

The Impact of Forest Roads on The Rate of Biological Invasion

Introduction

- Biological invasion of plant and tree species pose a major threat both to the ecosystem and economy [1].
- A significant amount of mathematical modelling has been conducted to simulate biological invasion [2].

Advantages of mathematical modelling:

- It is cheaper than field work
- Allows you to consider both small and large spatial scales of invasion
- It does not harm or disturb the environment in any way
 - It allows you to 'digitally' add an invasive species into an environment and investigate the consequences without introducing the invasive species into the real world
- We can predict (with reasonably good accuracy) the future spatial-temporal dynamics of the invasive species.

However, there is no 'universal' model of biological invasion.
Different models:

- Partial differential equation based models (reaction-diffusion framework):
 - Host-parasite
 - Prey-predator
 - Herbivore-grazer
- Integro-difference equation based models (dispersal kernel framework)

Dispersal kernel based models are ideal for modelling plants, as the model:

- provides a more realistic description of the real world by simulating the stages of a plant's growth and dispersal
- treats the growth and dispersal stage of the plant independently
- allows modelling of long distance dispersal



Fig 1: Japanese Knotweed is a highly problematic invasive species, by DEFRA it is estimated the cost of total control across Britain is approximately £1.56 Billion [3]. **Image:** rhs.org.uk

Model

We will be using a stage structured dispersal kernel based model N

$$N(\underline{r}^*)_{t+1} = \int_{\Omega} k(\underline{r}^*, \underline{r}) F(N_t(\underline{r})) d\underline{r} \quad (1)$$

- $\underline{r}^* = (x^*, y^*)$ - position where we can detect the invasive species
- $N(\underline{r}^*)_{t+1}$ - the population density at \underline{r}^* at time $t + 1$ (to be computed from the model)
- $N(\underline{r}^*)_t$ - the given population density at time t
- The population is stage-structured:

The growth stage - the population grows but there is no dispersal. The growth function is $F(N_t(\underline{r}))$.

The dispersal stage - the population disperses across the domain but does not grow. The dispersal kernel is $k(\underline{r}^*, \underline{r})$.

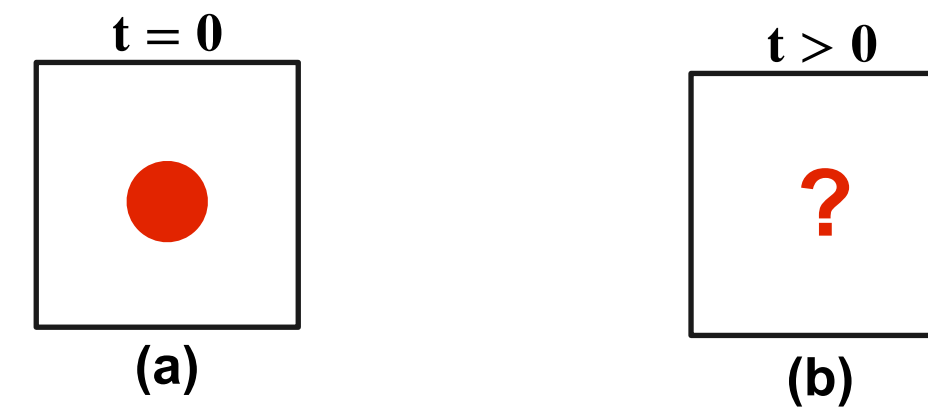


Figure 2: The summary of the invasion problem. **(a)** The initial distribution of the invasive species at time $t = 0$; the species is found in the small sub-domain (a red disk in the figure marks a region of non-zero density). The white colour in the panel corresponds to zero population density. **(b)** How will the invasive species spread over the domain as time progresses?

Previous Work

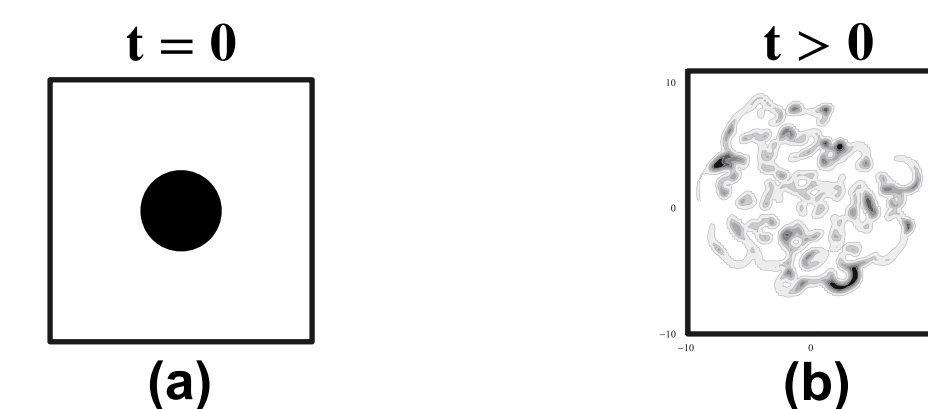


Figure 3: Example of spatio-temporal dynamics of the invasive species [4].

Stage structured dispersal kernel based models have already been used to model:

- Propagation of a single invasive species in 1 and 2 dimensions
- Interaction of an invasive species with another species (see Fig. 3)

However, the previous work assumes a simple domain geometry (no road!)

My Research

The Research Hypothesis:

- Roads provide an ideal environment for invasive species to spread [5].

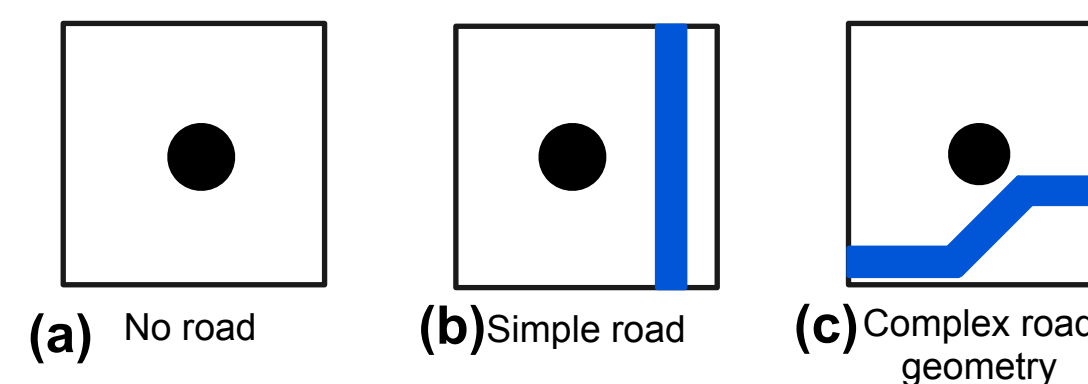


Figure 4: Various domain geometries (road is coloured in blue, a black disk marks a region of non-zero density).

Modelling: We use a different 'rule' of spread in the 'road' sub domain.

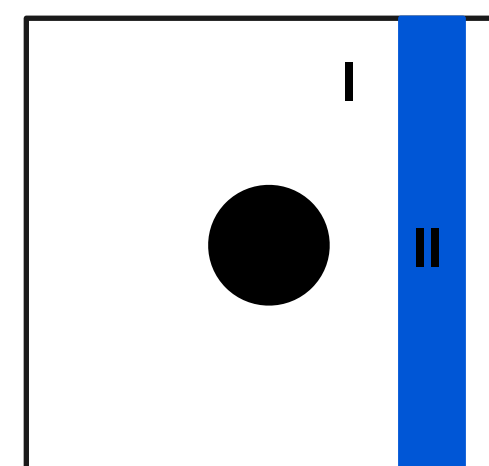
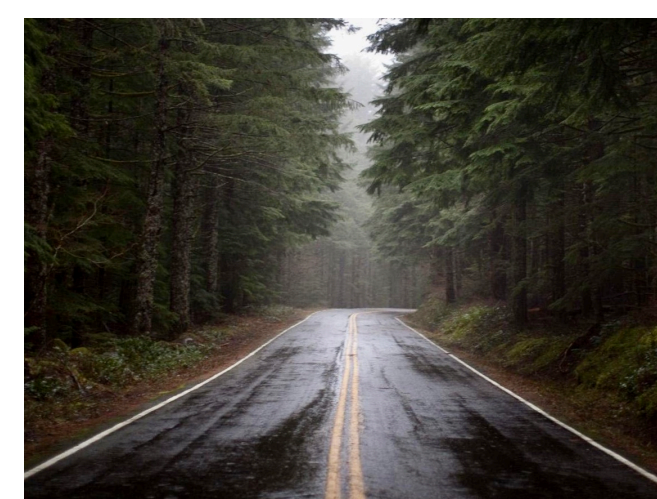


Figure 5: I - the forest sub-domain, II - the road sub-domain.

The different rule of spread is simulated by using different timescale and different dispersal kernels in the model

One-Dimensional Case

We first study the 1-D version of the model (1) to gain insight.

- The invasive species spreads along the interval $x \in [-L, L]$.
- The growth function is $F(N(y)) = 7N(y)\exp(-N(y))$
- The dispersal kernel is $k(x, y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-y)^2}{2\sigma^2}\right)$, where $\sigma = 0.3$

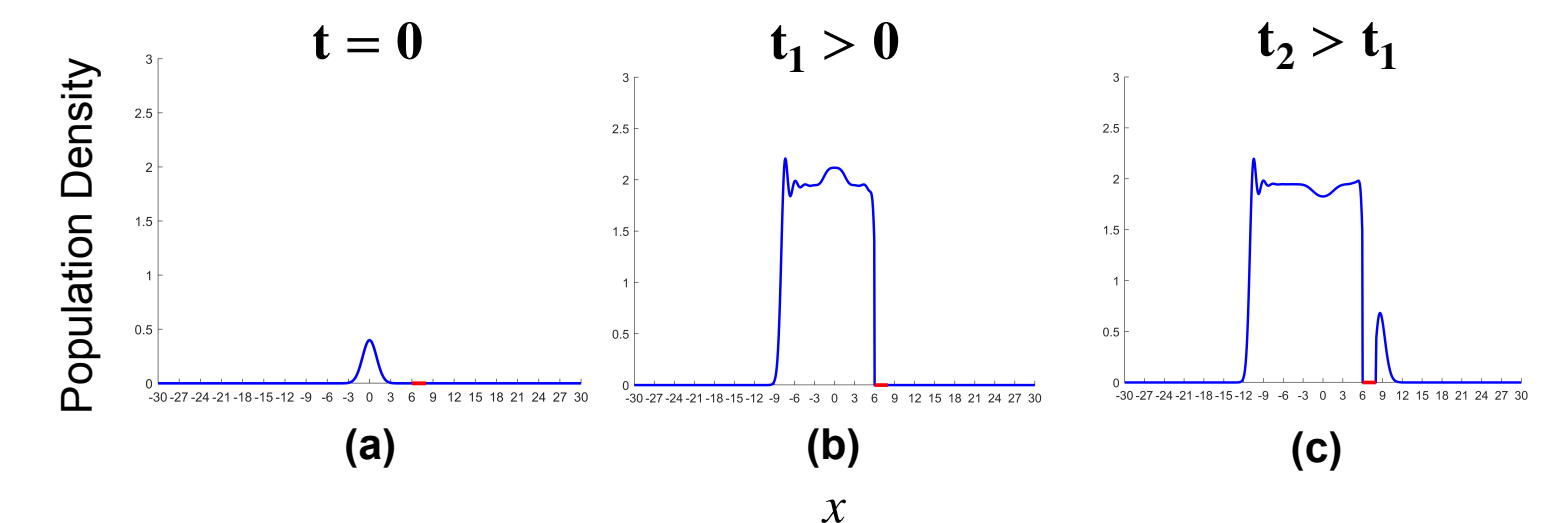


Figure 6: **(a)** Initial distribution of the invasive species, the 'road' is marked in red. **(b)** The population makes minimal interaction with the 'road'. The 'road' begins to make an impact on the spatial distribution of the invasive species: the overall symmetry of the population distribution has collapsed. **(c)** The area behind the road is now invaded.

Two-Dimensional Case

We then study the more realistic 2-D case using model (1).

- The invasive species spreads within the space $\underline{r} = (x, y)$, where $x, y \in [-L, L]$.
- The growth function is $F(N(\underline{r})) = 7N(\underline{r})\exp(-N(\underline{r}))$
- The dispersal kernel is $k(\underline{r}, \underline{r}^*) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{|\underline{r} - \underline{r}^*|^2}{2\sigma^2}\right)$, where $\sigma = 0.3$

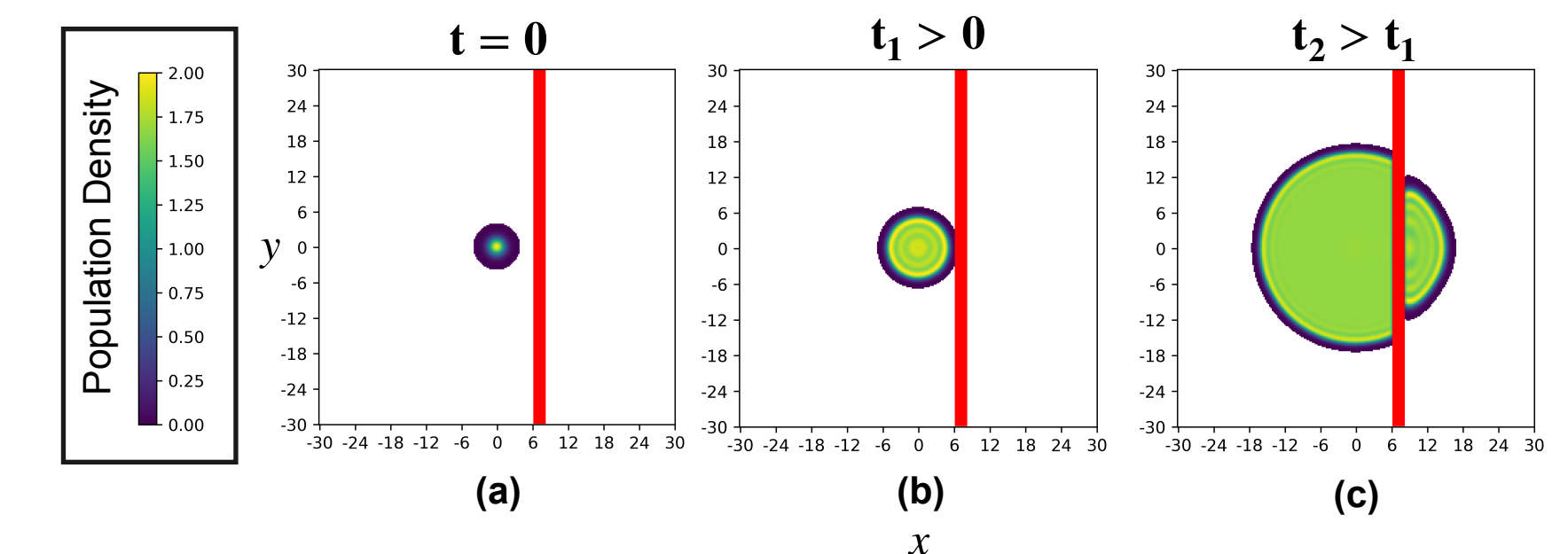


Figure 7: **(a)** Initial distribution of the invasive species. **(b)** The population reaches, but does not cross over the road. **(c)** The area behind the road is now invaded.

Current and Future Work

- Explore different growth functions, including Allee growth functions
- Investigate possible patterns of invasion
- Investigate the conditions needed for invasion to occur
- Study the 2-D problem
- Introduce complex road geometry



About the Author
My background is in applied mathematics. I am interested in the intersection of mathematics, computer science and nature.



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