta},\overline{\phi},\omega\right) \tag{97}$$

In particular:

$$\mu' = \overline{\mu}\left(\frac{\phi'-1}{\overline{\phi}-1}\right) - \tilde{\delta}\left(1-\frac{\phi'-1}{\overline{\phi}-1}\right) \tag{98}$$

Given our restrictions, this implies that $0 < -\tilde{\delta} < \mu' < \overline{\mu}$.

$\mathbb{V}()$ is strictly decreasing for $\mu > -\tilde{\delta}$, hence:

$$\mathbb{V}\left(\overline{\mu} + \tilde{\delta}, \overline{\phi}, \omega\right) = \mathbb{V}\left(\mu' + \tilde{\delta}, \phi', \omega\right) > \mathbb{V}\left(\overline{\mu} + \tilde{\delta}, \phi', \omega\right) \tag{99}$$

ii. *$0 > \tilde{\delta}^* > -\overline{\mu}$ defined by $\mathbb{V}\left(\underline{\mu} + \tilde{\delta}^*, \omega\right) = \mathbb{V}\left(\overline{\mu} + \tilde{\delta}^*, \omega\right)$ maximizes worst-case welfare for $\phi = \overline{\phi}$.*

$\mathbb{V}(\underline{\mu}, \omega) > \mathbb{V}(\overline{\mu}, \omega)$ and for $\underline{\mu} < 0$ and $\overline{\mu} > 0$ the welfare maximum $\mu_0 = 0$ is in the interior. This corresponds to case 1b of the proof of Lemma E.3, which implies that $\mu_0 - \overline{\mu} = -\overline{\mu} < \tilde{\delta}^* < 0$. □

E.4 Corollary: general applicability of the optimality result in Proposition 4.1

Corollary E.1. *A monetary policy rule with the same functional form as that in Proposition 4.1 can attain the same level of steady-state welfare as any other generic rule for a suitably high level of $\overline{\phi}$.*

Proof. We will focus on the symmetric case $\underline{\mu} = -\overline{\mu}$ but the logic is the same for cases in which the worst-case steady state corresponds to $\overline{\mu}$. Moreover, this sufficient condition can be derived even for $\delta = 0$, so we will assume that for expositional simplicity. Setting δ optimally will simply make the *suitably high level of* $\overline{\phi}$ somewhat lower.

Consider any policy plan delivering utility v_0 in steady state. Suppose that is welfare-superior to the policy currently in place $\mathbb{V}(-\overline{\mu}, \phi, \omega) < v_0 \leq \mathbb{V}(0, \phi, \omega)$, where the latter is the first-best allocation.

Proposition 2.2 ensures that welfare is strictly increasing and continous for $\mu < 0$ so there exists a μ', $-\overline{\mu} < \mu' \leq 0$, s.t. $\mathbb{V}(\mu', \phi, \omega) \geq v_0$. Lemma E.1 then ensures there also exists ϕ' s.t. $\mathbb{V}(-\overline{\mu}, \phi', \omega) = \mathbb{V}(\mu', \phi, \omega)$. So for any $\overline{\phi} \geq \phi'$ a policy rule of the form in equation (90) can attain at least the same level of steady state welfare as the generic alternative under consideration. □