PhD PROJECT PROPOSAL

PhD PROJECT TITLE

Topology optimization of mechanical structures with stability constraints.

PhD SUPERVISORY TEAM

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PhD PROJECT DETAILS

Project abstract

The aim of the project is to study and improve modelling and numerical techniques in topology optimization of mechanical structures under constraints on global stability of the optimal structure. Software based on the developed models and algorithms will be capable of finding optimal topology (material distribution) of complex three-dimensional elastic bodies.

Application of Topology

Topological design is the very topic of this project, in particular optimization of "topology" (material distribution) of mechanical structures.

DETAILED PROJECT DESCRIPTION

Nowadays, topology optimization of mechanical structures is a well-established technique that has been translated to commercial software and, through it, to industry (automotive, aircraft and many others). Despite this, there are still many challenges and open problems when it comes to reliable optimal design of complex three-dimensional structures. One of the main ones is the elastic stability of the optimal design.

"*Experience showed that structures may fail in some cases not on account of high stresses but owing to insufficient elastic stability.*" —S.P. Timoshenko

It was first shown in [1,2] that the problem of optimal topology design with constraints on elastic stability (linear buckling) can be formulated as a nonlinear nonconvex semidefinite

optimization problem (SDP). Although first formulated for truss topology optimization, in [3,4] it was shown that the formulation can be directly extended to topology optimization of solid structures discretised by the finite element method.

Solving nonlinear SDP is a challenging problem and, currently, there is only one available code PENNON [5]. This is, however, a general purpose software, unable to solve large-scale problems resulting from topology optimization.

The goal of the project is two-fold

1. **Topology optimization of solids with stability constraints by domain decomposition**.
The technique of domain decomposition of linear SDP problems arising from topology optimization was first investigated in [6]; it was shown that it allows to solve very large problems, as compared to the standard SDP software. The goal of this part is to extend the technique to nonlinear SDP formulation of topology optimization problem with stability constraints and to develop a first version of software for large scale 3D problems.

2. **Truss topology optimization with stability constraints as low-rank SDP**.
The solution of the SDP problem arising in topology optimization with stability constraints is a low-rank matrix, where the rank corresponds to the multiplicity of the buckling mode. Recently, it was shown that these problems (in linear setting) can be solved by penalty or barrier optimization algorithms [7]. In every iteration of such algorithm, one has to solve a system of linear equations. These equations can be solved by an iterative method with a preconditioner specifically using the low rank information. The goal of the project is to extend this technique to nonlinear SDP problems arising from topology optimization problems with stability constraint.

References:

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