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School of Mathematics Newsletter

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Farewell Peter

One of the longest serving members of the School, Professor Peter Butkovič, is retiring after 40 years as a university lecturer. Some of Peter's colleagues share their personal reflections.

Michal Kočvara

Peter was born in Czechoslovakia. During the communist regime lives of its citizens were heavily controlled and restricted. Gifted people in particular suffered from lack of opportunities for a rewarding career. Some of them found refuge in scientific research, especially in Mathematics, one of the least political disciplines. Lucky Mathematics. The Prague mathematical scene in the 60s and 70s was inhabited by names like Jindřich Nečas (nonlinear PDEs), Ivo Babuška (numerical analysis), Jaroslav Kurzweil (analysis; Henstock–Kurzweil integral), František Nožička (parametric optimization), Vlastimil Pták (analysis; linear algebra; M- and P-matrices) and Miroslav Fiedler (linear algebra; graph theory; Fiedler vector). Peter was one of the children of this productive era. As a PhD student of Fiedler, he gained thorough knowledge in discrete mathematics, linear algebra and graph theory. Very soon, still during his PhD studies, Peter found his passion: max-algebra (or tropical mathematics). It became the main topic of his scientific life. The fall of communism allowed Peter to travel abroad and he went directly to the source: in 1991 he joined one of the first pioneers of max-algebra Professor Raymond Cuninghame-Green in Birmingham. Starting as a Research Fellow, then all the way up to Professor in Applied Discrete Mathematics.

Both Peter and I studied at the Charles University in Prague. He was three years my senior so it is likely that we bumped into each other in the corridors of the Mathematics department. But we really met many years later in Birmingham in 2006. Since then I have always appreciated his high professional standards and valued his friendship. I wish him all the very best in the new stage of life and look forward to staying in touch.

Sergey Sergeev

My first contact with Peter occurred in 2005 when I wrote my first mathematical paper in English. It was then sent to Peter as an associated editor in the journal *'Linear Algebra and its Applications'*. Peter got interested in my work and invited me to join his collaboration with Hans Schneider. We started exchanging emails and this resulted in our first joint paper *'Generators, extremals and bases of max cones'*. In Autumn 2006, I got to know Peter in person when I visited him in Birmingham, together with Hans Schneider and Stephane Gaubert. I still remember how thrilling this first experience of our first joint mathematical discussion was for me, and new ideas and works that came out of it. That visit convinced me even more that I should continue working as a mathematician and, when the opportunity opened, I became a research fellow working with Peter on his research grant. As a research fellow I went with Peter to a number of international conferences and events. One of the earliest of them was the ILAS conference in Cancun (Mexico), where I got acquainted with a number of renowned researchers in Linear Algebra and saw the famous pyramids of Chichen-Itza.

Working with Peter was extremely fruitful. He generously shared with me many of his research ideas on a number of topics in max algebra (two-sided systems, diagonal similarity scaling, max-algebraic matrix powers and others). Besides that, he met with me regularly each week at a particular time. This made me feel more secure and helped us

to move our joint research forward. When I wrote something, Peter would read it with care and return it to me with innumerable but always essential questions and corrections to which I had to respond promptly and accurately.

Later Peter also introduced me to the art of grant writing, when he was applying for a grant and trusted me to read it and give my views. He would compare it to writing a science fiction book or trying to predict future. This experience also helped me when I applied for my first EPSRC grant during my probation years.

Peter also cared to make me and our frequent visitor Hans Schneider feel comfortable and enjoy our lives. Each time when Hans visited us Peter would take us to his favourite restaurants and places, of which Toby's Carvery and tea rooms in Brighton were perhaps the most enjoyable.

During the last ten years of my life Peter has been my teacher, my mentor and my friend. Now when Peter is retiring, I wish him to enjoy life and I am looking forward to his new works and hoping to continue my friendship and my collaboration with him.

Daniel Jones

My first experience of Peter was as a second year Undergraduate, back when he was lecturing Linear Programming. I knew as early as that that he had a gift for teaching and it was around this time that I decided I was going to focus on the Optimisation disciplines in the subsequent years of my studies. I continued to prioritise Peter's courses in the following years (Combinatorial Optimisation in the third year and Game Theory in the fourth) due, in no small part, to Peter's enthusiasm for his work.

I'd been unsure throughout most of my Undergrad years about what I should like to do at the end of it, but I knew at this point that I wanted to work with Peter – despite having no idea what his research actually entailed! He was kind enough to take me under his wing for my Master's year and I was introduced to the wonderful world of tropical algebra.

I sometimes wondered if it was the Maths that I loved but as time progressed through the PhD years and further still to today, I came to realise that it was, in fact, the combination of Peter's kind nature and high expectations that allowed me to flourish.

I owe a lot to Peter – he believed in me when others might not have and I hope I served him justice by producing a high quality of work that makes him proud. I'm excited to be part of a small group that will continue to build on his work in the coming generations and it's my aim not only to answer many of the exciting questions that Peter posed over the years but to do so in a way that is elegant, yet powerful – I know he would approve of that.

Peter is one of the most popular members of the School, among students and staff alike. The whole School wishes Peter well in his retired life, and hope to still see him regularly around the department!

Peter Butkovič

Welcome Marta!

Professor Marta Mazzocco recently joined the School of Mathematics from Loughborough University where she held a personal chair position since 2014, as well as the role of Director of Equality and Diversity for the School of Science. A hugely energetic mathematician, she delivers ambitious and imaginative academic leadership by anticipating research trends and attracting substantial external funding.

Professor Mazzocco is a specialist in the area of integrable systems, namely mathematical problems often motivated by mathematical physics that present unexpected beauty – a serendipity of geometric/algebraic/analytical structures that make a very complicated problem solvable.

Marta's research brings geometry, quantum algebra and analysis together to describe and tackle problems which have so far resisted all other methods. Speaking on her appointment, she said: "I look forward to start a new group in Geometry and Mathematical Physics here at Birmingham, an area of excellence in the UK originating from Sir Michael Atiyha's school that complements the existing outstanding expertise in School of Mathematics and in the School of Physics and Astronomy."

Professor Mazzocco is also a keen advocate of the importance of basic research and its role in so-

ciety. Commenting on this, she said: “I believe that in the current funding landscape that emphasises socio-economic impact, it is paramount to nurture interdisciplinary research as well as to raise the profile of the unique contribution that Mathematical Sciences brings to the overall research base, the economy and society”. This belief has driven her to become a member of the Council of the European Mathematical Society; of the Institute of Mathematics and Applications Research Committee; and of the London Mathematical Society Nominating Committee.

Summarising her ambitions for the school and its impact both for staff and research, she said:

“I am keen to make a difference at Birmingham University. My vision is that of a School that supports its staff to grow and thrive; widens the pool of talent both in student and staff recruitment; and is pace setting in its equality and diversity provision, as well as in its international research.”

Marta Mazzocco

ERC Advanced Grant success

Mason Chair, Professor Daniela Kühn has recently been awarded an ERC Advanced Grant to study *‘Extremal Combinatorics: existence, counting and typical structure’*. Professor Deryk Osthus is the co-investigator on the grant, which is valued at 1.8 million euros. Head of School, Professor Paul Flavell stated ‘This is an extremely exceptional achievement. Indeed there were only 9 Advanced grants awarded in mathematics throughout Europe this year. Daniela’s is the only one going to the UK.’

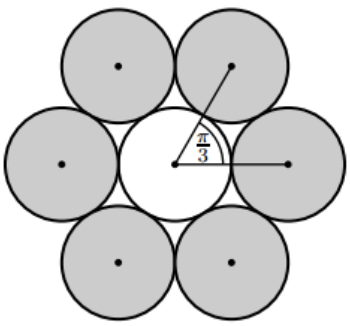
The ERC’s website states that their ‘mission is to encourage the highest quality research in Europe through competitive funding and to support investigator-driven frontier research across all fields, on the basis of scientific excellence’. In particular, the ERC Advanced Grant scheme is designed for ‘established, leading principal investigators who want long-term funding to pursue a ground-breaking, high-risk project’. In this project, Daniela and Deryk will consider questions concerning *existence, counting and typical structure* with a focus on inter-related topics involving combinatorial designs, decompositions, Latin squares as well as matchings in graphs and hypergraphs. The project themes have close connections e.g. to statistical physics, probability, algebra and theoretical computer science.

Spheres and kissing numbers

by FELIX JOOS AND WILL PERKINS

How densely can unit spheres be packed into d-dimensional Euclidean space? Whereas this question is not too difficult for $d = 2$, for $d = 3$ it is one of Hilbert’s celebrated 23 problems published in 1900. It was finally resolved by Hales in 1995 using a computer-assisted proof.

For general d, there is a very simple lower bound of 2^{-d}: take any saturated packing; doubling the radii of the spheres must cover all of $\mathbb{R}^d$, or else another center could be added. Around 1900, Minkowski improved this to $2\zeta(d) \cdot 2^{-d}$. The first result that improved asymptotic order of the lower bound is due to Claude Ambrose Rogers in 1947 (Annals of Mathematics, 48, 994–1002) who proved a lower bound of $\Theta(d \cdot 2^{-d})$. Strikingly, until today the best lower bound improves upon Rogers’ result only by a constant factor although the upper bound due to Kabatiansky and Levenshtein is significantly larger on an exponential scale. Interestingly, a few years later (1954–1958) Rogers was the Mason Professor of Mathematics at the University of Birmingham, the chair currently held by Daniela Kühn.



An optimal kissing and sphere packing configuration in dimension 2.

A closely related problem to sphere packing is the kissing number problem. It asks for the maximum number of non-overlapping unit spheres that can touch another central unit sphere. In 1694, Issac Newton and David Gregory disagreed on the kissing number in three dimensions and therefore the kissing number became known as the Newton number. It is easy to see that the kissing number problem can be equivalently phrased by asking how many non-overlapping spherical caps of angular radius $\pi/6$ can be placed onto the unit sphere (a specific case of *spherical codes*). Similarly as with spheres, by doubling the radii of the spherical caps we obtain a simple lower bound on the kissing number. This was first observed by Wyner and Shannon in the 1950's and 1960's. This simple lower bound had not been improved at all in the last 60 years, primarily because lattice-based approaches used for the sphere packing problem do not seem to work here. Closing the circle to Rogers, we together with Matthew Jenssen from Oxford, were able to improve this lower bound by a factor of d—exactly as Rogers did for the sphere packing problem (arXiv:1803.02702). Our proof uses the hard sphere model from statistical physics, a departure from previous methods of packing and covering problems.

Richard E(PSRC) Grant!

Recently, Dr Richard Mycroft was awarded a 3-year EPSRC grant to study 'Properties of extremal and random hypergraphs'. Below Richard describes what the project entails.

The broad focus of this research project is to generalise certain important recent developments in the theory of graphs to the hypergraph setting. Hypergraphs, like graphs, consist of vertices and edges, but whereas edges of a graph connect pairs of vertices, in a k-uniform hypergraph (k-graph for short) each edge consists of k vertices (so in particular a 2-graph is precisely a graph). The study of graphs is motivated by the way that edges of a graph represent common properties of a pair, such as being connected in a physical or abstract network, and in the same way a k-graph can be used to represent properties shared by collections of larger size. Such representations are often not as immediately apparent, but have been used to achieve numerous important advances both within mathematics and in applications to other areas. With this context in mind, this project aims to develop the theory of hypergraphs in two key areas, namely edit distances and embeddings of connected structures.

The edit distance between two graphs or k-graphs is a simple metric describing how different those graphs are: how many edges do you have to add and/or remove to transform one graph into the other (appropriately normalised)? Measuring distance like this arises naturally in various applications in biology and computer science. Our focus is on how far (in terms of edit distance) a k-graph can be from k-graphs satisfying a hereditary property P ('hereditary' means that if you delete vertices from a k-graph satisfying P then the resulting graph still satisfies P). In the graph case a seminal theorem of Alon and Stav states that the maximum distance is asymptotically attained with high probability by a random graph $G(n,p)$, which is formed on n vertices by including each pair as an edge with probability p independently of all other choices. However, even in the graph case there remain many open questions about the behaviour of this distance for various important graph properties, and in the hypergraph setting very little is known. Using recently-developed structural tools for hypergraphs I aim to generalise much of the known theory of edit distances for graphs to the hypergraph setting, with a key aim being to generalise the theorem of Alon and Stav for k-graphs.

The basic question of embeddings is whether you can find a copy of a given graph H within a given host graph G, or more widely what conditions on G guarantee the existence of a copy of H. The various forms of this question represent a vast and active area of research which over the past 20+ years has driven the development of important structural tools for graphs and more recently of hypergraphs. In the graph case this programme has been very successful, with advances such as the bandwidth theorem, which generalises many classical theorems of graph theory by giving very general conditions which ensure the presence of given spanning subgraphs within a large graph. However, our understanding for k-graphs is much more limited; for example, the number of edges in an n-vertex 3-graph which guarantees the presence of a com-

plete 3-graph on 4 vertices as a subgraph remains a key open problem. In particular, handling connected structures is a major hurdle to successfully generalising the full theory of embeddings for graphs to the hypergraph setting; the structural results developed so far for hypergraph embeddings fail to deal with connectivity in a satisfactory way. The second main focus of this project is develop a recent new approach I developed with Peter Allen, Julia Böttcher and Oliver Cooley to overcome this obstacle to embedding connected structures in k-graphs and so to generalise key known results about embeddings of graphs to the hypergraph setting.

Workshop on the role of mathematics in combatting antibiotic resistance and developing novel antibacterials

by SARA JABBARI

This workshop – held at the Fields Institute – brought together researchers from a variety of disciplines, united by a common application: a desire to use their expertise to understand and improve antimicrobials. Antimicrobial resistance (AMR) to current antibiotics, combined with a stagnation in novel antibiotic discovery in recent years, has resulted in a dramatic need to harness the power of multiple disciplines to tackle this problem. We brought together mathematicians, physicists, computer scientists and biologists to discuss and share approaches.

Furthermore, not only did our participants use a range of disciplines, they also adopt an array of different routes to tackle AMR. Topics covered included, amongst others, understanding and harnessing intracellular and intercellular mechanisms used by the bacteria themselves (e.g. transfer of plasmids containing AMR genes, gene expression levels), the interplay of bacteria in its various forms with the immune system, understanding the physical properties of biofilms and cell adhesion, manipulating other microorganisms to kill pathogenic bacteria (e.g. phage therapy and predatory bacteria), and the discovery of novel compounds for drug development.

Early career researchers mixed with scientists from academia and industry at all career stages from around the globe. We heard from experts in differential equation modelling, stochastic processes, biomechanics, machine learning, synthetic biology, chemical engineering, model parameterisation and microbiology, to name just a subset of the techniques covered. Crucially, our participants worked in groups dealing with experimental data for model development and prediction testing, either within their own laboratories or with the help of experimental collaborators. Such an approach thus optimises the reliability of the theoretical work and takes vital steps towards using the mathematical and physical sciences to tackle the worldwide problem that is AMR.

Maths Poster Competition

by ALBERTO ESPUNY DIAZ

On January 31st, the School hosted a Maths Poster Competition for our postgraduate students. A total of ten participants from all years entered from the different research groups of the School, presenting their posters to the more than fifty people who attended the session.

The main goal of this competition is to provide an opportunity for students to practice giving a poster presentation. This is a basic ability needed in academia, as posters are one of the main formats in which research is presented. The poster sessions are also a good chance for networking inside the School over some tea, coffee and biscuits, and this year's session was a great success in this respect too. The Maths Poster Competition will become an annual event, organised by the Postgraduate Student Committee.

On this first edition of the Maths Poster Competition, funded by the Postgraduate Development Fund, there was a first prize for the best poster in applied mathematics and a first prize for the best poster in pure mathematics. There were also combined second and third prizes. In addition to this, all posters are being displayed on the corridors of the Watson building, showcasing the excellent research undertaken by our PhD students.

The members of the jury who evaluated the presentations all agreed that the level of the posters was excellent overall. The prize for the best poster in pure mathematics was awarded to Padraig Condon, for his poster *'A bandwidth theorem for approximate decompositions'*. The prize for the best poster in applied mathematics was awarded to Leonardo Rocchi, for *'Efficient adaptive algorithms for elliptic PDEs with random data'*. The second prize was awarded

to Gianmarco Brocchi for *'Fourth order Schrodinger equation and Strichartz estimates: an extreme adventure'*. Finally, Tássio Naia got the third prize for his poster *'Inevitable patterns in directed networks'*.

All the information about this and future editions of the maths poster competition will be kept on the following website.

Analysis workshop

The Analysis Group recently hosted a one day meeting of the LMS Scheme 3 Harmonic Analysis and PDEs network. The event was co-funded by the School of Mathematics and the European Research Council. Currently the network includes Birmingham, Edinburgh, Madrid and Warwick as its nodes.

Around 20 people attended the workshop, including a good number of postgraduate students from the other nodes of the network. There were four talks, delivered by Michael Lacey (Georgia Institute of Technology), Giuseppe Negro (Instituto de Ciencias Matemáticas), Luz Roncal (Basque Center for Applied Mathematics), and Julien Sabin (Université Paris-Sud). The talks focused on exciting new developments in continuous and discrete harmonic analysis and the analysis of PDEs, including sharp Fourier restriction theory, extension problems, and sparse domination. They were typically followed by several questions and animated discussion.

This stimulating and exceptionally warm day ended with a nice, informal dinner in the historical Jewellery Quarter district in Birmingham.

News in Brief

- Congratulations go to Alex Bespalov who was recently promoted to Senior Lecturer, and to Nikolaos Fountoulakis who was promoted to Reader in Probabilistic Combinatorics.
- A reminder that there is still time to make use of the College Research Travel fund this financial year. Money can be spent on travel related to research (and could also pay for visitors to come here). Requests should be directed to the School Operations Manager, Hannah Roberts.

Professor Paul Flavell congratulating Professsor Chris Good at his inaugural lecture, Jan. 2018