Abstracts of Plenary Lectures

There will be nine plenary lectures, according to the following schedule.

Monday	9:15	Dan Král’	Vaughan Jeffries LT	R19
	13:55	Hendrik van Maldeghem	Vaughan Jeffries LT	R19
Tuesday	9:00	Penny Haxell	Vaughan Jeffries LT	R19
	13:30	Michael Krivelevich	Vaughan Jeffries LT	R19
Wednesday	9:00	Igor Pak	Vaughan Jeffries LT	R19
Thursday	9:00	Iain Moffatt	Vaughan Jeffries LT	R19
	13:30	Kristin Lauter	**Arts Main LT**	**R16**
Friday	9:00	Daniël Paulusma	Vaughan Jeffries LT	R19
	15:50	Gábor Tardos	Vaughan Jeffries LT	R19

The abstracts for these lectures are presented on the following pages.

Monday 9:15, Vaughan Jeffries LT

Analytic representations of large graphs

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MSC2000: 05C35

The theory of combinatorial limits provide analytic tools to represent and analyze large discrete objects. Such tools have found important applications in various areas of computer science and mathematics. They also led to opening new links between algebra, analysis, combinatorics, ergodic theory, group theory and probability theory. In this talk, we survey basic concepts concerning graph limits and particularly focus on links between dense graph limits and extremal combinatorics.

Monday 13:55, Vaughan Jeffries LT

Combinatorial constructions of exceptional buildings

Hendrik Van Maldeghem

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Ghent University

(This talk is based on joint work with Magali Victoor.)

MSC2000: 51E24

In this talk we first define the general concept of a building and survey some known combinatorial and geometric constructions of the exceptional buildings of types $\mathsf{F}_4, \mathsf{E}_6, \mathsf{E}_7, \mathsf{E}_8$. We zoom in on two construction, one for each of the types $\mathsf{E}_6, \mathsf{E}_7$, as intersections of quadrics, owing their existence to some peculiar combinatorial properties of two small generalised quadrangle (which are themselves quadrics in projective 4- and 5-space, respectively, over the field with two elements). Everything holds over an arbitrary field, finite or not, in any characteristic.

Tuesday 9:00, Vaughan Jeffries LT

Topological connectedness and independent sets in graphs

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MSC2000: 05C15, 05C35, 05C65, 05C69, 05C70

We describe a link between the topological connectedness of the independence complex of a graph and various other important graph parameters to do with colouring and partitioning. When the graph represents some other combinatorial structure, for example when it is the line graph of a hypergraph H, this link can be exploited to obtain information such as lower bounds on the matching number of H. Since its discovery there have been various applications of this phenomenon to other combinatorial problems. We also outline some of these applications.

Tuesday 13:30, Vaughan Jeffries LT

Expanders - how to find them, and how to use them

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MSC2000: 05C35, 05C38, 05C80, 05C83

A graph $G = (V, E)$ is called an expander if every vertex subset U of size up to $|V|/2$ has an external neighborhood comparable in size to that of U. Expanders have been a subject of intensive research for more than three decades and have become one of the central notions of modern graph theory.

We first discuss the above definition of an expander, its merits and alternatives. Then we present ways to argue that a given graph is an expander itself or contains a large expanding subgraph. Finally we consider properties of expanding graphs such as cycles and cycle lengths, embedding of large minors, and separators.

Wednesday 9:00, Vaughan Jeffries LT

Counting linear extensions and Young tableaux

Igor Pak

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I will survey various known and recent results on counting the number of linear extensions of finite posets. I will emphasize the asymptotic and complexity aspects for special families, where the problem is especially elegant yet remains #P-complete (or conjectured to be so). In the second half of the talk I will turn to posets corresponding to (skew) Young diagrams. This special case is important for many applications in representation theory and algebraic geometry. I will explain some surprising product formulas, connections to lozenge tilings, Selberg integrals and certain particle systems.

Thursday 9:00, Vaughan Jeffries LT

From graph duals to matrix pivots: a tour through delta-matorids

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MSC2000: 05B35, 05C10, 05C45, 5C83

This talk will consider a variety of everyday graph theoretical notions — duals, circle graphs, pivot-minors, Eulerian graphs, and bipartite graphs — and will survey how they appear in the theory of delta-matroids. The emphasis will be on exposing the interplay between the graph theoretical and matroid theoretical concepts, and *no prior knowledge of matroids will be assumed.*

Matroids are often introduced either as objects that capture linear independence, or as generalisations of graphs. If one likes to think of a matroid as a structure that captures linear independence in vector spaces, then a delta-matroid is a structure that arises by retaining the (Steinitz) exchange properties of vector space bases, but dropping the requirement that basis elements are all of the same size. On the other hand, if one prefers to think of matroids as generalising graphs, then delta-matroids generalise graphs embedded in surfaces. There are a host of other ways in which delta-matroids arise in combinatorics. Indeed, they were introduced independently by three different groups of authors in the 1980s, with each definition having a different motivation (and all different from the two above).

In this talk I'll survey some of the ways in which delta-matroids appear in graph theory. The focus will be on how the fundamental operations on delta-matroids appear, in different guises, as familiar and well-studied concepts in graph theory. In particular, I'll illustrate how apparently disparate pieces of graph theory come together in the world of delta-matroids.

Thursday 13:30, Arts Main LT

Supersingular isogeny graphs in cryptography (or how to keep your secrets in a post-quantum world)

Kristin Lauter

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As we move towards a world which includes quantum computers which exist at scale, we are forced to consider the question of what hard problems in mathematics our next generation of cryptographic systems will be based on. Supersingular Isogeny Graphs were proposed for use in cryptography in 2006 by Charles, Goren, and Lauter. Supersingular Isogeny Graphs are examples of Ramanujan graphs, which are optimal expander graphs. These graphs have the property that relatively short walks on the graph approximate the uniform distribution, and for this reason, walks on expander graphs are often used as a good source of randomness in computer science. But the reason these graphs are important for cryptography is that finding paths in these graphs, i.e. routing, is hard: there are no known subexponential algorithms to solve this problem, either classically or on a quantum computer. For this reason, cryptosystems based on the hardness of problems on Supersingular Isogeny Graphs are currently under consideration for standardization in the NIST Post-Quantum Cryptography (PQC) Competition. This talk will introduce these graphs, the cryptographic applications, and the various algorithmic approaches which have been tried to attack these systems.

Clique-Width for Hereditary Graph Classes

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MSC2000: 05C15, 05C75, 05C85

Clique-width is a well-studied graph parameter owing to its use in understanding algorithmic tractability: if the clique-width of a graph class $\mathcal{G}$ is bounded by a constant, a wide range of problems that are NP-complete in general can be shown to be polynomial-time solvable on $\mathcal{G}$. For this reason, the boundedness or unboundedness of clique-width has been investigated and determined for many graph classes. We survey these results for hereditary graph classes, which are the graph classes closed under taking induced subgraphs. We then discuss the algorithmic consequences of these results, in particular for the Colouring and Graph Isomorphism problems. We also explain a possible strong connection between results on boundedness of clique-width and on well-quasi-orderability by the induced subgraph relation for hereditary graph classes.

Friday 15:50, Vaughan Jeffries LT

The extremal theory of vertex or edge ordered graphs

Gábor Tardos

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MSC2000: 05C35

We enrich the structure of finite simple graphs with a linear order on either the vertices or the edges. Extending the standard question of Turán-type extremal graph theory we ask for the maximal number of edges in such a vertex or edge ordered graph on n vertices that does not contain a given pattern (or several patterns) as subgraph. The forbidden subgraph itself is also a vertex or edge ordered graph, so we forbid a certain subgraph with a specified ordering, but we allow the same underlying subgraph with a different (vertex or edge) order. This allows us to study a large number of extremal problems that are not expressible in the classical theory. This survey talk reports on ongoing research and includes a large selection of open problems.

Abstracts of Minisymposia Talks

This conference features six minisymposia, each comprising between four and six individual talks, with two or three talks in parallel in each time slot. These will take place according to the schedule below, in different rooms in the Arts building (R16). The plan on page **??** indicates where the rooms are located in relation to each other.

Tuesday	10:30–11:35	Extremal Combinatorics	Arts Main LT
		Designs and Latin Squares	Arts LR1
	15:00–16:05	Probabilistic Combinatorics	Arts Main LT
		Graph Colouring	Arts LR1
		Designs and Latin Squares	Arts LR3
	16:10–17:15	Probabilistic Combinatorics	Arts Main LT
		Graph Colouring	Arts LR1
Thursday	10:30–11:35	Extremal Combinatorics	Arts Main LT
		Designs and Latin Squares	Arts LR1
	15:00–17:15	Ramsey Theory	Arts Main LT
		Additive Combinatorics	Arts LR1

The abstracts for the minisymposia talks are presented on the following pages, ordered by the time they will take place. This is intended to allow easy comparison of the talks within a given time slot. To find an individual speaker please use the index on pages 172–173.

Hypergraph Lagrangians

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(This talk is based on joint work with Vytautas Gruslys and Natasha Morrison.)

MSC2000: 05C35, 05C65

Frankl and Füredi conjectured (1989) that any r-uniform hypergraph, whose edges form an initial segment of length m in the colex ordering, maximises the Lagrangian among all r-uniform hypergraphs with m edges, for all r and m. We prove this conjecture for $r = 3$ (and large m), and disprove it for all larger r (and a wide range of m). In the talk I will explain the notion of Lagrangians and focus on the counterexamples to the conjecture.

Generalised transversals of Latin squares

Ian Wanless

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(This talk is based on joint work with Nick Cavenagh.)

MSC2000: 05B15

A k-plex in a Latin square is a selection of entries which has exactly k representatives from each row, column and symbol. The 1-plexes are *transversals* and have been studied since Euler. In [2], I conjectured that for all even orders $n > 4$ there is a Latin square that has 3-plexes but no transversal. Here is an example of order 6, with a 3-plex highlighted:

$$\begin{bmatrix} \colorbox{lightgray}{1} & 2 & 3 & \colorbox{lightgray}{4} & 5 & \colorbox{lightgray}{6} \\ \colorbox{lightgray}{2} & 1 & 4 & \colorbox{lightgray}{3} & \colorbox{lightgray}{6} & 5 \\ 3 & \colorbox{lightgray}{5} & 1 & 6 & \colorbox{lightgray}{2} & \colorbox{lightgray}{4} \\ 4 & 6 & \colorbox{lightgray}{2} & 5 & \colorbox{lightgray}{3} & \colorbox{lightgray}{1} \\ \colorbox{lightgray}{5} & \colorbox{lightgray}{4} & \colorbox{lightgray}{6} & 2 & 1 & 3 \\ 6 & \colorbox{lightgray}{3} & \colorbox{lightgray}{5} & \colorbox{lightgray}{1} & 4 & 2 \end{bmatrix}$$

Much more recently, in joint work with Nick Cavenagh [1], we proved the aforementioned conjecture. We also showed that there are super-exponentially many Latin squares without transversals. I will discuss these two papers and the intervening history.

[1] N. J. Cavenagh and I. M. Wanless, Latin squares with no transversals, *Electron. J. Combin.* **24(2)** (2017), #P2.45.

[2] I. M. Wanless, A generalisation of transversals for Latin squares, *Electron. J. Combin.*, **9(1)** (2002), #R12.

A robust Corrádi-Hajnal Theorem

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(This talk is based on joint work with Julia Böttcher, Jan Corsten, Ewan Davies, Matthew Jenssen, Patrick Morris, Barnaby Roberts, Jozef Skokan.)

MSC2000: 05C35,05C80

The Corrádi-Hajnal Theorem states that if an n-vertex graph G, where $3|n$, has minimum degree $\delta(G) \geq \frac{2n}{3}$, then G contains a perfect triangle factor, i.e. a collection of $\frac{n}{3}$ pairwise vertex-disjoint triangles. In a celebrated breakthrough, Johansson, Kahn and Vu proved that, for some (moderate) $C > 0$, if $p \geq Cn^{-2/3}(\log n)^{1/3}$ then the binomial random graph $G(n, p)$ with high probability contains a perfect triangle factor. Both results are sharp (up to the value of C in the latter).

We prove a common almost-generalisation of these results: given any $\gamma > 0$ there is C such that if $\delta(G) \geq \left(\frac{2}{3} + \gamma\right)n$, and $p \geq Cn^{-2/3}(\log n)^{1/3}$, then with high probability the graph G_p contains a perfect triangle factor. Here G_p denotes the graph obtained from G by including edges of G independently with probability p (and including no edges which are not in G). This is sometimes called a 'robust' version of the Corrádi-Hajnal Theorem, after Krivelevich, Lee and Sudakov.

It seems that this result, surprisingly, does not follow from the method of Johansson, Kahn and Vu; we use a (somewhat) alternative proof of their theorem developed with Böttcher, Davies, Jenssen, Kohayakawa and Roberts. I will try to explain how the method works and where it gains over the Johansson-Kahn-Vu method for this problem.

Substitutes for the non-existent square lattice designs for 36 treaments

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(This talk is based on joint work with Peter J. Cameron, Leonard H. Soicher and E. R. Williams.)

MSC2000: 05B05, 05E30, 62K05, 62K10

A Latin square of order n can be used to make an incomplete-block design for n^2 treatments in $3n$ blocks of size n. The cells are the treatments, and each row, column and letter defines a block. Any pair of treatments concur in 0 or 1 blocks. These designs, which are called *square lattice designs*, were introduced in [5] and shown in [2] to be A-optimal for these parameters.

If there are mutually orthogonal Latin squares of order n, then the process can be continued, eventually giving an affine plane. But there are no mutually orthogonal Latin squares of order 6, so what should we do if we need a design for 36 treatments in 30 blocks of size 6?

A computer search reported in [3] found a design for 36 treatments in 24 blocks of size 6 whose measure of A-optimality is very close to the unachievable upper bound given by the non-existent square lattice design. Forty years later, a series of mistakes and wrong turnings in a search for sesqui-arrays in [1] led to the discovery of designs for 36 treatments in up to 8 replicates of 6 blocks of 6 treatments. They are all very good on the A-criterion. Their construction uses the distance-regular graph known as the Sylvester graph. This led to a repeat of the computer search, finding designs in up to 8 replicates. The semi-Latin squares in [4] can also be used to get such designs. For 8 replicates, the three methods give non-isomorphic designs with the same value of the A-criterion.

[1] Bailey R. A., Cameron P. J., Nilson T. Sesqui-arrays, a generalisation of triple arrays. *Australasian Journal of Combinatorics*, **17** (2018), 427–451.

[2] Cheng C.-S., Bailey R. A. Optimality of some two-associate-class partially balanced incomplete-block designs. *Annals of Statistics*, **19** (1991), 1667–1671.

[3] Patterson H. D., Williams E. R. A new class of resolvable incomplete block designs. *Biometrika*, **63** (1976), 83–92.

[4] Soicher L. H. Optimal and efficient semi-Latins squares. *Journal of Statistical Planning and Inference*, **143** (2013), 573–582.

[5] Yates F. A new method of arranging variety trials involving a large number of varieties. *Journal of Agricultural Science*, **226** (1936), 424–455.

Successive shortest paths in complete graphs with random edge weights

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(This talk is based on joint work with P. Balister, B. Mezei, G. Sorkin.)

MSC2000: 05C80, 68Q87, 05C85

The cost of the shortest path P_1 in a complete graph K_n with independent random edge weights $U(0,1)$ is known to converge in probability to $\ln n/n$. We define a second shortest path P_2 to be the cheapest path edge-disjoint from P_1, and consider more generally the cheapest path P_k edge-disjoint from all earlier paths. We show that the cost of P_k converges in probability to $(2k + \ln n)/n$ uniformly for all $k \leq n - 1$.

If we change the edge weights so that they are distributed exponentially with mean 1, then we get very precise results for constant k.

Revisiting a theorem by Folkman on graph colouring

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(This talk is based on joint work with P. Charbit, O. Defrain, G. Joret, A. Lagoutte, V. Limouzy, L. Pastor, and J.-S. Sereni.)

MSC2000: 05C15

We give a short proof of the following theorem due to Folkman (1969).

Theorem 1 (Folkman [1]). *For every graph G,*

$$\chi(G) \leq \max_{H \subseteq G}(|V(H)| - 2 \cdot (\alpha(H) - 1)).$$

We also discuss possible interpretations and generalisations of this result.

[1] Jon H. Folkman. *An upper bound on the chromatic number of a graph.* In Combinatorial theory and its application, II, (Proc. Colloq., Balatonfüred, 1969), 437–457. North-Holland, Amsterdam, 1970.

Designs and Decompositions

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MSC2000: 05B05, 05C70

In this talk I will give an introduction to Design Theory from the combinatorial perspective of (hyper)graph decompositions. I will survey some recent progress on the existence of decompositions, with particular attention to triangle decompositions of graphs, which provide a simple (yet still interesting) illustration of more general results.

Independent sets in the hypercube revisited

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University of Oxford

(This talk is based on joint work with Will Perkins.)

MSC2000: 05A16, 82B20, 05D40

We revisit Sapozhenko's classical proof on the asymptotics of the number of independent sets in the discrete hypercube $\{0,1\}^d$ and Galvin's follow-up work on weighted independent sets. We interpret Sapozhenko's proof in terms of the cluster expansion, a tool from statistical physics, which allows us to obtain considerably sharper asymptotics and to obtain detailed probabilistic information about the typical structure of (weighted) independent sets in the hypercube. These results refine those of Korshunov and Sapozhenko and Galvin, and answer several of questions of Galvin.

What can we do with the number of colours Hadwiger's Conjecture gives us?

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MSC2000: 05C15, 05C83

Hadwiger's Conjecture (1943) asserts that every graph without the complete graph K_{t+1} as a minor has a proper vertex-colouring using at most t colours. In spite of a lot of effort we seem to be very far from a proof of the conjecture in general. For instance, the best known upper bound on the number of colours needed for a graph without K_{t+1}-minor is $O\big(t\sqrt{\log t}\big)$; a number that hasn't changed since the 1980's.

In this talk we look at the following question: If all we have are just t colours, what kind of colouring can we guarantee for a graph without K_{t+1}-minor? We will concentrate on improper colourings (where not all edges are monochromatic) and on partial colourings (where only some of the vertices receive a colour).

[1] H. Hadwiger. Über eine Klassifikation der Streckenkomplexe. *Vierteljschr. Naturforsch. Ges. Zürich*, 88(2):133–142, 1943.

Design Theory and Unconditionally Secure Authentication

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MSC2000: 94C30

Cryptography is not just about providing secrecy or confidentiality. There are many applications where what is needed is *authentication,* to provide assurance that you are communicating with the person you think you are communicating with, and not an impostor. Design theory has long had a key role to play in the study of authentication in an *unconditionally secure* setting where no assumptions are made about the computational power of the adversary. In this talk we give examples of the use of designs in providing unconditionally secure authentication, highlight some natural definitions of generalisations of designs that have arisen from the study of authentication codes, and discuss related recent joint work with Sophie Huczynska on collections of disjoint subsets of finite groups satisfying particular difference properties.

Covering random graphs by monochromatic trees and Helly-type results for hypergraphs

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(This talk is based on joint work with D. Korándi and B. Sudakov.)

MSC2000: 05D05,05D10,05D40

How many monochromatic paths, cycles or general trees does one need to cover all vertices of a given r-edge-coloured graph G? These problems were introduced in the 1960's and were intensively studied by various researchers over the last 50 years. In this paper, we consider the question of covering random graphs using monochromatic trees, introduced by Bal and DeBiasio. Surprisingly, it is closely connected to the following, independently interesting, Helly-type problem about vertex covers of hypergraphs. Roughly speaking, the question is how large a cover of a hypergraph H can be if any subgraph of H with few edges has a small cover. We prove good bounds for the hypergraph problem and use them to estimate quite accurately the number of monochromatic trees needed to cover a random graph. Our results provide some very surprising answers to several questions in the area asked by Bal and DeBiasio, Kohayakawa, Mota and Schacht, Lang and Lo and Girão, Letzter and Sahasrabudhe.

Fractional colouring and the hard-core model

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(This talk is based on joint work with Ewan Davies, Rémi de Joannis de Verclos, and François Pirot.)

MSC2000: 05C15, 05C35, 05D10

A classic result of Shearer (1983), building on the seminal work of Ajtai, Komlós and Szemerédi (1981), showed that any triangle-free graph of maximum degree Δ on n vertices contains an independent set of size $\frac{1}{1+\varepsilon}\frac{\log\Delta}{\Delta}\cdot n$ for all Δ large enough. In a recent breakthrough that significantly strengthens Shearer's result, Molloy (2019) using the entropy compression method proved any triangle-free graph of maximum degree Δ has chromatic number at most $(1+\varepsilon)\frac{\Delta}{\log\Delta}$ for all Δ large enough. We discuss a simple proof of a slightly weaker version of Molloy's theorem, namely the same statement but with fractional chromatic number. The proof method uses basic local occupancy properties of the hard-core model and a simple greedy colouring procedure. The method underlies a still-developing intuition for chromatic structure in sparse graphs via the hard-core model.

The talk will touch on content from both http://arxiv.org/abs/1812.01534 and https://arxiv.org/abs/1812.11152.

Odd cycles in subgraphs of sparse pseudorandom graphs

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(This talk is based on joint work with Sören Berger and Mathias Schacht.)

MSC2000: 05C35, 05C38, 05C80

We answer two extremal questions about odd cycles that naturally arise in the study of sparse pseudorandom graphs. Let Γ be an (n, d, λ)-graph, i.e., an n-vertex, d-regular graph with all nontrivial eigenvalues in the interval $[-\lambda, \lambda]$. Krivelevich, Lee, and Sudakov conjectured that, whenever $\lambda^{2k-1} \ll d^{2k}/n$, every subgraph G of Γ with $(1/2 + o(1))e(\Gamma)$ edges contains an odd cycle C_{2k+1}. Aigner-Horev, Hàn, and Schacht proved a weaker statement by allowing an extra polylogarithmic factor in the assumption $\lambda^{2k-1} \ll d^{2k}/n$, but we completely remove it and hence settle the conjecture. This also generalises Sudakov, Szabo, and Vu's Turán-type theorem for triangles.

Secondly, we obtain a Ramsey multiplicity result for odd cycles. Namely, in the same range of parameters, we prove that every 2-edge-colouring of Γ contains at least $(1 - o(1))2^{-2k}d^{2k+1}$ monochromatic copies of C_{2k+1}. Both results are asymptotically best possible by Alon and Kahale's construction of C_{2k+1}-free pseudorandom graphs.

On Local and MAD Versions of Reed's Conjecture

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(This talk is based on joint work with T. Kelly.)

MSC2000: MSC 05C

One of the most well-known conjectures in graph coloring is Reed's conjecture that $\chi(G) \leq \left\lceil \frac{\Delta(G)+1+\omega(G)}{2} \right\rceil$. In 1998, Reed proved the chromatic number is at most some nontrivial convex combination of the two bounds. In this talk, we discuss two generalizations of this result. First we prove under some mild assumptions that the 'local list version' of Reed's result holds. Second, we used this local version to prove the maximum average degree version holds. As an application, we discuss how the latter results provide the current best known bounds for the number of edges in critical graphs without large cliques and for Hadwiger's conjecture.

On the Brown–Erdős–Sós problem

David Conlon

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MSC2000: 05C35

Let $f_r(n, v, e)$ be the maximum number of edges in an r-uniform hypergraph on n vertices which contains no induced subgraph with v vertices and at least e edges. The Brown–Erdős–Sós problem of determining $f_r(n, v, e)$ is a central question in extremal combinatorics, with surprising connections to a number of seemingly unrelated areas. For example, the result of Ruzsa and Szemerédi that $f_3(n, 6, 3) = o(n^2)$ implies Roth's theorem on the existence of 3-term arithmetic progressions in dense subsets of the integers. As a generalisation of this result, it is conjectured that

$$f_r(n, e(r-k) + k + 1, e) = o(n^k)$$

for any fixed $r > k \geq 2$ and $e \geq 3$. The best progress towards this conjecture, due to Sárközy and Selkow, says that

$$f_r(n, e(r-k) + k + \lfloor \log e \rfloor, e) = o(n^k),$$

where the log is taken base two. In this talk, we will discuss a recent improvement to this bound.

On strictly Neumaier graphs

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(This talk is based on joint work with Rhys Evans and Dmitry Panasenko.)

MSC2000: 05C69, 05E30

For a positive integer m, a clique in a regular graph is called *m-regular*, if every vertex that doesn't belong to the clique is adjacent to precisely m vertices from the clique. A regular clique can be equivalently viewed as a clique which is a part of an equitable 2-partition (see [3, 7]), or a completely regular code of radius 1 (see [8] and [2, p. 345]). It is well known that a clique in a strongly regular graph is regular if and only if it is a Delsarte clique (see [1]; [2, Proposition 1.3.2(ii)]; [2, Proposition 4.4.6]).

An edge-regular graph is called a *Neumaier graph* if it contains a regular clique. A Neumaier graph is called a *strictly Neumaier graph* if it is not strongly regular. In this talk we discuss recent results on strictly Neumaier graphs presented in [5], [6] and [4].

[1] S. Bang, A. Hiraki, J.H. Koolen, *Delsarte clique graphs*, Europ. J. Combin., 28 (2007) 501–516.

[2] A. E. Brouwer, A. M. Cohen, and A. Neumaier, *Distance-Regular Graphs*, Springer-Verlag, Berlin (1989).

[3] A. E. Brouwer and W. H. Haemers, *Spectra of Graphs*, Springer-Verlag, New York (2012).

[4] R. J. Evans, S. V. Goryainov, D. I. Panasenko, *The smallest strictly Neumaier graph and its generalisations*, to appear in The Electronic Journal of Combinatorics, https://arxiv.org/abs/1809.03417

[5] G. R. W. Greaves, J. H. Koolen, *Edge-regular graphs with regular cliques*, Europ. J. Combin., 71, 194–201 (2018).

[6] G. R. W. Greaves, J. H. Koolen, *Another construction of edge-regular graphs with regular cliques*, Discrete Mathematics, https://doi.org/10.1016/j.disc.2018.09.032

[7] C. Godsil, G. Royle, *Algebraic Graph Theory*, Springer-Verlag, New York (2001).

[8] A. Neumaier, *Completely regular codes*, Discrete Math., 106/107, 353–360 (1992).

Universality for bounded degree spanning trees in randomly perturbed graphs

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(This talk is based on joint work with Jie Han, Yoshiharu Kohayakawa, Richard Montgomery, Olaf Parczyk, Yury Person.)

MSC2000: 05C35, 05C05

Dirac-type questions in extremal graph theory concern asymptotically optimal conditions on the minimum degree of an n-vertex graph G to contain a given spanning graph F_n. On the other hand, a large branch of the theory of random graphs studies when random graphs $G(n,p)$ typically contain a copy of a given spanning graph F_n. Combining these two themes one can ask when F_n is typically contained in a *randomly perturbed graph* with linear minimum degree, that is, the graph $G_\alpha \cup G(n,p)$ where G_α is any n-vertex graph with minimum degree αn for some constant $\alpha > 0$.

We obtain the following universality type result for spanning bounded degree trees: With high probability the graph $G_\alpha \cup G(n, C/n)$ contains copies of all spanning trees with maximum degree at most Δ simultaneously, where C depends only on α and Δ. In the talk I will outline the proof of this result, which uses an absorbing strategy. This result is asymptotically optimal for $0 < \alpha < \frac{1}{2}$.

Latin set-theoretic solutions of the quantum Yang-Baxter equation

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(This talk is based on joint work with M. Bonatto, D. Stanovský, and P. Vojtěchovský.)

MSC2000: 20N05, 16T25

Given a set X, a map $S = (S_\ell, S_r) : X \times X \to X \times X$ is said to be a set-theoretic solution of the quantum Yang-Baxter equation (YBE) if (i) for each $x \in X$, the mappings $y \mapsto S_\ell(x, y)$ and $y \mapsto S_r(y, x)$ are bijections, and (ii) $(S\times\mathrm{id}_X)(\mathrm{id}_X\times S)(S\times\mathrm{id}_X) = (\mathrm{id}_X\times S)(S\times\mathrm{id}_X)(\mathrm{id}_X\times S)$. To the combinatorially minded, condition (i) seems pretty close to assuming that the binary operations S_ℓ, S_r are quasigroups, the algebraic counterpart of latin squares. Indeed, *latin solutions*, where one or both of S_ℓ, S_r is a quasigroup, are of considerable interest.

I will discuss two classes of quasigroups that can be interpreted as latin solutions. One of them is the classical variety of (left) distributive quasigroups, which nowadays have the fashionable name of *latin quandles*. The other class is defined by the identity $(xy)(xz) = (yx)(yz)$. Since the significance of this identity in the YBE context was first observed by W. Rump, we call these quasigroups *latin rumples*. They have some similarities to latin quandles but many differences. I will report on some initial results on latin rumples, which lead to some interesting open problems.

Some new variations on the Ramsey theme

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(This talk is based on joint work with students of BSM Research Experience classes.)

MSC2000: 05B07,05C55,05D10

- *Chromatic Ramsey number of trees, 2015 Spring.* For an r-uniform tree T, $\chi(T,k)$ is the smallest m ensuring that in every k-coloring of the edges of any m-chromatic r-uniform hypergraph, there is a monochromatic copy of T. The 3-uniform two-edge star is the simplest interesting case: In every k-coloring of the edges of any $(k+1)$-chromatic 3-uniform hypergraph, there are two edges of the same color intersecting in one vertex.

- *Ramsey number in Steiner triple systems, 2016 Summer.* A configuration C (partial Steiner triple system) is k-Ramsey if for all admissible $n > n_0 = n_0(C,k)$, every k-coloring of the triples of any $STS(n)$ contains a monochromatic copy of C. Among the 16 configurations with four triples, two are not even 1-Ramsey (because they are avoidable) but 13 are k-Ramsey for every k.

- *Ramsey number of triangles, 2018 Fall.* For $n \geq 3, n \neq 5$, in any coloring of $P(n)$, the power set of $[n]$, with $2^{n-2}-1$ colors, there is a monochromatic triangle.

Distinct distances in finite fields

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(This talk is based on joint work with B. Murphy and M. Rudnev.)

MSC2000: 52C10, 52C30, 52C35, 11T30

The Erdős distance problem is to find a bound on the minimum number of distinct distances that a point set can determine in the real plane. This was resolved (up to a logarithmic factor and to much acclaim) by Guth and Katz over the real plane.

One can also ask this question over other fields, in particular over finite fields $\mathbb{F}_p$. Here, not all of the techniques used by Guth and Katz are applicable because we no longer have the topology of the reals. Using the *bisector energy* of (amongst others) Lund and Petridis, we are able to obtain an improved bound. Here, we employ an incidence bound between points and planes in $\mathbb{F}^3$. In the case where no isotropic lines are present (i.e. when $p \equiv 3 \pmod 4$), we can "complexify" the field to obtain a further advantage, benefiting from point-line incidence bounds in both $\mathbb{F}^2$ and in $\mathbb{F}^3$.

I will sketch the main ideas we use to obtain this improvement.

Long monochromatic paths in random graphs

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MSC2000: 05D10, 05C80

Recall that the *size-Ramsey number* of F, $\hat{r}(F, r)$, is the smallest integer m such that there exists a *host graph* G with m edges such that any r-edge coloring of G yields a monochromatic copy of F. In this talk, we are concerned with the size-Ramsey number of the path P_n on n vertices. First, we explore some recent developments regarding $\hat{r}(P_n, r)$. Next, we study a Turán problem involving random graphs $G(N, p)$, the best-known host graphs for P_n. To that end, we consider the random variable $\mathrm{ex}(G(N, p), P_n)$, the maximum number of edges in a P_n-free subgraph of $G(N, p)$. The latter is joint work with József Balogh and Lina Li.

Polynomial bound for the partition rank vs the analytic rank of tensors

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MSC2000: 11B30, 15A69

A tensor defined over a finite field $\mathbb{F}$ has low analytic rank if the distribution of its values differs significantly from the uniform distribution. An order d tensor has partition rank 1 if it can be written as a product of two tensors of order less than d, and it has partition rank at most k if it can be written as a sum of k tensors of partition rank 1.

Kazhdan and Ziegler and (independently) Lovett proved that the partition rank is bounded below by the analytic rank. In the other direction, Green and Tao showed that if the analytic rank of an order d tensor is at most r, then its partition rank is at most $f(r, d, |\mathbb{F}|)$, where f is an Ackermann-type function in its parameters. In this talk I will sketch a proof that under the same assumptions the partition rank is in fact at most $g(r, d, |\mathbb{F}|)$, where, for fixed d and $\mathbb{F}$, g is a polynomial in r. Our result implies a similar improvement to the bounds for the quantitative inverse theorem for Gowers norms for polynomial phase functions of degree d.

Rainbow Spanning Tree Decompositions

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(This talk is based on joint work with Daniela Kühn, Richard Montgomery and Deryk Osthus.)

MSC2000: 05C70

A subgraph of an edge-coloured graph is called rainbow if all its edges have distinct colours. We present our recent result that, given any optimal colouring of a sufficiently large complete graph K_{2n}, there exists a decomposition of K_{2n} into isomorphic rainbow spanning trees. This settles conjectures of Brualdi–Hollingsworth (from 1996) and Constantine (from 2002) for large graphs.

Rado's criterion for squares and higher powers

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(This talk is based on joint work with Sam Chow & Sean Prendiville.)

MSC2000: 11B30, 11D72, 11L15

An equation is said to be partition regular if any finite colouring of the integers has a monochromatic solution, that is, a solution where all variables receive the same colour. A classical result of Rado fully characterises which linear equations are partition regular. We show that the same classification holds for sums of kth powers, provided the number of variables is large enough in terms of k.

In particular, we show that the equation

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = x_5^2$$

is partition regular over the integers. This is a small step towards answering a famous open problem of Erdős and Graham, which asks whether or not the Pythagorean equation

$$x^2 + y^2 = z^2$$

is partition regular.

Increasing paths in edge-ordered graphs

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(This talk is based on joint work with Matija Bucic, Matthew Kwan, Benny Sudakov, Tuan Tran, Adam Zsolt Wagner.)

MSC2000: 05C38

How long a monotone path can one always find in any edge-ordering of the complete graph K_n? This appealing question was first asked by Chvátal and Komlós in 1971, and has since attracted the attention of many researchers, inspiring a variety of related problems. The prevailing conjecture is that one can always find a monotone path of linear length, but recently now the best known lower bound was $n^{2/3-o(1)}$. This talk will be about how to almost close this gap, proving that any edge-ordering of the complete graph contains a monotone path of length $n^{1-o(1)}$.

Sumsets in several dimesions

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MSC2000: 11P70

A conjecture of Freiman establishes a lower bound on the cardinality of the sumset $A+A$ of a finite d–dimensional set $A \subset \mathbb{R}^d$ in terms of the cardinality $k = |A|$ of A and its volume. A set $A \subset \mathbb{R}^d$ is d–dimensional if it is not contained in a hyperplane. Its volume is the smallest volume of the convex hull of a set B which is Freiman isomorphic to A. The conjecture is equivalent to saying that the extremal sets for this problem are long simplices, consisting of a d–dimensional simplex and an extremal 1–dimensional set in one of the dimensions. In the talk we will survey the status of the conjecture and some related problems in connection with the Brunn–Minkowski inequality.

Abstracts of Contributed Talks

The contributed talks will take place at the following times, usually with six talks in parallel in each time slot, in six different rooms in the Arts building (R16). The plan on page **??** indicates where the rooms are located in relation to each other.

Monday	10:45–12:20	four consecutive slots
	15:25–17:25	five consecutive slots
Tuesday	11:40–12:00	one slot
Wednesday	10:30–12:05	four consecutive slots
Thursday	11:40–12:00	one slot
Friday	10:30–12:05	four consecutive slots
	13:40–15:15	four consecutive slots

The abstracts for these talks are presented on the following pages, ordered by the time they will take place. This is intended to allow easy comparison of the talks within a given time slot. To find an individual speaker please use the index on pages 172–173.

Clumsy packings of graphs

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(This talk is based on joint work with Anika Kaufmann/Kaplan, Raphael Yuster.)

MSC2000: 05C70

For graphs H and G, a packing of H in G is a set $\mathcal{F}$ of edge-disjoint copies of H in G. A packing $\mathcal{F}$ is maximal if any copy of H in G shares an edge with a member of $\mathcal{F}$. A packing is clumsy if it is maximal of smallest possible size. I.e., a clumsy packing corresponds to a most inefficient way to pack a graph. Using recent breakthrough results on perfect packings of dense graphs, we determine asymptotically the size of a clumsy packing of any graph in a sufficiently large complete graph. We also address clumsy packings of hypercubes.

Codes for correcting erasures with sporadic errors

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(This talk is based on joint work with Y. Tsunoda.)

MSC2000: 94B05 (94B25)

A binary error-correcting code of length n and minimum distance d is a set $\mathcal{C} \subseteq \mathbb{F}_2^n$ of n-dimensional vectors over the finite field $\mathbb{F}_2$ of order 2 such that $\min\{\text{dist}(\boldsymbol{c}, \boldsymbol{c}') \mid \boldsymbol{c}, \boldsymbol{c}' \in \mathcal{C}, \boldsymbol{c} \neq \boldsymbol{c}'\} = d$, where $\text{dist}(\boldsymbol{c}, \boldsymbol{c}')$ is the Hamming distance between $\boldsymbol{c}, \boldsymbol{c}' \in \mathbb{F}_2^n$. It is well-known in coding theory that a binary error-correcting code of length n and minimum distance d can, in principle, correct any combination of s bit flips and t erasures as long as $2s + t + 1 < d$. However, it is not trivial how to efficiently perform error-erasure correction. In this talk, we give a probabilistic construction for binary error-correcting codes $\mathcal{C}$ of length n and minimum distance $d = \Theta\left(\sqrt{\frac{n}{\log n}}\right)$ with $|\mathcal{C}| = \Theta(n)$ that allow for polynomial-time decoding under the assumption that the number s of errors is a constant. Our probabilistic construction provides such a code with high probability, that is, the probability that our randomized construction successfully provides such a code can be made arbitrarily close to 1. For small s, the time complexity of decoding our codes is nearly quadratic.

Combinatory Classes of Compositions with Higher-Order Conjugation

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MSC2000: 05A17, 05A15

The classical development of the theory of integer compositions by P. A. MacMahon (1854-1929) has recently been extended to compositions with conjugates of higher orders. This discussion will be based on certain classes of compositions possessing conjugates of a prescribed order - their enumeration and some identities they satisfy. These compositions specialize to standard results in a natural way. We will also give a generalization of MacMahon's identities for inverse-conjugate compositions and discuss inverse-reciprocal compositions.

The maximum length of K_r-Bootstrap Percolation

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(This talk is based on joint work with József Balogh, Alexey Pokrovskiy, and Tibor Szabó.)

MSC2000: 05C35, 05D99, 82B43

Graph-bootstrap percolation, also known as weak saturation, was introduced by Bollobás in 1968. In this process, we start with an initial "infected" set of edges E_0, and we infect new edges according to a predetermined rule. Given a graph H and a set of previously infected edges $E_t \subseteq E(K_n)$, we infect a non-infected edge e if it completes a new copy of H in $G = ([n], E_t \cup e)$. Formally, denote by $n_H(G)$ the number of copies of H in a graph G. Let

$$G_t = G_{t-1} \cup \{e \in E(K_n) \mid n_H(G_{t-1} \cup \{e\}) > n_H(G_{t-1})\}$$

and $E_t = E(G_t)$.

A question raised by Bollobás asks for the maximum time the process can run before it stabilizes, taken over all starting graphs. That is, we want to estimate the following parameter

$$M_H(n) = \max\{t \mid \exists G_0 \subseteq K_n \text{ such that } G_t \neq G_{t-1} \text{ in the } H\text{-bootstrap process}\}.$$

In 2015, Bollobás, Przykucki, Riordan, and Sahasrabudhe considered this problem for the most natural case where $H = K_r$. They answered the question for $r \leq 4$ and gave a lower bound for every $r \geq 5$. In their paper, they also conjectured that the maximal running time is $o(n^2)$ for every integer r. Here we disprove their conjecture for every $r \geq 6$ and we give a better lower bound for the case that $r = 5$. In the proof of the case $r = 5$ we use the Behrend construction.

Alternating Signed Bipartite Graph Colourings

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(This talk is based on joint work with R. Quinlan, K. Jennings.)

MSC2000: 05C15, 05C50

In this talk, we develop a theme of Brualdi et al. [1] by investigating a class of bipartite graphs that arise from alternating sign matrices. An *alternating sign matrix (ASM)* is a $(0, 1, -1)$-matrix in which the non-zero elements in each row and column alternate in sign, beginning and ending with 1.

An *alternating signed bipartite graph (ASBG)* is a graph G corresponding to an ASM A, with a vertex for each row and column of A. Vertex r_i is connected to vertex c_j by a blue edge if $A_{ij} = 1$ and by a red edge if $A_{ij} = -1$. In this talk, we present results on when a given graph G admits an edge colouring c such that the coloured graph G^c is an alternating signed bipartite graph.

[1] R. Brualdi, K. Kiernan, S. Meyer, M. Schroeder. *Patterns of Alternating Sign Matrices*. Linear Algebra and its Applications, **438**(10): 3967-3990, 2013.

MST of the Inner Dualist of Honeycomb Graphs

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(This talk is based on joint work with Prof. Faqir M Bhatti.)

MSC2000: 05C85, 68R10, 68W40

The family of honeycomb graphs is a well-known class of graphs and is an active area of research. Its inner dual is also an active area of research. The classical graph representations of such graphs do not incorporate the angle at which any of the edges are present. Moreover the fact that the inner dual does not characterize the hexagonal system completely was established in 1968 and 1969 and it was repeatedly discussed in several research papers, review papers and in a book.

It was noted that if the angle was somehow preserved in the inner dual then it can completely characterize the hexagonal system. This was the rationality behind the "characteristic graph". This is more widely known in the literature as the "inner dualist graph" or "dualist" and some researchers still use the term "characteristic graph".

He and He put forward the idea of the He-matrix in which the edges of the dualist graph are weighted and the weights define the angle of the edge. In the He-Matrix representation there can be six possible orientations of a graph, after reflections and rotations through fixed orientations. In the He-Matrix, edges parallel to the x-axis are given a weight of 1, edges having an angle of 60 degrees are given a weight of 2, and edges with an angle of 120 degrees are assigned a weight of 3.

In this talk I will discuss how edges of the inner dualist of honeycomb graphs can be partitioned into different classes according to specific angles 0, 60°, 120° with the x-axis (horizontal line). Subsets from these classes of edges are combined to give the minimum spanning tree.

During the presentation I will present a linear time algorithm for finding the orientation that gives the least of all minimum spanning trees among all orientations. This improves on classical algorithms by Prims and Kruskal and other more recent algorithms for calculating minimum spanning tree as they take more than linear time in terms of number of edges to find minimum spanning tree in any graph. In the talk, the derivation of this algorithm and related calculations will be discussed.

This talk will also cover some interesting observations for example some of these six orientations cannot contain the least weight minimum spanning tree. Most of the results are based upon the cardinality of elements in different classes of partitions.

The generalised Oberwolfach problem

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(This talk is based on joint work with P. Keevash.)

MSC2000: 05C70

Recently, much progress has been made on the general problem of decomposing a dense graph into a given family of sparse graphs (e.g. Hamilton cycles or trees). I will present a new result of this type: that any quasirandom dense large graph in which all degrees are equal and even can be decomposed into any given collection of two-factors (2-regular spanning subgraphs). A special case of this result gives a new proof of the Oberwolfach problem for large graphs.

Monday 11:10, Arts LR1

On the Extendability of Quaternary Linear Codes

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(This talk is based on joint work with H. Kanda and M. Shirouzu.)

MSC2000: 94B05, 94B27

An $[n, k, d]_q$ code is a linear code of length n, dimension k and minimum weight d over the field of q elements $\mathbb{F}_q$. An $[n, k, d]_q$ code $\mathcal{C}$ is called *extendable* if there exists an $[n + 1, k, d + 1]_q$ code $\mathcal{C}'$ which gives $\mathcal{C}$ as a punctured code, and $\mathcal{C}'$ is an *extension* of $\mathcal{C}$. As for the known results on the extendability of linear codes over $\mathbb{F}_4$, see [2] and [3]. Kanda [1] recently proved that an $[n, k, d]_3$ code with $k \geq 3$, $d \equiv -1$ or $-2 \pmod 9$, is extendable if $A_i = 0$ for all $i \not\equiv 0, -1, -2 \pmod 9$, where A_i denotes the number of codewords of $\mathcal{C}$ with weight i. We consider the extendability of an $[n, k, d]_4$ code with $k \geq 3$, $d \equiv -3 \pmod{16}$ satisfying $A_i = 0$ for all $i \not\equiv 0, -1, -2, -3 \pmod{16}$.

[1] H. Kanda, A new extension theorem for ternary linear codes and its application, submitted for publication.

[2] H. Kanda, T. Maruta, On the 3-extendability of quaternary linear codes, Finite Fields Appl. 52 (2018) 126–136.

[3] T. Maruta, M. Takeda, K. Kawakami, New sufficient conditions for the extendability of quaternary linear codes, Finite Fields Appl. 14 (2008) 615–634.

Contractible edges on longest cycles in a 3-connected graph

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MSC2010: 05C40

In this talk, we consider only finite, simple, undirected graphs with no loops and no multiple edges. Let $G = (V(G), E(G))$ be a graph. A graph G is called 3-*connected* if $|V(G)| \geq 4$ and $G - S$ is connected for any subset S of $V(G)$ having cardinality 2. Let G be a 3-connected graph. An edge e of G ($|V(G)| \geq 5$) is called *contractible* if the graph which we obtain from G by contracting e into one vertex (and replacing each of the resulting pairs of parallel edges by a simple edge) is 3-connected. We let $E_c(G)$ denote the set of contractible edges of G. In [2], Ota made a conjecture that there exists a constant $\alpha > 0$ such that if G is a 3-connected graph of order at least 5, then G has a longest cycle G such that $|E(C) \cap E_c(G)| \geq \alpha|E(C)|$. In [1], Fujita showed that such a constant exists if G is a 3-connected graph of order at least 5:

Theorem 1. *Let G be a 3-connected graph of order at least 5. Then there exists a longest cycle C of G such that $|E(C) \cap E_c(G)| \geq \lceil \frac{1}{7}|E(C)| + 1 \rceil$.*

In this talk, we prove the following theorem, which is a refinement of Theorem 1:

Theorem 2. *Let G be a 3-connected graph of order at least 5. Then there exists a longest cycle C of G such that $|E(C) \cap E_c(G)| \geq \lceil \frac{1}{6}|E(C)| + \frac{5}{6} \rceil$.*

Let α_0 denote the supremum of those real numbers α for which the aforementioned conjecture of Ota is true. Theorem 1 shows $\alpha_0 \geq \frac{1}{7}$, and we obtain $\alpha_0 \geq \frac{1}{6}$. On the other hand, $\alpha_0 \leq \frac{1}{3}$. To see this, let G be the line graph of a graph obtained from a 3-regular 3-connected graph by subdividing all edges once. Then G is 3-connected, and $|E(C) \cap E_c(G)| = \frac{|E(C)|}{3}$ for every longest cycle C of G.

[1] K. Fujita, Lower bounds on the maximum number of contractible edges on longest cycles of a 3-connected graph, *Far East J. Appl. Math.* **22** (2006), 55–86.

[2] K. Ota, Non-critical subgraphs in k-connected graphs, *PH. D. Dissertation, University of Tokyo* (1989).

Some results in 1-independent percolation

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(This talk is based on joint work with A. N. Day and V. Falgas-Ravry.)

MSC2000: 05D40, 60K35, 05C40, 05C38

We obtain an improved lower bound for the threshold probability p_c, for which every 1-independent measure with bond density $p > p_c$ percolates on the lattice $\mathbb{Z}^2$. We also present further results motivated by 1-independent percolation: for any connected graph G, let $f_G(p)$ be the infimum over all 1-independent measures μ with bond density p of the probability that a μ-random graph is connected. We obtain lower bounds for $f_G(p)$ for paths, ladders, complete graphs and cycles, and provide constructions giving matching upper bounds for paths, complete graphs and small cycles.

Monday 11:10, Arts LR4

Digraphs with Hermitian spectral radius at most 2

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(This talk is based on joint work with A. Munemasa.)

MSC2000: 05C50

Smith [7] and Lemmens and Seidel [4] showed that a connected simple graph whose $(0,1)$-adjacency matrix has spectral radius at most 2 is a subgraph of one of the extended Dynkin diagrams of the irreducible root lattices of types A, D, and E. We generalize this result to the class of digraphs (both arcs and undirected edges are allowed) with respect to their Hermitian adjacency matrices, which were recently introduced by Liu and Li [5] and independently by Guo and Mohar [2].

Guo and Mohar [3] studied digraphs whose Hermitian adjacency matrix has spectral radius *less* than 2. Using the classification of integer cyclotomic matrices by McKee and Smyth [6], and the classification of Hermitian cyclotomic matrices over the Gaussian integers by Greaves [1], we classify maximal digraphs whose Hermitian spectral radius is at most 2. In doing so, we find a counterexample to the statement of [3, Lemma 4.8(b)], leading to an omission in [3, Theorem 4.15]. We thus also complete the statement of [3, Theorem 4.15].

[1] Gary Greaves, Cyclotomic matrices over the Eisenstein and Gaussian integers. *J. Algebra* **372** (2012) 560–583.

[2] Krystal Guo, Bojan Mohar, Hermitian Adjacency Matrix of Digraphs and Mixed Graphs. *Journal of Graph Theory* **85**(1) (2017) 217–248.

[3] Krystal Guo, Bojan Mohar, Digraphs with Hermitian spectral radius below 2 and their cospectrality with paths. *Discrete Mathematics* **340**(11) (2017) 2616–2632.

[4] P.W.H. Lemmens, J.J. Seidel, Equiangular lines. *Journal of Algebra* **24**(3) (1973) 494–512.

[5] Jianxi Liu, Xueliang Li, Hermitian-adjacency matrices and hermitian energies of mixed graphs. *Linear Algebra and its Applications* **466** (2015) 182–207.

[6] James McKee, Chris Smyth, Integer symmetric matrices having all their eigenvalues in the interval $[-2, 2]$. *Journal of Algebra* **317** (2007) 260–290.

[7] John H. Smith, Some properties of the spectrum of a graph. In *Combinatorial Structures and their Applications (Proc. Calgary Internat. Conf., Calgary, Alta., 1969)*, Gordon and Breach, New York, 1970, 403–406.

Chordal graphs are easily testable

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MSC2000: 05C80

A graph G on n vertices is ϵ-*far* from satisfying a property $\mathcal{P}$ if one has to add or delete at least ϵn^2 edges to G to obtain a graph satisfying $\mathcal{P}$. A hereditary class $\mathcal{P}$ of graphs is *testable with query complexity* m_ϵ if for every fixed $\epsilon > 0$ the following holds. If G is ϵ-far from $\mathcal{P}$ then a set $X \subseteq V(G)$ sampled uniformly at random among all subsets of $V(G)$ of size m_ϵ induces a graph $G[X]$ that is not in $\mathcal{P}$ with probability at least $\frac{1}{2}$. We prove that property of being chordal is testable with a query complexity $m_\epsilon = O(1/\epsilon^c)$ that is a polynomial in $1/\epsilon$. This answers a question of Gishboliner and Shapira.

Monday 11:35, Arts Main LT

An approximate version of Jackson's conjecture

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(This talk is based on joint work with Anita Liebenau.)

MSC2000: 05C20, 05C45

In 1981 Jackson showed that the diregular bipartite tournament (a complete balanced bipartite graph whose edges are oriented so that every vertex has the same in and out-degree) contains a Hamilton cycle, and conjectured that in fact the edge set of it can be partitioned into Hamilton cycles. We prove an approximate version of this conjecture: for every $c > 1/2$ and every $\varepsilon > 0$, for large n every cn-regular balanced bipartite digraph on $2n$ vertices contains $(1 - \varepsilon)cn$ edge-disjoint Hamilton cycles.

New extremal Type II $\mathbb{Z}_4$-codes of length 32 obtained from Hadamard designs

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(This talk is based on joint work with S. Ban, D. Crnković and M Mravić.)

MSC2000: 94B05, 05B20, 05B05

For every Hadamard design with parameters $2-(n-1, \frac{n}{2}-1, \frac{n}{4}-1)$ having a skew-symmetric incidence matrix we give a construction of 54 Hadamard designs with parameters $2-(4n-1, 2n-1, n-1)$. This is a generalization of the construction given in [2].

For the case $n = 8$ we construct doubly-even self-orthogonal binary linear codes from the corresponding Hadamard matrices of order 32. From these binary codes we construct five new extremal Type II $\mathbb{Z}_4$-codes of length 32, using the method given in [3]. The constructed codes are the first examples of extremal Type II $\mathbb{Z}_4$-codes of length 32 and type $4^{k_1}2^{k_2}$, $k_1 \in \{7, 8, 9, 10\}$ whose residue codes have minimum weight 8. Further, correcting the results from [1] we construct 5147 extremal Type II $\mathbb{Z}_4$-codes of length 32 and type $4^{14}2^4$.

[1] K. H. Chan, *Three New Methods for Construction of Extremal Type II Z4-Codes*, PhD Thesis (University of Illinois at Chicago, 2012)

[2] D. Crnković, S. Rukavina, Some Symmetric (47,23,11) Designs, *Glas. Mat. Ser. III*, **38 (58)** (2003) 1–9.

[3] V. Pless, J. S. Leon, J. Fields, All Z_4 codes of Type II and length 16 are known, *J. Combin. Theory Ser. A*, **78** (1997) 32–50.

Global rigidity of linearly constrained frameworks

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(This talk is based on joint work with Hakan Guler and Bill Jackson.)

MSC2000: 52C25, 05C10

A (bar-joint) framework (G, p) in $\mathbb{R}^d$ is the combination of a graph G and a map p assigning positions to the vertices of G. A framework is rigid if the only edge-length-preserving continuous motions of the vertices arise from isometries of $\mathbb{R}^d$. The framework is globally rigid if every other framework with the same edge lengths arises from isometries of $\mathbb{R}^d$. Both rigidity and global rigidity, generically, are well understood when $d = 2$. A linearly constrained framework in $\mathbb{R}^d$ is a generalisation of a framework in which some vertices are constrained to lie on one or more given hyperplanes. Streinu and Theran characterised rigid linearly constrained generic frameworks in $\mathbb{R}^2$ in 2010. In this talk I will describe an analogous result for the global rigidity of linearly constrained generic frameworks in $\mathbb{R}^2$.

Maker-Breaker percolation games

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(This talk is based on joint work with Victor Falgas-Ravry.)

MSC2000: 91A24, 91A46, 82B43

The (p, q)-percolation game is a Maker-Breaker game played by two players, Maker and Breaker, on the edge set of the two-dimensional integer lattice. On each of her turns Maker claims p different unclaimed edges, while on each of his turns Breaker claims q different unclaimed edges. Informally speaking, Maker's aim is to build an arbitrarily long path from the origin of the lattice, while Breaker's aim is to prevent this from happening. More formally speaking, we say that Maker wins the game if she can guarantee on every turn that, among the set of unclaimed edges and edges that Maker has claimed, there is always a path from the origin escaping to infinity. The (p, q)-crossing game is similar to the above Maker-Breaker percolation game, except that it is played on a finite two dimensional grid and Maker's aim is to create a path of edges that crosses from the left side of the grid to the right side, while Breaker's aim is to prevent this from happening.

In this talk we will give a number of results for such Maker-Breaker games, and discuss their relations to each other, as well as the well known combinatorial games of Hex, Bridg-it and the Shannon switching game.

Bound for $(r, w]$-consecutive-disjunct matrices

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MSC2000: 05D40

A binary matrix is $(r, w]$-*disjunct* if the union of any r columns does not contain the intersection of any other w columns. In combinatorial group testing, a disjunct matrix generates a nonadaptive algorithm by regarding its rows as tests and its columns as items. In this talk, we focus on a variation of a disjunct matrix. Some columns of a matrix M of n columns are said to be *cyclically consecutive* if their indices in M are consecutive in the cyclic order $(1, 2, \ldots, n)$. A binary matrix is $(r, w]$-*consecutive-disjunct* if the union of any r cyclically consecutive columns does not contain the intersection of any other w cyclically consecutive columns. For group testing to be effective, we want to minimize the number of tests. Therefore, the number $t(n, r, w]$ of rows of a minimum $(r, w]$-consecutive-disjunct matrix of n columns is of special interest. The sharpest known upper bound on $t(n, r, w]$ is given in [1]. They employed the well-known Lovász Local Lemma, which is a powerful tool of the probabilistic method [2].

Theorem 1 ([1]). *For any positive integers $r, w,$ and n with $n \geq 3r + 3w$ and $r \geq w$,*

$$t(n,r,w] \leq (r+1)^w \left(1+\frac{1}{r}\right)^r \cdot \left(\ln\left((4r+4w-4)n - 6(r^2+w^2) - 13rw + 12r + 13w - 5\right) + 1\right).$$

We prove the following theorem by applying the alteration method in probabilistic combinatorics.

Theorem 2. *For any positive integers $r, w,$ and n with $n \geq r + w$,*

$$t(n,r,w] \leq \min_{t\in\mathbb{N}} \left\{ t + \left\lfloor n(n-r-w+1)\left(1-\left(\frac{w}{r+w}\right)^w\left(1-\frac{w}{r+w}\right)^r\right)^t \right\rfloor \right\}.$$

The above bound is tighter than the known bound in many cases.

[1] H. Chang, Y.-C. Chiu, and Y.-L. Tsai, "A Variation of Cover-Free Families and Its Applications," *Journal of Computational Biology*, vol. 22, no. 7, pp. 677 – 686, 2015.

[2] N. Alon and J. H. Spencer, *The Probabilistic Method*, 4th ed., John Wiley & Suns, 2016.

Monday 11:35, Arts LR5

On Monotonicity of Minimum Cost Inert Node Searching

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(This talk is based on joint work with Janka Chlebíková.)

MSC2000: 68R10, 05C99

Graph searching problems are well-studied with various real-world applications in addition to offering a better understanding of some structural parameters of graphs. Inert Node Searching is the problem of sweeping all the vertices of a graph using a set of searchers with the aim of capturing a fugitive while optimising certain resources. At a given time the fugitive resides on a vertex, and is defined to be fast, invisible, omniscient and lazy. We first consider Inert Node Searching as defined in [2] and show that the cost of search under this definition is not related to the minimum fill-in parameter as claimed in the paper. Next, we refine Inert Node Searching of [1] and introduce a new cost parameter called *guard cost*. We prove a relation between the minimum fill-in parameter, and guard cost of a graph as desired. Finally, by showing that guard cost of a graph can be minimised with a monotone strategy we close an open problem on the monotonicity of the cost parameter.

[1] Dariusz Dereniowski and Adam Stański. On tradeoffs between width- and fill-like graph parameters. *Theory of Computing Systems*, 63(3):450–465, 2019.

[2] Fedor V. Fomin, Pinar Heggernes, and Jan Arne Telle. Graph searching, elimination trees, and a generalization of bandwidth. *Algorithmica*, 41(2):73–87, 2005.

Monday 12:00, Arts Main LT

The Brown-Erdős-Sós conjecture in groups

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(This talk is based on joint work with Rajko Nenadov and Benny Sudakov.)

MSC2000: 05C65 Hypergraphs

The conjecture of Brown, Erdős and Sós from 1973 states that, for any $k \geq 3$, if a 3-uniform hypergraph H with n vertices does not contain a set of $k+3$ vertices spanning at least k edges then it has $o(n^2)$ edges. The case $k = 3$ of this conjecture is the celebrated $(6,3)$-theorem of Ruzsa and Szemerédi which implies Roth's theorem on 3-term arithmetic progressions in dense sets of integers.

Solymosi observed that, in order to prove the conjecture, one can assume that H consists of triples (a, b, ab) of some finite quasigroup Γ. Since this problem remains open for all $k \geq 4$, he further proposed to study triple systems coming from finite groups. In this case he proved that the conjecture holds also for $k = 4$.

We completely resolve the Brown-Erdős-Sós conjecture in groups, for all values of k. Moreover, we prove that the hypergraphs coming from groups contain sets of size $\Theta(\sqrt{k})$ which span k edges, which is best possible.

Monday 12:00, Arts LR1

On varieties defined by the intersection of many quadrics

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(This talk is based on joint work with Valentina Pepe.)

MSC2000: 51E21, 15A03, 94B05, 05B35

Let U be a subspace of quadrics defined on $\mathrm{PG}(k-1,\mathbb{F})$ with the property that U does not contain reducible quadrics. Let $V(U)$ be the variety of points of $\mathrm{PG}(k-1,\mathbb{F})$ which are zeros of all quadrics in U. In this talk I will consider the possibilities for $V(U)$ given that the dimension of U is large. A theorem of Castelnuovo from 1889 states that if the dimension of U is at least $\binom{k-1}{2}$ and $V(U)$ spans the space and has size at least $2k+1$ then $V(U)$ is contained in a normal rational curve.

If the dimension of U is $\binom{k-1}{2}-1$ and $V(U)$ spans the space and there is a group G which fixes U and no line of $\mathrm{PG}(k-1,\mathbb{F})$ then we can prove that any hyperplane of $\mathrm{PG}(k-1,\mathbb{F})$ is incident with at most k points of $V(U)$. If $\mathbb{F}$ is a finite field then the linear code generated by the matrix whose columns are the points of $V(U)$ is a k-dimensional linear code of length $|V(U)|$ and minimum distance at least $|V(U)|-k$. A linear code with these parameters is an MDS code or an almost MDS code.

I will give some examples of such subspaces U and groups G and present a conjecture that if $V(U)$ is large enough then the projection of $V(U)$ from any $k-4$ points is contained in the intersection of two linearly independent quadrics. This would be a strengthening of a theorem of Fano from 1894.

The 5-cube Cut Number Problem: A Short Proof for a Basic Lemma

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(This talk is based on joint work with R. Arce Nazario.)

MSC2000: 52B55, 52C99

The hypercube cut number $S(d)$ is the minimum number of hyperplanes in the d-dimensional Euclidean space $\mathbb{R}^d$ that slice all the edges of the d-cube. The problem was originally posed by P. O'Neil in 1971. B. Grünbaum, V. Klee, M. Saks and Z. Füredi have raised the problem at different times. In 2000, Sohler and Ziegler obtained a computational solution to the 5-cube problem. However finding a short proof for the problem, independent of computer computations, remains a challenging problem. We present a vertex coloring on the hypercube that simplifies proofs and looks to be a promising approach for the 5-cube problem. In particular, we give a short proof for the result presented by Emamy-Uribe-Tomassini in Hypercube 2002 based on Tomassini's Thesis. The proof here is substantially shorter than the original proof of 60 pages.

Monday 12:00, Arts LR3

The Namer–Claimer game

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MSC2000: 05D10

In each round of the Namer–Claimer game,

- *Namer* names a distance $d \in \mathbb{N}$;
- *Claimer* claims a subset of $[n]$ not containing any two points at distance d.

How quickly can Claimer claim subsets covering $[n]$ if Namer is trying to slow them down?

In this talk I won't give the direct argument showing that the answer is $O(\log \log n)$. Instead, I'll highlight connections with the Ramsey theory of Hilbert cubes and pose some generalisations of this problem.

A spectral characterization of the s-clique extension of the square grid graphs

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(This talk is based on joint work with Jack Koolen and Muhammad Riaz.)

MSC2000: 05C50, 05C75, 05E30

In this talk, I will discuss a result which shows that for integers $s \geq 2$, $t \geq 1$, any co-edge-regular graph which is cospectral with the s-clique extension of the $t \times t$-grid is the s-clique extension of the $t \times t$-grid, if t is large enough. Gavrilyuk and Koolen used a weaker version of this result to show that the Grassmann graph $J_q(2D, D)$ is characterized by its intersection array as a distance-regular graph, if D is large enough. The results have recently been published by European Journal of Combinatorics [1].

[1] S. Hayat, J.H. Koolen, M. Riaz, A spectral characterization of the s-clique extension of the square grid graphs, *European Journal of Combinatorics*, 76 (2019) 104–116.

Monday 12:00, Arts LR5

Some more recent applications of partial rejection sampling

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(This talk is based on joint work with Heng Guo and Jingcheng Liu.)

MSC2000: 68W20 05C80 05C85

Rejection sampling, sometimes called the acceptance-rejection method, is a simple, classical technique for sampling from a conditional distribution given that some desirable event occurs. The idea is to sample from the unconditioned distribution (assumed to be simple, for example a product distribution), accept the sample if the desirable event occurs and reject otherwise. This trial is repeated until the first acceptance. Rejection sampling in this form is rarely a feasible approach to sampling combinatorial structures, as the acceptance probability is generally exponentially small in the size of the problem instance. However, some isolated cases were known where an exact sample is obtained by resampling only that part of the structure that “goes wrong”, an example being the “sink-popping” algorithm of Cohn, Pemantle and Propp for sampling sink-free orientations in an undirected graph.

The situations in which this shortcut still yields an exact sample from the desired distribution can be characterised, and are related to so-called extreme instances for the Lovász Local Lemma. With this insight, it is possible to discover further applications. A couple of these will be presented. For the benefit of those who have seen these ideas before, the examples will be of a more recent vintage.

3-choosability of planar graphs of girth at least five, using the discharging method

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(This talk is based on joint work with Marthe Bonamy, Michelle Delcourt, František Kardoš, and Luke Postle.)

MSC2000: 05C15

A majority of questions about list coloring of planar graphs are resolved via the discharging method. However, there are several notable exceptions, for instance Thomassen's proof of 3-choosability of planar graphs of girth at least five [1]. It is natural to ask whether this result can be proven via discharging, or whether this is prevented by some fundamental obstructions. We show that the former is the case, proving by discharging a stronger claim: planar graphs of girth at least five have Alon-Tarsi number at most three.

[1] C. Thomassen. A short list color proof of Grötzsch's theorem. *J. Combin. Theory Ser. B*, 88:189–192, 2003.

Prime Strictly Concentric Magic Squares

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(This talk is based on joint work with S. Perkins and P.A. Roach.)

MSC2000: 05A99, 05B99

A Magic Square of order n is an n by n grid into which n^2 unique integers are placed such that all rows, columns and diagonals sum to the same value, termed the magic constant. This talk concerns specifically Prime Strictly Concentric Magic Squares (PSCMS). PSCMS are Magic Squares in which every integer in the square is a prime number, and for which every subsquare of lower order conforms to the constraints of a Magic Square. The number of minimum Prime Concentric Magic Squares (PSCMS) of odd order 5 to 19 has been calculated computationally, and presented without proof (Makarova, 2015). This talk presents relevant general definitions, examples and important properties of PSCMS. A minimum PSCMS of order 5 is defined. A construction is given and the number of PSCMS of order 5 is mathematically obtained.

[1] N. Makarova, Concentric magic squares of primes, http://primesmagicgames.altervista.org/wp/forums/topic/concentric-magic-squares-of-primes/, 2015. Last accessed 19/02/2019.

Counting spanning trees on the graph's complement

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MSC2000: 05C30,05C31

Counting spanning trees in a connected graph is a classic theme in Combinatorics. There are many techniques to compute this number, and while some of these are very much in use today, some have been forgotten. For this talk, I want to recover one technique and place it in the language of a very general algebraic invariant associated to a graph. The invariant is the U-polynomial which was introduced in 1999 by Noble and Welsh. I exemplify the technique with the n-star, the graph that is the union of n independent edges, and the n-path. In this latter case, there is a relation to the Chebyshev polynomials.

Rainbow Hamiltonian Cycles

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Given graphs $G_1, ..., G_n$ on the same vertex set of size n, each graph having minimum degree at least $\frac{n}{2}$, a recent conjecture of Ron Aharoni asserts that there exists a rainbow Hamiltonian cycle i.e. a cycle with edge set $\{e_1, ..., e_n\}$ such that $e_i \in E(G_i)$ for $1 \leq i \leq n$. This can be seen as a rainbow variant of the famous Dirac's theorem. In this paper, we prove this conjecture asymptotically. In fact, we show that for every $\varepsilon > 0$, there exists an integer $N > 0$, such that when $n > N$ for any graphs $G_1, ..., G_n$ on the same vertex set of size n, each graph having minimum degree at least $(\frac{1}{2} + \varepsilon)n$, there exists a rainbow Hamiltonian cycle.

Monday 15:25, Arts LR4

Intriguing sets in projective and polar geometries

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(This talk is based on joint work with Aart Blokhuis, Jozefien D'haeseleer, Morgan Rodgers, Leo Storme and Andrea Svob.)

MSC2000: 51A50, 51E20, 05B25, 05C50, 05E30

An intriguing set of a regular graph is a vertex set S such that for each vertex x of the graph $|\Gamma(x) \cap S|$ only depends on whether $x \in S$ or not. Intriguing sets have mainly been investigated for graphs arising from geometries. The Grassmann scheme of k-spaces the vector space $\mathbb{F}_q^n$ consists of $k+1$ graphs $\Gamma_0, \ldots, \Gamma_k$: two vertices corresponding to the k-spaces π and π' are adjacent in Γ_i if $\dim(\pi \cap \pi') = k - i$. The intriguing sets for the graphs Γ_i, $i = 1, \ldots, k$, generalise the concept of Cameron-Liebler line classes in $\mathrm{PG}(n, q)$ [2]. In the first part of the talk we will discuss several equivalent characterisations and the classification of intriguing sets of k-spaces in a vector space with a small parameter, based on the results in [1].

In the second part of the talk I will discuss intriguing sets of generators (maximals) of classical polar spaces, which are defined using the graphs where the intersection dimension of two generators determines the adjacency relation. These intriguing sets (generalising Cameron-Liebler sets and tight sets of generalised quadrangles) were introduced in [4] and studied further in [3]. The characterisation results vary depending on the type of the polar space. I will present these characterisation results and some classification results.

[1] A. Blokhuis, M. De Boeck, J. D'haeseleer. Cameron-Liebler sets of k-spaces in $\mathrm{PG}(n, q)$. Des. Codes Cryptogr., DOI: 10.1007/s10623-018-0583-1, 2018.

[2] P.J. Cameron and R.A. Liebler. Tactical decompositions and orbits of projective groups. Linear Algebra Appl. 46:91–102, 1982.

[3] M. De Boeck and J. D'haeseleer. Equivalent definitions for (degree one) Cameron-Liebler classes of generators in finite classical polar spaces. Submitted to Discrete Math., 2019.

[4] M. De Boeck, M. Rodgers, L. Storme and A. Švob. Cameron-Liebler sets of generators in finite classical polar spaces. Submitted to J. Combin. Theory Ser. A, 2018.

Choose your own adventure in parameterised graph algorithms

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MSC2000: 05C85

This talk will be on the general theme of parameterised graph algorithms, but I will ask the audience to choose which of four specific topics they would most like to hear about. These topics will be:

1. Reducing reachability in temporal networks;
2. Model-checking in multi-layer structures;
3. Approximately counting with an FPT decision algorithm;
4. Efficient parameterised algorithms for graph modularity.

Asymptotically good local list edge colourings

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(This talk is based on joint work with Marthe Bonamy, Michelle Delcourt, and Luke Postle.)

MSC2000: 05C15

We study list edge colourings under local conditions. Our main result is an analogue of Kahn's theorem in this setting. More precisely, we show that, for a graph G with sufficiently large maximum degree Δ and minimum degree $\delta \geq \ln^{25} \Delta$, the following holds. Suppose that lists of colours $L(e)$ are assigned to the edges of G, such that, for each edge $e = uv$,

$$|L(e)| \geq (1 + o(1)) \cdot \max \{\deg(u), \deg(v)\} .$$

Then there is an L-edge-colouring of G. We also provide extensions of this result for hypergraphs and correspondence colourings, a generalization of list colouring.

Heffter arrays with compatible and simple orderings

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(This talk is based on joint work with E.S. Yazici, D. Donovan and K. Burrage.)

MSC2000: O5B10

In the last 20 years biembedding pairs of designs and cycle systems onto surfaces has been a much-researched topic (see the 2007 survey "Designs and Topology" by Grannell and Griggs). In particular, in a posthumous work (2015), Archdeacon showed that biembeddings of cycle systems may be obtained via Heffter arrays. Formally, a Heffter array $H(m, n; s, t)$ is an $m \times n$ array of integers such that:

- each row contains s filled cells and each column contains t filled cells;
- the elements in every row and column sum to 0 in $\mathbb{Z}_{2ms+1}$; and
- for each integer $1 \leq x \leq ms$, either x or $-x$ appears in the array.

If we can order the entries of each row and column satisyfing two properties (compatible and simple), a Heffter array yields an embedding of two cycle decompositions of the complete graph K_{2ms+1} onto an orientable surface. Such an embedding is face 2-colourable, where the faces of one colour give a decomposition into s-cycles and the faces of the other colour give a decomposition into t-cycles. Thus as a corollary the two graph decompositions are *orthogonal*; that is, any two cycles share at most one edge. Moreover, the action of addition in $\mathbb{Z}_{2ms+1}$ gives an automorphism of the embedding. We give more detail about the above and present a new result: the existence of Heffter arrays $H(n, n; s, s)$ with compatible and simple orderings whenever $s \equiv 3 \pmod 4$ and $n \equiv 1 \pmod 4$.

Zero-free regions of graph polynomials and computational counting

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(This talk is based on joint work with Ewan Davies, Ferencs Bencs and Guus Regts.)

MSC2000: 05C15 05C85 82B20

Computational counting is the area of mathematics where one seeks to find polynomial-time algorithms to (approximately) count certain combinatorial objects such as independent sets, proper colourings, or matchings in a graph. More generally, each combinatorial counting problem has an associated generating function, namely the independence polynomial for independent sets, the chromatic and more generally Tutte polynomial for proper graph colourings, and the matching polynomial for matchings. Such graph polynomials are studied in mathematics and computer science, but also in statistical physics where they are normally referred to as partition functions. A fundamental question asks for which graphs and at which numerical values can one approximately evaluate these polynomials efficiently.

In this work we establish an intimate connection between the locations of the zeros of the graph polynomials and the locations at which these graph polynomials can be approximately evaluated by a polynomial-time algorithm. Our result is quite general and can be applied to a large class of functions that includes the independence, matching and Tutte polynomials and many more. It also allows complex evaluations, whereas previous methods have often been restricted to real or even integer evaluations.

Combining our method with results in the literature about the locations of zeros of certain graph polynomials immediately gives new regions of the complex plane where where they can be approximated in polynomial time. In addition we have found a new zero-free region for the partition function of the antiferromagnetic Potts model that allows us to approximately compute the number of proper k-colourings in graphs of maximum degree Δ for some improved values of $k = k(\Delta)$ compared to what was previously known.

A rainbow blow-up lemma for almost optimally bounded edge-colourings

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(This talk is based on joint work with Stefan Glock and Felix Joos.)

MSC2000: 05C35 (05C15, 05C78)

A subgraph of an edge-coloured graph is called rainbow if all its edges have different colours. We prove a rainbow version of the blow-up lemma of Komlós, Sárközy and Szemerédi that applies for almost optimally bounded colourings. A corollary of this is that there exists a rainbow copy of any bounded-degree spanning subgraph H in a quasirandom host graph G, assuming that the edge-colouring of G fulfills a boundedness condition that can be seen to be almost best possible.

This has many interesting applications beyond rainbow colourings, for example to graph decompositions. There are several well-known conjectures in graph theory concerning tree decompositions, such as Kotzig's conjecture and Ringel's conjecture. We adapt these conjectures for general bounded-degree subgraphs, and provide asymptotic solutions using our result on rainbow embeddings.

Open problems in finite projective spaces

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(This talk is based on joint work with J. A. Thas.)

MSC2000: 11G20, 51E21, 94B27

Apart from being an interesting and exciting area in combinatorics with beautiful results, finite projective spaces or Galois geometries have many applications to coding theory, algebraic geometry, design theory, graph theory, cryptology and group theory. As an example, the theory of linear maximum distance separable codes (MDS codes) is equivalent to the theory of arcs in $\mathrm{PG}(n, q)$.

Finite projective geometry is essential for finite algebraic geometry, and finite algebraic curves are used to construct interesting classes of codes, the Goppa codes, now also known as algebraic geometry codes. Many interesting designs and graphs are constructed from finite Hermitian varieties, finite quadrics, finite Grassmannians and finite normal rational curves. Further, most such structures have an interesting group; the classical groups and other finite simple groups appear in this way.

Unsolved problems in some of the following topics are considered:

(1) k-arcs;
(2) k-caps;
(3) Hermitian curves and unitals;
(4) maximal arcs;
(5) blocking sets;
(6) ovoids and spreads;
(7) algebraic curves over a finite field.

Subgraph Counting in Practice

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MSC: 05C85,05C60

Given a pair of graphs G and H, the subgraph counting problem asks how many copies of H are contained in G. The graphs G and H are referred to as the host graph and the pattern graph respectively. Subgraph counting is an effective means of comparing structural similarities between two or more real-world networks, or of a single network over time.

The problem of counting subgraphs is NP-complete in general. However, many real world networks have only a small number of vertices of high degree. Crucially, small subgraphs can be counted efficiently in networks with this structure. In the language of computational complexity, we say that subgraph counting is fixed parameter tractable (FPT) for host graphs with almost bounded degree, parameterised by the number of vertices in the pattern graph.

In this work, we implement an FPT algorithm for subgraph counting and compare its performance against that of a more general constraint programming approach on a variety of real-world instances. The purpose of these experiments is to determine how structured the host graph must be for the FPT algorithm to perform favourably.

Monday 16:15, Arts Main LT

Measurable version of Vizing's theorem

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(This talk is based on joint work with Jan Grebík.)

MSC2000: 05C85, 54H05, 68W15

We present a randomised local algorithm that properly colours most edges of a graph of maximum degree d using $d+1$ colours. This is applied to descriptive combinatorics to prove that every graphing of maximum degree d admits a measurable proper edge-colouring with $d+1$ colours, thus answering a question posed by Miklós Abért.

Monday 16:15, Arts LR1

Intercalates in double and triple arrays

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MSC2000: 05B30, 05B15, 05B10

A $r \times c$ double array on ω symbols is an array in which each symbol occurs κ times, $\kappa < r, c$, without any repeats in rows or columns, and the number of symbols common to two rows or two columns are (possibly different) constants. A triple array is a double array in which also the number of symbols common to a row and a column is a constant.

The main interest lies in triple arrays and their existence. Agrawal's Conjecture [1] says, in the canonical case, that there is a triple array if and only if there is a symmetric balanced incomplete block design with corresponding parameters. Besides a number of sporadic examples there are two infinite families of triple arrays known, one proved by Preece et al. [3] and the other by Nilson and Cameron [2] which also gives many families of proper double arrays.

Two double arrays of the same size and on the same set of symbols are said to be isomorphic to one another if one can be obtained from the other by a combination of the operations permuting rows, columns and symbols. Therefore, it can be useful to count the number of intercalates (embedded 2×2 Latin squares) as this number is invariant under such operations, when wanting to classify arrays or decide if two arrays are isomorphic.

In this talk we count intercalates in double and triple arrays, especially in arrays given in [2]. For example, we prove the existence of an infinite family of triple arrays in which every two occurrences of an entry lie in an intercalate.

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[2] T. Nilson and P. J. Cameron. Triple arrays from difference sets. *J. Combin. Des.*, 25, no. 11:494–506, 2017.

[3] D. A. Preece,W. D. Wallis and J. L. Yucas. Paley triple arrays. *Australas. J. Combin.*, 33:237–246, 2005.

The critical problem for binary matroids

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(This talk is based on joint work with Tatsuya Maruta.)

MSC2000: 05B35, 94B05, 51E21

The Critical Problem (Crapo and Rota, 1970) is the problem of finding the maximum dimension of a subspace which does not intersect a fixed subset in a vector space over a finite field, or in the original statement of the problem, of finding the least number of hyperplanes whose intersection contains no element of the subset (cf. [1]). This least number was introduced in the context of matroid theory where it has attracted attention as the critical exponent of a representable matroid over a finite field.

In this talk, we mainly focus on the problem for binary matroids. In particular, we will discuss the Walton-Welsh Conjecture (1980), an upper bound on the critical exponent of a loopless binary matroid having no minor isomorphic to a matroid of the complete graph K_5, from the perspective of blocking sets in binary projective spaces.

No knowledge of matroids will be assumed in this talk.

[1] J.P.S. Kung, Critical problems, in: Matroid Theory, Seattle, WA, 1995, *Contemporary Mathematics*, **197**, American Mathematical Society, Providence, RI, pp. 1–127, 1996.

[2] N. Bono, T. Maruta, K. Shiromoto and K. Yamada, On the non-trivial minimal blocking sets in binary projective spaces, *submitted*, 2019.

Rainbow Independent Sets in Proper Classes

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(This talk is based on joint work with Ron Aharoni, Jinha Kim, and Minki Kim.)

MSC2000: 05C69,05C35,05C55

Suppose $\mathcal{F} = (I_1, I_2, \ldots, I_N)$ is a collection of sets on the same ground set V, where sets may overlap and repeat. A set $S \subset V$ is said to be *rainbow* (with respect to $\mathcal{F}$) if it has an injective colouring $f : S \to \{1, \ldots, N\}$ respecting the sets, namely each $x \in I_{f(x)}$.

Drisko's Theorem states that if $M_1, \ldots, M_{2n-1}$ are matchings of size n in a bipartite graph then there exists a rainbow matching of size n. Equivalently, in any line graph of a bipartite graph, $2n-1$ independent sets of size n are enough to guarantee a rainbow independent set of size n.

In this talk I will discuss variants of this extremal problem for other classes of graphs - in particular, induced H-free graphs, and graphs of maximum degree $\leq \Delta$.

Monday 16:15, Arts LR4

Projective planes with polar spaces

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(This talk is based on joint work with Jeroen Schillewaert, Hendrik Van Maldeghem, Magali Victoor.)

MSC2000: 51B25

We will discuss the following question.

"What are the geometries consisting of points and polar spaces—such that each polar space is a convex subspace in the induced point-line geometry—that are similar to projective planes, i.e., each pair of points is contained in at least one such polar space, and each pair of such polar spaces intersects in a nonempty singular subspace of both?"

Apart from partial linear spaces, it turns out that there are essentially only five such geometries, including the exceptional Lie incidence geometry $\mathsf{E}_{6,1}(k)$ over any field k.

We continue with an interesting extension of this result, which has a surprising, strong link to the geometries of the Freudenthal-Tits magic square.

[1] A. De Schepper, J. Schillewaert, H. Van Maldeghem, M. Victoor, On exceptional Lie geometries, submitted, 43pp.

The Width of Minimum Cost Tree Decompositions

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(This talk is based on joint work with Kitty Meeks and Puck Rombach.)

MSC2000: 05C85, 68Q25

Tree decompositions have been very successfully employed in the design of parameterized graph algorithms. Typically an upper bound on the running time of such algorithms depends on the width of the decomposition provided, i.e., the size of its largest bag. For this reason much effort has been directed towards finding tree decompositions with minimum width. However, this is not the right way of constructing an 'algorithmically best' tree decomposition because the width of a tree decomposition which minimizes the running time of some algorithm is not always minimum. The intuition behind this phenomenon is that it is sometimes better to allow a few large bags in order to accommodate many small bags.
This talk will will address progress related to the question:

"is the width of an 'algorithmically best' tree decomposition bounded with respect to treewidth?"

Strong chromatic index of K_4-minor free graph

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(This talk is based on joint work with W.F. Wang, Q. Wang.)

MSC2000: 05C15

The strong chromatic index $\chi'_S(G)$ of a graph G is the smallest integer k such that G has a proper edge k-colouring with the condition that any two edges at distance at most 2 receive distinct colours. In this paper, we prove that if G is a K_4-minor free graph with maximum degree $\Delta \geq 3$, then $\chi'_S(G) \leq 3\Delta - 2$. The result is best possible in the sense that there exist K_4-minor free graphs G with maximum degree Δ such that $\chi'_S(G) = 3\Delta - 2$ for any given integer $\Delta \geq 3$. We shall also outline a polynomial algorithm based on the proof.

Ranking Connected Block Designs on A-optimality criterion using Concurrence and Levi graphs; a unified approach

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(This talk is based on joint work with Dr. Rosemary Bailey, University of St. Andrews.)

MSC2000: 05B05

In our quest towards the designs with minimum possible variance of the parameter estimates, we consider different optimality criterion. One such criterion is the A-optimality criterion, that is, ranking the designs in a given class of designs according to the minimum sum of variances of the estimates of the differences between two treatments or pairwise differences. In other words, A-optimality criterion maximises the harmonic mean of the non-trivial eigenvalues of the information matrix of the design in the given class. We have employed a unified approach towards the use of the concurrence and Levi graphs associated to designs.

Monday 16:40, Arts LR2

Excluded minors for classes of binary functions

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(This talk is based on joint work with G. Farr & K. Morgan.)

MSC2000: 05B35, 05B99, 15A03, 94B60

A *binary function* over a finite ground set E is a function $f : 2^E \to \mathbb{R}$, where $f(\emptyset) = 1$.

Given a matrix over $GF(2)$ with columns indexed by E, the indicator function of the rowspace is a binary function. Such a matrix represents a binary matroid, whose rank function can be expressed in terms of the associated binary function. This motivates our investigation of binary functions and their associated rank functions.

In [1], every binary function f is assigned a *rank function* $Qf : 2^E \to \mathbb{R}$ by the transform Q, given by

$$Qf(X) := \log_2 \left(\frac{\sum_{Y \subseteq E} f(Y)}{\sum_{Y \subseteq E \setminus X} f(Y)} \right).$$

Likewise, any rank function $r : 2^E \to \mathbb{R}$ (with $r(\emptyset) = 0$) has a corresponding binary function. The standard matroid operations of contraction and deletion are extended to binary functions in [1].

In this talk, we examine some minor-closed classes of binary functions, and determine their excluded minors. These include the classes of matroids and polymatroids, in the setting of binary functions. This approach yields a new proof of Tutte's excluded minor characterisation of binary matroids, by looking at excluded minors in the more general class of binary functions.

[1] G. E. Farr, A generalization of the Whitney rank generating function, *Math. Proc. Cambridge. Phil. Soc.* **113** (1993) 267–280.

Monochromatic Cycle Partitioning

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(This talk is based on joint work with A. Lo.)

MSC2000: 05C65, 05C70

Lehel conjectured that every red/blue edge-colouring of the complete graph admits a vertex partition into a red cycle and a blue cycle. This conjecture was proved by Bessy and Thomassé in 2010.

We consider a generalisation of Lehel's conjecture to hypergraphs. In particular we prove that every red/blue edge-colouring of the complete 4-uniform hypergraph contains a red and a blue tight cycle such that their union covers almost all vertices.

On non-singular Hermitian varieties of $PG(4, q^2)$

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(This talk is based on joint work with Francesco Pavese.)

MSC2000: 51E15, 51E21

A non-singular Hermitian variety of $PG(r, q^2)$, that is the set of absolute points of a Hermitian polarity of $PG(r, q^2)$, is projectively equivalent to the hypersurface of $PG(r, q^2)$ of degree $q + 1$ having equation

$$X_0^{q+1} + \cdots + X_r^{q+1} = 0;$$

see [1]. In [3, 4] it has been proved that if $\mathcal{X}$ is a hypersurface of degree $q+1$ in $PG(r, q^2)$, $r \geq 3$ odd, such that it has $(q^{r+1} + (-1)^r)(q^r - (-1)^r)/(q^2 - 1)$ rational points and does not contain linear subspaces of dimension greater than $\frac{r-1}{2}$, then $\mathcal{X}$ is a non-singular Hermitian variety of $PG(r, q^2)$. This result generalizes the characterization obtained in [2] for the Hermitian curve of $PG(2, q^2)$, $q \neq 2$.

Here, we deal with the 4-dimensional projective case. Our main result is achieved by combining geometric and combinatorial arguments with algebraic geometry.

Theorem 1. *Let H be a hypersurface of $PG(4, q^2)$, $q > 3$, defined over $GF(q^2)$, without $GF(q^2)$-hyperplane components and not containing planes. If the degree of H is $q + 1$ and the number of its rational points is $q^7 + q^5 + q^2 + 1$, then every plane of $PG(4, q^2)$ meets H in at least q^2+1 rational points. If there is at least a plane π such that $N_{q^2}(\pi \cap H) = q^2+1$, then H is a non-singular Hermitian variety of $PG(4, q^2)$.*

[1] J. W. P. Hirschfeld, *Projective geometries over finite fields*, Oxford Mathematical Monographs. The Clarendon Press, Oxford University Press, New York, 1998.

[2] J. W. P. Hirschfeld, L. Storme, J. A. Thas, J. F. Voloch, A characterization of Hermitian curves, *J. Geom.* **41** (1991), n. 1-2, 72–78.

[3] M. Homma, S. J. Kim, The characterization of Hermitian surfaces by the number of points, *J. Geom.* **107** (2016), 509–521.

[4] M. Homma, S. J. Kim, Number of points of a nonsingular hypersurface in an odd-dimensional projective space, *Finite Fields Appl.* **48** (2017), 395–419.

Monday 16:40, Arts LR5

Firebreaking: one-shot fire control on a graph

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(This talk is based on joint work with Kathleen D. Barnetson, Andrea C. Burgess, Jared Howell, David A. Pike, and Brady Ryan.)

MSC2000: 05C40,05C85

Suppose we have a network that is represented by a graph G and a fire (or other type of contagion) erupts at some vertex of G. We are able to respond to this outbreak by establishing a firebreak at k other vertices of G, so that the fire cannot pass through these fortified vertices. The question that now arises is which k vertices will result in the greatest number of vertices being saved from the fire, assuming that the fire will spread to every vertex that is not fully behind the k vertices of the firebreak. This is the essence of the Firebreak decision problem. I will mention several complexity results on this problem, stating that it is intractable on split graphs and bipartite graphs, and will outline how it can be solved in polynomial time when restricted to graphs having constant-bounded treewidth, permutation graphs, or the intersection graphs of paths in a tree with few leaves.

Separating tree-chromatic number from path-chromatic number

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(This talk is based on joint work with F. Barrera-Cruz, S. Felsner, P. Micek, H. Smith, L. Taylor, W.T. Trotter.)

MSC2000: 05C15, 05C75

We apply Ramsey theoretic tools to show that there is a family of graphs which have tree-chromatic number at most 2 while the path-chromatic number is unbounded. This resolves a problem posed by Seymour.

Group Divisible Designs with Block Size 4

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MSC2000: 05B05

A *group divisible design*, K-GDD, of type $g_1^{u_1} \dots g_r^{u_r}$ is an ordered triple $(V, \mathcal{G}, \mathcal{B})$ such that: (i) V is a base set of cardinality $u_1 g_1 + \cdots + u_r g_r$; (ii) $\mathcal{G}$ is a partition of V into u_i subsets of cardinality g_i, $i = 1, \dots, r$, called *groups*; (iii) $\mathcal{B}$ is a collection of subsets of V with cardinalities $k \in K$, called *blocks*; and (iv) each pair of elements from distinct groups occurs in precisely one block but no pair of elements from the same group occurs in any block. If k is a number, we abbreviate $\{k\}$-GDD to k-GDD.

Group divisible designs are useful and important structures that provide the main ingredients for establishing the existence of infinite classes of various combinatorial objects by way of a standard technique known as Wilson's Fundamental Construction. Although general existence problem for GDDs is not solved, considerable progress has been achieved when the block size is a small constant. In particular, the existence spectrum for k-GDDs has been completely determined in the following cases: (i) 2-GDDs (trivial); (ii) 3-GDDs of type g^u (Hanani, 1975); (iii) 3-GDDs of type $g^u m^1$ (Colbourn, Hoffman, Rees, 1992); (iv) 4-GDDs of type g^u (Brouwer, Schrijver, Hanani, 1977); (v) 4-GDDs of type $(3a)^4 b^1 (6a)^1$ (Rees, Stinson, 1989; Wang, Shen, 2008; F, 2019).

The next case one would naturally consider is that of 4-GDDs of type $g^u m^1$, but here only a partial solution has been achieved (Ge, Rees, 2002, 2004; Ge, Rees, Zhu, 2002; Ge, Ling, 2004, 2005; Schuster, 2010, 2014; Wei, Ge, 2013, 2014, 2015).

I shall be reporting on recent progress towards the determination of the existence spectrum of 4-GDDs of type $g^u m^1$ as well as type $g^d b^1 (gd/2)^1$ for $d = 5, 6, 7$.

I shall also present some joint work with Terry Griggs (Open University) concerning G-designs, where G is a 6-vertex graph with 8, 9 or 10 edges. A *G-design of order n* is an edgewise decomposition of the complete graph K_n into copies of G.

On Reliability Roots of Simplicial Complexes and Matroids.

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(This talk is based on joint work with J. I. Brown.)

MSC2000: 05B35, 05E99

Assume that the vertices of a graph G are always operational, but the edges of G fail independently with probability $q \in [0, 1]$. The all-terminal reliability of G is the probability that the resulting subgraph is connected. The all-terminal reliability is a polynomial in q, and it was conjectured that all the roots of (nonzero) reliability polynomials fall inside the closed unit disk. It has since been shown that there exist some connected graphs which have their reliability roots outside the closed unit disk, but these examples seem to be few and far between, and the roots are only barely outside the disk. In this talk we generalize the notion of reliability to simplicial complexes and matroids and investigate when, for small simplicial complexes and matroids, the roots fall inside the closed unit disk.

Partitioning 2-coloured complete 3-graphs into two monochromatic tight cycles

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(This talk is based on joint work with Frederik Garbe, Richard Lang, Allan Lo, Richard Mycroft.)

MSC2000: 05C38,05C65,05D10

As a variant on the traditional Ramsey-type questions, there has been a lot of research about the existence of spanning monochromatic subgraphs in complete edge-coloured graphs and hypergraphs. One of the central questions in this area was proposed by Lehel around 1979, who conjectured that the vertex set of every 2-edge-coloured complete graph can be partitioned into two monochromatic cycles of distinct colours. This was answered in the affirmative by Bessy and Thomassé in 2010.

We generalise the question of Lehel to the setting of 3-uniform hypergraphs (3-graphs). More precisely, we show that every sufficiently large 2-edge-coloured complete 3-graph admits a vertex partition into two monochromatic tight cycles, possibly of the same colour. We also present examples showing that (in contrast to the graph case) it is not always possible to find partitions into two monochromatic tight cycles of different colours.

From our proof, we also show that in the same setting (large 2-edge-coloured complete 3-graphs) we can always find a vertex partition into a tight cycle and a tight path, both of which are monochromatic and have different colours; or cover all but at most 2 vertices with two vertex-disjoint monochromatic tight cycles of different colours. This answers questions of Gyárfás and of Bustamante, Hàn and Stein.

The Number of Cubic Surfaces with 27 Lines Over a Finite Field

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Tekirdag Namık Kemal University

(This talk is based on joint work with Anton Betten.)

MSC2010: 05B25, 05E18, 14E05, 14J26, 51E25

In 1849, Cayley and Salmon showed that a smooth cubic surface has 27 lines. Later, Clebsch considered maps from the surface to the plane which are birational.

There is a fundamental relationship between the cubic surfaces with 27 lines and the sets of 6 points in a plane in general position, which we call nonconical arcs. In 1967, Hirschfeld determined the number of nonconical 6-arcs in $\mathrm{PG}(2, q)$.

In this talk, we give a formula to count the number of cubic surfaces with 27 lines in $\mathrm{PG}(3, q)$. This formula is only depend on q.

Monday 17:05, Arts LR5

Burn, baby, burn: Mathematical firefighting to reduce potential disease spread

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(This talk is based on joint work with J. Enright.)

MSC2000: 05C85, 92D30, 91A46

The Firefighter game offers a simple, discrete time model for the spread of a perfectly infectious disease and the effect of vaccination. A fire breaks out on a graph at time 0 on a set F of f vertices. At most d non-burning vertices are then defended and cannot burn in the future. Vertices once either burning or defended remain so for the rest of the game. At each subsequent time step, the fire spreads deterministically to all neighbouring undefended vertices and then at most d more vertices can be defended. The game ends when the fire can spread no further. Determining whether k vertices can be saved is NP-complete. I focus on finding maximal minimal damage (mmd) graphs - graphs which have the least burning if the fire starts in the worst place and the defenders defend optimally. I shall present some new and old results linking mmd graphs to optimal graphs for the Resistance Network Problem of finding graphs where all F-sets of vertices have limited neighbourhoods; a new framework for proving graphs are mmd and a new algorithm for optimal defense of a graph under certain conditions.

Tuesday 11:40, Arts Main LT

A degree sequence Komlós theorem

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(This talk is based on joint work with H. Liu and A. Treglown.)

MSC2000: 05C07, 05C70

Given graphs G and H, we define an *H-tiling in G* to be a collection of vertex-disjoint copies of H in G. Let $\varepsilon > 0$. We call an H-tiling *perfect* if it covers all of the vertices in G and *ε-almost perfect* if it covers all but at most an ε-proportion of the vertices in G. An important theorem of Komlós [1] provides the minimum degree of G which ensures an ε-almost perfect H-tiling in G. We present a degree sequence strengthening of this result. This is joint work with Hong Liu and Andrew Treglown.

Using the aforementioned theorem of Komlós [1], Kühn and Osthus [2] determined the minimum degree of G that ensures a perfect H-tiling in G. We present a degree sequence version of their result as an application of our degree sequence Komlós theorem. This is joint work with Andrew Treglown.

[1] J. Komlós, *Tiling Turán Theorems*, Combinatorica, **20**, (2000), 203-218.

[2] D. Kühn and D. Osthus, The minimum degree threshold for perfect graph packings, *Combinatorica* **29** (2009), 65-107.

Hall–Paige and synchronization

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(This talk is based on joint work with J. N. Bray, Q. Cai, P. Spiga and H. Zhang.)

MSC2000: 05E30, 20B15, 20M35

The Hall–Paige conjecture, made by Marshall Hall Jr. and Lowell J.Paige in 1955, states that the following four conditions on a finite group G are equivalent: G has a complete mapping; the Cayley table of G has a transversal; the Cayley table of G has an orthogonal mate; G has trivial or non-cyclic Sylow 2-subgroups. In particular, by Burnside's Transfer Theorem, the conjecture implies that non-abelian simple groups satisfy these conditions. The conjecture was proved by Stewart Wilcox, Anthony Evans, and John Bray in 2009; the last step of the proof has just been published.

From any Latin square one can construct a strongly regular Latin square graph. For groups of order greater than 2, the above conditions are equivalent to the statement that the Latin square graph of the Cayley table of the group G has clique number equal to chromatic number.

Diagonal groups are one of the classes of primitive group arising in the O'Nan–Scott Theorem. They are a little hard to describe in general, but the diagonal group whose socle is the product of three copies of a non-abelian simple group S is just the automorphism group of the Latin square graph of the Cayley table of S. So the Hall–Paige conjecture implies that this diagonal group is *non-synchronizing*, and indeed can be used to show that any diagonal group with more than two factors in its socle is non-synchronizing.

J. N. Bray, Q. Cai, P. J. Cameron, P. Spiga and H. Zhang, The Hall–Paige conjecture, and synchronization for affine and diagonal groups, *J. Algebra*, on-line ahead of print; doi: https://doi.org/10.1016/j.jalgebra.2019.02.025

Progress on the Ubiquity Conjecture

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(This talk is based on joint work with N. Bowler, C. Elbracht, J. Erde, J.P. Gollin, M. Pitz and M. Teegen.)

MSC2000: 05C83

A classic result of R. Halin about infinite graphs says the following: If a graph G contains n disjoint *rays*, i.e., one-way infinite paths, as subgraphs for every $n \in \mathbb{N}$, then G already contains infinitely many disjoint rays as subgraphs.

While the above statement trivially remains true if we ask for any finite graph H instead of a ray, it is not true for arbitrary infinite graphs instead of rays. So let us define the following: a graph G is called *ubiquitous w.r.t. the subgraph relation* if any infinite graph containing n disjoint copies of G as subgraphs for every $n \in \mathbb{N}$ also contains infinitely many disjoint copies of G as subgraphs. Similarly, we define being ubiquitous w.r.t. other relations between graphs, such as the minor or topological minor relation.

Probably one of the most fundamental conjectures about infinite graphs is the following one due to T. Andreae, called the *Ubiquity Conjecture*.

Ubiquity Conjecture. *Every locally finite connected graph is ubiquitous with respect to the minor relation.*

In a series of four papers [1, 2], partially still in preparation, we have made progress on the Ubiquity Conjecture. This includes sufficient conditions for a graph to be ubiquitous with respect to the minor relation, for example being countable and having bounded tree-width. Moreover, we proved that all trees – irrespective of their cardinality – are ubiquitous w.r.t. the topological minor relation.

In this talk, I will first give an overview about the Ubiquity Conjecture and explain how ends of graphs are related to it. Then I will discuss some of our results and the involved proof strategies.

[1] N. Bowler, C. Elbracht, J. Erde, J.P. Gollin, K. Heuer, M. Pitz and M. Teegen. Ubiquity in graphs I: Topological ubiquity of trees. arXiv:1806.04008, 2018.

[2] N. Bowler, C. Elbracht, J. Erde, J.P. Gollin, K. Heuer, M. Pitz and M. Teegen. Ubiquity in graphs II: Ubiquity of graphs with non-linear end structure. arXiv:1809.00602, 2018.

On the Ramsey number for trees versus wheels of small order

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(This talk is based on joint work with Yusuf Hafidh.)

MSC2000: 05C15, 05C55, 05D10

Let G and H be arbitrary graphs. The *Ramsey number* $R(G, H)$ is the smallest positive integer r such that for any graph F of order r, either F contains G or $\bar{G}$ contains H, where $\bar{G}$ is the complement of G. The problem of determining the Ramsey number $R(G, H)$ if G is a tree T_n on n vertices and H is a wheel W_m on $m + 1$ vertices has been extensively investigated. E.T. Baskoro, Surahmat, S.M. Nababan, and M. Miller (2002) showed that $R(T_n, W_4) = 2n + a$ with $a = +1$ if T_n is a star and n is even, and $a = -1$ otherwise; and $R(T_n, W_5) = 3n-2$, for any $n \geq 3$. Next, Y.J. Chen, Y.Q. Zhang, and K.M. Zhang (2004) derived the Ramsey number $R(T_n, W_6)$ in the case where the maximum degree of T_n is at least $n - 3$. The Ramsey number $R(T_n, W_6)$ and $R(T_n, W_7)$ for all trees T_n were then completely determined. For odd wheels, Y. Zhang, H. Broersma, Yaojun Chen (2016) proved that $R(T_n, W_m) = 3n - 2$ for odd $m \geq 3$ and $n \geq m - 2$, and T_n being a tree for which the Erdős-Sós Conjecture holds. However, the problem of finding $R(T_n, W_m)$ is far from completely solved in general. It has been conjectured that $R(T_n, W_m) = 2n-1$ if the maximum degree of T_n is small and m is even. For a tree T_n with large maximum degree, the $R(T_n, W_m)$ is also unknown in general if m is even. In this talk, we shall determine the Ramsey number $R(T_n, W_8)$ for all trees T_n of order n with maximum degree at least $n-3$.

Keywords: Ramsey number, tree, wheel.

A new approach to residues

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MSC2000: 05E40, 13F25

Residues are an important ingredient in generatingfunctionology or the method of coefficients. There are analytic, formal and cohomology approaches to residues. We would like to provide a new foundation for residues using only basic language of commutative algebra.

Recall that analytic residues are defined on meromorphic differentials by certain integration

$$\operatorname{res}\frac{g dz_1 \wedge \cdots \wedge dz_n}{f_1 \cdots f_n} = \left(\frac{1}{2\pi\sqrt{-1}}\right)^n \int_\Gamma \frac{g dz_1 \wedge \cdots \wedge dz_n}{f_1 \cdots f_n}.$$

There are constraints on analytic residues hidden in the domain of convergence and also in the region of integration. Although these analytic constraints are not essential in actual applications, they still need to be taken care of.

Formal residues, also called the "coefficient of" operator, defined on power series enjoy similar properties of analytic residues. In particular,

$$[z_1^{i_1} \cdots z_n^{i_n}]f = a_{i_1 \cdots i_n}$$

for power series $f = \sum a_{i_1 \cdots i_n} z_1^{i_1} \cdots z_n^{i_n}$ regardless of convergence. However the effect of changes of variables are not transparent, since differentials are overlooked in the formal side.

Cohomology residues defined on generalized fractions are a part of Grothendieck duality. They are convenient to use as formal residues, while keeping the rich meaning of analytic residues. Cohomology residues are characterized by certain algebraic laws together with the formula

$$\operatorname{res}\begin{bmatrix} dz_1 \wedge \cdots \wedge dz_n \\ z_1^{i_1}, \ldots, z_n^{i_n} \end{bmatrix} = \begin{cases} 1, & \text{if } i_1 = \cdots = i_n = 1; \\ 0, & \text{otherwise.} \end{cases}$$

It is plausible to construct a new framework using the formula and laws as defining rules. In the talk, we show that this is indeed the case by filling up the details without the machinery of homological algebra. The new approach consists of a pairing for differentials and systems of parameters. The pairing can be considered as an algebraic analogue of the integration of a differential form on a manifold. Lagrange inversion formulas are built into the framework.

The critical window in random digraphs

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MSC2000: 05C80, 05C20, 60C05

We consider the component structure of the random digraph $D(n,p)$ inside the critical window $p = n^{-1} + \lambda n^{-4/3}$. We show that the largest component $\mathcal{C}_1$ has size of order $n^{1/3}$ in this range. In particular we give explicit bounds on the tail probabilities of $|\mathcal{C}_1|n^{-1/3}$.

Cruse's theorem for partial symmetric $(\nu, \ldots, \nu_n)$-Latinized squares

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(This talk is based on joint work with Sibel Özkan.)

MSC2000: 05B15

In 1974 Cruse gave necessary and sufficient conditions for an $r \times r$ partial symmetric latin square R to be extended to a $p \times p$ symmetric latin square on the same symbols. In 2011 Bobga, Goldwasser, Hilton and Johnson showed that Cruse's theorem could be re-expressed in the form that a symmetric version of Hall's Condition would be necessary and sufficient for R to be extendable.

Here we extend both these variations of Cruse's theorem to $(\nu_1, \ldots, \nu_n)$-latinized squares. We have n symbols $\sigma_1, \ldots, \sigma_n$ where $n \geq p$ and n positive integers $\nu_1, \nu_2, \ldots, \nu_n$ with $1 \leq \nu_i \leq p$ $(1 \leq i \leq n)$ and $\sum_{i=1}^{n} \nu_i = p^2$. An $r \times r$ partial $(\nu_1, \ldots, \nu_n)$-latinized square is an $r \times r$ matrix with cells filled from $\{\sigma_1, \ldots, \sigma_n\}$ in such a way that each symbol σ_i occurs at most once in any row and at most once in any column, and at most ν_1 times altogether $(1 \leq i \leq n)$. If $r = p$ then we have a $(\nu_1, \ldots, \nu_n)$-latinized square, and if $n = p$ then we have a latin square of order n.

We first give a direct generalization of Cruse's theorem. We give necessary and sufficient conditions for an $r \times r$ partial symmetric $(\nu_1, \ldots, \nu_n)$-latinized square to be extendable to a $p \times p$-symmetric $(\nu_1, \ldots, \nu_n)$-latinized square.

We then give an alternative version in which we show that R can be extended if and only if it satisfies a suitable symmetric version of Hall's Condition.

There is an added interest in this in view of the fact that Ryser's theorem of 1951 for extending an $r \times s$ latin rectangle to an $n \times n$ latin square which does have a Hall analogue (proved by Hilton and Johnson in 1990) cannot be generalized satisfactorily to a similar theorem for $(\nu_1, \ldots, \nu_n)$-latinized rectangles, but its Hall analogue can be so extended.

New graphs of high girth and high chromatic number

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MSC2000: 05D40, 05C80, 05C68

Mycielski, in 1959, asked whether there exist graphs of arbitrarily large girth and large chromatic number. The intuition behind this question is that such graphs, if they exist, must be hard to construct since given any graph of large girth we can colour every large local part of it using just 2 colours.

Soon after, Erdős answered this question affirmatively. Instead of constructing such graphs, he used probabilistic arguments to show their existence. Since then, there have been several explicit constructions.

In the talk we will begin by recalling Erdős' probabilistic proof. We will then present a new probabilistic proof based on random Cayley graphs and comment on the differences between the two approaches.

An introduction to the Online Graph Atlas

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(This talk is based on joint work with Graham Farr, Kerri Morgan, and Paul Bonnington.)

MSC2000: 68R10, 05C99

In this talk we introduce a new research tool, an online interactive repository of graphs called the Online Graph Atlas (OLGA). The repository is designed to enable efficient retrieval of information about graphs and to enable queries based on combinations of standard graph parameters. Parameters include chromatic number, chromatic index, domination number, independence number, clique number, matching number, vertex cover number, size of automorphism group, vertex connectivity, edge connectivity, eigenvalues, treewidth, and genus.

Inspired by Read and Wilson's book, *An Atlas of Graphs (OUP,* 1998*)* [1], Barnes, Bonnington and Farr developed the first prototype of OLGA in 2009, which was extended by Sio, Farr and Bonnington in 2010 by adding more parameters. We changed its design to make it more flexible and extendable. We also introduced new parameters such as chromatic index, chromatic number, degeneracy, eigenvalues, size of automorphism group, treewidth and Tutte polynomial. OLGA now stores over 20 standard parameters for each graph of up to 10 vertices. We used recursive algorithms and exact algorithms for parameter computation. OLGA is not limited to the role of a search engine for graphs. We demonstrate how to use OLGA as a tool to explore conjectures and theorems involving the parameters.

[1] R. C. Read and R. Wilson, *An Atlas of Graphs*, OUP, 1998.

Ramsey theory on infinite graphs

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MSC2000: 05D10, 05C55, 05C15, 05C05, 03C15, 03E75

The Infinite Ramsey Theorem states that given $n, r \geq 1$ and a coloring of all n-sized subsets of $\mathbb{N}$ into r colors, there is an infinite subset of $\mathbb{N}$ in which all n-sized subsets have the same color. Extensions of Ramsey's Theorem to ultrahomogeneous structures have been studied for several decades, in particular infinite graphs. In this setting, one colors all copies of a finite graph within a given infinite graph, the goal being to find an infinite graph isomorphic to the original one in which as few colors as possible are used. Analogously to the Kechris-Pestov-Todorcevic correspondence between the Ramsey property and extreme amenability of universal minimal flows, Zucker recently found a correspondence between Ramsey properties for infinite structures and completion flows. This additionally motivates the search for infinite structures with good Ramsey properties.

We present a method of using trees with certain distinguished nodes to code vertices in a graph, developed in [1]. This builds on work of Sauer in his analysis of the Ramsey theory of the Rado graph, but has added the benefit of being able to capture the essence of forbidden configurations. We will provide an overview of how these trees are utilized to procure Ramsey theory of the Henson graphs, the universal homogeneous k-clique-free graphs in [1] and [3], answering questions of Kechris-Pestov-Todorcevic and of Sauer. These methods are currently being applied to find Ramsey theory for new ultrahomogeneous structures. We will conclude with some current and future directions for the developing field of Ramsey theory on infinite structures, including a result on infinite dimensional Ramsey theory on copies of the Rado graph in [2].

[1] Natasha Dobrinen, *The Ramsey theory of the universal homogeneous triangle-free graph*, (2017), 65 pp, Submitted. arXiv:1704.00220v5.

[2] ———, *Borel sets of Rado graphs and Ramsey's theorem*, (2019), 25 pp, Submitted. arXiv:1904.00266v1.

[3] ———, *Ramsey theory of the universal homogeneous k-clique-free graph*, (2019), 68 pp, Preprint. arXiv:1901.06660.

Realization of digraphs in Abelian groups

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(This talk is based on joint work with Zs. Tuza.)

MSC2000: 05C20, 05C25, 05C78

Suppose that there exists a mapping ψ from the arc set $E(\overrightarrow{G})$ of $\overrightarrow{G}$ to a finite Abelian group Γ such that if we define a mapping φ from the vertex set $V(\overrightarrow{G})$ of G to Γ by

$$\varphi_\psi(x) = \sum_{y \in N^+(x)} \psi(yx) - \sum_{y \in N^-(x)} \psi(xy), \quad (x \in V(G)),$$

then φ_ψ is injective. In this situation, we say that $\overrightarrow{G}$ is *realizable* in Γ.

Let $\overrightarrow{G}$ be a directed graph of order n with no component of order less than 3. So far the problem of realization of digraphs was considered only in case of elementary Abelian groups [1, 2]. In this talk we will show that $\overrightarrow{G}$ is realizable in any finite Abelian group Γ such that $|\Gamma| \geq 4n$. Moreover if n is sufficiently large for fixed $\varepsilon > 0$ ($n \geq n_0(\varepsilon)$) then $\overrightarrow{G}$ is realizable in any Γ such that $|\Gamma| > (1+\varepsilon)n$.

[1] Y. Egawa, *Graph labelings in elementary abelian 2-groups*, Tokyo Journal of Mathematics 20 (1997) 365–379.

[2] Y. Fukuchi, *Graph labelings in elementary abelian groups*, Discrete Mathematics 189 (1998) 117–122.

Wednesday 10:30, Arts LR5

Characterising inflations of monotone grid classes of permutations

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(This talk is based on joint work with Michael Albert and Robert Brignall.)

MSC2000: 05A05, 06A07

We characterise those permutation classes whose simple permutations are monotone griddable. This characterisation is obtained by identifying a set of nine substructures, at least one of which must occur in any simple permutation containing a long sum of 21s.

Covers and partial transversals of Latin squares

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(This talk is based on joint work with Darcy Best, Rebecca J. Stones, and Ian M. Wanless.)

MSC2000: 05B15

The topic of transversals within Latin squares became of interest through the study of mutually orthogonal Latin squares (MOLS). The connection to such useful objects is strong, as a pair of Latin squares are a pair of MOLS if and only if each of the Latin squares decomposes into transversals. Along with this connection to MOLS, transversals have been of interest in the literature recently and have had a number of papers study them in their own right. It is well known that some Latin squares contain no transversals, but what is not so clear is how *close* a Latin square is to containing a transversal. Previously, this problem has been studied by studying partial transversals of Latin squares and observing how large we can make such a partial transversal. In this presentation, we will discuss an alternate substructure of a Latin square that can also be considered to be *close* to a transversal, which we call a cover.

A *cover* of a Latin square is a subset of entries of the Latin square such that each row, column, and symbol is represented at least once in the set of entries. We will present the results we have found regarding covers of minimum size within a Latin square, which shows a kind of duality between covers and partial transversals. We also will present work on minimal covers, which shows a clear distinctiveness between covers and partial transversals. After this, we will demonstrate a few other results of interest on this topic.

Fractional Coloring with Local Demands

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(This talk is based on joint work with Luke Postle.)

MSC2000: 05C15, 05C69, 05C70

In fractional coloring, we assign vertices of a graph subsets of the [0,1]-interval and adjacent vertices receive disjoint subsets. We investigate fractional colorings where vertices "demand" varying amounts of "color," determined by local parameters such as the degree of a vertex. By Linear Programming Duality, all of the problems we study have an equivalent formulation as a problem concerning weighted independence numbers. Many well-known results concerning the fractional chromatic number and independence number have natural generalizations in this new paradigm. We discuss several such results as well as open problems. In particular, we prove that if G is a graph and $f(v) \leq 1/(d(v)+1/2)$ for each $v \in V(G)$, then either G contains a clique K such that $\sum_{v \in K} f(v) > 1$ or G has a fractional coloring in which each vertex v receives a subset of measure at least $f(v)$. This result is the "local demands" version of Brooks' Theorem; it considerably generalizes the Caro-Wei Theorem and also implies new bounds on the independence number.

Some problems suggested by the Online Graph Atlas project

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(This talk is based on joint work with Srinibas Swain, Paul Bonnington and Kerri Morgan.)

MSC2000: 05C99, 68R10, 68Q25

The Online Graph Atlas (OLGA) is an electronic repository of all graphs (up to isomorphism) up to some order (currently 10), together with values of several important parameters for each graph. It has been built at Monash University and was originally inspired by the printed repository of Read and Wilson [1]. In a separate talk, Srinibas Swain will describe and demonstrate the current version of the system. In this talk, we present some graph-theoretic questions raised by this project.

One family of questions we consider is the following. Let f be a nonnegative integer-valued graph parameter, invariant under isomorphism. We seek to calculate f recursively, using its values on subgraphs obtained by deleting single vertices or single edges. Sometimes — e.g., if f is circumference — this can be done exactly, and this is useful as it helps us calculate the values of f efficiently for all graphs in the repository. At other times, we do not know of an exact recurrence, but it is often the case that we can still find simple recursive upper and lower bounds. For example, in some situations — such as if f is tree-width or degeneracy — we may be able to show that, for all $v \in V(G)$,

$$\max\{f(G-v) \mid v \in V(G)\} \leq f(G) \leq 1 + \min\{f(G-v) \mid v \in V(G)\}.$$

For such an f, we are especially interested in how often these bounds coincide, because in such cases we know the value of f exactly. When the bounds do not coincide, we must resort to some other method to compute f, and this may be costly.

Let the proportion of n-vertex graphs for which the bounds coincide be $\mu_f(n)$. We ask:

1. Do there exist constants $L_f > 0$ and $U_f < 1$ such that $L_f \leq \mu_f(n) \leq U_f$ for all sufficiently large n?
2. Does $\lim_{n\to\infty} \mu_f(n)$ exist?

These questions may be asked for any f. We discuss some specific parameters and report some computational results for them.

[1] R. C. Read and R. Wilson, *An Atlas of Graphs*, OUP, 1998.

Ramsey upper density of infinite graphs

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MSC2000: 05D10, 05C63

Let H be an infinite graph. In a two-coloring of the edges of the complete graph on the natural numbers, what is the densest monochromatic subgraph isomorphic to H that we are guaranteed to find? We measure the density of a subgraph by the upper density of its vertex set. This question, in the particular case of the infinite path, was introduced by Erdős and Galvin. Following a recent result for the infinite path, we present bounds on the maximum density for other choices of H, including exact values for a wide class of bipartite graphs.

Connectivity of non-commuting graphs for finite rings

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(This talk is based on joint work with W. Maneerut and B.Khuhirun.)

MSC2000: 05C25

Let R be a non-commutative ring. The non-commuting graph of R, denoted by Γ_R, is a simple graph with a vertex set of elements in R except for its center. Two distinct vertices x and y are adjacent if $xy \neq yx$. In this paper, we study the vertex-connectivity and edge-connectivity of a non-commuting graph associated with a finite non-commutative ring R, denoted by $\kappa(\Gamma_R)$ and $\lambda(\Gamma_R)$, respectively. We prove a lower bound for $\kappa(\Gamma_R)$ and $\lambda(\Gamma_R)$. We show that the edge-connectivity of Γ_R is equal to $\delta(\Gamma_R)$, the minimum degree of Γ_R. In particular, we consider the relation between $\kappa(\Gamma_R)$, $\lambda(\Gamma_R)$ and $\delta(\Gamma_R)$. Finally, for a ring of order p^n, we determine $\kappa(\Gamma_R)$ and $\lambda(\Gamma_R)$ where p is a prime number, and $n \in \{2, 3, 4, 5\}$.

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[2] Chartrand G, Lesniak L, Zhang P. Graphs and Digraphs. 6th eds. New York: Chapman and Hall; 2016.

[3] Dutta J, Basnet DK. On non-commuting graph of a finite ring. Preprint.

[4] Eldridge KE. Orders for finite noncommutative rings with unity. The American Mathematical Monthly. 1968; 75(5): 512−514.

[5] Erfanian A, Khashyarmanesh K, Nafar Kh. Non-commuting graphs of rings. Discrete Mathematics, Algorithms and Applications. 2015; 7(3): 1550027-1−1550027-7.

[6] Saluke JN. On commutativity of finite rings. Bulletin of the Marathwada Mathematical Society. 2012; 13(1): 39−47.

[7] Vatandoost E, Ramezani F. On the commuting graph of some non-commutative rings with unity. Journal of Linear and Topological Algebra. 2016; 05(04): 289−294.

The minimum Manhattan distance of a permutation

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(This talk is based on joint work with Cheyne Homberger and Peter Winkler.)

MSC2000: 05A05

Let π be a permutation of $\{1, 2, \ldots, n\}$. If we identify a permutation with its graph, namely the set of n dots at positions $(i, \pi(i))$, it is natural to consider the minimum L^1 (Manhattan) distance, $d(\pi)$, between any pair of dots.

A conjecture due to Bevan, Homberger and Tenner [1] (motivated by permutation patterns) states that when d is fixed and $n \to \infty$, the probability that $d(\pi) \geq d + 2$ tends to e^{-d^2-d}. In this talk, I will discuss our proof of this conjecture, and how the expected value (and higher moments) of $d(\pi)$ can be computed when π is chosen uniformly at random and $n \to \infty$.

[1] David Bevan, Cheyne Homberger and Bridget Eileen Tenner, Prolific permutations and permuted packings: downsets containing many large patterns. *J. Combinatorial Theory, Series A* **153** (2018), 98–121.

[2] Simon R. Blackburn, Cheyne Homberger and Peter Winkler, 'The minimum Manhattan distance and minimum jump of permutations', *J. Combinatorial Theory, Series A* **161** (2019), 364–386.

Limits of Sequences of Latin Squares

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(This talk is based on joint work with R. Hancock, J. Hladký, and M. Sharifzadeh.)

MSC2000: 05C99, 05B15, 60C05

We introduce a limit theory for sequences of Latin squares paralleling the ones for dense graphs and permutations. The limit objects are certain distribution valued two variable functions, which we call *Latinons*, and left-convergence is defined via densities of $k \times k$ subpatterns of Latin Squares. The main result is a compactness theorem stating that every sequence of Latin squares of growing orders has a Latinon as an accumulation point. Furthermore, our space of Latinons is minimal, as we show that every Latinon can be approximated by Latin squares. This relies on a result of Keevash about combinatorial designs. We also introduce an analogue of the cut-distance and prove counterparts to the counting lemma, sampling lemma and inverse counting lemma.

The Glauber dynamics for edges colorings of trees

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(This talk is based on joint work with Marc Heinrich and Guillem Perarnau.)

MSC2000: 60J10, 68W20, and 05C15

We initiate the study of Glauber dynamics for edge colorings of graphs by bounding the mixing time for the dynamics on edge-colorings of trees on n vertices with maximum degree Δ. We show that for $k \geq \Delta + 1$ the Glauber dynamics for k-edge-colorings of T mixes in polynomial time; (this is best possible as the chain is not even ergodic for $k \leq \Delta$). Our proof uses a recursive decomposition of the tree into subtrees; we bound the relaxation time of the original tree in terms of the relaxation time of its subtrees using block dynamics and chain comparison techniques.

Spectral bounds for graph parameters – Part I

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(This talk is based on joint work with Pawel Wocjan.)

MSC2000: 05C15, 05C50

This talk is the first of a pair of consecutive talks by Clive Elphick and Pawel Wocjan on their joint work on spectral bounds for graph parameters, that lie between the clique number, $\omega(G)$, and the chromatic number, $\chi(G)$. This first talk will provide a non-technical overview, without discussion of proofs. The second talk will provide examples of our proof techniques, for example for bounds for the quantum chromatic number.

Spectral bounds typically use the eigenvalues of matrix representations of graphs to bound graph parameters. For example a well known bound due to Hoffman states that:

$$1 + \frac{\mu_1}{|\mu_n|} \leq \chi(G),$$

where μ_1 and μ_n are the largest and smallest eigenvalues of the adjacency matrix. Spectral bounds can also use the *inertia* of a graph, which is the numbers of positive (n^+), zero (n^0) and negative (n^-) eigenvalues of the adjacency matrix.

In this pair of papers we investigate to what extent known spectral lower bounds for the chromatic number are in fact lower bounds for parameters that lie between $\omega(G)$ and $\chi(G)$. For example we prove that all spectral lower bounds for $\chi(G)$ are lower bounds for the quantum chromatic number, $\chi_q(G)$, where for some graphs $\chi_q(G) \ll \chi(G)$.

We also investigate upper bounds for the independence number, $\alpha(G)$. A well known upper bound for the independence number is that:

$$\alpha(G) \leq n^0 + \min{(n^+, n^-)}.$$

We prove that:

$$\alpha(G) \leq \alpha_q(G) \leq n^0 + \min{(n^+, n^-)},$$

where $\alpha_q(G)$ is the quantum independence number. Many graphs have $\alpha = n^0 + \min{(n^+, n^-)}$, so for these graphs $\alpha_q = \alpha$. The talk will conclude with open questions.

Ramsey numbers of edge-ordered graphs

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(This talk is based on joint work with Martin Balko.)

MSC2000: 05D10

We introduce and study a variant of Ramsey numbers for *edge-ordered graphs*, that is, graphs with linearly ordered sets of edges. The *edge-ordered Ramsey number* $\overline{R}_e(\mathfrak{G})$ of an edge-ordered graph $\mathfrak{G}$ is the minimum positive integer N such that there exists an edge-ordered complete graph $\mathfrak{K}_N$ on N vertices such that every 2-coloring of the edges of $\mathfrak{K}_N$ contains $\mathfrak{G}$ as an edge-ordered subgraph.

The edge-ordered Ramsey number $\overline{R}_e(\mathfrak{G})$ is finite for every edge-ordered graph $\mathfrak{G}$ with a primitive recursive upper bound. We have better estimates for special classes of edge-ordered graphs. In particular, we prove $\overline{R}_e(\mathfrak{G}) \leq 2^{O(n^3 \log n)}$ for every bipartite edge-ordered graph $\mathfrak{G}$ on n vertices. We also introduce a natural class of edge-orderings, called *lexicographic edge-orderings*, for which we can prove much better upper bounds on the corresponding edge-ordered Ramsey numbers.

Cyclic cycle systems of the complete multipartite graph

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(This talk is based on joint work with Francesca Merola and Tommaso Traetta.)

MSC2000: 05B30

In this talk we consider decompositions of the complete multipartite graph $K_m[n]$ with m parts of size n into cycles. While necessary and sufficient conditions for the existence of a decomposition of $K_m[n]$ into cycles of given lengths are known in the case $n \in \{1, 2\}$, in general even the question of ℓ-cycle decomposition of $K_m[n]$ remains open.

We will consider cycle decompositions with additional algebraic structure. Note that we may view $K_m[n]$ as a Cayley graph $\mathrm{Cay}(G; G \setminus N)$, where the vertex set G is a group of order mn and N is a subgroup of order n. Given a cycle $C = (c_0, c_1, \ldots, c_{\ell-1})$ in $K_m[n]$ and an element $g \in G$, we define the cycle $C + g = (c_0 + g, c_1 + g, \ldots, c_{\ell-1} + g)$. A cycle decomposition $\mathcal{D}$ of $K_m[n] = \mathrm{Cay}(G; G \setminus N)$ is *G-regular* if $C + g \in \mathcal{D}$ for every $C \in \mathcal{D}$ and $g \in G$. In the case $G = \mathbb{Z}_{mn}$ and $N = m\mathbb{Z}_{mn}$, a $\mathbb{Z}_{mn}$-regular cycle system of $K_m[n]$ is called *cyclic*.

We give necessary and sufficient conditions for the existence of a cyclic ℓ-cycle decomposition of $K_m[n]$ when $2\ell \mid (m-1)n$; this is a natural case as it allows us to construct cyclic decompositions using difference families. Moreover, we give additional necessary conditions for the existence of a G-regular cycle decomposition of $K_m[n]$ for an arbitrary group G of order mn.

Correlation for Permutations

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(This talk is based on joint work with Imre Leader and Eoin Long.)

MSC2000: 05D99, 60C05

Let $X = \{1, 2, \ldots, n\}$. A family $\mathcal{F}$ of subsets of X is an *up-set* if every superset of a member of $\mathcal{F}$ is also a member of $\mathcal{F}$. That is, for every $F \in \mathcal{F}$ and $i \in X$ we have $F \cup \{i\} \in \mathcal{F}$. The well-known (and very useful) Harris-Kleitman inequality says that any two up-sets are positively correlated. In other words, if $\mathcal{A}, \mathcal{B} \subseteq \mathcal{P}(X)$ are both up-sets then

$$\frac{|\mathcal{A} \cap \mathcal{B}|}{2^n} \geq \frac{|\mathcal{A}|}{2^n} \times \frac{|\mathcal{B}|}{2^n}.$$

Our aim in this talk is to explore analogues of the Harris-Kleitman inequality for families of *permutations* of X. It turns out that there are two natural notions of what it means for a family of permutations to be an up-set (corresponding to the strong and weak Bruhat orders) and surprisingly the correlation that occurs in the two cases is quite different.

We show that, in the strong Bruhat order on S_n, up-sets are positively correlated. Thus, for example, for a (uniformly) random permutation π, the event that no point is displaced by more than a fixed distance d and the event that π is the product of at most k adjacent transpositions are positively correlated.

In contrast, for the weak Bruhat order we show that this completely fails: perhaps surprisingly, there are two up-sets each of measure $1/2$ whose intersection is arbitrarily small.

We also prove an analogous correlation result for a family of non-uniform measures which includes the Mallows measures and discuss some applications and open problems.

On counting problems related to orthogonal Latin squares

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(This talk is based on joint work with S. Das and T. Szabó.)

MSC2000: 05B15

A *Latin square of order* n is an $n \times n$ array with entries in $[n]$ such that each integer appears exactly once in every row and every column. Two Latin squares L and L' are said to be *orthogonal* if, for all $x, y \in [n]$, there is a unique pair (i, j) such that $L(i, j) = x$ and $L'(i, j) = y$; a set of Latin squares is *mutually orthogonal* if any two of them are orthogonal. The motivation to study orthogonal Latin squares comes both from their connections to other combinatorial structures and from their importance in practical applications.

After the question of existence, a natural and important problem is to determine how many structures satisfying given properties there are. In this talk, we will present an upper bound on the number of ways to extend a system of k mutually orthogonal Latin squares to a system of $k + 1$ mutually orthogonal Latin squares and discuss some applications, comparing the resulting bounds to previously known lower and upper bounds.

Structures of edge colored complete bipartite graphs without proper colored cycles of specified length

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MSC2000: 05C15, 05C20, 05C38

Let G be a graph. A mapping $c : E(G) \to \mathbb{N}$ is called an *edge-coloring* of G and $c(e)$ is called the *color* of an edge e. A graph with an edge-coloring map is called an edge-colored graph and denoted by (G, c). A subgraph H of G is called *rainbow* if every pair of edges in H have distinct colors and H is said to be *properly colored*, or *PC*, if any two adjacent edges have different colors.

In this talk, we will first consider the structures of edge-colored complete bipartite graphs $(K_{n,m}, c)$ without PC cycles of length four, and next the number of disjoint PC cycles in $(K_{n,m}, c)$ will be discussed related with the Bermond-Thomassen Conjecture. Finally several problems and results around this topic will be given.

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[3] R. Cada, A. Kaneko, Z. Ryjacek and K. Yoshimoto, Rainbow cycles in edge-colored graphs, Discrete Mathematics **339** (2016) 1387-1392

[4] R. Cada, K. Ozeki and K. Yoshimoto, A complete bipartite graph without properly colored cycles of length four, *submitted.*

[5] S. Fujita, R. Li and S. Zhang, Color degree and monochromatic degree conditions for short properly colored cycles in edge colored graphs, J. Graph Theory **87** (2018) 362–373.

[6] T. Gallai, Transitiv orientierbare graphen, Acta Math. Acad. Sci. Hungar, **18** (1967) 25–66.

Wednesday 11:45, Arts LR2

SPECTRAL BOUNDS FOR GRAPH PARAMETERS – PART II

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(This talk is based on joint work with Clive Elphick.)

MSC2000: 05C15, 05C50

This talk is the second of a pair of consecutive talks by Clive Elphick and Pawel Wocjan on their joint work on spectral bounds for graph parameters, that lie between the clique number $\omega(G)$, and the chromatic number, $\chi(G)$. The second talk will provide more technical details.

Let $[c] = \{0, \dots, c-1\}$ for some positive integer c and let I_d and 0_d denote the identity and zero matrices acting on $\mathbb{C}^d$. A quantum c-coloring of an undirected graph $G = (V, E)$ without loops and without multiple edges is a collection of orthogonal projectors

$$\{P_{v,k} : v \in V, k \in [c]\}$$

acting on $\mathbb{C}^d$ for some $d > 0$ such that

- for all vertices $v \in V$

$$\sum_{k\in[c]} P_{v,k} = I_d \qquad \text{completeness}$$

- for all edges $vw \in E$ and for all $k \in [c]$

$$P_{v,k}P_{w,k} = 0_d \qquad \text{orthogonality}$$

The quantum chromatic number, χ_q, is the smallest c for which the graph G admits a quantum c-coloring. The classical chromatic number is a special case when $d = 1$.

We will establish that the existence of a quantum c-coloring implies the existence of a unitary matrix U acting on $\mathbb{C}^n \otimes \mathbb{C}^d$ such that

$$\sum_{\ell=1}^{c-1} U^\ell (A \otimes I_d)(U^*)^\ell = -A \otimes I_d,$$

where A is the adjacency matrix of G and n its number of vertices.

We will show that applying the majorization result for maximal eigenvalues of Hermitian matrices X and Y, $\mu_{\max}(X)+\mu_{\max}(Y) \geq \mu_{\max}(X+Y)$, to the above matrix equality yields the Hoffman bound for the quantum chromatic number: $\chi_q(G) \geq 1 + \mu_{\max(A)}/|\mu_{\min}(A)|$.

Similarly, we will show that applying the rank inequality for positive semidefinite matrices X and Y with $X - Y$ positive semidefinite, $\text{rank}(X) \geq \text{rank}(Y)$, to the above matrix inequality yields the inertial lower bound on the quantum chromatic number: $\chi_q(G) \geq 1 + \max\{n^+(A)/n^-(A), n^-(A)/n^+(A)\}$, where $n^\pm(A)$ denote the number of positive and negative eigenvalues of A.

The connected size Ramsey number for matchings versus small disconnected graphs

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(This talk is based on joint work with B. Rahadjeng, E.T. Baskoro.)

MSC2000: 05D10, 05C55

Let F, G, and H be simple graphs. The notation $F \to (G, H)$ means that if all the edges of F are arbitrarily colored red or blue, then there always exists either a red subgraph G or a blue subgraph H. The size Ramsey number of graph G and H, denoted by $\hat{r}(G, H)$ is the smallest integer k such that there is a graph F with k edges satisfying $F \to (G, H)$. In this research, we study a modified size Ramsey number, namely the connected size Ramsey number. In this case, we only consider connected graphs F satisfying the above properties. This connected size Ramsey number of G and H is denoted by $\hat{r}_c(G, H)$. In this talk we will discuss an upper bound on $\hat{r}_c(nK_2, H), n \geq 2$ where H is $2P_m$ or $2K_{1,t}$, and the exact values of $\hat{r}_c(nK_2, H)$, for some fixed n.

The Edge Metric Dimension of Cayley Graphs $\Gamma(\mathbb{Z}_n \oplus \mathbb{Z}_2)$ and its Barycentric Subdivisions

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(This talk is based on joint work with Nida Siddiqui.)

MSC2000: 05C12; 05C76; 05C90

The main objective of this study is to determine the edge metric dimension (EMD) of the Cayley graphs $\Gamma(\mathbb{Z}_n \oplus \mathbb{Z}_2)$ and its barycentric subdivision. It is proved that the Cayley graphs and its subdivisions have constant EMD and its edge metric generator (EMG) set contains only three vertices to resolve all the edges of Cayley graphs $\Gamma(\mathbb{Z}_n \oplus \mathbb{Z}_2)$ and its barycentric subdivision also. In particular EMD remains invariant under the barycentric subdivision of $\Gamma(\mathbb{Z}_n \oplus \mathbb{Z}_2)$. On the contrary, in [1] it was proved that the metric dimension of the Cayley graphs $\Gamma(\mathbb{Z}_n \oplus \mathbb{Z}_2)$ does not remain invariant under its barycentric subdivision.

[1] Imran, M., On the Metric Dimension of Barycentric Subdivision of Cayley Graphs, Acta Math. Appl. Sin., 32(4), 1067-1072, 2016.

Independent set permutations and matching permutations

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(This talk is based on joint work with T. Ball, K. Hyry and K. Weingartner.)

MSC2000: 05C69, 05A05

In 1987 Alavi, Malde, Schwenk and Erdős [1] showed that the independent set sequence of a graph is unconstrained in terms of its pattern of rises and falls, in the following sense: for any $m \in \mathbb{N}$ and any permutation π of $\{1, \dots, m\}$ there is a graph with largest independent set having size m, and with

$$i_{\pi(1)} \leq i_{\pi(2)} \leq \cdots \leq i_{\pi(m)},$$

where i_k is the number of independent sets of size k in the graph. Their construction yielded a graph with around m^{2m} vertices, and they raised the following question:

> Determine the smallest order large enough to realize every permutation of order m as the sorted indices of the vertex independent set sequence of some graph.

We answer this question exactly.

Alavi et al. also observed that the *matching* sequence of a graph is, by contrast, quite constrained — at most 2^{m-1} permutations of $\{1, \dots, m\}$ can be realized as the sorted indices of the matching sequence of some graph. They asked whether the upper bound of 2^{m-1} was optimal; we show that it is not.

Many open problems remain in this area.

[1] Y. Alavi, P. Malde, A. Schwenk, P. Erdős, The vertex independence sequence of a graph is not constrained, *Congr. Numer.* **58** (1987), 15–23.

Cycle lengths in graphs of given minimum degree and chromatic number

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(This talk is based on joint work with J. Gao, Q. Huo and C. Liu.)

MSC2000: 05C38

We prove a tight minimum degree condition in general graphs for the existence of paths between two given endpoints, whose lengths form a long arithmetic progression with common difference one or two. Using this as a primary tool, we obtain several exact and optimal results on cycle lengths in graphs of given minimum degree, connectivity or chromatic number.

Weakly self-orthogonal designs and related codes

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(This talk is based on joint work with I. Novak.)

MSC2000: 05B30, 94B05

A 1-design is weakly self-orthogonal if all the block intersection numbers have the same parity. If both the parameter k and the block intersection numbers are even then a 1-design is called self-orthogonal and its incidence matrix generates a self-orthogonal code. We analyze extensions, of the incidence matrix and an orbit matrix of a weakly self-orthogonal 1-design, that generate a self-orthogonal code.

Additionally, we study methods for constructing LCD codes by extending the incidence matrix and an orbit matrix of a weakly self-orthogonal 1-design.

Ramsey Numbers of Brauer Configurations

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(This talk is based on joint work with S. Prendiville.)

MSC2000: 11B30

A classical theorem of van der Waerden states that, for any positive integers $k, r \geqslant 2$, there exists a positive integer $\mathrm{W}(r, k)$ such that for any partition of $\{1, 2, ..., \mathrm{W}(r, k)\}$ into r parts, one part of the partition contains an arithmetic progression of length k. Obtaining quantitative bounds for the *van der Waerden* numbers $\mathrm{W}(r, k)$ is a notoriously difficult problem. In 2001, Gowers obtained the current best upper bound

$$\mathrm{W}(r, k) \leqslant 2^{2^{r^{2^{2^{k+9}}}}}.$$

In this talk we consider a generalisation of van der Waerden's theorem due to Brauer. Brauer established the existence of a quantity $\mathrm{B}(r, k)$ such that for any partition of $\{1, 2, ..., \mathrm{B}(r, k)\}$ into r parts, there exists an arithmetic progression of length k whose terms and common difference all belong to the same part of the partition. The problem of obtaining bounds on *Brauer numbers* $\mathrm{B}(r, k)$ has the additional difficulty that the *Brauer configurations* $\{x, d, x + d, ..., x + (k - 1)d\}$ are not translation invariant. By modifying Gowers' methods to overcome this obstacle, we show that one can obtain a bound for the Brauer numbers which is comparable to Gowers' bound for the van der Waerden numbers. Explicitly, we derive a bound of the form

$$\mathrm{B}(r, k) \leqslant 2^{2^{r^{C(k)}}},$$

where $C(k) > 0$ is a positive constant which depends on k only.

Finding perfect matchings in random regular graphs in linear expected time.

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(This talk is based on joint work with Alan M. Frieze.)

MSC2000: 05C80, 05C85

In a seminal paper on finding large matchings in sparse random graphs [1], Karp and Sipser proposed two algorithms for this task. The second algorithm has been intensely studied, but due to technical difficulties, the first algorithm has received less attention. Empirical results suggest that the first algorithm is superior. We show that this is indeed the case, at least for random regular graphs graphs. We show that w.h.p. the first algorithm will find a matching of size $n/2 - O(\log n)$ on a random r-regular graph ($r = O(1)$). We also show that the algorithm can be adapted to find a perfect matching w.h.p. in $O(n)$ time, as opposed to $O(n^{3/2})$ time for the worst-case.

[1] R.M. Karp and M. Sipser, *Maximum matchings in sparse random graphs*, Proceedings of the 22nd Annual IEEE Symposium on Foundations of Computing (1981) 364-375.

Thursday 11:40, Arts LR4

Sums of linear transformations in higher dimensions

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MSC2000: 11B13, 11B30, 11P70

Given a finite subset A of integers and coprime natural numbers q, s, we consider the set $q \cdot A + s \cdot A$, that is, the sum of dilates of A. In recent years, finding suitable lower bounds for the cardinality of such sets in terms of $|A|, q$ and s has seen considerable activity. In 2014, Balog and Shakan found sharp estimates for the same, that were tight in both the main term as well as the error term. Subsequently, they considered this problem in higher dimensional integer lattices. In this talk, we present a short survey of these results including our own improvement in the higher dimensional setting.

Linear Programming complementation and its application to fractional graph theory

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(This talk is based on joint work with George Mertzios and Viktor Zamaraev.)

MSC2000: 90C05, 05C65, 05C69, 05C70

In this talk, we introduce a new kind of duality for Linear Programming (LP), that we call LP complementation. We prove that the optimal values of an LP and of its complement are in bijection (provided the original LP has an optimal value greater than one).

The main consequence of the LP complementation theorem is for hypergraphs. We introduce the complement of a hypergraph and we show that the fractional packing numbers of a hypergraph and of its complement are in bijection; similar results hold for fractional matching, covering and transversal numbers.

This hypergraph complementation theorem has several consequences for fractional graph theory. We consider the following particular problem: let G be a graph and b be a positive integer, then how many vertex covers of G, say $S_1, \ldots, S_{t_b}$, can we construct such that every vertex appears at most b times in total? The integer b can be viewed as a budget we can spend on each vertex, and given this budget we aim to cover all edges for as long as possible (up to time t_b). We then prove that $t_b \sim \frac{\chi_f}{\chi_f - 1} b$, where χ_f is the fractional chromatic number of G.

The Structure of Connected Hypergraphs without long Berge Paths

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(This talk is based on joint work with E. Győri, O. Zamora.)

MSC2000: 05C35, 05C38, 05C65, 05D05

A problem, first considered by Erdős and Gallai in 1959, was to determine Turán number of paths and families of long cycles. Recently numerous mathematicians started investigating similar problems for r-uniform hypergraphs. We generalize a result of Balister, Győri, Lehel and Schelp [1] for r-uniform hypergraphs. We determine the unique extremal structure of an n-vertex, r-uniform, connected, hypergraph with the maximum number of hyperedges, without a Berge-path of length k, for all $n \geq N_{k,r}$, $k \geq 2r + 13 > 17$. We also generalise these results for a broader class of hypergraphs, where the size of each hyperedge is at most r and the set of hyperedges is a Sperner family. The following is a main theorem from the manuscript.

Theorem 1. *For all n, k, r integers, such that $n > N_{k,r}$, $r \geq 3$ and $k \geq 2r + 13$, we have*

$$ex_r^{conn}(n, BP_k) = \binom{\lfloor \frac{k-1}{2} \rfloor}{r-1}\left(n - \left\lfloor \frac{k-1}{2} \right\rfloor\right) + \binom{\lfloor \frac{k-1}{2} \rfloor}{r} + 1_{2|k}\binom{\lfloor \frac{k-1}{2} \rfloor}{r-2}$$

and the extremal hypergraph is unique, see Figure 1.

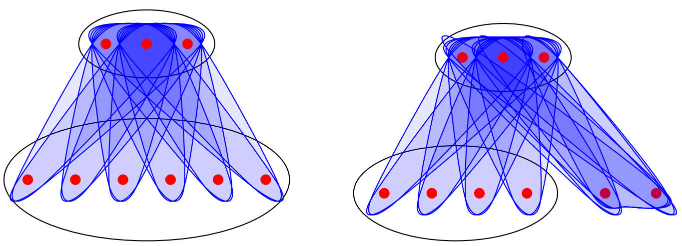

Figure 1: Extremal Constructions of Theorem 1

[1] P. N. Balister, E. Győri, J. Lehel, R. H. Schelp.Connected graphs without long paths. *Discrete Mathematics* **308(19)** (2008): 4487-4494.

Maximum hittings by maximal left-compressed intersecting families

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(This talk is based on joint work with Richard Mycroft.)

MSC2000: 05D05

The celebrated Erdős-Ko-Rado Theorem states that for all integers $r \leq n/2$ and every family $\mathcal{A} \subseteq [n]^{(r)}$, if $\mathcal{A}$ is intersecting (meaning that no pair of members of $\mathcal{A}$ are disjoint), then $|\mathcal{A}| \leq \binom{n-1}{r-1}$. For $r < n/2$, the star is the unique family to achieve equality. In this talk we consider the following variant, asked by Barber: for integers r and n, where n is sufficiently large, and for a set $X \subseteq [n]$, what are the maximal left-compressed intersecting families $\mathcal{A} \subseteq [n]^{(r)}$ which achieve maximum hitting with X (i.e. have the most members which intersect X)? We answer the question for every X, extending previous results by Borg and Barber which characterise those sets X for which maximum hitting is achieved by the star.

Friday 10:30, Arts LR2

From EDT0L grammars to generating functions

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(This talk is based on joint work with my student Kevin Martinsen.)

MSC2000: 05A15

A combinatorial technique due to Delest, Schützenberger and Viennot (referred to as the DSV method) finds a bijection between a suitable stratified combinatorial structure and a context-free language, determines an unambiguous context-free grammar for the language, then transforms the grammar into a system of equations whose solution is the ordinary generating function for the growth of the combinatorial structure. A classic application of this technique is finding the growth of words in the free monoid on {a,b,A,B} that represent the identity in the free group on two generators (see for example chapter 12 of [1]).

Of course such a "suitable" combinatorial structure is highly restrictive because the DSV method described above produces only algebraic generating functions. In the 2013 paper [2] we showed that the DSV method can be extended to the wider class of indexed languages, producing transcendental generating functions.

Lindenmayer systems (L-systems) constitute a widely studied class of formal languages used for modeling growth of biological structures, chemical reactions, mathematical fractals, etc. Unlike the standard serial grammars most used in computer science or natural languages, L-systems are highly parallel rewriting systems. Surprisingly, one of the most robust classes of L-systems, the "extended, table, 0-interaction, Lindenmayer" (ET0L) languages is contained within the indexed languages.

In this presentation, we show a translation between the deterministic sub-class EDT0L and indexed grammars, after which the extended DSV method can be applied. It turns out that every example in [2] is in fact EDT0L. However, the translation of one particularly intractable DSV process from [2] into an EDT0L system sheds new light on the underlying ordinary generating function.

[1] Office Hours with a Geometric Group Theorist. Edited by Clay and Margalit. *Princeton University Press*, Princeton, NJ, 2017. ISBN 978-0-691-15866-2

[2] Adams, Freden, Mishna. From indexed grammars to generating functions. *RAIRO Theor. Inform. Appl.* **47** (2013), no. 4, 325–350

Friday 10:30, Arts LR3

The growth of the Möbius function on the permutation poset

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MSC2000: 05A05, 06A07

We show that the growth of the principal Möbius function, $\mu[1, \pi]$, on the permutation poset is at least exponential in the length of the permutation.

The problem of the Möbius function on the permutation pattern poset was first raised by Wilf [1]. Earlier work by Smith [2] showed that the growth was at least $O(n^2)$, and recently Jelínek, Kantor, Kynčl and Tancer [3] improved this lower bound to $O(n^7)$. In the other direction, Brignall, Jelínek, Kynčl and Marchant [4] show that the proportion of permutations of length n with principal Möbius function equal to zero is asymptotically bounded below by $(1 - 1/e)^2 \geq 0.3995$.

To demonstrate that the growth is at least exponential, we define a way to construct a permutation of length $n+4$ from a permutation of length n using a technique that we call "ballooning", illustrated in Figure 1. This allows us to define a sequence of permutations $\pi_1, \pi_2, \pi_3 \ldots$ with lengths $n, n+4, n+8, \ldots$ and we then show that $\mu[1, \pi_{i+1}] = 2\mu[1, \pi_i]$, giving us exponential growth.

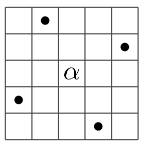

Figure 1: The 2413-balloon of the permutation α.

[1] H. S. Wilf. The patterns of permutations. *Discrete Mathematics*, 257(2):575–583, 2002.

[2] J. P. Smith. On the Möbius function of permutations with one descent. *Electronic Journal of Combinatorics*, 21(2):Paper 2.11, 19pp., 2014.

[3] V. Jelínek, I. Kantor, J. Kynčl and M. Tancer. On the growth of the Möbius function of permutations. *http://arxiv.org/abs/1809.05774*, 2018.

[4] R. Brignall, V. Jelínek, J. Kynčl and D. Marchant. Zeros of the Möbius function of permutations. *http://arxiv.org/abs/1810.05449*, 2018.

Friday 10:30, Arts LR4

Graphs with Power Domination at most 2

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(This talk is based on joint work with Najibeh Shahbaznejad.)

MSC2000: O5C35, 05C69

Let G be a connected graph and S a subset of its vertices. Let $C[S]$ be the set obtained from S as follows. First, put into $C[S]$ the vertices from the closed neighborhood of S. Then, repeatedly add to $C[S]$ vertices $w \in V(G) \setminus C[S]$ that have a *private* neighbor v in $C[S]$, i.e., vertices such that all the other neighbors of v are already in $C[S]$. After no such vertex w exists, the set $C[S]$ monitored by S has been constructed. The set S is called a *power dominating set* of G if $C[S] = V(G)$ and the *power domination number* $\gamma_P(G)$ is the minimum cardinality of a power dominating set [1,2].

Notice that $1 \leq \gamma_P(G) \leq \gamma(G)$, since every dominating set is power dominating. Observe also that if either $|V(G)| \leq 5$ or $\gamma(G) = 1$ or $G \in \{P_n, C_n\}$, then $\gamma_P(G) = 1$. In [2], it was shown that if T is a tree, then $\gamma_P(T) = 1$ if and only if T is a spider. In [5], it was proven that if G is a planar graph of diameter 2, then $\gamma_P(G) \leq 2$. Given any two graphs G and H, a set of necessary and sufficient conditions to guarantee that $\gamma_P(G \Box H) = 1$ were displayed in [3,4].

In this talk, we present a number of new results similar and/or related to the previous ones, involving other graph families and graph operations.

[1] T. L. Baldwin, L. Mili, M. B. Boisen Jr., R. Adapa, Power system observability with minimal phasor measurement placement, *IEEE Trans. Power System.* 82 (2) (1993) 707–715.

[2] T. W. Haynes, S. M. Hedetniemi, S. T. Hedetniemi, M. Henning, Domination in graphs applied to electric power networks, *SIAM J. Discrete Math.* 15 (4) (2002) 519–529.

[3] K. W. Soh, K. M. Koh, Recent results on the power domination numbers of graph products, *New Zealand J. Math.*. 48 (2018) 41–53.

[4] S. Varghese, A. Vijayakumar, On the power domination number of graph products, *Algor. and Discr. App. Math., Lect. Notes in Comp. Sci.* 9062 (2016) 357–367.

[5] M. Zhao, L. Kang, Power domination in planar graphs with small diameter, *J. Shanghai Univ.* 11 (3) (2007) 218–222.

Friday 10:30, Arts LR5

Graph Isomorphism for (H_1, H_2)-free Graphs: An Almost Complete Dichotomy

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(This talk is based on joint work with Marthe Bonamy, Matthew Johnson and Daniël Paulusma.)

MSC2000: 05C60, 05C75

The Graph Isomorphism problem, which is that of deciding whether two given graphs are isomorphic, is a central problem in algorithmic graph theory. Babai [1] recently proved that the problem can be solved in quasi-polynomial time, but it is not known if this can be improved to polynomial-time on general graphs.

We consider the Graph Isomorphism problem restricted to classes of graphs characterized by two forbidden induced subgraphs H_1 and H_2. By combining old and new results, Schweitzer [4] settled the computational complexity (polynomial-time solvable or GI-complete) of this problem restricted to (H_1, H_2)-free graphs for all but a finite number of pairs (H_1, H_2), but without explicitly giving the number of open cases. Grohe and Schweitzer [3] proved that Graph Isomorphism is polynomial-time solvable on graph classes of bounded clique-width. By combining previously known results for Graph Isomorphism with known results for boundedness of clique-width, we reduce the number of open cases to 14. By proving a number of new polynomial-time and GI-completeness results, we then further reduce this number to seven.

[1] L. Babai. Graph isomorphism in quasipolynomial time [extended abstract]. *Proc. STOC 2016*, pages 684–697, 2016.

[2] M. Bonamy, K. K. Dabrowski, M. Johnson, and D. Paulusma. Graph isomorphism for (H_1, H_2)-free graphs: an almost complete dichotomy. *Proc. WADS 2019, LNCS*, to appear. arXiv:abs/1811.12252.

[3] M. Grohe and P. Schweitzer. Isomorphism testing for graphs of bounded rank width. *Proc. FOCS 2015*, pages 1010–1029, 2015.

[4] P. Schweitzer. Towards an isomorphism dichotomy for hereditary graph classes. *Theory of Computing Systems*, 61(4):1084–1127, 2017.

The Structure of Hypergraphs without long Berge cycles

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(This talk is based on joint work with E. Győri, N. Lemons, N. Salia.)

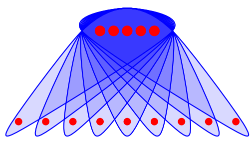
MSC2000: 05C65, 05C38, 05C35

We study the structure of r-uniform hypergraphs containing no Berge cycles of length at least k for $k \leq r$, and determine some properties of such structure. In particular with our method we determine the extremal number of $\mathcal{BC}_{\geq k}$-free hypergraphs for every value of n, giving an afirmative answer to the conjectured value when $k = r$ and giving a new and simple solution to a recent result of Kostochka-Luo when $k < r$. One of the main results is the following

Theorem 1. *Let $r > 2$ and n be positive integers, then*

$$ex_r(n, \mathcal{BC}_{\geq r}) = \max \left\{ \left\lfloor \frac{n-1}{r} \right\rfloor (r-1), n-r+1 \right\}.$$

When $n - r + 1 > \frac{n-1}{r}(r-1) + 1$ the only extremal graph is $\mathcal{S}_n^{(r)}$.



[1] Győri, E., Lemons, N., Salia, N., Zamora, O. (2018). The Structure of Hypergraphs without long Berge cycles. arXiv preprint arXiv:1812.10737.

[2] P. Erdős, T. Gallai. On maximal paths and circuits of graphs. *Acta Math. Acad. Sci. Hungar.* **10** (1959): 337–356.

[3] B. Ergemlidze, E. Győri, A. Methuku, N. Salia, C. Tompkins and O. Zamora. Avoiding long Berge cycles, the missing cases $k = r + 1$ and $k = r + 2$ *arXiv preprint* arXiv:1808.07687 (2018)

[4] Z. Füredi, A. Kostochka, R. Luo. Avoiding long Berge cycles. *arXiv preprint* arXiv:1805.04195 (2018).

[5] A. Kostochka, and R. Luo. On r-uniform hypergraphs with circumference less than r. *arXiv preprint* arXiv:1807.04683 (2018).

VC dimension and a union theorem for set systems

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(This talk is based on joint work with António Girão, Ross J. Kang.)

MSC2000: 05D05

Fix positive integers k and d. In [1] we show that, as $n \to \infty$, any set system $\mathcal{A} \subset 2^{[n]}$ for which the VC dimension of $\{\triangle_{i=1}^{k} S_i \mid S_i \in \mathcal{A}\}$ is at most d has size at most $(2^{d \bmod k} + o(1))\binom{n}{\lfloor d/k \rfloor}$. Here $\triangle$ denotes the symmetric difference operator. This is a k-fold generalisation of a result of Dvir and Moran [2], and it settles one of their questions.

A key insight is that, by a compression method, the problem is equivalent to an extremal set theoretic problem on k-wise intersection or union that was originally due to Erdős and FranklÀ [4].
Using induction, we determine the maximum size of a family $\mathcal{F} \subset 2^{[n]}$ that is k-wise $(n-d)-$intersecting for n large enough.
This work is very closely related to early work of Frankl [3]

We also give an example of a family $\mathcal{A} \subset 2^{[n]}$ such that the VC dimension of $\mathcal{A} \cap \mathcal{A}$ and of $\mathcal{A} \cup \mathcal{A}$ are both at most d, while $|\mathcal{A}| = \Omega(n^d)$. This provides a negative answer to another question of Dvir and Moran [2].

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An interpretation for the Tutte polynomial at (2,-1)

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MSC2000: 05A19, 05C30, 05C31

For a graph G, its Tutte polynomial $t(G; x, y)$ has several combinatorial interpretations when it is evaluated at certain points, for example it is known that the number of spanning forests equals $t(G; 2, 1)$, the number of acyclic orientations equals $t(G; 2, 0)$, and the number of spanning trees equals $t(G; 1, 1)$ if G is connected.

The evaluation $t(G; 2, -1)$ has been studied recently since C. Merino [2] proved that $t(K_{n+2}; 1, -1) = t(K_n; 2, -1)$. Later this identity was generalized in [1] where the authors describe sufficient conditions for a graph G to have the property that there exist two vertices u, w such that $t(G; 1, -1) = t(G \setminus \{u, w\}; 2, -1)$; also, a combinatorial interpretation for $t(K_n; 2, -1)$ is given as the number of spanning even increasing forests of a complete graph. Surprisingly, a new combinatorial approach was found in [3, 4] where $t(G; 2, -1)$ reappears when G is a circle graph, this time counting non-intersecting chord diagrams associated to the chord diagram for G. Following these ideas we prove that the number of spanning even increasing forests of G equals $t(G; 2, -1)$ when the graph G is of the form $K_n + \overline{K_2}$ or $K_n + \overline{K_3}$.

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Longest increasing subsequences in random pattern-avoiding permutations

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(This talk is based on joint work with Toufik Mansour.)

MSC2000: 05A05, 05A15

Let S_n denote the set of all permutations of length n on the set $[n] := \{1, 2, \cdots, n\}$. For $\tau = \tau_1\tau_2 \cdots \tau_k \in S_k$ and $\sigma = \sigma_1\sigma_2 \cdots \sigma_n \in S_n$, it is said that τ appears as a *pattern* in σ if there exists a subset of indices $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ such that $\sigma_{i_s} < \sigma_{i_t}$ if and only if $\tau_s < \tau_t$ for all $1 \leq s, t \leq k$. For example, the permutation 213 appears as a pattern in 24315 because it has the subsequences $2--15$, $-43-5$ or $--315$. If τ does not appear as a pattern in σ, then σ is called a τ-*avoiding* permutation. We denote by $S_n(\tau)$ the set of all τ-avoiding permutations of length n. For a given $\sigma \in S_n$, we denote by $L_n(\sigma)$ the length of a *longest increasing subsequence* in σ, that is,

$$L_n(\sigma) = \max\{k \in [n] : \text{there exist } 1 \leq i_1 < i_2 < \cdots < i_k \leq n \text{ and } \sigma_{i_1} < \sigma_{i_2} < \cdots < \sigma_{i_k}\}.$$

The problem of determining the asymptotic behavior of L_n on S_n under the uniform probability distribution has a long and interesting history [5]. It is known that $E(L_n) \sim 2\sqrt{n}$ and $n^{-1/6}(L_n - E(L_n))$ converges in distribution to the Tracy-Widom distribution as $n \to \infty$ [1]. In this talk, we will present some exact and asymptotic formulas for $E(L_n)$ on some pattern-avoiding permutation classes under the uniform probability distribution. Specifically, our results determine the asymptotic behavior of $E(L_n)$ on $S_n(\tau^1, \tau^2) = S_n(\tau^1) \cap S_n(\tau^2)$ with $\tau^1 \in S_3$ and $\tau^2 \in S_4$ for all possible cases. The earliest results in this direction were obtained for $S_n(\tau)$ with $\tau \in S_3$ in [2]. The case $S_n(\tau^1, \tau^2)$ with $\tau^1, \tau^2 \in S_3$ is studied for all possible cases in [3]. Hence, our results make some new contributions to this research program [4].

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Multiset Dimensions of Trees

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(This talk is based on joint work with Yusuf Hafidh, Suhadi Saputro, Presli Siagian, Steven Tanujaya, and Saladin Uttunggadewa.)

MSC2000: 05C12, 05C05

Let G be a connected graph and W be a set of vertices of G. The representation multiset of a vertex v with respect to W, $r_m(v|W)$, is defined as a multiset of distances between v and the vertices in W. If $r_m(u|W) \neq r_m(v|W)$ for every pair of distinct vertices u and v, then W is called an m-resolving set of G. If G has an m-resolving set, then the cardinality of a smallest m-resolving set is called the multiset dimension of G, denoted by $md(G)$; otherwise, we say that $md(G) = \infty$.

In this talk, we show that for a tree T other than a path, if $md(T) < \infty$, then $3 \leq md(T) \leq n - 1$. We shall also characterize trees with multiset dimension 3 and provide necessary and sufficient conditions for caterpillars and lobsters having finite multiset dimension.

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Connectivity in Hypergraphs

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(This talk is based on joint work with Megan Dewar and John Proos.)

MSC2000: 05C65, 05C40, 68Q17

We consider two natural notions of connectivity for hypergraphs: weak and strong. The strong deletion of a vertex v entails removing v from the vertex set of the hypergraph as well as removing from the edge set every edge that contains v. In contrast, the weak deletion of a vertex v merely entails removing v from each edge that contains v, as well as removing v from the vertex set of the hypergraph. Define $\kappa_S(H)$ (resp. $\kappa_W(H)$) to be the least number of vertices whose strong (resp. weak) deletion from a hypergraph H results in a disconnected hypergraph. Additionally, define $\kappa'_W(H)$ to be the least number of edges whose removal from the edge set of H results in a disconnected hypergraph.

We generalise a result of Whitney [1] about connectivity of graphs and prove that $\kappa_S(H) \leq \kappa'_W(H) \leq \delta(H)$ for any nontrivial hypergraph H. We also show that determining a minimum strong vertex cut is NP-hard for general hypergraphs, and, moreover, this problem remains NP-hard when restricted to hypergraphs with maximum edge size at most 3.

Hypergraph Saturation Irregularities

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MSC2000: 05C65, 05C35

For a fixed graph F, Turan's number $\text{ex}(F,n)$ is the maximum number of edges in an F-free graph on n vertices. Note that in any maximal F-free graph, adding any new edge must create a copy of F as a subgraph. This inspires the following definition: we say a graph G is *F-saturated* if it does not contain any copies of F but adding any new edge creates some copy of F. Then Turan's number can be defined equivalently as

$$\text{ex}(F,n) = \max\{e(G) : G \text{ is } F\text{-saturated and } |G| = n\}.$$

Replacing maximum by minimum gives the saturation number

$$\text{sat}(F,n) = \min\{e(G) : G \text{ is } F\text{-saturated and } |G| = n\},$$

which forms an interesting counterpoint to the Turan number – the saturation number is in many ways less well-behaved. For example, we know that the Turan density $\lim_{n\to\infty} \text{ex}(F,n)/n^2$ exists. Tuza [1] conjectured that $\text{sat}(F,n)/n$ must tend to a limit as n tends to infinity, but this conjecture is still open.

The definition of saturation extends to families of graphs. Pikhurko [2] disproved a strengthening of Tuza's conjecture by finding a finite family $\mathcal{F}$ of graphs such that $\text{sat}(\mathcal{F},n)/n$ does not converge as n tends to infinity.

Pikhurko then asked whether a similar behaviour can occur for families of r-uniform hypergraphs. I resolve this question by exhibiting for all r a finite family of r-uniform hypergraphs $\mathcal{F}$ such that $\text{sat}(\mathcal{F},n)/n^{r-1}$ does not converge as n tends to infinity, thus settling a generalisation of Tuza's conjecture to families of hypergraphs.

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The poset of graphs ordered by induced containment

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MSC2000: 06A07, 05C99

We study the poset $\mathcal{G}$ of all unlabelled graphs, up to isomorphism, with $H \leq G$ if H occurs as an induced subgraph in G. We present some results on the Möbius function of $\mathcal{G}$, where the *Möbius function* of a poset P is defined recursively by $\mu_P(a, a) = 1$ for all a, $\mu_P(a, b) = 0$ if $a \not\leq b$ and if $a < b$ then $\mu_P(a, b) = -\sum_{c \in [a,b)} \mu_P(a, c)$. The poset $\mathcal{G}$ has a countably infinite number of elements and is locally finite, so we focus our attention on the *intervals* $[a, b] = \{z \in \mathcal{G} \,|\, a \leq z \leq b\}$, see Figure 1 for an example of an interval of $\mathcal{G}$ and its Möbius function.

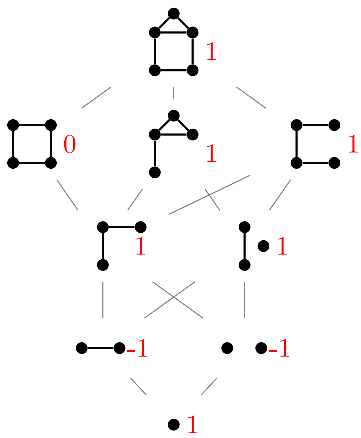

Figure 1: The interval $[K_1, H]$ in $\mathcal{G}$, where H is the house graph, with $\mu(K_1, X)$ in red.

We begin with some simple results on the Möbius function between well known graphs such as complete graphs K_n, cycle graphs C_n, empty graphs $\overline{K}_n$ and bipartite graphs. We then consider intervals between graphs consisting of disjoint paths. Let P_x^n be n disjoint copies of the path of length x. By applying an inductive proof, we show that $\mu(P_1^2, P_x^2)$ is given by the Fibonacci numbers. Moreover, using Discrete Morse theory we show that $\mu(P_1^n, P_5^n)$ is given by the Catalan numbers.

We finish with two conjectures on the Möbius function of $\mathcal{G}$. Firstly, we claim that $\mu(K_1, H^n)$ is also given by the Catalan numbers, where H^n is n copies of the house graph, that is, the graph at the top of Figure 1. Secondly, we conjecture that $\mu(P_1^n, P_5^x P_4^y)$ is given by the Schröder numbers, where $P_x^n P_y^m$ is the disjoint union of P_x^n and P_y^m.

The Expansion of a Chord Diagram and the Genocchi Numbers

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MSC2000: 05A15

A chord diagram E is a set of chords of a circle such that no pair of chords has a common endvertex. Let $v_1, v_2, \ldots, v_{2n}$ be a set of vertices arranged in clockwise order along a circumference. A chord diagram $C_n = \{v_1v_{n+1}, v_2v_{n+2}, \ldots, v_nv_{2n}\}$ is called an n-crossing and a chord diagram $N_n = \{v_1v_2, v_3v_4, \ldots, v_{2n-1}v_{2n}\}$ is called an n-necklace. For a chord diagram E having a 2-crossing $S = \{x_1x_3, x_2x_4\}$, the chord expansion of E with respect to S is the replacement of E with $E_1 = (E \setminus S) \cup \{x_2x_3, x_4x_1\}$ or $E_2 = (E \setminus S) \cup \{x_1x_2, x_3x_4\}$. Beginning with a chord diagram E, by iterating chord expansions, we have a multiset $\mathcal{NCD}(E)$ of nonintersecting chord diagrams in the end.

It is known that the cardinality of $\mathcal{NCD}(E)$ as a multiset equals $T(G_E; 2, -1)$, where $T(G; x, y)$ is the Tutte polynomial of a graph G and G_E is the circle graph of E([3]). For a complete graph K_n, Merino showed that $T(K_n; 2, -1) = Eul_{n+1}$, where $\{Eul\}_{n\geq 1} = (1, 1, 2, 5, 16, 61, 272, \ldots)$, the Euler numbers([1]). Since the circle graph of C_n is K_n, the caridinality of $\mathcal{NCD}(C_n)$ corresponds to the Euler numbers. (See also [2].)

For a chord diagram E and a nonintersecting chord diagram F, let us denote the multiplicity of F in $\mathcal{NCD}(E)$ by $m(E, F)$. In this talk, it is shown that $\{m(C_n, N_n)\}_{n\geq 2} = (1, 1, 2, 3, 8, 17, 56, 155, 608, \ldots)$, which corresponds to the Genocchi number when n is odd and the median Genocchi number when n is even.

Acknowledgements. This work was supported by JSPS KAKENHI Grant Number 19K03607.

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Word-representability of split graphs generated by morphisms

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MSC2000: 05C62, 68R15

Letters x and y alternate in a word w if after deleting in w all letters but the copies of x and y we either obtain a word $xyxy\cdots$ (of even or odd length) or a word $yxyx\cdots$ (of even or odd length). A graph $G = (V, E)$ is word-representable if and only if there exists a word w over the alphabet V such that letters x and y alternate in w if and only if $xy \in E$. It is known that a graph is word-representable if and only if it admits a certain orientation called semi-transitive orientation.

Word-representable graphs generalise several important classes of graphs such as 3-colourable graphs, circle graphs, and comparability graphs. There is a long line of research in the literature dedicated to word-representable graphs that is summarised in [1, 2]. A particular research direction here is the study of word-representability of split graphs initiated in [3], where characterisations were obtained for a number of subclasses of split graphs.

Of our interest are subclasses of split graphs that can be defined by iteration of morphisms. Namely, we use the replacement of 0s and 1s by $k \times k$ matrices A and B, respectively, to generate larger adjacency matrices and to study word-representability of the corresponding graphs. Therefore, we introduce to the theory of word-representable graphs objects studied in combinatorics on words (i.e. morphisms), and enlarge our knowledge on word-representability of split graphs.

In this talk, I will discuss a number of general results and their applications to word-representability of split graphs generated by morphisms. In particular, I will give word-representability conditions on adjacency matrices in terms of permutations of rows and columns. Semi-transitive orientations play a key role in our results.

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Tree-homogeneous graphs

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MSC2000: 05C25,05E20

Let X be a class of graphs. A (simple undirected) graph Γ is X*-homogeneous* if for any graph isomorphism $\varphi : \Delta_1 \to \Delta_2$ between finite induced subgraphs Δ_1 and Δ_2 of Γ such that Δ_1 is isomorphic to a graph in X, there exists an automorphism of Γ that extends φ. For example, if $X = \{K_1\}$, then X-homogeneity is vertex-transitivity, and if $X = \{K_2\}$, then X-homogeneity is arc-transitivity. A graph is *tree-homogeneous* if it is X-homogeneous where X is the class of trees. We will discuss some recent progress on classifying the finite tree-homogeneous graphs, as well as some related work on a class of highly symmetric point-line incidence structures.

Friday 11:20, Arts LR5

A COMPLEXITY DICHOTOMY OF COLOURFUL COMPONENTS PROBLEMS IN k-CATERPILLARS AND SMALL-DEGREE PLANAR GRAPHS

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(This talk is based on joint work with Janka Chlebíková.)

MSC2000: 68R10, 05C40, 68Q17, 05C85

A connected component of a vertex-coloured graph is said to be *colourful* if all its vertices have different colours, and a graph is colourful if all its connected components are colourful. Given a vertex-coloured graph, the COLOURFUL COMPONENTS problem asks whether there exist at most p edges whose removal makes the graph colourful, and the COLOURFUL PARTITION problem asks whether there exists a partition of the vertex set with at most p parts such that each part induces a colourful component. We first study the problems on k-caterpillars (caterpillars with hairs of length at most k) and explore the boundary between polynomial and NP-complete cases. Both problems are known NP-complete on 2-caterpillars with unbounded maximum degree. We show that they remain NP-complete on binary 4-caterpillars and on ternary 3-caterpillars. This answers an open question regarding the complexity of the problems on trees with maximum degree at most 5 [1]. On the positive side, thanks to a simple preprocessing of the input graphs, we give a linear time algorithm for 1-caterpillars with unbounded degree, even if the backbone is a chordless cycle. This improves the previously known quadratic complexity on paths [2] and widens the class of graphs. Finally, we answer an open question regarding the complexity of COLOURFUL COMPONENTS on graphs with maximum degree at most 5 [1]. We prove that the problem remains NP-complete on 5-coloured planar graphs with maximum degree 4, and on 12-coloured planar graphs with maximum degree 3.

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Many cliques with few edges

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(This talk is based on joint work with Jamie Radcliffe.)

MSC2000: 05C30, 05C35, 05C69

The problem of maximizing the number of cliques has been studied within several classes of graphs. For example, among graphs on n vertices with clique number at most r, the Turán graph $T_r(n)$ maximizes the number of copies of K_t for each size t. Among graphs on m edges, the colex graph $\mathcal{C}(m)$ maximizes the number of K_t's for each size t.

In recent years, much progress has been made on the problem of maximizing the number of cliques among graphs with n vertices and maximum degree at most r. In this talk, we discuss the edge analogue of this problem: which graphs with m edges and maximum degree at most r have the maximum number of cliques? We prove in some cases that the extremal graphs contain as many disjoint copies of K_{r+1} as can fit, with the leftovers in another component. These remaining edges form a colex graph.

Minimum degree conditions for powers of cycles and paths

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MSC2000: 05C35, 05C38

The study of conditions on vertex degrees in a host graph G for the appearance of a target graph H is a major theme in extremal graph theory. The k^{th} power of a graph G is obtained from G by joining any two vertices at distance at most k. We study minimum degree conditions under which a graph G contains the k^{th} power of cycles and paths of arbitrary specified lengths. We determine precise thresholds, assuming that the order of G is large. This extends a result of Allen, Böttcher and Hladký [1] concerning the containment of squared paths and squared cycles of arbitrary specified lengths and settles a conjecture of theirs in the affirmative.

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Enumeration of Permutation Classes by Inflation of Independent Sets of Graphs

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(This talk is based on joint work with C. Bean and H. Ulfarsson.)

MSC2000: 05A15

Bean, Tannock and Ulfarsson [1] show a link between the permutations in Av(123) and Av(132) to independent sets of certain graphs. We extend their results to enumerate Av(2314, 3124), and moreover certain subclasses obtained by adding patterns of the form $1 \oplus \pi$ where π is skew-indecomposable. Extending the idea further allows us to get the enumeration of five classes and certain subclasses of these. Overall, this technique gives a unified way of enumerating a total of 48 classes avoiding patterns of length 4 and many more of longer length.

More precisely, we choose an independent set of size k in the graph U_n together with a list of k non-empty permutations in $\mathrm{Av}(2314, 3124, P)$ where P is a set of skew-indecomposable permutations. We establish a bijection between these objects and permutations in $\mathrm{Av}(2314, 3124, 1 \oplus P)$. From [1], we get the generating function, $F(x, y)$, where the coefficient of $x^n y^k$ gives the number of independent set of size k in U_n. We show that:

Theorem 1. *Let P be a set of skew-indecomposable permutations and $A(x)$ be the generating function of* $\mathrm{Av}(2314, 3124, P)$. *The generating function of* $\mathrm{Av}(2314, 3124, 1 \oplus P)$ *is $B(x) = F(x, A(x) - 1)$.*

This can be used to enumerate eight classes avoiding length 4 patterns, and many more avoiding longer patterns. Moreover, a similar theorem can be stated for the classes $\mathrm{Av}(2413, 3142, 1 \oplus P)$ where all π in P are sum-indecomposable. This can be used to enumerate eight more classes avoiding length 4 patterns.

We then describe new graphs and provide a closed formula for the generating function counting independent sets. We show how these can be used to enumerate $\mathrm{Av}(S, 1 \oplus P)$ where S is any subset of size 3 of $\{2314, 3124, 3142, 2413\}$ and P a set of indecomposable permutations (either skew or sum depending on the subset S chosen). This gives the enumeration of 32 new classes avoiding length 4 patterns.

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Fractional chromatic number, maximum degree and girth

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(This talk is based on joint work with Jean-Sébastien Sereni.)

MSC2000: 05C15, 05C69

It is well known that you can colour a graph G of maximum degree Δ greedily with $\Delta + 1$ colours. Moreover, this bound is tight, since it is reached by the cliques and odd cycles. Johansson proved with a pseudo-random colouring scheme that you can colour triangle-free graphs of maximum degree Δ with no more than $O(\Delta/\ln\Delta)$ colours. This result has been recently improved by Molloy to $(1+\varepsilon)(\Delta/\ln\Delta)$ for any $\varepsilon > 0$ when Δ is large enough. This is tight up to a multiplicative constant, since it is possible to pseudo-randomly construct a family of graphs of maximum degree Δ, arbitrary large girth, and (fractional) chromatic number larger than $\Delta/(2\ln\Delta)$. These results only settle the case of graphs of large degree, and there remains a lot to say for small degree graphs.

When the graphs are of small degree, it is interesting to consider the fractional chromatic number instead, since it has infinitely many possible values — note that if G is a subcubic graphs, then either $G = K_4$, G is bipartite, or $\chi(G) = 3$. It has already been settled that the maximum fractional chromatic number over the triangle-free subcubic graphs is 14/5. By taking advantage of the special properties of the so-called *hard-core distribution* on maximal independent sets, we prove that the fractional chromatic number of graphs of maximum degree d and girth at least 7 is at most

$$\min_{k\in\mathbb{Z}_{\geq 4}} \frac{2d + 2^{k-3} + k}{k}.$$

By focusing on d-regular graphs and using the uniform distribution on *maximum* independent sets — a special setting of the hard-core distribution — we can use a similar method with the help of a computer in order to derive new lower bounds on the independence ratio — or equivalently upper bounds on the *Hall ratio* — of d-regular graphs, with $d \in \{3, 4, 5\}$, of girth g varying between 6 and 12.

$d \backslash g$	6	7	8	9	10	11	12
3	2.727272		2.625224	2.604167	2.557176	2.539132	2.510378
4	3.153846		3.038497	3.017382	3		
5		3.6	3.5				

Table 1: Upper bounds on the Hall ratio.

Friday 11:45, Arts LR4

On a Property of Perfect Hash Families

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(This talk is based on joint work with Charles J. Colbourn.)

MSC2000: 05B15,05B30

A perfect hash family (PHF), denoted $PHF_\lambda(N; k, v, t)$, is an $N \times k$ array over v symbols such that every $N \times t$ array contains λ rows with all distinct symbols; a $PHF_\lambda(N; k, v, t)$ is optimal if there is no $PHF_\lambda(N-1; k, v, t)$. Computational one-row-at-a-time methods often produce PHFs with the first row consisting of each symbol appearing as equally often as possible. One then can ask the question of whether optimal PHFs always exist having some row with this property. When $k = v$, this is true; for "small" k such that $k \geq v+1$, under certain conditions of N and t, it is not, even for $\lambda = 2$. We give examples for the second statement, and a conjecture for when optimal PHFs have this property.

Recognizing Generating Subgraphs

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(This talk is based on joint work with Vadim E. Levit.)

MSC2000: 05C69

A graph G is *well-covered* if its maximal independent sets are of the same cardinality [4]. Assume that a function w is defined on its vertices. Then G is w*-well-covered* if all maximal independent sets are of the same weight. For every graph G, the set of weight functions w such that G is w-well-covered is a *vector space* [2], denoted $WCW(G)$ [1]. Deciding whether an input graph G is well-covered is **co-NP**-complete[3, 5]. Therefore, finding $WCW(G)$ is **co-NP**-hard.

A *generating subgraph* of a graph G is an induced complete bipartite subgraph B of G on vertex sets of bipartition B_X and B_Y, such that each of $S \cup B_X$ and $S \cup B_Y$ is a maximal independent set of G, for some independent set S. If B is generating, then $w(B_X) = w(B_Y)$ for every weight function $w \in WCW(G)$. Therefore, generating subgraphs play an important role in finding $WCW(G)$. The decision problem of whether a subgraph of an input graph is generating is **NP**-complete, even in the restricted case that the subgraph is $K_{1,1}$ [1].

In this talk we prove that recognizing generating subgraphs is **NP**-complete, even if the input is restricted to graphs without cycles of lengths 3 and 5, or to bipartite graphs with girth at least 6. On the other hand, we supply polynomial algorithms for recognizing generating subgraphs and finding $WCW(G)$, when the input graph is bipartite without cycles of length 6.

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[2] Y. Caro, N. Ellingham and G. F. Ramey, Local structure when all maximal independent sets have equal weight, *SIAM Journal on Discrete Mathematics* 11:644-654, 1998.

[3] V. Chvatal and P. J. Slater, A note on well-covered graphs, Quo Vadis, Graph Theory?, *Annals of Discrete Mathematics*, North Holland, Amsterdam 55:179-182, 1993.

[4] M. D. Plummer, Some covering concepts in graphs, *Journal of Combinatorial Theory* 8:91-98, 1970.

[5] R. S. Sankaranarayana and L. K. Stewart, Complexity results for well-covered graphs, *Networks* 22:247-262, 1992.

Resilient degree sequences with respect to Hamilton cycles and matchings in random graphs

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(This talk is based on joint work with Alberto Espuny Díaz, Jaehoon Kim, Daniela Kühn and Deryk Osthus.)

MSC2000: 05C45, 05C80

Pósa's theorem states that any graph G whose degree sequence $d_1 \leq \ldots \leq d_n$ satisfies $d_i \geq i+1$ for all $i < n/2$ has a Hamilton cycle. This degree condition is best possible. We show that a similar result holds for suitable subgraphs G of random graphs, i.e. we prove a 'resilient' version of Pósa's theorem: if $pn \geq C \log n$ and the i-th vertex degree (ordered increasingly) of $G \subseteq G_{n,p}$ is at least $(i + o(n))p$ for all $i < n/2$, then G has a Hamilton cycle. This is essentially best possible and strengthens a resilient version of Dirac's theorem obtained by Lee and Sudakov.

Chvátal's theorem generalises Pósa's theorem and characterises all degree sequences which ensure the existence of a Hamilton cycle. We show that a natural guess for a resilient version of Chvátal's theorem fails to be true. We formulate a conjecture which would repair this guess, and show that the corresponding degree conditions ensure the existence of a perfect matching in any subgraph of $G_{n,p}$ which satisfies these conditions. This provides an asymptotic characterisation of all degree sequences which resiliently guarantee the existence of a perfect matching.

Difference of forbidden pairs containing a claw

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(This talk is based on joint work with Guantao Chen, Michitaka Furuya, Songling Shan and Ping Yang.)

MSC2000: 05C75

Let $\mathcal{G}_1$ and $\mathcal{G}_2$ be two families of graphs, and let P be a certain property for graphs. We assume that every member of $\mathcal{G}_2$ satisfies P, and consider the problem of whether or not members of $\mathcal{G}_1$ satisfy P. If we suppose $\mathcal{G}_1 \subseteq \mathcal{G}_2$, then every member of $\mathcal{G}_1$ satisfies P. Now, we suppose a weaker condition than $\mathcal{G}_1 \subseteq \mathcal{G}_2$.

We first suppose that the family $\mathcal{G}_1 - \mathcal{G}_2$ is finite. Then every member of $\mathcal{G}_1$ satisfies P with finite exceptions. Since we can check whether finite members of $\mathcal{G}_1$ satisfy P or not in finite time, we can regard the desired problem as solved.

We next suppose that the members of $\mathcal{G}_1 - \mathcal{G}_2$ are characterizable (not necessarily finite). Then each member of $\mathcal{G}_1$ either satisfies P or is characterized. If the characterization has a simple structure, then we may be able to check whether such graphs satisfy P or not. Thus, in this case, it might be possible to solve the desired problem.

In this talk, we apply the above strategy for the forbidden subgraph problems. In particular, we show that all graphs in the following classes are characterizable.

1. connected $\{K_{1,3}, B_{1,2}\}$-free but not N-free graphs [2],
2. connected $\{K_{1,3}, Z_2\}$-free but not $B_{1,1}$-free graphs [1],
3. connected $\{K_{1,3}, B_{1,1}\}$-free but not P_5-free graphs [1], and
4. connected $\{K_{1,3}, B_{1,2}\}$-free but not P_6-free graphs [1].

In our talk, we also introduce applications of our characterizations, which gives new theorems and alternating proofs of known results.

[1] G. Chen, M. Furuya, S. Shan, S. Tsuchiya and P. Yang, Difference of forbidden pairs containing a claw, submitted.

[2] M. Furuya and S. Tsuchiya. Claw-free and $N(2,1,0)$-free graphs are almost net-free. *Graphs Combin.* **31** (2015) 2201–2205.

Census Coverage Adjustment Methodology

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MSC2000: 90C27

Although every effort is made to ensure that everyone is counted in a census, inevitably some people and households are missed. In order to produce a complete census dataset, we must statistically create extra records to fill in these gaps.

We estimate how many people we have missed using key demographic categories such as age-sex bands, ethnicity groups, and household size. The challenge, and the focus for this talk, is how to create new, plausible records which fit these constraints.

In this talk we consider two different methods: the statistical method used in 2011, and a potential alternative for future censuses using combinatorial optimisation.

Distance matching extension in star-free graphs

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(This talk is based on joint work with R.E.L. Aldred and Jun Fujisawa.)

MSC2000: 05C70

A matching M in a graph G is *extendable* if there exists a perfect matching in G which contains M. Matching extension is a field of research which aims to find a sufficient condition for a matching to be extendable. Traditionally, research has focused on the size of a matching. For a nonnegative integer k, a graph G is k-extendable if $|V(G)| \geq 2k+2$ and every matching of size k in G is extendable. The study of k-extendable graphs was initiated by Plummer [1], and it has been a central topic in matching extension.

While k-extendable graphs are still actively studied, some recent research looks at matching extension from a different point of view. They consider the distance between edges in a matching rather than its size. For a set of edges F, the minimum distance of F is the smallest distance between two distinct edges in F. (If $|F| = 1$, we define $d(F) = +\infty$.) A graph G is *distance d extendable* if every matching of minimum distance at least d is extendable in G. The topic dealing with distance d extendable graphs is called distance matching extension.

In this talk, we discuss distance matching extension in star-free graphs. We first report that for every integer $k \geq 3$, there exists an integer d such that every locally $(k-1)$-connected $K_{1,k}$-free graph of even order is distance d extendable. Then we raise/lower the local connectivity in the hypothesis and observe how it affects the conclusion. If time permits, we explain main ideas of the proofs.

[1] M. D. Plummer, On n-extendable graphs. *Discrete Math.*, **31** (1980), 201–210.

Smallest cyclically covering subspaces of $\mathbb{F}_q^n$

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(This talk is based on joint work with Peter Cameron and David Ellis.)

MSC2000: 05E18, 05B40

Let $\sigma : \mathbb{F}_q^n \to \mathbb{F}_q^n$ be the *cyclic shift* operator; the map which permutes the entries of each vector by shifting them cyclically one step clockwise. We say a subspace $U \leq \mathbb{F}_q^n$ is *cyclically covering* if the union of the cyclic shifts of U is equal to $\mathbb{F}_q^n$, i.e. $\bigcup_{r=0}^{n-1} \sigma^r(U) = \mathbb{F}_q^n$. This talk will investigate the problem of determining the minimum possible dimension of a cyclically covering subspace of $\mathbb{F}_q^n$. This is a natural generalisation of a problem posed in 1991 by Peter Cameron who investigated the binary case. Our results imply lower bounds for a well-known conjecture of Isbell, and a generalisation thereof, supplementing lower bounds due to Spiga. Using techniques from combinatorics, representation theory and the theory of finite fields we prove upper and lower bounds for each fixed q and answer the question completely for infinitely many values of q and n. Finally we consider the analogous problem for general representations of groups.

This is joint work with Peter Cameron and David Ellis.

Friday 14:05, Arts Main LT

Dirac's theorem for random regular graphs

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(This talk is based on joint work with P. Condon, A. Girão, D. Kühn and D. Osthus.)

MSC2000: 05C45, 05C80

We prove a 'resilience' version of Dirac's theorem in the setting of random regular graphs. More precisely, we show that, whenever d is sufficiently large compared to $\varepsilon > 0$, a.a.s. the following holds: any subgraph of the random n-vertex d-regular graph $G_{n,d}$ with minimum degree at least $(1/2 + \varepsilon)d$ is Hamiltonian.

This proves a conjecture of Ben-Shimon, Krivelevich and Sudakov. Our result is best possible: firstly, the condition that d is large cannot be omitted, and secondly, the minimum degree bound cannot be improved.

Is There a Simple Polyhedral Proof of the Celebrated Strong Perfect Graph Theorem?

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(This talk is based on joint work with Jingpeng Li.)

MSC2000: 05C17

The strong perfect graph theorem is the proof of the famous Berge's conjecture that the graph is perfect if and only if it is free of odd holes and odd anti-holes. The conjecture was settled after 40 years in 2002 by Maria Chudnovsky et. al. and the proof was published in 2006. However, the only known proof is lengthy and intricate, and uses a combinatorial approach. In this proposal we explore the possibility of a simple short proof of the strong perfect graph theorem using polyhedral methods. Our proposal emerges naturally from our work to calculate the capacity of wireless multihop networks. The problem of calculating the capacity of multihop wireless networks is directly linked to whether the graph is perfect or not.

[1] Chudnovsky, M., Robertson, N., Seymour, P., & Thomas, R. (2006). The strong perfect graph theorem. Annals of mathematics, 51-229.

Wang-Landau sampling for estimation of the reliability of physical networks

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(This talk is based on joint work with Pasin Marupanthorn.)

MSC2000: 05C80,68M10,90C35

Modern physical networks, for example in communication and transportation, can be interpreted as directed graphs. Network models are used to identify the probability that given nodes are connected, and therefore the effect of a failure at a given link. This is essential for network design, optimization, and reliability. In this study, we investigated three alternative ensembles for estimating network reliability using the Wang-Landau algorithm. The first performed random walks on a structure function having two possible states: connected and disconnected. The second used random walks on a reliability polynomial. The third combined random walks with the average of connecting probabilities. The accuracy and limitations of the three ensembles were compared by estimating the reliability of two network models: a bridge network and a ladder-type network. The simulation results showed that the use of a random walk on a structure function failed to estimate the reliability of a highly reliable network, whereas the other two approaches performed efficiently. The use of a random walk on a reliability polynomial, and combining this with the average of connecting probabilities yielded highly accurate estimates. However, the use of the average of connecting probabilities required less computation time when applied to a large network.

Antimagic Labeling of Biregular Bipartite Graphs

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(This talk is based on joint work with Yulin Chang, Guanghui Wang, Donglei Yang.)

MSC2000: 05C78

An antimagic labeling of a graph G with n vertices and m edges is a bijection from the set of edges of G to the integers $\{1, 2, \cdots, m\}$ such that all n vertex sums are pairwise distinct, where the vertex sum of a vertex is the sum of labels of all edges incident to it. A graph G is antimagic if G has an antimagic labeling. Hartsfield and Ringel conjectured that every connected graph other than K_2 is antimagic, which is commonly referred to as the Antimagic Labeling Conjecture. In this talk, I shall introduce antimagic labeling of biregular bipartite graphs and the main idea of our proof.

Finite groups with few automorphism orbits relative to their number of element orders

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(This talk is based on joint work with Michael Giudici and Cheryl E. Praeger.)

MSC2000: 20D60

The study of objects X that are 'highly symmetric' in the sense of certain transitivity assumptions on natural actions of the automorphism group $\mathrm{Aut}(X)$ has a rich history, both inside and outside of combinatorics. As examples, we mention vertex-transitive graphs [1, Definition 4.2.2, p. 85] and flag-transitive designs [2].

Another group-theoretic example is provided by Zhang's 1992 paper [3], in which he extensively studies so-called *AT-groups*, defined as finite groups G such that $\mathrm{Aut}(G)$ acts transitively on each subset of G consisting of all elements of a fixed order. Equivalently, if $\omega(G)$ denotes the number of $\mathrm{Aut}(G)$-orbits on G and $\mathrm{o}(G)$ denotes the number of distinct element orders in G, then G is an AT-group if and only if $\omega(G) = \mathrm{o}(G)$.

The purpose of this talk is to discuss recent results concerning the two parameters $\mathfrak{d}(G) := \omega(G) - \mathrm{o}(G) \geq 0$ and $\mathfrak{q}(G) := \omega(G)/\mathrm{o}(G) \geq 1$, each of which may be viewed as a measure for how far G is from being an AT-group. More precisely, the results state that the index $[G : \mathrm{Rad}(G)]$ of the largest soluble normal subgroup $\mathrm{Rad}(G)$ of G (a measure for how far G is from being soluble) can be bounded from above both by a function in $\mathfrak{d}(G)$ and by a function in $\mathfrak{q}(G)$ and $\mathrm{o}(\mathrm{Rad}(G))$.

The proofs of these results, which will be sketched briefly, combine various classical combinatorial results (e.g. concerning the asymptotics of some integer partition counting functions) with counting methods of a more group-theoretic flavour (keyword: coset-wise counting) and, of course, the classification of finite simple groups.

[1] N.L. Biggs and A.T. White, *Permutation Groups and Combinatorial Structures*, Cambridge University Press (London Mathematical Society Lecture Note Series, 33), Cambridge, 1979 (reprinted 2008).

[2] M. Huber, *Flag-transitive Steiner designs*, Birkhäuser (Frontiers in Mathematics), Basel, 2009.

[3] J. Zhang, On Finite Groups All of Whose Elements of the Same Order Are Conjugate in Their Automorphism Groups, *J. Algebra* **153**:22–36, 1992.

A proof of a conjecture on disjoint cycles in tournaments

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(This talk is based on joint work with Fuhong Ma.)

MSC2000: 05C20

It was conjectured in [2] that for given positive integers q at least 3 and k, any tournament with minimum out-degree at least $(q-1)k-1$ contains at least k disjoint cycles of length q. In this talk, we provide a proof of the conjecture. Our result is also an affirmative answer concerning tournaments to the conjecture of Bermond-Thomassen[1].

Keywords: Tournaments; Minimum out-degree; Disjoint cycles

[1] J. C. Bermond, C. Thomassen, Cycles in digraphs-a survey, J. Graph Theory 5 (1) (1981) 1-43.

[2] N. Lichiardopol, Vertex-disjoint directed cycles of prescribed length in tournaments with given minimum out-degree and in-degree, Discrete Math. 310 (19) (2010) 2567-2570.

Friday 14:30, Arts LR1

Spanning surfaces in 3-uniform hypergraphs

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(This talk is based on joint work with Agelos Georgakopoulos, Richard Montgomery and Bhargav Narayanan.)

MSC2000: 05E45 (05C65, 05C35)

I will discuss a topological extension of Dirac's theorem to 3-uniform hypergraphs, which solves a problem suggested by Gowers in 2005.

One of the most fundamental questions about global structure in a graph is whether there is a Hamilton cycle, that is, a cycle through all the vertices. This is one of Karp's original list of NP-complete problems, and there have been a wealth of extremal results, beginning with the classical theorem of Dirac giving the best possible minimum degree condition which will guarantee a Hamilton cycle.

Because of the central importance of this question, a number of different extensions to hypergraphs have previously been considered. However, all of these treat a Hamilton cycle in a hypergraph as a rigid pattern of interlocking edges with respect to some cyclic ordering of the underlying vertex set, and are consequently inherently one-dimensional notions. The best known of these is the so-called tight cycle, and Rödl, Ruciński and Szemerédi proved the analogue of Dirac's theorem for tight cycles in 3-uniform hypergraphs.

A more natural viewpoint is that a Hamilton cycle is a topological circle covering all vertices, and so for 3-uniform hypergraphs one would ask for a topological sphere covering all vertices. This gives a genuinely higher-dimensional question, which fits naturally with the recent body of work treating random hypergraphs as random simplicial complexes. Indeed, the sharp threshold for a spanning sphere to appear in the binomial random hypergraph was recently found by Luria and Tessler in this context.

I will address the extremal question of the minimum codegree needed to guarantee the existence of such a structure, giving an asymptotically tight bound. The result is not specific to the sphere, but applies to any given surface.

Rainbow matchings in edge-colored graphs with given average color degree

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MSC2000: 05C15

Let G be a simple edge-colored graph with n vertices. A subgraph of G is rainbow if all of its edges have distinct colors. The color degree $d^c(v)$ of $v \in V(G)$ is the number of distinct colors on the edges incident with v. Let $\overline{d^c}_G$ denote the average color degree of G, i.e., $\overline{d^c}_G = \sum_{v \in V(G)} d^c(v)/n$. As a classic problem, the existence of rainbow matchings has been widely studied in proper edge-colored graphs with given minimum color degree. Kritschgau[1] generalized this problem to edge-colored graphs with given average color degree. Kritschgau proved that every edge-colored graph G on $n \geq 3k^2 + 4k$ vertices with $\overline{d^c}_G \geq 2k$ contains a rainbow matching of size k. In this paper, we improve the result above and show that $n \geq 4k - 4$ is sufficient.

[1] Jürgen Kritschgau, Rainbow matchings of size m in graphs with total color degree at least $2mn$, arXiv:1810.05324v1, 2018.

Friday 14:30, Arts LR3

Graph Functionality

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(This talk is based on joint work with Aistis Atminas and Vadim Lozin.)

MSC2000: 05C62, 05C30

Let $G = (V, E)$ be a graph and A its adjacency matrix. We say that a vertex $y \in V$ is a function of vertices $x_1, \ldots, x_k \in V - y$ if there exists a Boolean function f of k variables such that for any vertex $z \in V - \{y, x_1, \ldots, x_k\}$, $A(y, z) = f(A(x_1, z), \ldots, A(x_k, z))$. The functionality $\mathrm{fun}(y)$ of vertex y is the minimum k such that y is a function of k vertices. The functionality $\mathrm{fun}(G)$ of the graph G is $\max_H \min_{y \in V(H)} \mathrm{fun}(y)$, where the maximum is taken over all induced subgraphs H of G.

In this talk, I will present some results regarding boundedness of functionality in various graph classes, and relating functionality to other graph parameters including clique-width. I will then discuss functionality in the context of graph representations, looking in particular at implicit representations. Finally, I will propose some open problems.

[1] A. Atminas, A. Collins, V. Lozin, and V. Zamaraev, Implicit representations and factorial properties of graphs, *Discrete Mathematics*, 338 (2015) 164–179.

[2] S. Kannan, M. Naor, S. Rudich, Implicit representation of graphs. *STOC* 1988: 334–343.

Friday 14:30, Arts LR5

Sequences in Groups with Distinct Partial Products

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(This talk is based on joint work with Kat Cannon-MacMartin, Jacob Hicks and John Schmitt.)

MSC2000: 20D60, 05B99

Let G be a group of order n with identity e. For a sequence of elements $(g_1, \ldots, g_k)$ of G, define its *partial products* $(h_0, \ldots, h_k)$ by $h_0 = e$ and $h_i = g_1 \cdots g_i$ for $1 \leq i \leq k$. For which subsets $S \subseteq G \setminus \{e\}$ is it possible to order the elements of S so that the partial products are distinct?

When $|S| = n-1$, this is the well-studied question of whether G is sequenceable, a question that remains open for many groups. For $|S| < n-1$ the question arises in relation to graph decompositions and Heffter systems. Alspach has conjectured that when G is cyclic it is always possible to order the elements of S in this way provided that the product of the elements in S is not the identity.

We show how the problem may be approached using Alon's Combinatorial Nullstellensatz when $|G| = mp$ for prime p and small m, translating the problem into one of finding monomials with non-zero coefficients in particular polynomials over $GF(p)$. Among other results, this lets us prove Alspach's Conjecture when $|S| \leq 10$ and $|G|$ is prime, and determine when a subset S, with $|S| \leq 9$, of the dihedral group of order twice an odd prime has the desired ordering.

Disjoint Cycles in a digraph with Partial Degree

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(This talk is based on joint work with Jin Yan.)

MSC2000: 05C20

Let $D = (V, A)$ be a digraph of order n and let W be a subset of V with $|W| \geq 2k$, where k is an integer. Suppose that every vertex of W has semi-degree at least $(3n-3)/4$ in D. Then for any k integers $n_1, \ldots, n_k$ with $n_i \geq 2$ $(1 \leq i \leq k)$ and $\sum_{i=1}^{k} n_i \leq |W|$, D contains k disjoint (directed) cycles $C_1, \ldots, C_k$ such that $|V(C_i) \cap W| = n_i$ for all $1 \leq i \leq k$. This result partially answers the question posed by Hong Wang [1]. Moreover, the condition on semi-degree is sharp.

Keywords: Disjoint cycles; Partial degree; Semi-degree conditions

[1] H. Wang, Partition of a subset into two directed cycles, manuscript.

A proof-theoretic analysis of the rotation lattice of binary trees

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MSC2000: 03F03, 05C30, 05C99

The classical Tamari lattice Y_n is defined as the set of binary trees with n internal nodes, with the partial ordering induced by the (right) rotation operation. It is not obvious why Y_n is a lattice, but this was first proved by Haya Friedman and Dov Tamari in the late 1950s. More recently, Frédéric Chapoton discovered another surprising fact about the rotation ordering, namely that Y_n contains exactly $\frac{2(4n+1)!}{(n+1)!(3n+2)!}$ pairs of related trees. (Even more surprisingly, this formula was already computed by Tutte in the early 1960s, just in a completely different context: enumeration of planar maps!)

In the talk I will describe a new way of looking at the rotation ordering motivated by old ideas in proof theory. This will lead us to systematic explanations of: 1. the lattice property of Y_n, and 2. the Tutte-Chapoton formula for the number of intervals in Y_n.

[1] H. Friedman and D. Tamari. Problèmes d'associativité: une structure de treillis finis induite par une loi demi-associative. *Journal of Combinatorial Theory*, 2:215–242, 1967.

[2] F. Chapoton. Sur le nombre d'intervalles dans les treillis de Tamari. *Séminaire Lotharingien de Combinatoire*, (B55f), 2006. 18 pp. (electronic).

[3] W. T. Tutte. A census of planar triangulations. *Canadian Journal of Mathematics*, 14:21–38, 1962.

[4] J. Lambek. The mathematics of sentence structure. *The American Mathematical Monthly*, 65(3):154–170, 1958.

[5] N. Zeilberger. A sequent calculus for a semi-associative law (extended version). *Logical Methods in Computer Science*, 15(1):1–23, 2019.

Minimizing the number of copies of K_r in a K_s-saturated graph

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(This talk is based on joint work with P. Loh.)

MSC2000: 05Dxx

Graph saturation is one of the oldest areas of investigation in extremal combinatorics. A graph G is called F-saturated if G does not contain a subgraph isomorphic to F, but the addition of any edge creates a copy of F. We resolve the most fundamental question of minimizing the number of cliques of size r in a K_s-saturated graph for all sufficiently large numbers of vertices, confirming a conjecture of Kritschgau, Methuku, Tait and Timmons. We further prove a corresponding stability result.

Mortality and synchronization in unambiguous automata

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(This talk is based on joint work with D. Perrin.)

MSC2000: 68Q45, 68Q70, 20M35, 05D99

Given a set $M_1, \ldots, M_k$ of $n \times n$ matrices over non-negative integers, the mortality problem asks for the shortest product of these matrices producing the zero matrix. Thus, we are looking for a shortest product of generators producing the zero matrix in the monoid $\mathcal{M}$ generated by $M_1, \ldots, M_k$. A special case which is important for coding theory and automata theory is where $\mathcal{M}$ is finite and transitive. The monoid $\mathcal{M}$ is *transitive* if for every pair i, j there is a matrix with a positive entry at the position (i, j). Finiteness and transitivity are equivalent to the fact that $\mathcal{M}$ is transitive and contains only $0, 1$-matrices. We concentrate on this case. One can consider a non-deterministic finite automaton (NFA) with the state set $Q = \{1, 2, \ldots, n\}$, alphabet $\Sigma = \{a_1, \ldots, a_k\}$ and the transition relation Δ repeating for the letter a_i the action of M_i on the set of row indices. Starting from a finite transitive monoid $\mathcal{M}$ of matrices, we then obtain a strongly connected unambiguous automaton $\mathcal{A}$. Strong connectivity means that for any pair p, q of states there is a word labeling a path from p to q. Unambiguity means that for any states p, q and any word w there is at most one path from p to q labeled by w. Mortal words then are exactly the words mapping every state to the empty set.

Another important concept is the notion of synchronizing words. A word is called *synchronizing* if it corresponds to a matrix of rank 1. Synchronizing words allow us to reset an unambiguous automaton and to control its behaviour. Let $\mathcal{A}$ be a n-state strongly connected unambiguous automaton. It is called *incomplete* if it admits a mortal word, otherwise it is called *complete*. It is called *synchronizing* if it admits a synchronizing word. Carpi (Theoretical Computer Science 60: 285–296, 1988) found a $\frac{n^3}{2}$ upper bound on the length of a shortest synchronizing word in $\mathcal{A}$ if it is complete and synchronizing. Kiefer and Mascle (STACS 2019) obtained a n^5 upper bound on the length of a shortest mortal word for $\mathcal{A}$ if it is incomplete. We show that there is a n^5 upper bound on the length of a shortest synchronizing word for $\mathcal{A}$ if it is synchronizing, not requiring it to be complete.

There is a natural correspondence between a strongly connected unambiguous automaton $\mathcal{A}$ and the star X^* of a variable length code X it decodes (Berstel, Perrin, Reutenauer, "Codes and Automata", Cambridge University Press). A word is mortal for $\mathcal{A}$ if and only if it is not a factor of some word in X^* (a word w is called a *factor* of w' if $w' = uwv$ for some words u, v). A word w is synchronizing for $\mathcal{A}$ if for any words u, v such that $uwv \in X^*$ we have $uw, wv \in X^*$. We show that if a n-state strongly connected unmabiguous automaton decodes the star of a finite code, the upper bound on the length of a shortest mortal word for it can be lowered to $n^4 \log n$.

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