Can DSGE Models Explain Real Exchange Rate Facts at All Frequencies?

Shifu Jiang
Room 103, 12 Southpark Terrace, Glasgow University, Glasgow, United Kingdom

Abstract
This paper evaluates the ability of a standard IRBC model augmented with input adjustment cost of imported goods in explaining different aspects of real exchange rate like the standard deviation, the autocorrelation function, the spectrum and integral correction mechanism. I find that the simple IRBC model with appropriate calibration can well capture all the features of real exchange rate. The input adjustment cost plays the key role. Comparing to the standard model, it implies a reversed impulse response of real exchange rate with a fast speed going back to steady state and introduces a long run cyclical movement in most macroeconomic variables. I find this particular impulse response helps explain the PPP puzzle.

Keywords: Real Exchange Rate, Autocorrelation function, Spectrum, integral correction mechanism, DSGE simulation

1. Introduction

The movement of real exchange rate is expected to be a stationary time series given the Purchasing Power Parity (PPP) hypothesis, which says that, once converted into a common currency, two countries should have the same price level in the long run. However the price level will generally not be constant. The question is asked by Rogoff (1996): If we take that liquidity effects drive real exchange rate highly volatile (triple more volatile than GDP), then the real exchange rate would be expected not to be very persistent. However the measurement using either half-life (about 2-5 years) or first order autocorrelation (roughly 0.91-0.97 for quarterly data) suggests the opposite. This problem, known as PPP puzzle, has been partially solved thanks to the work of Chari, Kehoe, and McGrattan (2002); Steinsson (2008); Corsetti, Dedola, and Leduc (2008); Rabanal, Rubio-Ramirez, and Tuesta (2011); Rabanal and Tuesta (2013, among others) in which their models can generate high volatility and/or persistence of real exchange rate at Business cycle (BC) frequencies. However, a recent study by Rabanal and Rubio-Ramirez (2015) documents that, by spectral analysis, the business cycle frequencies only account about 25% variance of real exchange rate and about 70% variance is assigned to frequencies lower than business cycle frequencies. Therefore, the current success of literature comes with a price for missing a large part of the story. Rabanal and Rubio-Ramirez (2015) studies a variety of international real business cycle (IRBC) model and goes one step ahead by fitting the spectrum of real exchange rate. They refer the problem of standard IRBC model as generating “excess persistence” in the sense that the standard model assigns too much volatility of real exchange rate to the BC frequencies when the model is calibrated to fit the standard deviation of un-filtered data. However, they do not answer the question asked by Rogoff (1996).

Another strand of literature addresses PPP puzzle by reduced form empirical models in which the real exchange rate contains certain types of dynamic. The typical aim is to find shorter half-life implied by the dynamic. One of the most popular types of dynamic features time-varying mean reversion speed, as suggested by Rogoff (1996). The literature (Lothian and Taylor 2008; Taylor, Peel, and Sarno 2001; Paya and Peel 2006, among others) has found strong evidence of such dynamic by assuming smooth transition (ST)
autoregression. However, [Jiang and Talmain, 2015] finds that this type of dynamic also has the problem of “excess persistence” since the autocorrelation function implied by this dynamic decay slowly relative to the data. Alternatively [Jiang and Talmain, 2015] employs another type of nonlinear dynamic called integral correction mechanism (ICM) whose performance often beats the smooth transition. The authors find that the possible origin of the ICM effect could be related to its power in explaining the spectrum and autocorrelation function of real exchange rate. They show that impulse response implied by ICM is the key to understand PPP puzzle. However, the microeconomic foundation behind ICM is unclear and structural interpretation of shocks in reduce form is impossible.

Given the limitation of working at business cycle frequencies, this paper asks whether the features of real exchange rate at all frequencies, which are robust across pairs of countries, can be generated in a single DSGE framework. The features of real exchange rate include the standard deviation, the autocorrelation function (ACF), the spectrum and the appearance of ICM. One aim of this paper is to link the theoretical literature using DSGE models to the empirical literature using reduced form model. According to my knowledge, the new findings in the empirical literature, such as shorter half-life and new dynamic, are often not seriously taken into account by the researches in which DSGE models are evaluated to fit data. Modeling real exchange rate at all frequencies facilitates this aim as the reduced form models are often applied to un-filtered data. The hope is that, if the DSGE model can generate all these features and in particular the ICM, it should suggest a way to understand the PPP puzzle. The framework I will employ is the IRBC model augmented with a input adjustment cost of imported good in the final good production function (workhorse model), which depends on the change of ratio between domestic and imported goods in current and next periods. It significantly changes both the inter-temporal and intra-temporal decisions made by the agents. This feature is first introduced in DSGE model by [Erceg et al., 2005] and plays the central role in [Rabanal and Rubio-Ramirez, 2015]. Since the input adjustment cost helps [Rabanal and Rubio-Ramirez, 2015] fit the spectrum of real exchange rate, I take it as a good starting point to capture other features of real exchange rate.

The findings of this paper are threefold. Firstly, I find that the model employed by [Rabanal and Rubio-Ramirez, 2015] over fits the spectrum in the sense that it misses other features of real exchange rate. A minor modification fixes the problem by capturing fairly well all the features of real exchange rate. Secondly, I study the impulse response function of a broad range of variables in a long horizon. It turns out that the influence of the size of input adjustment cost on the dynamic of all macroeconomic variables is not monotonous. The minor modification to the model of [Rabanal and Rubio-Ramirez, 2015] makes a big difference to the impulse responses, in which I highlight the inter-temporal distortion of input adjustment cost. Thirdly, the DSGE model provides a structural interpretation to the dynamic of ICM which helps better understand the PPP puzzle. At the final calibration, the input adjustment cost introduces a long lasting cyclical impulse response in most of the macroeconomic variables including real exchange rate, given a persistent positive shock to the home country productivity. Surprisingly I find the initial response of real exchange rate is negative and dying out quickly.

The rest of this paper is organized as follows. Section 2 discuss the features of real exchange rate that is aimed to explain. The next section introduces the IRBC model. In section 4 I investigate whether the IRBC model is able to generate the features of real exchange rate seen data and section 5 tries to understand the microeconomic foundation. The last section concludes.

2. Data features

This section describes the source of data and reports features of the real exchange rate from different aspects. I study the quarterly U.S. effective data. The real exchange rate is collected from federal reserve broad’s major index. This index is weighted average of the foreign exchange values of the U.S. dollar against the currencies circulated widely outside the country of issue, including Euro, Canada dollar, Japanese yen, British pound, Swiss franc, Australian dollar and Swedish krona. These seven countries are referred as the rest of world (R.W.). U.S. is treated as the home country and the R.W. is treated as the foreign country. All rest of data are collected from the U.S. bureau of economic analysis, except the Solow Residual taken from [Ahmad et al., 2013] has shown that this type of nonlinearity can be detected in an rich yet standard IRBC model with nontradable goods and sticky price. And as argued by [Jiang and Talmain, 2015], the smooth transition is unlikely to be the key feature of real exchange rate.
The real exchange rate is known to be highly volatile and persistent. The standard deviation of the sample is about 0.1095. As shown in the left panel of Figure 1, about 75% of the variance is at low frequency. The standard deviation at BC frequencies is 0.051, or 3.3 times to that of GDP. The degree of persistence is usually summarized by the first order autocorrelation, which is about 0.95. Standard tests for unit root such as Augmented Dickey-Fuller can not reject the unit root. Jiang and Talmain (2015) suggests investigating not only the first order autocorrelation but the autocorrelation function (ACF), which is natural to do because the autocorrelation function corresponds to spectrum one by one. As shown in the right panel of Figure 1, ACF goes down very fast to a negative level and cyclically move around zero with decaying magnitude. The ACF suggests that real exchange rate is persistent but not in a monotonous manner. The autocorrelations higher than order 7 are insignificant according to 90% confidence interval. However it may not be a good idea to take simply higher order autocorrelation as zero because in this case the fast decay of ACF at lower order says that real exchange rate is a smooth process. The higher order autocorrelation is also important to understand the shape of the spectrum. The low frequencies can account a significant proportion of volatility only if the real exchange rate is autocorrelated in the long run.

To study the hidden features of real exchange rate, I estimate a time series model with integral correction mechanism (ICM). The general form of such model is

$$err_t = \sum_{j=1}^{p} \beta_j^{ecm} err_{t-j} + \beta_j^{icm} IE_{t-1} + \epsilon_t,$$

where integral error \((IE)\) is defined by

$$IE_{t-1} = \sum_{i=0}^{t-1} err_i,$$

and \(err_t\) is defined by

$$err_{t-1} = q_{t-1} - q_{t-1}^E,$$

with \(q^E\) representing “equilibrium” value of real exchange rate (will be defined shortly) and \(\beta^{icm} \leq 0\) and \(\beta_j^{ecm}\) features a stationary AR(p) process. The key parameter of this model is \(\beta^{icm}\), which determines the

---

Notes:
1. The gray area in the panel A represents BC frequencies, defined from 8 to 32 quarters. Frequencies lower than BC will be referred as low frequencies.
2. The dotted lines represent 90% confidence intervals.

[1] Rabanal et al. (2011). The data span from 1973q2 to 2006q4 during which the estimate of Solow Residual is available.

[2] Through out this paper, business cycle is obtained by applying H-P filter.

[3] It is much faster than an AR(1) process using a AR coefficient equal to the sample first order autocorrelation.
appearance of the integral correction mechanism. When $\beta^{icm} < 0$, the model features a physical pendulum-like behavior. Let $err_t$ be a massive bob hung by a rod, the bottom position is the equilibrium in this model where $err_t = 0$. Given a shock, $err_t$ will move around this equilibrium position periodically because the term $IE_t$ can store the “momentum” until all momentum drained by air friction. $\beta^{ecm}$ play the role of air friction. The trajectory of the massive bob is the impulse response of $err_t$. To better the dynamic of ICM, see Figure 2 for the impulse response of real exchange rate when the model is estimated using U.S. data. In this model, $\beta^{ecm}$ controls the “conventional persistence”, e.g. how fast is the real exchange rate reverting to the equilibrium position. It also determines the period of cyclical part movement. $\beta^{icm}$ is analogous to the length of rope and gravity. It controls the magnitude of cyclical part of the movement, which is the “cyclical persistence”.

When estimating the model, I have to define the “equilibrium” value of real exchange rate $q^E$ first. This is done only loosely in the empirical literature by allowing channels through which macroeconomic fundamentals can affect the real exchange rate. Following Jiang and Talmain (2015), I defined the $q^E_t = a_0 - \alpha_1 rSR_t - \alpha_2 tot_t - \alpha_3 tb_t$. $rSR$ is the ratio of Solow residual between two countries, reflecting the Harrod-Balassa-Samuelson (HBS) effect. $tb$ is the trade balance to output ratio, and it is used to capture the net foreign assets effect. $tot$ is the control variable terms of trade. At last I select $p = 1$ based on first order partial autocorrelation. All parameters in equation 1-3 are estimated simultaneously by maximizing likelihood. I construct the distribution of each parameters using the bootstrap technique with the size equal to 5000. This will be used later to facilitate the comparison between ICM estimated from data and ICM generated from IRBC model. The results are reported in Table 1 where insignificant terms are dropped.

Significant $\beta^{icm}$ is found for U.S. real exchange rate, indicating the appearance of ICM. The frequency of significant $\beta^{icm}$ in the bootstrap sample is about 46%.

3. The DSGE framework

The DSGE framework I will use is the one developed by Rabanal and Rubio-Ramirez (2015). The world consists of two countries with one referred as Home and the other as Foreign. The two countries are

---

[6] The model does not pass the diagnostic test of function form. This problem may be caused by the missing nonlinearity in the form of, for example, smooth transition. Adding smooth transition mechanism allows the model pass diagnostic test but does not change the estimate of other parameters.

[7] Terms of trade is significant. However it enters the model only as a control variable to the HBS effect so it is dropped as well.
Table 1: Time series model estimation

<table>
<thead>
<tr>
<th>Estimated ICM</th>
<th>( \beta_{icm} )</th>
<th>( \beta_{ecm} )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 )</td>
<td>0.95699</td>
<td>-0.00841</td>
<td>0.03212</td>
</tr>
<tr>
<td>(1161.05)</td>
<td>(31.203)</td>
<td>(-3.247)**</td>
<td></td>
</tr>
<tr>
<td>[4.5466, 4.5600]</td>
<td>[0.8795, 0.9854]</td>
<td>[-0.0172, -0.0056]</td>
<td></td>
</tr>
<tr>
<td>( R^2 = 0.9141 )</td>
<td>LL = 270.603</td>
<td>ICM = 46.44%</td>
<td></td>
</tr>
<tr>
<td>AR(1) = 0.604</td>
<td>ARCH(1) = 0.903</td>
<td>Function form = 0.01</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. Numbers in parentheses are t-ratio, numbers in bracket are the bootstrap 90% interval, numbers in braces are P-values.
2. Stars next to the parentheses under \( \beta_{icm} \) represents significantly different from zero at level 10%, 5% and 1%. The critical value is obtained by Monte Carlo experiments because \( \theta > 0 \) makes model unstationary and the distribution of t-statistics under the null is non-standard.
3. AR(1), ARCH(1) and Function form represent the diagnostic score tests of neglected first order serial correlation, first order autoregressive heteroskedasticity and general nonlinearity respectively.
4. Numbers in braces of ICM gives the frequency \( \beta_{icm} \) is significant in the bootstrap sample.

Symmetric unless indicated otherwise. Each country produces tradable intermediate goods using local labor and capital. There is only one international asset: a real riskless bond. To save space, this section presents only the problems of Home country agents and the problems of Foreign country can be constructed in a similar way. Variables in Foreign country are denoted with superscript *.

### 3.1. Households

The representative household maximizes life time discounted utility

\[
E_t \sum_{j=t}^{\infty} \beta^{j-t} \left[ C_j^\tau (1 - L_j)^{1-\tau} \right]^{1-\sigma} \left[ 1 - \sigma \right],
\]

subject to the following budget constraint

\[
P_t (C_t + X_t) + P_{H,t} Q_t D_t \leq P_t (W_t L_t + R_t K_{t-1}) + P_{H,t} \left[ D_{t-1} - \Phi (D_t) \right],
\]

and the law of motion for capital

\[
K_t = (1 - \delta) K_{t-1} + X_t - \frac{\psi}{2} K_{t-1} - \left( \frac{X_t}{K_{t-1}} - \delta \right)^2.
\]

The notation follows Rabanal and Rubio-Ramirez (2015): \( \beta \in (0, 1) \) is the subjective discount factor, \( \tau \in (0, 1) \) is the weight on consumption in utility function, \( \sigma > 0 \) is the coefficient of relative risk aversion, \( \delta \in (0, 1) \) is the depreciation rate of capital and \( \psi \geq 0 \) controls the size of capital adjustment cost. \( L_t \) are the units of labor normalized on one, \( C_t \) are the units of consumption of final goods, \( X_t \) are the units of investment, and \( K_t \geq 0 \) is the capital stock at the end of period \( t \). \( P_t \) and \( P_{H,t} \) are the nominal price of final goods and Home intermediate goods respectively, measured by Home country currency. \( W_t \) is the real wage rate, \( R_t \) is the real rental return of capital. All real terms are measured in terms of final goods. \( D_t \) are the units of internationally traded riskless bond held by Home country household during period \( t \) and pays one unit of the Home intermediate goods at the beginning of period \( t + 1 \). The price of this bond measured by units of the Home intermediate goods is denoted by \( Q_t \). To induce stationarity of the model, Home country is subjected to a portfolio adjustment cost. The adjustment cost is measured by \( \Phi (D_t) = \frac{\psi}{2} A_t \left( \frac{D_t - D_{t-1}}{A_t} \right)^2 \) in units of Home intermediate goods where \( A_t \) is Home country productivity.

Solving the maximization problem gives the following first order condition with respect to \( C_t, L_t, X_t, K_t \) and \( D_t \):

\[
U_{C_t} = \lambda_t,
\]
$$-\frac{U_{L_t}}{U_{C_t}} = W_t,$$

$$\lambda_t = \mu_t \left[ 1 - \psi \left( \frac{X_t}{K_{t-1}} - \delta \right) \right],$$

$$\mu_t = \beta E_t \left[ \lambda_{t+1} R_{t+1} + \mu_{t+1} \left( 1 - \delta + \psi \left( \frac{X_t^2}{K_{t-1}} - \delta^2 \right) \right) \right],$$

$$Q_t = \beta E_t \left( \frac{\lambda_{t+1} P_{H,t+1}/P_{t+1}}{P_{H,t}/P_t} \right) - \frac{\partial \Phi}{\partial D_t},$$

where $U_L$ and $U_C$ is derivative of single period utility with respect to labor and consumption respectively, $\lambda_t$ is Lagrange multiplier of budget constraint and $\mu_t$ is the Lagrange multiplier of capital law of motion.

### 3.2. Intermediate goods firms

The market of domestic intermediate goods firms is perfectly competitive. The representative intermediate goods producer uses domestic labor and domestic capital in order to produce intermediate goods sold to both the Home and Foreign final goods producers. The firms maximize period-by-period profits by solving

$$\max_{L_t,K_{t-1}} P_{H,t} (Y_{H,t} + Y_{H,t}^*) - P_t (W_t L_t + R_t K_{t-1}),$$

subject to the production function

$$Y_{H,t} + Y_{H,t}^* = A_t^{1-\alpha} K_{t-1}^{\alpha-1} L_t^{1-\alpha},$$

where $Y_{H,t}$ and $Y_{H,t}^*$ denote respectively the Home intermediate goods demanded by Home and Foreign markets, $\alpha \in (0, 1)$ is capital share of output and $A_t$ is the Home country productivity cointegrated with the Foreign country productivity $A_t^*$

$$\left[ \frac{\Delta \ln A_t}{\Delta \ln A_t^*} \right] = \left[ \begin{array}{c} A \\ A^* \end{array} \right] \left( \begin{array}{c} \kappa \\ -\kappa \end{array} \right) \left( \ln A_{t-1} - \ln A_{t-1}^* \right) + \left[ \begin{array}{c} \epsilon_{A,t} \\ \epsilon_{A^*,t} \end{array} \right],$$

where $\omega \in (0, 1)$ denotes the degree of home bias or the degree of openness, $\sigma_{HF} > 0$ is a parameter determine the long run elasticity of substitution between domestic and imported intermediate goods, $\varphi_t = \left[ 1 - \frac{1}{2} \left( \frac{Y_{HF,t}}{Y_{HF,t-1}} - 1 \right) \right]$ is the input adjustment cost with $t$ the size of cost. This specification has two
implications. Given the time varying $\varphi_t$, the real cost of producing one unit of final goods will generally be time varying. Therefore it distorts the inter-temporal decision made by households. Whenever the final goods firms, or equivalently the households, find it optimal to change the ratio between domestic and imported goods, they must take the expected input adjustment cost next period into account. On the other hand, the input adjustment cost implies that the imported goods share in consumption is relatively unresponsive in the short run to the changes in its price. Therefore the short run price elasticity of imported goods will be lower then $\sigma_{HF}$. This is supported by the recent literature that estimates the trade elasticity; see for example, Hooper et al. (2000). In addition, note that the cost is due to the change of ratio between domestic and imported goods, which suggests that the level of imported goods is allowed to jump costlessly in response to changes in overall consumption demand. To give a quantitative example, when $t = 900$, change of 1% in the ratio will result in $\varphi_t = 0.955$ so Foreign intermediate goods being 4.5% less efficient in production.

Denote $\Omega_{t,j} = \beta^{t-j} \frac{\lambda_j}{\tau}$ as the stochastic discount factor, the final goods firm maximize the expected discount profits given by

$$E_t \sum_{j=t}^{\infty} \Omega_{t,j} (P_j Y_j - P_{H,j} Y_{H,j} - P_{F,j} Y_{F,j}),$$

subject to the production function. The first order condition with respect to $Y_{H,t}$ and $Y_{F,t}$ are

$$P_t \frac{\partial Y_t}{\partial Y_{H,t}} + E_t \left( \Omega_{t,t+1} P_{t+1} \frac{\partial Y_{t+1}}{\partial Y_{H,t}} \right) = P_{H,t},$$

$$P_t \frac{\partial Y_t}{\partial Y_{F,t}} + E_t \left( \Omega_{t,t+1} P_{t+1} \frac{\partial Y_{t+1}}{\partial Y_{F,t}} \right) = P_{F,t}.$$

The price index of final goods is defined accordingly as

$$P_t = \left[ \omega P_{H,t}^{1-\sigma_{HF}} + (1- \omega) P_{F,t}^{1-\sigma_{HF}} \right]^{1/1-\sigma_{HF}},$$

and the real exchange rate is, given Law of One Price,

$$RER_t = \frac{\omega P_{H,t}^{1-\sigma_{HF}} + (1- \omega) P_{F,t}^{1-\sigma_{HF}}}{\omega P_{H,t}^{1-\sigma_{HF}} + (1- \omega) P_{F,t}^{1-\sigma_{HF}}} = \frac{\left[ \left( \omega P_{H,t}^{1-\sigma_{HF}} + (1- \omega) P_{F,t}^{1-\sigma_{HF}} \right)^{1-\sigma_{HF}} \right]^{1/1-\sigma_{HF}}}{\left[ \left( \omega P_{H,t}^{1-\sigma_{HF}} + (1- \omega) P_{F,t}^{1-\sigma_{HF}} \right)^{1-\sigma_{HF}} \right]^{1/1-\sigma_{HF}}}.$$  (6)

3.4. Market clearing conditions

Finally the model is closed by final goods market clear condition

$$C_t + X_t = Y_t,$$

and international bond market clear condition

$$D_t + D_t^* = 0.$$

3.4.1. Benchmark calibration

The parameters of cointegration process are taken from the estimates by Rabanal et al. (2011) in which $A = 0.001$, $A^* = 0.006$, $\kappa = -0.007$, $\sigma_A = 0.0108$ and $\sigma_A^* = 0.088$. The subjective discount factor $\beta$, the consumption share in utility function $\tau$, the depreciation rate $\delta$ and the capital share in production $\alpha$ are set to the standard value in literature and they are equal to 0.99, 0.34, 2, 0.025 and 0.36 respectively. There is no standard value for the cost of holding international asset $\phi$ as the literature uses different function form of the cost. I set it to 0.03 so that the model get the standard deviation of trade balance about right. The rest of parameters are calibrated to fit the spectrum of real exchange rate by the strategy of Rabanal and Rubio-Ramirez (2015). I set the low home bias $\omega = 0.8$ and a high long-run elasticity of substitution $\sigma_{HF} = 3$, same as Rabanal and Rubio-Ramirez (2015). The parameter of input adjustment cost $\iota$ is set to 140.
Figure 3: Fit of ACF at different frequencies by baseline model

Note:
1. The 90CI denote the upper and lower 90% confidence interval of the sample ACF. It is calculated by the asymptotic normal distribution and standard deviation $SE(\rho_h) = \sqrt{\frac{1+2\sum_{i=1}^{h-1} \rho_i^2}{N}}$ where $\rho_h$ denote h order autocorrelation and N is the sample size.

4. Main Experiments

The main experiments conducted in this paper is to investigate whether the features of real exchange rate data reported in section 2 can be generated in the model described in section 3. The standard deviation, ACF and spectrum can be calculated analytically from the IRBC model and they are expected to be reasonably close to their counterparts from data. The integral correction mechanism is uncovered from the IRBC model by the following way. The IRBC model is solved up to first order approximation\(^8\). Artificial real exchange rate and other variables are simulated for the same length as data. Parameters in equation 1—3 are estimated by the artificial variables. The procedure is repeated 5000 times, which results in a Monte Carlo sample of each parameter in equation 1—3. When comparing the ICM generated by IRBC model to the ICM estimated in data, I compare the Monte Carlo sample to the Bootstrap sample described in section 2.

4.1. The baseline results

To facilitate comparison, I use a baseline model in which input adjustment cost is shut down by setting $\iota = 0$. The home bias is reset to 0.9 to match the import to output ratio in the steady state. It is well known that the substitution between domestic and imported goods is critical to the real exchange rate. Two values for the elasticity of substitution $\sigma_{HF}$ is considered. When $\sigma_{HF} = 0.55$ the model does a good job at the business cycle frequencies. It closely fit the standard deviation of H-P filtered real exchange rate relative to that of GDI\(^9\) (3.26 in model and 3.3 in data) although the standard deviation at all frequencies is 0.22, double to 0.1095 in the data. In terms of the persistence at BC frequencies, right panel of Figure 3 plots the ACF of the real exchange rate at BC frequencies for the data against the model. The data shows a fast initial decay to a negative value. The higher order of ACF is very noisy and insignificant but the ACF of order 10 has significant negative value. It is surprising to find that those unusual features of ACF at BC frequencies can be generated in the baseline model. However, the estimate of ICM on the model side only detect significant $\beta_{icm}$ in 8% of the Monte Carlo sample. This probability is only slightly larger than the size of the test, suggesting no or very small integral correction mechanism in the baseline model.

The other relevant value of $\sigma_{HF}$ is 0.75, which allows the model to fit standard deviation of the real exchange rate at all frequencies (0.1073 in the model and 0.1095 in the data). It also increases the frequency of ICM appearance to 16%. However, it only explain 30% of standard deviation at BC frequencies (0.015 in the model and 0.051 in the data) and assign most of the standard deviation to frequencies lower than BC.

\(^8\)However, all the results are robust to higher order approximation.

\(^9\)The moment of IRBC model at BC frequencies is obtained by applying H-P filter to the model.
### Table 2: ICM in Baseline model

<table>
<thead>
<tr>
<th></th>
<th>$\beta_{ecm}$ 90% int.</th>
<th>$\beta_{icm}$ 90% int.</th>
<th>$p(\text{icm})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td>[0.8795, 0.9854]</td>
<td>[-0.0172, -0.0056]</td>
<td>46%</td>
</tr>
<tr>
<td>$\sigma_{HF} = 0.55$</td>
<td>[0.9083 1.0000]</td>
<td>[-0.0067 -0.0003]</td>
<td>8%</td>
</tr>
<tr>
<td>$\sigma_{HF} = 0.75$</td>
<td>[0.9239 1.0052]</td>
<td>[-0.0060 -0.0003]</td>
<td>16%</td>
</tr>
<tr>
<td>$\sigma_{HF} = 1.5$</td>
<td>[0.9660 1.0208]</td>
<td>[-0.0056 -0.0000]</td>
<td>63%</td>
</tr>
<tr>
<td>$\sigma_{HF} = 3.0$</td>
<td>[0.9793 1.0312]</td>
<td>[-0.0060 -0.0000]</td>
<td>90%</td>
</tr>
</tbody>
</table>

Notes: $p(\text{icm})$ is the frequency of significant ICM in the Monte Carlo or Bootstrap sample.

This problem is referred as the excess persistence of the RER in [Rabanal and Rubio-Ramirez (2015)](https://doi.org/10.1111/j.1468-0262.2015.00579.x). The left panel of Figure 3 shows that too much persistence is generated by the model at all frequencies although the excess persistence problem is slightly eased comparing to $\sigma_{HF} = 0.55$ by noting that the ACF decreasing faster when $\sigma_{HF}$ increases. This experiment gives a general lesson that a model may completely miss the evolution of persistence even it get the first order autocorrelation about right.

Using ACF instead of the spectrum, I replicate the results discussed in [Rabanal and Rubio-Ramirez (2015)](https://doi.org/10.1111/j.1468-0262.2015.00579.x) which suggests that the performance of baseline model face trade off between low frequencies and BC frequencies. In order to fit the real exchange rate features at BC frequencies, the model generates extra persistence and volatile at low frequencies. On the other hand, if the model fits the features at all frequencies, the model underestimates the persistence and volatility at BC frequencies. Therefore the model is unable to fit the shape of the spectrum. The new finding here is the relationship between the substitution $\sigma_{HF}$ and the frequency of ICM appearance. To investigate this relationship further, I run another two experiments using $\sigma_{HF}$ equal to 1.5 and 3 respectively. Table 2 reports the estimates of ICM under different values of $\sigma_{HF}$. First note that the frequency of ICM occurrence ($p(\text{icm})$) is increasing along the $\sigma_{HF}$. However, given the estimate of $\beta_{icm}$ remaining at a small magnitude, the increasing of $p(\text{icm})$ is only caused by the smaller standard deviation in the residual. On the other hand, more volatility of real exchange rate is explained by higher $\beta_{ecm}$. Using the interpretation of $\beta_{ecm}$ and $\beta_{icm}$ discussed in section 2, higher substitution between domestic and imported intermediate goods increases the volatility of real exchange rate by increasing it conventional persistence while the cyclical persistence remains at small magnitude.

### 4.2. Workhorse model

Adding the input adjustment cost helps the model to reallocate the variance of the real exchange rate at different frequencies. The upper half of Figure 4 shows the fit of spectrum and ACF. Using the benchmark calibration, this model is therefore able to fit the spectrum very well. The ACF of the model at all frequencies now well lies within the confidence interval of data, fixing the excess persistence problem in the baseline model. It also has the cyclical movement. However the magnitude of the cyclical movement is negligible comparing to the data.

I now turn to the estimates of ICM using simulations of workhorse model. The 90% confidence interval of $\beta_{icm}$ is [-0.0170, -0.0017] in the Monte Carlo sample, covering the point estimate in data -0.0084. The 90% confidence interval of $\beta_{ecm}$ [0.8493, 0.9621] cover the point estimate in data 0.96 at the margin. The lower panel of Figure 4 compares the distribution of $\beta_{icm}$ and $\beta_{ecm}$ in IRBC Monte Carlo sample and data Bootstrap sample. It shows that the model explains the volatility of real exchange rate by underestimate the magnitude of cyclical persistence $\beta_{icm}$ and overestimate the magnitude of conventional persistence $\beta_{ecm}$. The frequency that ICM is found statistically significant is only 18% in the model while this probability is 46% in data. Nevertheless, the workhorse model is an improvement over the baseline model.

Can we push the workhorse model further? To answer this question, I first argue that the perfect fit of the spectrum as in [Rabanal and Rubio-Ramirez (2015)](https://doi.org/10.1111/j.1468-0262.2015.00579.x)’s strategy is likely to suffer from “over-fitting”. Unlike the theoretical ACF and spectrum, the sample ACF is not the exact correspondence to the sample spectrum hence fitting exactly one of them may miss the information that contained in the other. I propose a calibration strategy that fits both the spectrum and ACF reasonably well in the sense that IRBC model

---

\[10\] Since the critical value of t-statistics of $\beta_{icm}$ is obtained by Monte Carlo experiment with sample size equal to 135, there is no worry of the small sample effect.
captures the basic shape of the spectrum and ACF and will lie in the 90% interval of data. Given that the sample ACF and spectrum contain the similar but slightly different information, this strategy should ease the over-fitting problem. In my final calibration, I substantially increase the $\iota$ to 900. The home bias $\omega$ and the portfolio adjustment cost $\phi$ are slightly adjusted to 0.82 and 0.05 respectively. This calibration generates excess volatility for a wide range of variables. Therefore I replace the cointegration productivity process by a stationary VAR(1) process,

$$
\begin{bmatrix}
\log A_t \\
\log A_t^* 
\end{bmatrix} = \begin{bmatrix}
0.97 & 0.025 \\
0.025 & 0.97 
\end{bmatrix} \begin{bmatrix}
\log A_{t-1} \\
\log A_{t-1}^* 
\end{bmatrix} + \begin{bmatrix}
\epsilon_t \\
\epsilon_t^* 
\end{bmatrix},
$$

$$
\begin{bmatrix}
\epsilon_t \\
\epsilon_t^* 
\end{bmatrix} \sim N\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \Sigma \right),
$$

$StD(\epsilon_t) = 0.0073, StD(\epsilon_t^*) = 0.0044, Corr(\epsilon_t, \epsilon_t^*) = 0.29,$

estimated by [Heathcote and Perri (2002)](http://example.com). The standard deviation of the real exchange rate is now 0.1075, same as the benchmark calibration and data. Figure 5 reports the good fits of ACF and spectrum. The frequency of ICM occurrence is 41.68%. The magnitude of $\beta_{ecm}$ and $\beta_{icm}$ is much closer to the data. Comparing to the benchmark calibration, the 90% interval of $\beta_{icm}$ increase (in absolute) to [-0.0166, -0.0031] and the 90% interval of $\beta_{ecm}$ increase to [0.8637, 0.9728]. Hence, the new calibration explains volatility real exchange rate using larger conventional and cyclical persistence and smaller standard deviation in the residual. To summary, the model developed by [Rabanal and Rubio-Ramirez (2015)](http://example.com) is able to capture a wide range of real exchange rate features with appropriate calibration and stationary productivity.

---

11 In fact, the cointegration technology is introduced by [Rabanal et al. (2011)](http://example.com) to increase volatility of real exchange rate at BC frequencies. There is no evidence supporting cointegration technology if one's aim is to explain real exchange rate at all frequencies. Regarding other aspects of the model, the baseline model and workhorse model with benchmark calibration both underestimate the standard deviation of GDP growth rate while this final calibration overestimate it.
5. Microeconomic foundation of integral correction mechanism

So far it is clear that the baseline model can only generate highly volatile real exchange rate by increasing it conventional persistence so it is unable to capture the cyclical persistence. The workhorse model fixes this problem by adding input adjustment cost. However, the mechanism of real exchange rate dynamic is still unknown at the micro level. To investigate this problem, I study the impulse responses to a positive Home productivity shock in the workhorse model with final calibration and different size of the cost. I highlight the role played by the input adjustment cost by varying its size $\iota$. There are three domains of $\iota$ matter here. The first one is of course 0 by which the cost is shut down. The second domain is $(0, 245)$ during which the higher size of cost the higher volatility of real exchange rate. When $\iota > 245$, the volatility of real exchange rate starts to decrease. In additional, the basic shape of impulse responses remain within each domain but vary across the domains. The benchmark calibration falls into the second domain and the final calibration falls into the third. Rabanal and Rubio-Ramirez (2015) argues that the cost make elasticity of substitution between domestic and imported goods time varying. The analytically expression of short run elasticity is difficult to calculate so they propose to calculate the short run elasticity defined as

$$\sigma_{Pseudo}^{HF,k} = \frac{\hat{y}_{H,t+k} - \hat{y}_{F,t+k}}{\hat{p}_{H,t+k} - \hat{p}_{F,t+k}},$$

where $\hat{y}$ and $\hat{p}$ denote log deviation from steady state. They report the impulse responses of key variables, including $\sigma_{Pseudo}^{HF,k}$, only in 40 periods. In their case, the short run substitution well behaves in the sense that it reverting monotonously from small positive number to the long run substitution determined by $\sigma_{HF}^{12}$. However, 40 periods is only slightly longer then BC frequencies hence the dynamic at low frequencies can hardly be seen. I plot the impulse responses for 200 periods and find that in the final calibration (and actually in most reasonable calibration) the behavior of short run substitution is “crazy”$^{13}$. It jumps between large positive number to large negative number and can be as high as 363. The long run substitution is not only affected

---

$^{12}$They report the impulse response of $\sigma_{Pseudo}^{HF,k}$ using a calibration that does not match spectrum or real exchange rate. And they do not update the impulse response after they report their final calibration.

$^{13}$This is probably due to the local approximation when solving the model. Higher order approximation will not help.
Note: The notations follow section 3 except that prices in this graph are defined relative to the final good price.
by $\sigma_{HF}$ but also affected by the size of cost $\iota$. In the benchmark calibration it takes 152 quarters for $\sigma_{HF,k}^{\text{pseudo}}$ to reach its long run value 3.62. Given the difficulty in interpreting the behavior of $\sigma_{HF,k}^{\text{pseudo}}$, alternatively I study the impulse response of $\varphi_t = \left[1 - \frac{\iota}{2} \left( \frac{P_{F,t}}{P_{H,t-1}} - 1 \right)^2 \right]$ which is referred as input adjustment cost in order to be distinguished from $\iota$, the size of cost. Figure 6 reports the impulse responses of workhorse models by 3*2 blocks. Each row of the blocks is reported for $\iota$ equal to 0, 150 and 900 respectively. The left column of blocks contains the intra-temporal variables and the right column contains inter-temporal variables. All price are defined relative to the local final goods price.

In the following, I focus on the impulse responses of Home country. Let’s start with a baseline model in which $\iota = 0$ shut down the cost and standard results of IRBC model apply. At the period in which shock occur, the right panel shows that the higher productivity in Home country pushes up the aggregate demand by raising the capital return. Consumption is higher due to Euler equation and the high capital return attracts more investment. The left panel plots the components of aggregate demand. Higher capital return (together with the associated higher wage) and higher productivity have opposite effects on the marginal cost of producing Home intermediate goods. Since the perfect competitive intermediate goods firms set price $P_H$ equal to the marginal cost, the decrease in $P_H$ indicates that the effect of higher capital return is smaller than the effect of higher productivity. $P_F$ increases because the capital return in Foreign country has to grow to comply with the uncovered interest rate parity, which in turn pushes up the marginal cost of production in Foreign country. From equation 6, $RER$ is a monotonous decreasing function of $\frac{P_H}{P_F}$ so real exchange rate increase. The real exchange rate also features a hump shape because the spillover of Home country productivity to Foreign country has a lag, which causes asynchronous movements between capital return in Home and Foreign country, see the discussion of Steinsson (2008).

There are two effects of the input adjustment cost on the model. Firstly, it makes the final goods generally more expensive because the final goods production using imported goods is more costly. Therefore the inter-temporal substitution of final goods is distorted because the input adjustment cost in each period is generally different depends on the choice of household. Secondly, the cost change the intra-temporal substitution between Home and Foreign goods.

When the input adjustment cost is introduced but not very strong, increasing $\iota$ in the second domain strengthens the response of input adjustment cost $\varphi$. This result suggests that with relatively small size of cost the households find it optimal not to to largely reduce $\varphi$ by carefully smoothing the ratio between domestic and imported goods. The households in Home country foresee a relatively larger (in absolute, so as below) input adjustment cost after the shock so the aggregate demand now decreases in order to finance more expensive final goods in the future. The capital return drops accordingly. Compare to the case of $\iota = 0$, a relatively small size of cost reverses the impulse response direction of Home country inter-temporal variables on the right panel. In the meantime, the impulse response direction of Home country net foreign assets is not reversed (not shown). Households in Foreign country still borrow from Home country in order to invest in the Home country as the case of $\iota = 0$. Given the higher capital return in Foreign country, this will happen only if the Foreign households expect a fast drop in the real exchange rate. As being confirmed shortly, the fast movement of real exchange rate (either increase or decrease, depending on $\iota$) is a crucial feature of real exchange rate dynamic when input adjustment cost exists, regardless the size of cost.

Turning to the left panel, the response of $Y_{H,t}$ is much larger because the low capital return further reduces the marginal cost. By the same reasoning, the response of $Y_F$ is also larger due to the larger response of Foreign capital return. As a result, the response of real exchange rate is amplified. The appearance of input adjustment cost enlarges the standard deviation. Also note that the hump shape response of real exchange rate disappears because of the more synchronized movement of capital return in each country. When the input adjustment cost rapidly dying out, the distortion of inter-temporal relationship becomes smaller so the behavior of inter-temporal variables go back rapidly to the path in the baseline situation. Home aggregate demand and capital return increase back to its level seen in the first row of the figure. Such behaviors, by the reasoning above, transmit to the real exchange rate, which features “over correction” by going below steady state after reaching it.

If the cost is higher than 245, an increase in $\iota$ will make the response of $\varphi$ smaller at equilibrium so as the volatility of real exchange rate. This result suggests that the size of the cost is so big that the households
have to reduce it by keeping the changes in $\frac{Y_{F,t}}{Y_{H,t}}$ smaller. Given the expectation of future $\varphi$, the impulse response of most inter-temporal variables has the same directions but larger magnitudes comparing to the case of $\iota = 0$. However, the direction of intermediate goods price and real exchange rate are reversed. $P_H$ increases because the increase in capital return is so large that overwhelm the effect of higher productivity on the marginal cost. The same reasoning applies to $P_F$. When the input adjustment cost $\varphi$ decreases in absolute and reaches the steady state by the first time, the productivity has not reverted to steady state so the households will change $\frac{Y_{F,t}}{Y_{H,t}}$ with smaller or even zero cost. This activity, however, increases the input adjustment cost again so all variables in the economy have cyclical movement. The magnitude of the cycle decay along time because the driving force, productivity, is weaker. Also note that cyclical movement is more significant than the case of $\iota = 150$.

To summarize, the impulse responses of real exchange rate is very close to the one implied by ICM and resemble the shape of ACF. It helps explain the PPP puzzle further because, in the presence of input adjustment cost, the impulse response of real exchange rate to a persistent AR(1) productivity process features a fast reversion to steady state in the short run and a cyclical movement in the long run. This particular dynamic reduces the variance in the short run (higher frequencies) and increase the variance in the long run (low frequencies). By contrast, the conventional belief is that high volatility is mainly driven by nominal shocks and the persistence is caused by persistent real shocks. Another finding in this section is that the size of cost can alter the direction of real exchange rate initial response.

6. Summary

This paper aims to examine the ability of IRBC model in explaining several features of real exchange rate, in particular the integral correction mechanism found by Jiang and Talmain (2015). I document that, using U.S. effect data, real exchange rate features high volatility, cyclical movement of ACF, the importance of low frequencies, and a significant integral correction mechanism. I find that standard IRBC model is able to fit one or two features but face trade off to fit all of them. The workhorse model is able to closely match all the features seen in data. All these features, together with the PPP puzzle, can be understood by a relative big input adjustment cost of imported goods in the final goods production. The impulse response of real exchange rate to a persistent real shock has three new features comparing to that in standard model: 1) The initial response to a positive shock is negative 2) The deviation from steady state decay very fast in the short run 3) There are persistent cyclical movement in the long run. These findings suggest that ICM is a fairly good reduced form single equation representation of the role played by input adjustment cost. It is interesting in the further researches to find empirical evidence of such real exchange rate behavior probably in a Bayesian structural VAR context and to examine the impulse response of real exchange rate to a nominal shock.

Reference


