A discrete geometry model of fire propagation in urban areas

L-1 norms and fire propagation

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- $L = \mathbb{Z}^2$... the integer lattice
- A_k... the set of points in $x \in L$ to which we can propagate in at most k time steps, $k \geq 1$ (and homogeneity of the system)
- $\tau_k(x) = \begin{cases} \text{propagation time to } x \text{ in at most } k \text{ steps, if } x \in A_k \\ +\infty, o.w. \end{cases}$
- $v(x) = \tau^*(x) = \lim_{k\to\infty} \tau_k(x)$... propagation time to x
- $$f_k \equiv co(\tau_k) : dom(co(\tau_k)) \cup \mathbb{R}^2 \to \mathbb{R}$$
  $$f_k\left(\sum_{i\in I} \lambda_i x_i\right) = \sum_{i\in I} \lambda_i \tau_k(x_i),$$
  where $(\forall i \in I)\, x_i \in A_k$ and $\sum_{i\in I} \lambda_i x_i$ is a convex combination

Lemma

Let $x \in dom(f_k)$, *then*

$$f_k(x) = kf\left(\frac{x}{k}\right).$$

Lemma

The lower boundary of $k \times epi(f)$ *is equal to* f_k, *where* $k \times epi(f)$ *denotes the* k*th Minkowski sum of* $epi(f)$.

Lemma

$$k \times epi(x \to f(x)) = epi\left(x \to kf\left(\frac{x}{k}\right)\right)$$

Lemma

$$kf\left(\frac{x}{k}\right) \leq \tau_k\left(x\right) \leq kf\left(\frac{x}{k}\right) + constant.$$

It follows...

$$\inf_{k\geq 1} kf\left(\frac{x}{k}\right) \leq v \leq \inf_{k\geq 1}\left(kf\left(\frac{x}{k}\right) + \text{constant}\right)$$

Lemma

$$\inf_{k\geq 1} kf\left(\frac{x}{k}\right) = f'\left(0; x\right) = \sup_{p\in\partial f(0)} < p, x >$$

Fire propagation is a polyhedral norm...

Theorem

$$\lim_{s \to \infty} \frac{v(sx)}{s} = \sup_{p \in \delta f(0)} < p, x > .$$

The long-term geometry of the fire front depends simply on the immediate propagation directions, A_1 (since A_k are Minkowski sums of A_1)

Important example, the Von-Neumann neighbourhood...

$$A = \{(1,0), (0,1), (-1,0), (0,-1)\},$$

with corresponding times $\tau_1, \tau_2, \tau_3, \tau_4$ respectively

- Radiative heating between large surface areas
- The polyhedral norm is a deformed L_1 ball

L_1 balls

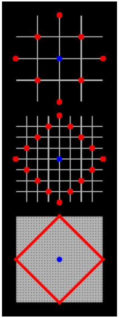

Figure: L_1 norm

- Q... set of discrete states
- $Q = \{0, 1\}$ (ignited or not)... purely geometric
- For a simulation, we use the states used in the paper of Zhao

- 0 - original state (white)
- 1 - ignition
- 2 - flashover (self-developing)
- 3 - full development
- 4 - collapse
- 5 - extinguished

Show video 1

Non-perfect lattices and extra factors (wind or changing urban geometries)... deformed L_1 balls

Figure: L_1 balls

- Multiple sources... union of deformed L_1 balls (show video)
- Changing geometry across the urban environment... Finsler geometry

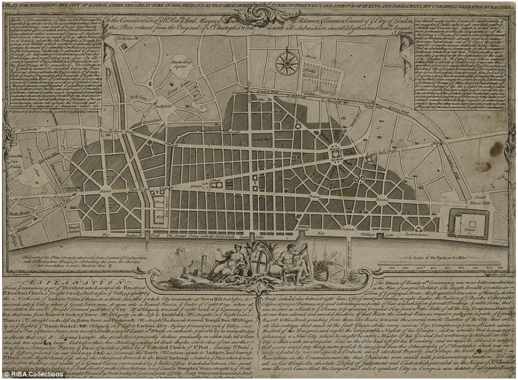

Figure: octagonal polyhedral norm

- change of wind on third day
- octagonal geometry on extreme edge of fire

Figure: 'dips' after crossing low density areas

- small 'dips' in the expected straight edges of the L1 ball
- modelled by a so-called 'Finsler-geometry'
- 'cell densities' can be incorporated into the model

future work

- 3D models for non-flat cities
- It may be possible to reverse engineer to find the ignition point
- Rome 64 AD, emperor Nero
- Different deformation of L_1 ball in different parts of the city
- Analytic formulas for propagation speeds to specific buildings
- Easier to interpret and modify than the applied model of Zhao
- Incorporate stochasticity (randomness) into the model (implies rounder corners of the fire front)

Thank you for your attention!

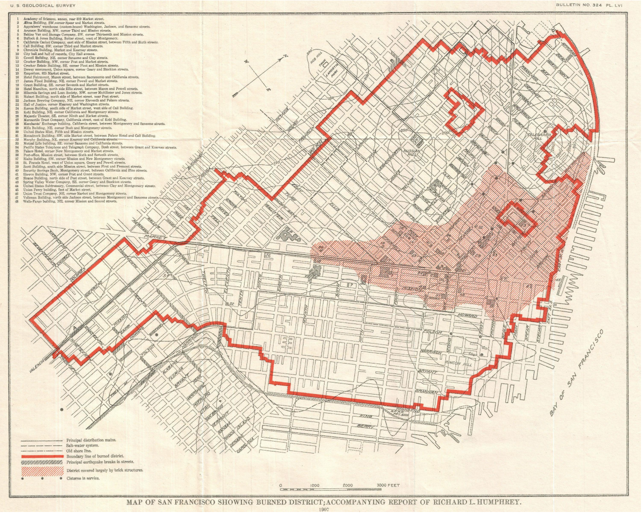

Figure: deformed L_1 ball with boundaries